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Uniqueness of the relativistic nucleon state

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Within the basic concepts of the constituent-quark model formulated in the light-cone Fock approach I examine the general symmetry properties of the relativistic nucleon wave function. With these symmetry restrictions I develop an economical parametrization of the nucleon wave function expanding it in terms of known spinor amplitudes multiplied by certain unknown momentum wave functions. I also present a simple model of a relativistic spin wave function which fixes the expansion coefficients and leaves us with the nucleon-ground-state-model wave function uniquely determined. Such a relativistic model serves as a basis for a unified description of low- and high-momentum-transfer nucleon properties presented in the following paper.

I. INTRODUCTION

The constituent-quark-model (CQM) description of the nucleon, and any other light hadron, is based on the following three concepts.

(i) *Valence-quark dominance.* The Fock expansion for a hadronic state can be approximately saturated by the valence-quark configuration.

(ii) *Universal hadronic scale.* There is a universal hadronic scale of ≈ 1 fm relevant to all static hadron properties.

(iii) *Constituent quark.* QCD, as the underlying quark-gluon field theory, can be cut off at some scale of the order of the universal hadronic scale. This produces an effective field theory of the constituent quark with parameters, e.g., mass (the constituent-quark mass) and form factors (the anomalous moments) which are to be determined from the experiment.

It is becoming widely accepted that the world of light-quark hadrons is intrinsically relativistic, even in the CQM picture. As has been convincingly argued[1] this fact can be traced back to the existence of the universal hadronic scale. Fortunately, there is a natural and consistent framework for describing the composite system with a fixed number of constituents. This framework is the light-cone formalism[2] which provides an extremely useful scheme for a relativistic CQM approach.

Unfortunately, at the moment our theoretical understanding of QCD dynamics is in a very rudimentary state. The central unknown of the theory is, in fact, its nonperturbative nature responsible for the confinement of quarks in color-singlet states. In particular, in spite of the progress in perturbative QCD studies, for most purposes of the light-cone description of the composition of the hadron we must continue to rely on models[3] of the wave function for guidance. There are, in such circumstances, two key tools to be used in model building.

(1) These are the relativistic invariance and all the other symmetry properties which already imply a large number of useful relations and restrictions on the hadronic wave functions. Many of these relations and restrictions can be used without any detailed knowledge of the quark dynamics. Others need some additional arguments of a dynamical nature but result from some simple and general properties of the interactions.

(2) If the permissible form of the valence wave function is known, then a large number of phenomena can be used to provide rigorous constraints on the form of the wave function.

To appreciate the power of the first part of the procedure let us invoke the case of the nonrelativistic description of the nucleon ground state as an example. As shown first by Franklin[4] and then by Rolnick,[5] the nucleon spin state is uniquely determined if we consider three quarks in a color singlet, obeying Fermi statistics, and orbiting all in the same s state. Therefore, the symmetry properties together with some simple dynamical assumptions (i.e., quarks are all in the same s state) provide an economical parametrization of the nucleon ground state in terms of a single totally symmetric space (or momentum) wave function.

On the basis of these encouraging results, the purpose of this paper is to give a generalization of the Franklin-Rolnick procedure from the nonrelativistic to the covariant light-cone description of the nucleon ground state. The outline of the paper is as follows. In Sec. II the light-cone description of the nucleon quark state is reviewed. We study the kinematical constraints following from the proper Poincaré group invariance. Having done this, in Sec. III, we discuss model-independent restrictions which follow from the other kinematical symmetries (viz., parity, isospin, and color). In general, for any nucleon-valence-quark state, there are 16 helicity-momentum amplitudes. After having imposed various kinds of the kinematical restrictions we are left with four independent amplitudes. There, we also develop an economical parametrization of the nucleon wave functions, expressing them in terms of known spinor amplitudes multiplied by certain unknown momentum wave functions. This spinor basis is especially suited to an approach which uses some additional dynamical arguments to determine the wave function.

