Exotic quarks in superstring models: Implications on *CP* violation and heavy-meson mixing

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At low energies, superstring models lead to an effective E_6 theory containing exotic charge $-\frac{1}{3}$ quarks, denoted h. Here we work out the consequences of the mixing of h with the b quark, on *CP* violation, and heavy-neutral-meson systems. This mixing is found to remove the lower bound on m_t from ϵ and significantly reduces the prediction for ϵ'/ϵ. While B^0-$\bar{B}^0$ mixing remains unaltered, there is a considerable enchancement of D^0-$\bar{D}^0$ mixing from supersymmetric contributions.

I. INTRODUCTION

Superstring theories can have interesting phenomenological consequences at low energies. Compactification of the heterotic string leads to the emergence of an E_6 supersymmetric (SUSY) grand unified theory (GUT) in four dimensions.[1] The massless string excitation modes are arranged in 27-plets of E_6. The GUT symmetry can be broken by Wilson loops related to the non-simply-connected nature of the Calabi-Yau manifold leading to a low-energy gauge group of at least rank five. If there are no intermediate mass scales, the whole **27** representation survives down to low energies and each generation will have, in addition to the usual 16 fermions, 11 others. Among these, we are interested in the color-triplet charge $-\frac{1}{3}$ h-quark states which can mix with the d-type quarks when the electroweak gauge symmetry is broken. Here we study the effects of this mixing on *CP* violation and heavy-neutral-meson systems.

Recent precise evaluation of the Kobayashi-Maskawa (KM) matrix (V) elements give[2] $|V_{ud}|=0.9729 \pm 0.0012$, $|V_{us}|=0.221\pm 0.002$, and $|V_{ub}|<0.0067$ (90% C.L.). These values imply a small deviation from unitarity which has often been taken as an indication for the existence of a fourth generation.[3] In the context of superstring models this deviation has been attributed to the mixing of h with d-type quarks.[4] Neglecting intergenerational mixing there are three possibilities: h_d-d, h_s-s, h_b-b mixing. Of these, h_d-d mixing has previously been studied[5] with reference to neutral currents, h-quark decay, top-quark mass, and sneutrino vacuum expectation value (VEV). From unitarity, $V_{uh_d}^2+V_{uh_s}^2+V_{uh_b}^2 \simeq 0.06-0.07$. Since V_{ud}, V_{us} are much larger, h_d-d and h_s-s mixings are suppressed and in contrast to h_b-b mixing have no significant effect on *CP* violation. Henceforth, we drop the subscript b and consider only h-b mixing. This mixing turns out to be rather large and gives rise to significant effects which are discussed in the next few sections. Some brief accounts of this work have already been presented elsewhere.[6]

Our paper is structured as follows. In Sec. II we discuss *CP* violation and the B^0-$\bar{B}^0$ system in the light of h-b mixing. In the next section we consider D^0-$\bar{D}^0$ mixing. We end in Sec. IV with our conclusions.

II. *CP* VIOLATION AND *h*-*b* MIXING

In the standard three-generation model, quark mixing is represented through the (3×3) KM matrix V:

$$V=\begin{bmatrix} c_1 & -s_1c_3 & -s_1s_3 \\ s_1c_2 & c_1c_2c_3-s_2s_3e^{i\delta} & c_1c_2s_3+s_2c_3e^{i\delta} \\ s_1s_2 & c_1s_2c_3+c_2s_3e^{i\delta} & c_1s_2s_3-c_2c_3e^{i\delta}\end{bmatrix}, \quad (1)$$

where $c_i=\cos\theta_i$, $s_i=\sin\theta_i$ $(i=1,2,3)$.

