Relativistic model of nucleon and pion structure: Static properties and electromagnetic soft form factors

Zbigniew Dziembowski

Institute of Theoretical Physics, Warsaw University, Hoza 69, PL-00-681 Warszawa, Poland

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I have studied the valence-quark system in a light-cone version of the constituent-quark model. A relativistic description is derived by applying light-cone boosts to model wave functions at rest which describe a valence system with the standard quark-model J^P assignments, the usual constituent-quark mass, and a universal hadronic scale. With the scale fixed by static properties at ≈ 320 MeV, we find that the relativistic constituent-quark model offers an excellent description of the hadron electromagnetic form factors up to $Q^2 \approx \text{few GeV}^2$, but at larger scales is invalid.

I. INTRODUCTION

In previous papers^{1,2} a light-cone wave-function model was presented with the aim of giving the quark-model explanation of the surprising features of hadron structure as recently revealed by QCD sum rules^{3,4} and lattice calculations.⁵ It also has been suggested that to account for the high-momentum-transfer behavior of elastic form factors one needs a quite simple, albeit very different from nonrelativistic-potential-model intuition, picture of the valence-quark structure. Namely, it is the standard quark-model spin-parity assignments together with a large momentum scale of the hadron valence wave function (i.e., with quarks which are highly relativistic in the bound state) that gives the perturbative QCD power-law behavior for the pion electric and nucleon magnetic form factors consistent in sign and magnitude with experiment at Q^2 beyond a few GeV². Here we look at the hadron form factors in the low- Q^2 region. The motivation for this work is (i) to connect in the relativistic wave function the low- and high- Q^2 regions and (ii) to check the domain of validity of the constituent-quark-model (CQM) picture.

The main result is that our simple quark-modelinspired relativistic wave function, but now with a small momentum scale of the order of the constituent-quark mass, represents dominant physics in the region below $Q^2 \approx$ few GeV² where the onset of the leading power law occurs. On the other hand, at larger Q^2 we observe that the soft form factors (i.e., the ones with the CQM contributions) fall off much faster than indicated by experiments. Thus, in contrast with some previous claims, it shows that there is no way to postpone the region of validity of perturbative QCD predictions.⁶ Notice that for the pion case a similar conclusion has been reached by Jacob and Kisslinger in Ref. 7. In the conclusion of this paper speculations are made about the physics behind the existence of two different momentum scales (i.e., those valid for the low- and high-momentum-transfer picture of hadron structure).

II. RELATIVISTIC NUCLEON AND PION WAVE-FUNCTION MODEL

Current-quark phenomenology describes a picture of the hadron as having a decomposition into Fock-space states which in general consists of not only the valence configuration $q\bar{q}$ or qqq, but also a sea of quarkantiquark pairs and gluons. QCD, as the underlying quark-gluon field theory, when cut off at some scale μ , produces an effective field theory of quarks with parameters, e.g., quark mass, that depend on the scale μ . The COM corresponds to the effective field theory which is cut off at a scale of the order of 1 GeV with the effective quark mass (the constituent mass) of the order of 330 MeV. At this scale the Fock expansion can be approximately saturated by the valence-quark configuration. To complete the usual CQM (i.e., low- Q^2) picture of the light-hadron structure one assumes the existence of a universal hadronic scale, defined by the constituentquark mass and relevant to all low- Q^2 hadron properties.⁸

Following this basic physics we can make a reasonable guess for a relativistic wave function for any groundstate hadron. Derivations would be analogous to those given in Refs. 1 and 2. We nevertheless briefly describe them again here for completeness.

