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Inflation as a transient attractor in R^2 cosmology

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Using the equivalence between the curvature-squared gravity theory and the Einstein theory with a scalar field, we show that the potential in the latter system has a very flat plateau. It turns out that inflation is quite natural but transient in the R^2 cosmology. All anisotropic Bianchi types of space-time except IX approach the de Sitter solution as an attractor, followed by the Friedmann universe after sufficient inflation. We find a similar behavior in higher- $(4<D<10)$ dimensional theories, in which inflation is not exponential-type but power-law-type. The dilaton coupling to the R^2 term is also investigated. The coupling destroys the inflationary solution.

I. INTRODUCTION

Inflation is now very desirable in modern cosmology.[1,2] It may solve the horizon, flatness, and monopole problems. In resolution of such problems, we usually assume an isotropic and homogeneous Friedmann-Robertson-Walker (FRW) space-time. In order to see, however, whether inflation really solves those problems, we should start from an anisotropic and inhomogeneous space-time. We are also interested in whether or not inflation is natural. If inflation is an attractor for a wide range of initial conditions or for a large class of models, we easily understand why our present Universe is *at the present state.* In particular, assuming a cosmological constant, if the de Sitter solution is a unique attractor for any initial conditions (we call this the *no-hair conjecture*[3]), inflation becomes quite natural.

Recently, several authors have attempted to show that inflation really occurs even in an anisotropic (or inhomogeneous) space-time and it isotropizes (or homogenizes) initial anisotropy (or inhomogeneity).[4-10] For some models, they have shown that inflation is really an attractor. One of the most important works was done by Wald,[5] in which he showed that all Bianchi types of space-time except IX with a cosmological constant approach the de Sitter solution as an attractor and its anisotropy disappears within one Hubble expansion time. Generalizations of this work to a chaotic inflationary model[6] and to an inhomogeneous case[7] have been done. Starobinskii also found a set of general solutions starting with inhomogeneous initial conditions, under which all space-times approach the locally de Sitter solution.[9] From those analyses, the no-hair conjecture seems to be true.

We know, so far, two types of inflation: one is due to a flat potential appearing, e.g., in grand-unified-theory (GUT's) phase transitions (type I) (Ref. 1), and the other due to a scalar curvature-squared (R^2) term (type II) proposed by Starobinskii.[2] For type-II inflation, there are few attempts to prove the no-hair conjecture. Starobinskii and Schmidt[10] have found the same result in the type-II model as Starobinskii did in the type-I model. A natural question arises: Is Wald's important result true in R^2 theory also? We will show in this paper that inflation in this model is always a transient attractor, finding the de Sitter solution within one Hubble expansion time for all Bianchi models except IX.

Another point of interest is what happens in higher dimensions. Recently, higher-dimensional theory has been taken seriously as a realistic model, including string theory.[11,12] Hence, it may be interesting to see whether an inflationary solution in higher dimensions also becomes an attractor. We will show that there is a critical dimension (ten) below which a power-law inflationary solution exists and it is really an attractor in the FRW model.

In the superstring theory, a dilaton field, which is responsible for conformal invariance, appears.[13] The dilaton coupling to the other fields gives rise to very important effects on the dynamics of the system. For example, Boulware and Deser[14] found the de Sitter solution in ten-dimensional Einstein theory with a Gauss-Bonnet curvature-squared term, which however disappears in the $N=1$, ten-dimensional supergravity with a Gauss-Bonnet term coupled to a dilaton. In the four-dimensional model reduced from the ten-dimensional gravity theory, we found similar results.[15,16] For the case with a dilaton coupled to electromagnetic fields, the dynamical structure of charged black holes also changes drastically.[17] In this paper we investigate what happens in theories with a scalar curvature-squared term coupled to a dilaton field and will show that the dilaton coupling destroys an inflationary solution such as the Boulware-Deser case.