The phase δ is responsible for *CP* violation. V_{ud} and V_{us} are well known. V_{ub} and V_{cb} are determined[7] from the bottom lifetime τ_B and the ratio $\bar{R}=\Gamma(b\to ul\nu)/\Gamma(b\to cl\nu)$ through the formulas

$$\Gamma(b\to u)=\frac{G_F^2 m_b^5}{192\pi^3}|V_{ub}|^2\gamma_2 , \quad (2)$$

$$\Gamma(b\to c)=\frac{G_F^2 m_b^5}{192\pi^3}|V_{cb}|^2\gamma_1 , \quad (3)$$

where $\gamma_1=3.2$ and $\gamma_2=6.92$ subsume various phase-space suppression factors and QCD corrections. Since $\bar{R}$ is small ($\bar{R}\approx 0.04$) and τ_B large ($\tau_B>1$ psec), it is found, using Eqs. (2) and (3), that θ_2 and θ_3 are constrained to rather small values. This affects the *CP* predictions of the model quite strongly. In fact, from the usual K^0-$\bar{K}^0$ box-diagram calculation one finds[7]

$$|\epsilon|=3.82\times 10^4 B(\mathrm{Im}\lambda_t \mathrm{Re}\lambda_t)s(m_t)\eta_2 , \quad (4)$$

where B—the so-called "bag factor"—is an estimation of the uncertainty in the matrix element $\langle K^0|H_{\mathrm{eff}}^{\Delta S=2}|\bar{K}^0\rangle$, $\lambda_t=V_{td}^*V_{ts}$, and

$$s(m_t)=x_t[\tfrac{1}{4}+\tfrac{9}{4}(1-x_t)^{-1}-\tfrac{3}{2}(1-x_t)^{-2}] + \frac{3}{2}\left[\frac{x_t}{x_t-1}\right]^3 \ln x_t ,$$

with $x_t=(m_t/M_W)^2$ is a monotonically increasing function of m_t, and η_2 is a QCD correction factor. In (4), terms involving m_c and m_u (which contribute negligibly) have been dropped. For a fixed τ_B and $\bar{R}$, $\mathrm{Im}\lambda_t \mathrm{Re}\lambda_t$ peaks for a certain value of δ. This determines the minimum value of m_t required to fit the experimental value of ϵ (Ref. 8).

With the KM matrix elements fixed, and m_t taken to be in the range 30–60 GeV, the other CP-violation parameter ϵ'/ϵ is extracted from the penguin diagram and is given by[7]

$$\epsilon'/\epsilon=15.6Hs_2c_2s_3s_\delta \ , \tag{5}$$

with $H=0.54$.

Turning now to the E_6 model, h-b mixing generalizes V to a 3×4 matrix:

$$V=\begin{bmatrix} c_1 & -s_1c_3 & -s_1s_3c_\alpha & s_1s_3s_\alpha \\ s_1c_2 & c_1c_2c_3-s_2s_3e^{i\delta} & (c_1c_2s_3+s_2c_3e^{i\delta})c_\alpha & -(c_1c_2s_3+s_2c_3e^{i\delta})s_\alpha \\ s_1s_2 & c_1s_2c_3+c_2s_3e^{i\delta} & (c_1s_2s_3-c_2c_3e^{i\delta})c_\alpha & -(c_1s_2s_3-c_2c_3e^{i\delta})s_\alpha \end{bmatrix} . \tag{6}$$

The most general (3×4) mixing matrix—h mixing with d, s, and b—requires six angles and three phases. In (6), we have retained only the dominant h-b mixing (see Sec. I) which requires one extra mixing angle, α.

Our strategy is as follows: We determine θ_1 and θ_3 from V_{ud} and V_{us}. θ_3 is found to be at least 1 order larger than the estimation in the standard model. Then Eq. (2) is used to determine V_{ub} as a function of $\bar{R}$ and τ_B from which α can be extracted. Figure 1 shows the variation of $\cos\alpha$, from which it is evident that h-b mixing can be rather large.

Using Eqs. (2) and (3) one now gets

$$\theta_2=\tfrac{1}{2}\{\cos^{-1}[c/(a^2+b^2c_\delta^2)^{1/2}] + \sin^{-1}[bc_\delta/(a^2+b^2c_\delta^2)^{1/2}]\} \ , \tag{7}$$

where

$$a=c_3^2-c_1^2s_3^2, \quad b=-2c_1s_3c_3 \ ,$$
$$c=c_3^2+s_3^2(c_1^2-0.231/\bar{R}) \ .$$

θ_2 is also found to be much larger than in the previous case.