Our calculations are based on the use of the light-cone Fock-state-basis approach.⁹ With the valence-dominance assumption, any hadron state with momentum $p^{\mu} = (p^{+}, p^{-}, \vec{p}_{\perp}) = (p^{0} + p^{3}, (m_{H}^{2} + p_{\perp}^{2})/p^{+}, \vec{p}_{\perp})$ is determined by the light-cone wave function

$$\psi_{\mathrm{val}}(x_i, \vec{\mathbf{k}}_{\perp i}, \lambda_i), \quad \sum x_i = 1, \quad \sum \vec{\mathbf{k}}_{\perp i} = 0$$
 (1)

This represents the amplitude for finding constituents with momentum $p_i^+ = x_i p_i^+$, $\vec{p}_{\perp i} = x_i \vec{p}_{\perp} + \vec{k}_{\perp i}$, and helicity λ_i , which is invariant under all kinematical Lorentz transformations, that contains the Lorentz boost along the three-direction. Hence, it is determined if it is known at rest. This feature greatly simplifies formfactor calculations where one should know the hadron wave function in different frames as an input.

We start with the valence ground state described by the product of the momentum-harmonic-oscillator wave function ϕ and static spin part χ^{H} . In the hadron rest frame (denoted by c.m.), $\sum \mathbf{k}_{i} = 0$,

$$\psi_{\lambda}(\mathbf{k}_{i},\lambda_{i}) = \phi_{\mathrm{c.m.}}(\mathbf{k}_{i})\chi_{\lambda}^{H}(\lambda_{i}) , \qquad (2)$$

where

$$\phi_{\rm c.m.}(\mathbf{k}_i) = \begin{cases} A_{\pi} \exp\left[-\sum_{i=1}^{2} \mathbf{k}_i^2 / 4\beta^2\right] & \text{for pion ,} \\ \\ A_{N} \exp\left[-\sum_{i=1}^{3} \mathbf{k}_i^2 / 2\alpha^2\right] & \text{for nucleon ,} \end{cases}$$

and

$$\chi^{\pi}(\lambda_i) = \chi^{\dagger}_{\lambda_1} \sigma_2 \chi_{\lambda_2} , \qquad (3a)$$

$$\chi_{\uparrow}^{N}(\lambda_{i}) = \chi_{\lambda_{\uparrow}}^{\dagger} \sigma_{2} \chi_{\lambda_{2}} \chi_{\lambda_{3}}^{\dagger} \chi_{\uparrow} , \qquad (3b)$$

for the pion and nucleon, respectively. In the above expressions χ_{λ} are two-component Pauli spinors. We keep flavor and color implicit. With $\alpha \approx \beta \approx$ the universal hadronic scale, the equal-t wave functions are known to give a reasonable first-approximation description of static pion and nucleon properties.¹⁰

Now, in order to get the Lorentz-invariant light-cone wave function (1), we first use the Brodsky-Huang-Lepage prescription¹¹ for the harmonic wave functions which leads to the identification

$$\phi_{\rm cm}(\mathbf{k}_i) \rightleftharpoons \phi_{\rm LC}(\mathbf{x}_i, \mathbf{k}_{\perp i})$$

where

$$\phi_{\rm LC}(x_i, \vec{k}_{\perp i}) = \begin{cases} A_{\pi} \exp\left[-\sum_{i=1}^2 \frac{k_{\perp i}^2 + m_i^2}{x_i} / 8\beta^2\right], \\ A_N \exp\left[-\sum_{i=1}^3 \frac{k_{\perp i}^2 + m_i^2}{x_i} / 6\alpha^2\right], \end{cases}$$
(4a)

for pion and nucleon, respectively.

Then we need an approximation to deal with the problem of the angular momentum in light-cone dynamics. For this purpose, we use a light-cone analog of the mock-hadron method by $Isgur^{12}$ together with the Melosh transformation¹³ which relates the equal-t wave functions in Eq. (3) and light-cone spin states of free spin- $\frac{1}{2}$ constituents. With these relations, we get the following model for the Lorentz-invariant light-cone wave function (1):

$$\psi_{\lambda}(\mathbf{x}_{i},\vec{\mathbf{k}}_{\perp i},\lambda_{i}) = \phi_{\mathrm{LC}}(\mathbf{x}_{i},\vec{\mathbf{k}}_{\perp i})\chi_{\lambda}^{H}(\mathbf{x}_{i},\vec{\mathbf{k}}_{\perp i},\lambda_{i}) \Big/ \left[\prod_{i} \mathbf{x}_{i}\right]^{1/2},$$
(5)