II. MODEL LAGRANGIAN AND THE EQUIVALENT SYSTEM

We start with the action

$$S_{R^2}=\int d^DX\frac{\sqrt{-g}}{2\kappa^2}[R(g)+\alpha e^{\beta\Phi}R^2-(\nabla\Phi)^2]\ , \quad (1)$$

where D is the dimensionality of space-time, $R(g)$ is a scalar curvature of metric $g_{\mu\nu}$, and Φ is a dilaton field. α and β are coupling constants. Taking variations of the action (1), we find the basic equations

$$G_{\mu\nu}=\frac{1}{1+2\alpha e^{\beta\Phi}R}\{\nabla_\mu\Phi\nabla_\nu\Phi-\tfrac{1}{2}(\nabla\Phi)^2g_{\mu\nu}-2\alpha[\tfrac{1}{4}g_{\mu\nu}e^{\beta\Phi}R^2-\nabla_\mu\nabla_\nu(e^{\beta\Phi}R)+\Box(e^{\beta\Phi}R)g_{\mu\nu}]\}\ , \tag{2}$$

$$\Box\Phi+\tfrac{1}{2}\alpha\beta e^{\beta\Phi}R^2=0\ . \tag{3}$$

For the case without a dilaton field ($\Phi=0$), Starobinskii analyzed those equations, showing that there is an unstable de Sitter solution followed by the present Friedmann era after sufficient inflation.[2] This model has been investigated by many authors, especially intensively by Mijic, Morris, and Suen.[18] In their paper, the equivalence between the curvature-squared theory and the Einstein gravity theory with a scalar field[19] has been applied to analyze the model without showing the explicit form of potential.

What we are interested in here is whether inflation is natural in the present model. For this purpose also, it is much more convenient to use the above equivalence. The equivalence is easily generalized for our D-dimensional model with a dilaton as follows.

Let us consider the following Weyl conformal transformation:

$$\bar{g}_{\mu\nu}=[1+2\alpha e^{\beta\Phi}R(g)]^{2/(D-2)}g_{\mu\nu}\ . \tag{4}$$

This conformal factor is uniquely determined if the gravitational action of $\bar{g}_{\mu\nu}$ is forced to be the Einstein-Hilbert one. Introducing a new dynamical variable (scalar field) Ψ defined by

$$\Psi=\left[\frac{D-1}{D-2}\right]^{1/2}\ln[1+2\alpha e^{\beta\Phi}R(g)] \tag{5}$$

instead of $R(g)$, we can easily show that the action (1) [or the basic equations (2) and (3)] is equivalent to the following action:

$$S_{\text{scalar}}=\int d^DX\frac{\sqrt{-\bar{g}}}{2\kappa^2}[R(\bar{g})-(\bar{\nabla}\Psi)^2-e^{-\sqrt{(D-2)/(D-1)}\Psi}(\bar{\nabla}\Phi)^2-2V(\Psi,\Phi)]\ , \tag{6}$$

$$V(\Psi,\Phi)=\frac{1}{8\alpha}e^{-\beta\Phi}\exp\left[\frac{(D-4)\Psi}{\sqrt{(D-1)(D-2)}}\right]\times\left\{1-\exp\left[\left[\frac{D-2}{D-1}\right]^{1/2}\Psi\right]\right\}^2, \tag{7}$$

where variables with an overbar denote those with respect to $\bar{g}_{\mu\nu}$. This system is just the Einstein gravity theory (the metric $\bar{g}_{\mu\nu}$) with two scalar fields, Ψ and Φ, whose potential is $V(\Psi,\Phi)$. The dilaton field Φ has a noncanonical kinetic term like a nonlinear sigma model. The basic equations in this system are explicitly written down as

$$\bar{G}_{\mu\nu}=e^{-\sqrt{(D-2)/(D-1)}\Psi}[\bar{\nabla}_\mu\Phi\bar{\nabla}_\nu\Phi-\tfrac{1}{2}(\bar{\nabla}\Phi)^2\bar{g}_{\mu\nu}]+\bar{\nabla}_\mu\Psi\bar{\nabla}_\nu\Psi-\tfrac{1}{2}(\bar{\nabla}\Psi)^2\bar{g}_{\mu\nu}-V\bar{g}_{\mu\nu}\ , \tag{8}$$

$$\bar{\Box}\Psi+\left[\frac{D-2}{D-1}\right]^{1/2}e^{-\sqrt{(D-2)/(D-1)}\Psi}(\bar{\nabla}\Phi)^2-\frac{\partial V}{\partial\Psi}=0\ , \tag{9}$$

$$\bar{\Box}\Phi-\left[\frac{D-2}{D-1}\right]^{1/2}(\bar{\nabla}\Psi)(\bar{\nabla}\Phi)-\frac{\partial V}{\partial\Phi}=0\ . \tag{10}$$

We shall discuss this model for three cases: (i) $\Phi=0$, $D=4$ (Sec. III); (ii) $\Phi=0$, $D>4$ (Sec. IV); and (iii) $\Phi\neq0$ (Sec. V), separately.