Now with the KM angles much larger, the lower bound on m_t, coming from ϵ, is removed, e.g., if $\bar{R}=0.04$, $B=\frac{1}{3}$, $\tau_B=1.0$ psec, and $\cos\alpha=0.12$, m_t must be greater than just 1 GeV.

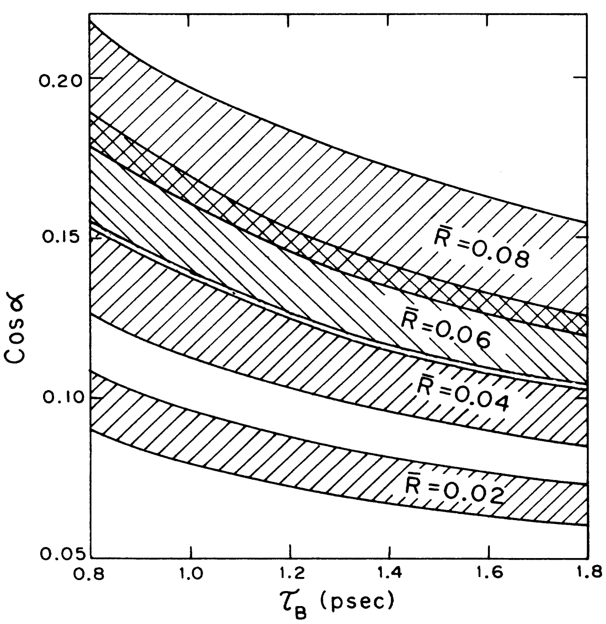

FIG. 1. Variation of $\cos\alpha$ with τ_B for different $\bar{R}$. The shaded regions are allowed when V_{us} ranges from 0.219 to 0.223.

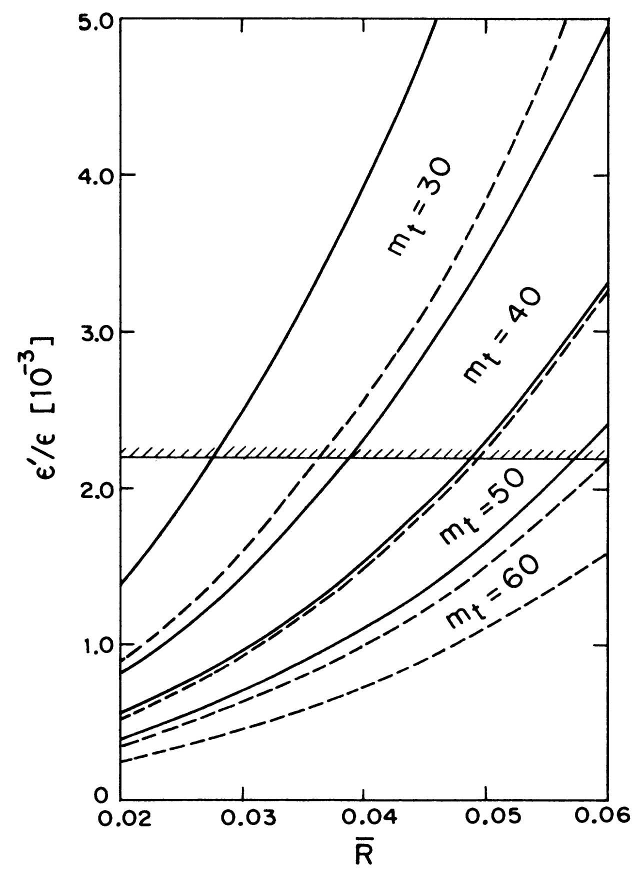

FIG. 2. ϵ'/ϵ as a function of $\bar{R}$ for different top masses. Solid (dashed) lines correspond to $V_{us}=0.223$ (0.219).