where

$$\chi^{\pi}(\mathbf{x}_{i}, \mathbf{k}_{\perp i}, \lambda_{i}) = \overline{u}_{\lambda_{1}}(m_{\pi} + p_{\mu}\gamma^{\mu})\gamma_{5}v_{\lambda_{2}} \text{ for pion }, \qquad (6a)$$

$$\chi^{N}_{\uparrow}(\mathbf{x}_{i},\vec{\mathbf{k}}_{\perp i},\lambda_{i}) = J_{\uparrow}(\hat{1},\hat{3},\hat{2}) + J_{\uparrow}(\hat{2},\hat{3},\hat{1}) \text{ for nucleon }, \quad (6b)$$

with

$$J_{\uparrow}(\hat{1},\hat{2},\hat{3}) = \overline{u}_{\lambda_{1}}(m_{N} + p_{\mu}\gamma^{\mu})\gamma_{5}v_{\lambda_{2}}\overline{u}_{\lambda_{3}}u_{\uparrow}$$

 $\hat{1}, \hat{2}, \text{ and } \hat{3}$ are collective momentum and helicity indices, $(x_i, \vec{k}_{\perp i}, \lambda_i), i=1,2,3.$ u_{λ} and v_{λ} are the light-cone spinors of Ref. 14. The relativistic nonstatic spin wave functions χ^H are given in Tables I and II.

Note in (5) the coupling between the relative momenta and the quark helicities. The transformation to the infinite-momentum frame,

c.m.:
$$\psi_{\lambda}(\mathbf{k}_{i},\lambda_{i}) \rightleftharpoons LC: \psi_{\lambda}(x_{i},\mathbf{k}_{\perp i},\lambda_{i})$$

leads to the nonstatic spin wave function χ^H which turns out to be crucial for the interpretation (presented in Refs. 1 and 2) of some surprising features of the valence structure of light hadrons as revealed by the QCD sumrule approach and the successful description of elastic form factors in the region below $Q^2 \approx \text{few GeV}^2$ [as discussed after Eq. (8)].

This remark completes the specification of our model and leaves us ready to determine its parameters. Other works related to the subject of the light-cone description of hadron structure are given in Ref. 15. The main novel element of the present approach is the fact that our wave functions, at least approximately (i.e., in the weakbinding limit), are constrained to be eigenstates of J^P .

III. STATIC PROPERTIES OF THE VALENCE-QUARK CONFIGURATION

According to the basic assumptions of the CQM the constituent-quark degrees of freedom are sufficient for a satisfactory description of the hadron spectrum and the hadron low-energy properties. However, the validity of this assumption has only been confirmed in some potential nonrelativistic models. As has been convincingly argued¹⁶ the universal hadronic scale invalidates the non-relativistic approximation usually used in the CQM. Our light-cone wave functions (5) offer us a chance of a fully relativistic, albeit model-dependent, calculation of the nucleon and pion properties.

It is known that electromagnetic (EM) and weak form factors have exact expressions¹⁷ in terms of the lightcone wave function ψ_{val} . The matrix element of a hadronic EM current $j^+=j^0+j^3$ is diagonal in the Fockstate basis if one chooses the following Drell-Yan coordinate system:¹⁸

$$p^{\mu} = (p^{+}, m_{H}^{2} / p^{+}, \vec{0}_{\perp}) ,$$

$$q^{\mu} = (0, 2p \cdot q / p^{+}, \vec{q}_{\perp}) ,$$

Then, the only x^+ -ordered diagrams which contribute to the matrix element $\langle p+q | j^+ | p \rangle$ are the ones where the photon attaches directly to the EM current of the constituent quarks. If we neglect the quark anomalous magnetic moments, then we get

TABLE I. The pion light-cone spin wave function $\chi^{H}(\hat{1},\hat{2})$ with $a_i = m_{\pi} x_i + m$, $k_i^{L,R} = k_i^1 \mp i k_i^2$.