III. INFLATION IN R^2 THEORY (Ref. 20)

We consider the case without a dilaton field in four dimensions ($\Phi=0$, $D=4$). The potential $V(\Psi)$ in the equivalent system is

$$V(\Psi)=\frac{1}{8\alpha}(1-e^{-\sqrt{2/3}\Psi})^2\ , \tag{11}$$

whose shape is drawn in Fig. 1. It is trivially seen from Fig. 1 that the potential has a very long and flat plateau. The height of this plateau, which is determined by the coefficient α as $\kappa^{-2}V(\infty)\approx m_{\text{Pl}}^4(8\alpha/\kappa^2)^{-1}$, is of the order $10^{-13}-10^{-17}\times m_{\text{Pl}}^4$, because α/κ^2 is constrained to be of the order $10^{12}-10^{16}$ from the density perturbation.[2,18]

First we consider an anisotropic but homogeneous Bianchi-type space-time. The potential V is almost constant $V_\infty\equiv V(\infty)=1/8\alpha$ on the plateau, which behaves

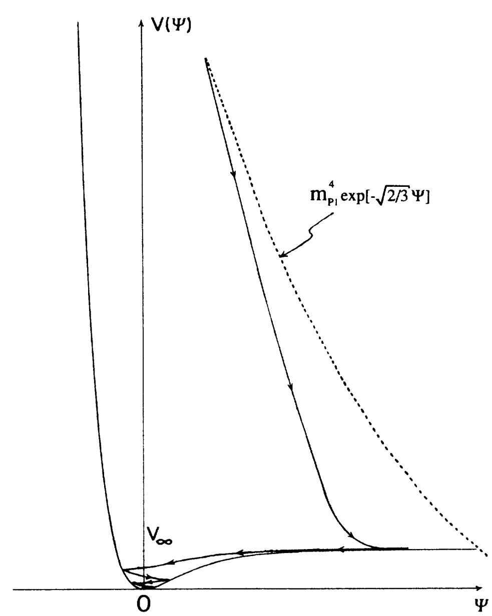

FIG. 1. The potential in the equivalent system to the curvature-squared gravity theory. The height of the plateau is $V_\infty=1/8\alpha\sim10^{-13}-10^{-17}$ in Planck units. The Planck limit in the fictitious $\bar{g}$ world is shown by the dashed line. The Universe evolves along the solid line with arrows and first reaches the plateau. The Friedmann era follows after sufficient inflation.

as a cosmological constant. The energy-momentum tensor without the potential term, $T_{\mu\nu} \equiv \overline{\nabla}_\mu \Psi \overline{\nabla}_\nu \Psi - \frac{1}{2}(\overline{\nabla}\Psi)^2 \overline{g}_{\mu\nu}$, satisfies the strong and dominant energy conditions. Therefore, we can follow Wald's method. From the Einstein equations (8) and (9) and the equation of motion for the scalar field (10), we find the equations

$$\tfrac{1}{3}K^2 = \tfrac{1}{2}(\sigma_{\mu\nu}\sigma^{\mu\nu} - R^{(3)} + \dot{\Psi}^2) + V \ , \tag{12}$$

$$\dot{K} = -\tfrac{1}{3}K^2 - \sigma_{\mu\nu}\sigma^{\mu\nu} - \dot{\Psi}^2 + V \ , \tag{13}$$

$$\dot{E}_\Psi = -K\dot{\Psi}^2 \ , \tag{14}$$

where $R^{(3)}$ is the scalar curvature of $t=$const hypersurface, an overdot denotes a time derivative, and E_Ψ is the *energy* of the scalar Ψ defined by

$$E_\Psi \equiv \tfrac{1}{2}\dot{\Psi}^2 + V(\Psi) \ . \tag{15}$$

K and $\sigma_{\mu\nu}$ are the expansion and the shear of the normal vector n_μ to the $t=$const homogeneous hypersurface, respectively.

Using Eqs. (12) and (13), we reach the same results as Wald's as follows:

$$\sqrt{3V_\infty} \le K \le \frac{\sqrt{3V_\infty}}{\tanh(t/\tau)} \ , \tag{16}$$

$$\sigma_{\mu\nu}\sigma^{\mu\nu} \le \frac{2V_\infty}{\sinh^2(t/\tau)} \ , \tag{17}$$

$$\dot{\Psi}^2 \le \frac{2V_\infty}{\sinh^2(t/\tau)} \ , \tag{18}$$

where $\tau \equiv \sqrt{3/V_\infty} = \sqrt{24\alpha} \approx 10^6 - 10^8 t_{\rm Pl}$.