For the other CP-violation parameter ϵ'/ϵ, the results are shown in Fig. 2. The experimental bound of ϵ'/ϵ, obtained by combining the results of Refs. 9 and 10 is also indicated. The curves do not vary with τ_B since θ_3 is fixed from the experimental value of V_{us} while θ_2, obtained from Eq. (7), depends only on $\bar{R}$ but not on τ_B. It is seen from the figure that ϵ'/ϵ is now much smaller and there is no inconsistency with the existing experimental results, but an order-of-magnitude improvement on the present bound will put this model under pressure.

So far we have discussed the implications of this E_6 model on the kaon system only. For B^0-$\bar{B}^0$ mixing the t-quark exchange in the internal lines dominates in the box-diagram calculation.[11] To obtain the predictions of this model for B_d^0-$\bar{B}_d^0$ (B_s^0-$\bar{B}_s^0$) mixing, one simply has to replace the value of $\lambda_t^d = V_{td}^* V_{tb}$ ($\lambda_t^s = V_{ts}^* V_{tb}$) of the standard model with those of this model. $|\lambda_t^d|$ and $|\lambda_t^s|$ for different values of $\bar{R}$ are presented in Table I. These are of the same order of magnitude as those of the standard model. Hence we can conclude that h-b mixing has no significant effect on the B^0-$\bar{B}^0$ system.

The SUSY nature of this model has not yet been considered. It is well known that in these theories gluino exchange gives rise to flavor violation[12] because of a mismatch between squark and quark mass matrices. Ignoring left-right squark mixing, flavor violation originates through the left-handed squark mass matrix

$$M^2(\tilde{D}_L) = \mu_L^2 I + \hat{M}_d^2 + cV^+\hat{M}_u^2 V , \quad (8)$$

where we have chosen a basis in which the d-quark mass matrix is diagonal $\hat{M}_d$. μ_L is a flavor-blind SUSY-breaking parameter. The model-dependent constant c is negative and ~ 1 and is a measure of flavor violation arising through radiative corrections involving the up-type Yukawa couplings. Incorporating the $\tilde{h}$ squark gives a 4×4 matrix in place of Eq. (8). We have checked that the imaginary part of $\lambda_t = V_{td}^* V_{ts}$—which is at the root of CP violation—remains roughly unchanged after the inclusion of h. However, as we will discuss in the following section, SUSY contributions to D^0-$\bar{D}^0$ mixing with the h quark is actually dominant.

III. D^0-$\bar{D}^0$ MIXING

In the standard model, D^0-$\bar{D}^0$ mixing is generated through box diagrams with d-type quarks in the internal lines and c and u quarks in the external lines. Since $m_c \gg m_s, m_d$ one cannot now neglect the momenta in the external lines of the box diagram. One finds[13]

TABLE I. $|\lambda_t^d|$ and $|\lambda_t^s|$ are tabulated for two values of V_{us}.

V_{us} \ $\bar{R}$	0.02	0.04	0.06	0.08
		$\lvert\lambda_t^d\rvert$ (10^{-3})		
0.219	6.29	5.63	4.13	2.69
0.223	7.22	5.83	4.19	2.70
		$\lvert\lambda_t^s\rvert$ (10^{-2})		
0.219	3.88	5.18	5.58	5.80
0.223	4.73	5.55	5.82	5.94

$$\langle \bar{D}^0 | H_{\text{eff}} | D^0 \rangle = -\frac{G_F}{\sqrt{2}}(\alpha f_D^2 m_D B/6\pi \sin^2\theta_W) \times (V_{cb}^* V_{ub})^2 (I_1^b + \tfrac{5}{8} I_2^b m_D^2) \quad (9)$$

with

$$I_1^b = -(m_b^2/2M_W^2) - (m_c^2/6M_W^2)\ln(m_b^2/m_c^2) - (m_c^2/6M_W^2)$$

$$I_2^b = -(\tfrac{1}{3}M_W^2)\ln(m_b^2/m_c^2) ,$$

where we have retained only the dominant b-quark contributions. Using this formula Δm_D ($\Delta m_D = 2\,\text{Re}M_{12}$) turns out to be of order 10^{-17}–10^{-18} GeV—well below the experimental upper limit, viz., 6.5×10^{-13} GeV (Ref. 14).