λ_1	λ_2	$\chi^{\pi}(\hat{1},\hat{2})\sqrt{x_1x_2}$
t	t	$a_1k_2^L - a_2k_1^L$
Ť	Ļ	$a_1 a_2 + k_1^L k_2^R$
Ļ	Ť	$-a_1a_2-k_1^Rk_2^L$
Ļ	Ļ	$a_1k_2^R - a_2k_1^R$

 λ_1	λ_2	λ3	$\chi_1^N(\widehat{1},\widehat{2},\widehat{3})\sqrt{x_1x_2x_3}$
t	t	t	$-2a_1a_2k_3^L+a_3(a_1k_2^L+a_2k_1^L)$
t	Ţ	t	$a_1a_2a_3 + k_2^R(2a_1k_3^L - a_3k_1^L)$
Ļ	Ť	t	$a_1a_2a_3 + k_1^R(2a_2k_3^L - a_3k_2^L)$
t	t	Ļ	$-2a_1a_2a_3-k_3^R(a_1k_2^L+a_2k_1^L)$
Ļ	t	Ļ	$k_1^R k_2^L k_3^R + a_2 (2a_3 k_1^R - a_1 k_3^R)$
t	Ļ	Ļ	$k_1^L k_2^R k_3^R + a_1 (2a_3 k_2^R - a_2 k_3^R)$
Ļ	Ļ	Ť	$-2k_{1}^{R}k_{2}^{R}k_{3}^{L}-a_{3}(a_{1}k_{2}^{R}+a_{2}k_{1}^{R})$
Ļ	Ļ	Ļ	$-2a_3k_1^Rk_2^R+k_3^R(a_1k_2^R+a_2k_1^R)$

TABLE II. The nucleon light-cone spin wave function $\chi_1^N(\hat{1}, \hat{2}, \hat{3})$ with $a_i = m_N x_i + m$, $k_i^{L,R} = k_i^{1} \mp i k_i^{2}$.

$$\left\langle p+q;H\lambda_{f}\left|\frac{J^{+}}{p^{+}}\right|p;H\lambda_{i}\right\rangle =\sum_{n}\int \left[dx\right]\left[d^{2}k\right]\psi_{\lambda_{f}}^{H\dagger}(x_{m},\vec{k}_{\perp m},\lambda_{m})\psi_{\lambda_{i}}^{H}(x_{m},\vec{k}_{\perp m},\lambda_{m})\frac{\bar{u}_{n}}{\sqrt{k_{h}^{+}}}Q_{n}\gamma^{+}\frac{u_{n}}{\sqrt{k_{n}^{+}}},\qquad(7)$$

where Q_n is the charge of the struck constituent with momentum $\vec{k}_{\perp n} = \vec{k}_{\perp n} + (1 - x_n)\vec{q}_{\perp}$. The spectator quarks have momenta $\vec{k}_{\perp m} = \vec{k}_{\perp m} - x_m \vec{q}_{\perp}$. The calculation of the (axial) weak form factor is formally identical to that of the EM matrix element, except for the replacement $\gamma^+ \rightarrow \gamma_5 \gamma^+$. The corresponding form factors and their moments can be identified from the standard nucleon and pion vertex parametrization.

We start at low $Q^2 \ll 1$ GeV² and fit the static properties of the nucleon and pion using wave functions of the form (5) where the quarks are given their constituent masses m = 330 MeV. Our results are given in Tables III and IV. We are not surprised that with the universal scale $\alpha \approx \beta \approx 320$ MeV all static properties of the nucleon and pion but the charge radius of the neutron are described to an accuracy of 10%. It is interesting to note that relativistic kinematics in the three-quark nucleon state yield a contribution to $\langle r^2 \rangle_n^{1/2}$ of the correct sign but a factor of 2 too small in size. However, it is known that $\langle r^2 \rangle_n^{1/2}$ is a sensitive measure of the interquark interactions and receives sizable contributions from spin forces in the dynamics¹⁹ or from the pion cloud.²⁰ With the use of the dynamical approach of Ref. 21 effects of this kind can be accommodated to the light-cone description.