Hence, when the Universe is on the plateau, the expansion rate K approaches $\sqrt{3V_\infty}$ within one Hubble expansion time τ. Anisotropy $\sigma_{\mu\nu}\sigma^{\mu\nu}$ and the kinetic energy of the scalar field $\dot{\Psi}^2$ disappear also within τ. Any anisotropic but homogeneous space-time except Bianchi type IX approaches the isotropic de Sitter solution ($\propto e^{t/\tau}$) in one Hubble expansion time. The Universe is completely isotropized because the Universe must be rolling down on the potential for at least 60 times the Hubble expansion time in order to solve the horizon problem, etc. This result can be easily generalized to inhomogeneous cases satisfying some conditions such that the three-dimensional scalar curvature $R^{(3)}$ of $t=$const hypersurface is always negative in a synchronous reference systems, as discussed in the type-I inflationary model by Jensen and Stein-Schabes.[7]

From the above analysis, we may have the following scenario: If the Universe starts more or less at the Planck scale, the initial energy $E_{\Psi,0}$ is naively given by $e^{-\sqrt{2/3}\Psi} m_{\rm Pl}^4$. Because, of the conformal transformation (4), the energy in the physical g world is given by $(1+2\alpha R)E_\Psi$, which is limited to the Planck scale $m_{\rm Pl}^4$ initially. From such an initial stage, the Universe easily evolves onto the plateau as shown in Fig. 1. The probability that the Universe reaches directly the true minimum ($\Psi=0$) may be extremely small. If the Universe is initially expanding, K is always positive because of Eq. (12) with $R^{(3)}<0$. The system of the scalar field is always dissipative ($\dot{E}_\Psi<0$). Losing the *energy* E_Ψ, the Universe evolves onto the plateau as in Fig. 1. Once the Universe reaches the plateau, the Universe is going to roll over on the plateau either in the positive or in the negative direction depending on initial conditions. Even if the Universe is initially rolling over in the positive direction as in Fig. 1, the scalar field Ψ gradually loses its velocity because of the friction term $H\dot{\Psi}$ and the potential gradient $\partial V/\partial\Psi$, and eventually changes its direction. In any case, therefore, the scalar field is rolling down finally in the negative direction and the Universe is isotropized within one Hubble expansion time, finding sufficient inflation as discussed above. The Universe reaches the potential minimum ($\Psi=0$), at which a cosmological constant automatically vanishes. In this sense, an inflationary phase is always a transient attractor, and then inflation is a quite natural phenomenon in R^2 gravity theory.

We may need one remark. The above attractor property has been discussed in the equivalent system ($\overline{g}_{\mu\nu}$ system). Because of the conformal transformation, it is not, in general, trivial that inflation in the Weyl-rescaled ($\overline{g}_{\mu\nu}$) system always guarantees inflation in the original ($g_{\mu\nu}$) system. In our case, however, we can easily show that it is true. Because when the Universe is undergoing inflation in the Weyl-rescaled system, the scalar field Ψ is changing very slowly. While we have the relation (4), i.e.,

$$g_{\mu\nu} = \exp\left[-\frac{2\Psi}{\sqrt{(D-1)(D-2)}}\right]\overline{g}_{\mu\nu} \ . \tag{19}$$

This guarantees inflation in the original curvature-squared theory also.

IV. HIGHER-DIMENSIONAL R^2 THEORY

We consider a D-dimensional space-time. The potential of the scalar field Ψ in the present system is

$$V = \frac{1}{8\alpha}\exp\left[\frac{(D-4)\Psi}{\sqrt{(D-1)(D-2)}}\right] \times \left\{1 - \exp\left[-\left[\frac{D-2}{D-1}\right]^{1/2}\Psi\right]\right\}^2 \ . \tag{20}$$

The difference from the four-dimensional case appears only in the potential shape. The potential has a very flat plateau in the four-dimensional model, while the potential in a higher- (>4) dimensional model diverges exponentially as $\Psi \to \infty$, i.e.,

$$V \approx \frac{1}{8\alpha}\exp\left[\frac{(D-4)}{\sqrt{(D-1)(D-2)}}\Psi\right] \quad \text{as } \Psi \to \infty \ . \tag{21}$$

It seems that there is no inflationary solution unless $D=4$, because there is no plateau in the potential. However, if a potential of a scalar field has an exponential function form, there is another type of inflation that is called a power-law inflation proposed by Abbott and Wise[21] and intensively analyzed by Lucchin and Matarrese.[22] The expansion law of a scale factor a is not exponential but power law, whose exponent is greater than 1 so that the Universe expands faster than the horizon.