With the inclusion of the h quark, there are additional contributions to Δm_D. Diagrams involving the h quark ($m_h > 20$ GeV) will dominate (like the b-quark dominance in the previous case) and in analogy with (9) gives

$$\langle \bar{D}^0 | H_{\text{eff}} | D^0 \rangle |_h = -\frac{G_F}{\sqrt{2}}(\alpha f_D^2 m_D B/6\pi \sin^2\theta_w) \times (V_{cb}^* V_{ub})^2 \tan^2\alpha (I_1^h + \tfrac{5}{8} I_2^h m_D^2) . \quad (10)$$

The integrals $I_{1,2}^h$ give an enhancement of ~ 10 which taken together with the factor $\tan^2\alpha$ gives Δm_D of order 10^{-14}–10^{-15} GeV, still much smaller than the experimental bound.

In view of this situation it is of interest to look into SUSY contributions to D^0-$\bar{D}^0$ mixing through gluino exchange. The up-squark mass matrix, which is relevant in this case, is

$$M^2(\tilde{U}_L) = \mu_L^2 I + \hat{M}_u^2 + c' V \hat{M}_d^2 V^+ . \quad (11)$$

The model-dependent constant c' is of ~ 1. When the gluino-exchange box diagrams are calculated it is legitimate to ignore the momenta in the external lines since the gluino and the squarks are much heavier than the u and c quarks. One finds[15]

$$M_{21} = -\frac{\alpha_s^2 m_D f_D^2 B}{108 m_{\tilde{g}}^2} \sum_{j,m=1}^{6} \Gamma^{ju*}\Gamma^{jc}\Gamma^{mu*}\Gamma^{mc} \times (4I_{jm} + 11K_{jm}) , \quad (12)$$

where

$$I_{jm} = \frac{1}{z_j - z_m}\left[\frac{z_j \ln z_j}{(1-z_j)^2} + \frac{1}{1-z_j} - (z_j \leftrightarrow z_m)\right] ,$$

$$K_{jm} = \frac{1}{z_j - z_m}\left[\frac{z_j^2 \ln z_j}{(1-z_j)^2} + \frac{1}{1-z_j} - (z_j \leftrightarrow z_m)\right] ,$$

with

$$z_j = (\tilde{m}_j / m_{\tilde{g}})^2 .$$

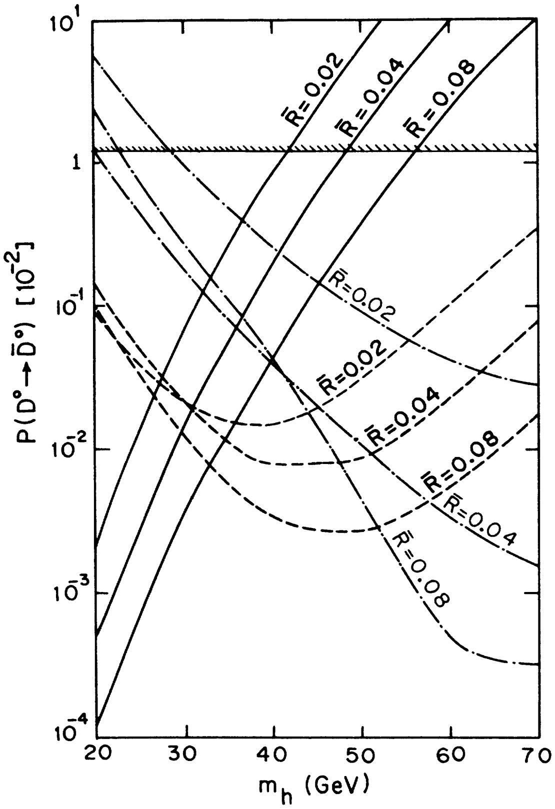

FIG. 3. $P(D^0 \rightarrow \bar{D}^0)$ as a function of m_h. Solid, dashed, and dashed-dotted lines are for $\mu_L=50$, $m_{\tilde{g}}=40$, $\mu_L=70$, $m_{\tilde{g}}=70$, and $\mu_L=100$, $m_{\tilde{g}}=100$, respectively (all masses in GeV).