IV. EM FORM FACTORS

Our next objective is to compute the pion and nucleon form factors in the same CQM approximation (m $=m_{\rm const}=330$ MeV). We emphasize the point made earlier that the light-cone wave functions have the form (5) which is valid in an arbitrary frame. Then the Drell-Yan form (7) at $Q^2 = -q_1^2$ gives an exact expression for the valence-constituent-quark contribution. It leads to a parameter-free prediction if the momentum scale factors are chosen to fit the static properties, i.e., $\alpha \approx \beta \approx 320$ MeV. Thus we find that the resulting so-called soft contributions reproduce the data extremely well up to the scale $Q_0^2 \approx 3$ and 2 GeV², for the nucleon and pion case, respectively.

The relativistic CQM yields results (see Figs. 1 and 2) which are practically identical with the empirical parametrization give by the scaling law and multipole formulas:

$$G_E^p(Q^2) = G_M^p(Q^2) / \mu_p = G_M^n(Q^2) / \mu_n$$

= (1+Q²/0.71 GeV²)⁻²

 $F_{\pi}(Q^2) = (1+Q^2/0.46 \text{ GeV}^2)^{-1}$

(Ref. 22). The observed relatively wide range of validity of the relativistic CQM description is due to a proper boost treatment inherent in our light-cone approach. To demonstrate this feature we use as an example the resultant formula for the pion form factor:

TABLE III. Nucleon magnetic moments (in nucleon magnetons), charge radii (in fm), and g_A/g_V ; Experiment: $\mu_p = 2.793$, $\mu_n = -1.913$, $\langle r^2 \rangle_p^{1/2} = 0.84$ (Ref. 26), $\langle r^2 \rangle_n^{1/2} = -0.34$ (Ref. 26), $\sigma_A = -1.23$

$\underline{g_A}/\underline{g_V} = 1.25.$							
	200	240	280	320	360	400	
μ_p	2.91	2.89	2.85	2.80	2.76	2.70	
μ_n	-1.87	-1.83	- 1.79	-1.73	-1.68	-1.63	
$\langle r^2 \rangle_p^{1/2}$	1.14	1.00	0.90	0.83	0.78	0.73	
$\langle r^2 \rangle_n^{1/2}$	-0.11	-0.13	-0.14	-0.15	-0.16	-0.17	
gA/gV	1.47	1.38	1.30	1.20	1.10	0.99	

TABLE IV. Pion charge radius (in fm) and f_{π} (in MeV). Experiment: $\langle r^2 \rangle_{\pi}^{1/2} = 0.66$ (Ref. 27), $f_{\pi} = 93$.

	200	240	280	320	360	400
$\langle r^2 \rangle_{\pi}^{1/2}$	0.82	0.74	0.69	0.64	0.60	0.57
f_{π}	91	98	99	93	82	64

$$F_{\pi}(Q^{2}) = N_{\pi}^{2} \int_{0}^{1} \frac{dx}{x(1-x)} \exp[-(m^{2}+\xi^{2})/x(1-x)] \times \{a^{2}-2a[\xi^{2}+x(1-x)]-b[\xi-x(1-x)]+\xi^{4}+2(x-x^{2})^{2}\},$$
(8)

where $a = a_1a_2$, $b = (a_1 + a_2)^2$, $\xi^2 = (1 - x)^2Q^2/4$, and all masses and momenta are scaled by 2β . The Gaussian Q^2 dependence is strongly weakened by (i) the term in curly brackets following from the nonstatic spin wave function, and (ii) the combination $(1 - x)^2Q^2$ which is due to relativistic kinematics described in Eq. (7) (Ref. 23). One can check that the above-mentioned softening mecha-



FIG. 1. (a) Pion form factor calculated in the present work with the pion wave function of Eq. (5); $m_{\rm const} = 330$ MeV, $\beta = 320$ MeV. The data are from Ref. 22. (b) Pion form factor calculated in the present work with the pion wave function of Eq. (5); $m_{\rm const} = 330$ MeV, $\beta = 320$ MeV. The data are from Ref. 28.

nism for the pion case is in operation up to $Q^2 \leq 16\beta^2 \approx 1.6 \text{ GeV}^2$.