Assuming a space-time being the flat FRW type in D dimensions,

$$d\bar{s}_D^2 \equiv \bar{g}_{\mu\nu} dX^\mu dX^\nu = -d\bar{t}^2 + \bar{a}(\bar{t})^2 d\mathbf{x}^2 , \quad (22)$$

and the potential of a scalar field having an exponential form as $\exp(\lambda\Psi)$, we find the basic equations

$$\frac{(D-1)(D-2)}{2}\bar{H}^2 = \tfrac{1}{2}\dot{\Psi}^2 + V(\Psi) , \quad (23)$$

$$(D-2)\dot{\bar{H}} = -\dot{\Psi}^2 , \quad (24)$$

$$\ddot{\Psi} + (D-1)\bar{H}\dot{\Psi} = \frac{\partial V}{\partial \Psi} , \quad (25)$$

where $\bar{H} \equiv \dot{\bar{a}}/\bar{a}$ and an overdot denotes the derivative with respect to $\bar{t}$.

Setting $\bar{H} = p/\bar{t}$, i.e., $\bar{a} \propto \bar{t}^p$ and $\dot{\Psi} = q/\bar{t}$, we find a power-law solution as

$$p = \frac{4}{\lambda^2(D-2)} \quad \text{and} \quad q = -\frac{2}{\lambda} . \quad (26)$$

In order for inflation of power-law type to occur, p should be larger than 1; hence, the coefficient of the potential λ should be smaller than the critical value $2/\sqrt{D-2}$. This yields from Eq. (21) that the dimensionality of space-time should be smaller than 10. When the dimensionality is lower than 10, hence, we may have power-law inflation. It is interesting to note that the same critical dimensionality appears from the conditions of whether general inhomogeneous solutions have chaotic behavior near the singularity in a higher-dimensional Einstein system.[23] The critical dimension $D=10$ is marginal both for power-law inflation to occur in the present model and for chaotic behavior to be found in their model, although the critical dimension $D=10$ itself is not included for inflation but included for chaotic behavior.

It is worth noting that power-law inflation in the Weyl-rescaled ($\bar{g}_{\mu\nu}$) system always guarantees power-law inflation in the original ($g_{\mu\nu}$) system as follows. The metric in the original system is

$$ds_D^2 = -dt^2 + a(t)^2 d\mathbf{x}^2 = \Omega^2[-d\bar{t}^2 + \bar{a}(\bar{t})^2 d\mathbf{x}^2] , \quad (27)$$

where

$$\Omega \equiv \exp\left[-\frac{\Psi}{\sqrt{(D-1)(D-2)}}\right] \propto \bar{t}^m \quad (28)$$

with

$$m \equiv \frac{2}{\lambda\sqrt{(D-1)(D-2)}} , \quad (29)$$

for the power-law inflationary solution (26). Then, we find the power-law solution in the original system as

$$a \equiv \Omega\bar{a} \propto \bar{t}^{(p+m)} \propto t^{(p+m)/(1+m)} \quad (30)$$

with $t \propto \bar{t}^{(1+m)}$, showing that the power in the original system is also larger than unity if $p > 1$.

We can execute the same analysis as that by Halliwell, by which he showed that the power-law inflationary solution is always the attractor for open ($k=-1$) and flat ($k=0$) FRW models.[24] The analysis is exactly the same as in four dimensions , so we do not repeat it here. For some inhomogeneous models, however, this power-law solution may not be an attractor as discussed by Barrow.[8] Then, this inflationary solution would not be favored.

V. DILATON COUPLING

If there is a dilaton field coupled to the other fields such as that in a superstring model, the dynamics may change drastically as shown for the ten-dimensional de Sitter solution,[14] four-dimensional de Sitter solution,[15,16] and charged black holes.[17]

Boulware and Deser showed that, although the de Sitter solution does exist in ten-dimensional Einstein gravity with the Gauss-Bonnet combination of curvature-squared terms, it disappears in the $N=1$, ten-dimensional supergravity model with the Gauss-Bonnet term coupled to the dilaton field. The inflationary solution is destroyed by the dilaton coupling. We shall consider the similar coupling in our model.