In Sec. IV we conjecture, following current light-cone phenomenology,[3(b)] that the ground state is described by

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a single, totally symmetric momentum wave function. Then the spinor basis expansion contains only three unknown numerical coefficients which must be taken from the experiment or from another dynamical ansatz. The resultant parametrization can serve as an analogue of the Franklin-Rolnick theorem.

In Sec. V we present a simple model of the relativistic spin wave function which fixes the expansion coefficients and leaves us with the nucleon-model wave function uniquely determined. A more detailed comparison of the prediction with the experimental data will be given in the subsequent papers.

II. LIGHT-CONE NUCLEON-QUARK WAVE FUNCTION

Since Dirac's work,[6] there is a natural covariant framework for describing the composite system with a fixed number of constituents. This framework is the light-cone formalism[2] in which the hadron is described by a momentum-space Fock basis defined at equal $x^+=t+z$ rather than the more familiar equal-t wave functions, i.e.,

$$|0\rangle,\ |\mathbf{p}_i\sigma_i\rangle=b^\dagger(\mathbf{p}_i,\sigma_i)|0\rangle,\ldots,$$

where $b^\dagger(\mathbf{p}_i,\sigma_i)$ is the creation operator of the ith constituent quark with momentum $\mathbf{p}_i=(p_i^+,\vec{p}_{\perp i})$, $p^+\equiv p^0+p^3$ (Ref. 7) and internal quantum numbers $\sigma_i=(\lambda_i,t_i,c_i)$. Here, the indices are for the light-cone helicity, isospin, and color, respectively. The light-cone description of single-particle momentum-helicity states is briefly reviewed in Appendix A.

With the valence-quark-dominance assumption (i), any nucleon state ψ with momentum $\mathbf{p}=(p^+,\vec{p}_\perp)$ and helicity λ is described by

$$|\mathbf{p}\lambda\psi\rangle=\sum_{\sigma_i}\int\left[\prod_{i=1}^{3}\frac{dp_i^+d^2p_{\perp i}}{16\pi^3\sqrt{p_i^+}}\right]\frac{1}{(p^+)^{3/2}}\times\Psi_{\mathbf{p}\lambda}(\mathbf{p}_i,\sigma_i)b^\dagger(\mathbf{p}_1,\sigma_1)\times b^\dagger(\mathbf{p}_2,\sigma_2)b^\dagger(\mathbf{p}_3,\sigma_3)|0\rangle\ , \tag{1}$$

where $\Psi_{\mathbf{p}\lambda}(\mathbf{p}_i,\sigma_i)=\langle\mathbf{p}_1\sigma_1,\mathbf{p}_2\sigma_2,\mathbf{p}_3\sigma_3|\mathbf{p}\lambda\psi\rangle$. Now, it is useful to recall some suitable features of the light-cone approach. It will allow us to simplify the general expression (1) and give a starting point for the discussion of wave-function-symmetry properties. We will leave untouched all complications due to the nontrivial structure of the QCD vacuum (e.g., gluon and quark condensates, chiral-symmetry breaking, and the like) even if some of them can be incorporated into the light-cone formulation of QCD (Ref. 8). At the present level of discussion we will take it for granted that all effects due to them can be absorbed into the effective parameters of the CQM.

One of the unique features of the equal-x^+ quantization approach is the fact that it enables the complete separation of the kinematical and dynamical features of Poincaré invariance.[9] Ten generators of Poincaré algebra can be replaced by two commuting algebras $\mathcal{K}$ and $\mathcal{D}$. The kinematical algebra $\mathcal{K}=(P^+,\vec{P}_\perp,\ \vec{E}_\perp,K^3)$ consists of three components of momentum and three boosts.