When form factors are calculated at $Q^2 \gtrsim Q_0^2$ their falloff with Q^2 is much faster than indicated by experiments (see Figs. 3 and 4). Hence, as expected, by the characteristic (quark) scale $\mu^2 \gg Q_0^2 / N \approx 1 \text{ GeV}^2$ (N=the number of constituents) our soft form-factor formulas are not at all good representations of the data. In this region the hadron form factors are calculated in perturbative QCD (Ref. 14) as a special case of exclusive reactions. The amplitude for scattering is a convolution of a hardscattering amplitude and the quark distribution amplitude within the initial and final hadrons. The hardscattering amplitude, containing the point interactions of N valence quarks, leads to the power falloff at large Q^2 . As mentioned above the hadron wave function (5) with light highly relativistic quarks (i.e., with the large momentum scale) gives the distribution amplitudes with the strong asymmetry making the perturbative QCD prediction for the pion and nucleon consistent with the data (see Figs. 3 and 4).

V. SUMMARY

The prescription adopted here allows the rest-frame wave function of the quark model to be used to con-



FIG. 2. (a) Proton magnetic form factor calculated in the present work with the nucleon wave function of Eq. (5); $m_{\text{const}} = 330 \text{ MeV}, \alpha = 320 \text{ MeV}$. The data are from Ref. 29.



FIG. 3. Soft contribution to $F_{\pi}(Q^2)$ calculated as in Fig. 1 compared to hard contribution (Ref. 3) and to experiment (Ref. 28).

struct a light-cone wave function valid in any reference frame. Starting with the basic concepts of the CQM we first calculate the so-called static properties (i.e., quantities proportional to q_{\perp}^n , n=0,1,2) which are nevertheless sensitive to the proper boost treatment. The calculation leads to results which are quite satisfactory and consistent with the CQM assumptions. Using the momentum scale factor which fits the pion and nucleon, we find the relativistic wave functions to be remarkably successful in providing accurate form factors over a wide range of Q^2 . In order to realize that the results are far from being trivial one should compare them with the output of a typical calculation which uses the standard equal-*t* formalism (see Ref. 24).

Improvements of the present approach will involve the use of more realistic spin-momentum wave functions with details of the interquark interactions taken into account or/and more complicated nonvalence configurations (e.g., meson clouds, etc.). Some amount of more complicated configurations is expected to provide a complete explanation of the neutron charge radius.

In concluding, we note that the relativistic wave function provides a link between the low- and high-



FIG. 4. Soft contribution to $G_{M}^{\mu}(Q^{2})$ calculated as in Fig. 2 compared to hard contribution (Ref. 3) and to experiment (open circles, Ref. 29; solid circles, Ref. 30).

momentum-transfer picture of the hadron structure. To make the link union of the CQM and parton-quark model one must clarify the status of the quark mass, since the quark-parton model assumes that quarks are pointlike and essentially massless. The meaning of the two momentum scales also remains to be provided. There is some indication that the concept of a scale-dependent effective quark mass²⁵ may serve the purpose. If the characteristic scale increases then the effective quark mass starts to run at $\mu \approx m_{\text{const}}$, making the valence system more relativistic. The effect can be partly simulated in the CQM with the following substitution: (the running quark mass, the universal scale) \rightarrow (the constituentquark mass, the effective scale > the universal scale) made in the relativistic wave function. Details of the latter idea will be reported upon in the near future.

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