In order to investigate this system, it is also convenient to use the equivalence discussed in Sec. II. The potential $V(\Psi,\Phi)$ in the equivalent system ($D=4$) is shown in Fig. 2. As seen from Fig. 2, there is no flat plateau. The plateau which appears in the the model in Sec. III is no longer a plateau, because the potential is not flat *in the dilaton* (Φ) *direction*. Consequently, we can conclude that the dilaton coupling destroys the de Sitter solution in the same way as in the model by Boulware and Deser.

We may, however, find a power-law-type inflation as discussed in Sec. IV. Because, from Eq. (7), the asymptotic forms of the potential $V(\Psi,\Phi)$ are exponential functions as

$$\text{(i) } V \approx \frac{1}{8\alpha} e^{-\beta\Phi} \exp\left[\frac{(D-4)}{\sqrt{(D-1)(D-2)}}\Psi\right] \quad \text{for } \Psi \to \infty , \quad (31)$$

$$\text{(ii) } V \approx \frac{1}{8\alpha} e^{-\beta\Phi} \exp\left[\frac{-D}{\sqrt{(D-1)(D-2)}}\Psi\right] \quad \text{for } \Psi \to -\infty . \quad (32)$$

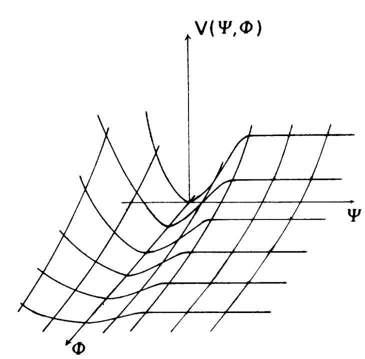

FIG. 2. The potential in the equivalent system to the curvature-squared theory with a dilaton coupling in four dimensions. The potential drops monotonically in the dilaton (Φ) direction. This destroys the inflationary solution which exists in the case without a dilaton.

For example, we consider the simplest case ($D=4$). For $\Psi \gg 1$, $\Psi \approx \Psi_0$(const) is a solution because of the flatness of the potential in the Ψ direction. Only Φ rolls down over the potential V in the positive direction. Normalizing the kinetic term of Φ as $(2\kappa^2)^{-1}(\tilde{\nabla}\phi)^2$ in the action (6), we find

$$V \approx \frac{1}{8\alpha} \exp(-\beta e^{\Psi_0/\sqrt{6}}\phi) \quad \text{for } \phi \to \infty \tag{33}$$

with

$$\phi \approx \exp(-e^{\Psi_0/\sqrt{6}})\Phi \ . \tag{34}$$

In order for power-law inflation to occur, the coefficient in an exponential potential should be smaller than $\sqrt{2}$. This yields the condition

$$|\beta| < \sqrt{2} \exp\left[-\frac{\Psi_0}{\sqrt{6}}\right] \ll 1 \ . \tag{35}$$

Hence, although the dilaton coupling destroys the de Sitter solution, the power-law type may still remain. In the superstring model, however, β may not be an arbitrary parameter; rather, it may be fixed as $\beta = 1/\sqrt{2}$ (if the coupling exists[13]) and then even the power-law type is not possible.

Both models (Boulware and Deser's and our own) are two extreme cases in most general models with curvature-squared terms. We might find the same conclusion for more general cases from those two extreme examples.

VI. CONCLUSION

We have shown that inflation is always a transient attractor in the R^2 gravity theory using the equivalence between the R^2 theory and Einstein theory with scalar fields. Any anisotropic space-time except Bianchi type IX (or inhomogeneous space-time with $R^{(3)} < 0$) approaches an isotropic de Sitter space-time in one Hubble expansion time, followed by the Friedmann universe after sufficient inflation.

The equivalence is quite useful for any analysis in the R^2 theory, because we usually know well the Einstein system with ordinary matter (or scalar fields) than a higher-derivative dynamical system.

In higher ($D > 4$) dimensions, there is also a transient attractor in FRW models if $D < 10$; however, it is power-law inflation which may not be an attractor for some inhomogeneous models.

As for a coupling with dilaton, although we do not know yet which combination of the curvature-squared terms appears in a superstring model, it seems that any combination does not allow an inflationary solution, because two extreme cases, with the Gauss-Bonnet combination and with the scalar curvature-squared term, do not allow inflation unless the dilaton field is fixed at some finite point.

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