Γ is the unitary matrix which diagonalizes $M^2(\tilde{U}_L)$. Since every term in $\hat{M}_u^2$ is much larger than the corresponding term in $\hat{M}_d^2$, flavor violation in Eq. (11) is not very pronounced, and Δm_D is of order 10^{-18}–10^{-21} GeV.[16]

Flavor violation will be enhanced in the E_6 model because in this case $\hat{M}_d$ also includes m_h. Using the 3×4 matrix V (explicitly determined in Sec. I) and Eq. (12), we can now calculate SUSY contributions to Δm_D in this case. In Fig. 3 we have plotted $P(D^0 \rightarrow \bar{D}^0)$ given by

$$P(D^0 \rightarrow \bar{D}^0)=\frac{1}{2}\left[\frac{\Delta m_D}{\Gamma}\right]^2 \tag{13}$$

(Γ being the D^0 decay width$=1.53\times 10^{-12}$ GeV) as a function of m_h for different values of $\bar{R}$. We have also studied the dependence of P on different squark ($\simeq 50, 70, 100$ GeV) and gluino ($\simeq 40, 70, 100$ GeV) masses. For this calculation we have chosen $f_D=0.2$ GeV, $B=\frac{1}{3}$, and the SUSY-flavor-violation parameter $c'=-0.5$.

Experimentally $P(D^0 \rightarrow \bar{D}^0)$ is obtained from like-sign dilepton data and has been measured[17] in proton-Fe collisions, neutrino production, and muon scattering. In Fig. 3 we have shown the most stringent bound $[P(D^0 \rightarrow \bar{D}^0) \leq 0.012]$ obtained from the last mentioned experiment. It is seen that for light squarks and gluinos, $m_h > 50$ GeV is inconsistent with the above experimental number. For heavier superparticles, this bound is probably at a much higher m_h than the range which we have scanned. Of course, if $B=1$, the theoretical numbers increase by a factor of 9. In that case the bounds on m_h are more stringent.

We have also examined the variation of $P(D^0 \rightarrow \bar{D}^0)$ with m_t in the range 30–60 GeV. We find the dependence to be very weak when $m_h=70$ GeV and it gets weaker as m_h decreases. Thus irrespective of the top-quark mass $P(D^0 \rightarrow \bar{D}^0)$ can provide a positive test for h-b mixing.

IV. CONCLUSION

In the E_6 models, which are the low-energy remnants of the heterotic superstring, there are extra charge $-\frac{1}{3}$ quarks, h. We studied the CP-violation consequences of the mixing of an h quark with the b quark. The lower bound on m_t from ϵ is removed in this case and ϵ'/ϵ is generally within the current experimental limits even for the most unfavorable choice of the bag factor B. For B_d^0-$\bar{B}_d^0$ and B_s^0-$\bar{B}_s^0$ mixing there is not much difference between the predictions of this model and the standard model. In particular the recent observation of B_d^0-$\bar{B}_d^0$ mixing by the ARGUS Collaboration[18] requires a heavier m_t in this model also.

Investigating the effects of this mixing on the D^0-$\bar{D}^0$ system, the contribution to Δm_D from SUSY box diagrams is considerably enhanced, and in some cases exceeds the experimental upper limit. Thus, one can set an upper bound on m_h from D^0-$\bar{D}^0$ mixing for a particular choice of superparticle masses.

In conclusion, h-b mixing has rather striking effects on CP violation and heavy-meson mixing. Future experimental measurements of ϵ'/ϵ and $P(D^0 \rightarrow \bar{D}^0)$ will serve to test this hypothesis more closely.

Note added. While this paper was being prepared we received Ref. 19 which includes some related work on flavor mixing in the standard model incorporating a fourth down quark.

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