(a) The boost along the three-direction (K is a positive multiplier), i.e.,

$$K^3:\ p^+\to Kp^+,\ \ p^-\to K^{-1}p^-,\ \ \vec{p}_\perp\to\vec{p}_\perp\ . \tag{2a}$$

(b) The transverse boosts

$$\vec{E}_\perp:\ p^+\to p^+,\ \ p^-\to p^-+2\vec{p}_\perp\cdot\vec{v}_\perp+p^+\vec{v}_\perp^{\,2}\ ,\qquad \vec{p}_\perp\to\vec{p}_\perp+p^+\vec{v}_\perp\ . \tag{2b}$$

The dynamical algebra $\mathcal{D}=(M,\vec{\mathcal{J}}_\perp,\mathcal{J}^3)$ is generated by the mass operator M, the transverse spin operator $\vec{\mathcal{J}}_\perp$, and the light-cone helicity operator $\mathcal{J}^3$. In fact, the latter is also the kinematical operator but is needed here to close the dynamical algebra. Let us mention that the component P^- of the total four-momentum operator is related to the mass operator by

$$P^-=(M^2+\vec{P}_\perp^{\,2})/P^+\ . \tag{3}$$

Along with the decomposition $\mathcal{K}\oplus\mathcal{D}$ there is a factorization of basis nucleon states into the kinematical and dynamical parts, viz.,

$$|\mathbf{p}\ \text{int}\rangle=|\mathbf{p}\rangle\otimes|\text{int}\rangle\ , \tag{4}$$

where int labels the internal dynamical variables needed to distinguish different states at rest ($p^+=m_N,\vec{p}_\perp=0$). This factorization is a complete one because the kinematical algebra acts only on the momentum vector $\mathbf{p}$ in a manner which is independent of the internal variables int. In particular, it means that there is no Wigner-type rotation in the approach [compare Eq. (A5)].

There is a suitable choice of variables for a description of a composite system with a finite number of elementary constituents. To accomplish the factorization (4), instead of the individual momenta $\mathbf{p}_i=(p_i^+,\vec{p}_{\perp i})$ we can use the following variables: the total momentum $\mathbf{p}=(p^+,\vec{p}_\perp)$, $p^+=\sum p_i^+$, $\vec{p}_\perp=\sum\vec{p}_{\perp i}$, the fractions $x_i=p_i^+/p^+$, and the individual transverse momenta at rest $\vec{k}_{\perp i}=\vec{p}_{\perp i}-x_i\vec{p}_\perp$. It is easy to check that the variables $(x_i,\vec{k}_{\perp i})$ are invariant under the kinematical transformation (2). With the normalization convention of Ref. 7, eigenstates of the momenta P^+ and $\vec{P}_\perp$ are of the form

$$\Psi_{\mathbf{p}\lambda}(\mathbf{p}_i,\sigma_i)=16\pi^3p^+\delta^3\left[\mathbf{p}-\sum\mathbf{p}_i\right]\psi_\lambda(x_i,\vec{k}_{\perp i},\sigma_i)\ . \tag{5}$$

This seemingly trivial expression warrants some comments.

(i) The wave function ψ is independent of the nucleon's momentum $\mathbf{p}=\sum\mathbf{p}_i$. Thus, Eq. (5) formulates the complete separation of the overall center-of-mass-system motion in momentum space. As shown by Leutwyler and Stern in Ref. 9 it is a consequence of the transitivity of the kinematical group in momentum space. Such a separation is not possible in the equal-t approach. In the latter case, the kinematical group generated by $(\vec{P},\vec{J})$ is transitive only in coordinate space. However, the x- and the p-space bases are related in an interaction-dependent way.

(ii) The wave function ψ is invariant under all kinematical Poincaré transformations, i.e., translations, three boosts (2), and the rotation about the three-direction. Hence, it is determined if it is known at rest. Both features bring great simplification to model building as well as to matrix element calculations where one should know the nucleon wave function in different frames as an input.

We return our attention to the general formula of the state vector (1). Using Eq. (5) we find

$$|\mathbf{p}\lambda\psi\rangle=\sum_{\sigma_i}\int\frac{[dx][d^2k_\perp]}{\sqrt{x_1x_2x_3}}\psi_\lambda(x_i,\vec{k}_{\perp i},\sigma_i)\times\prod_{i=1}^{3}b^\dagger(x_ip^+,x_i\vec{p}_\perp+\vec{k}_{\perp i},\sigma_i)|0\rangle\ ,\qquad(6)$$

where

$$[dx]=\prod_{i=1}^{3}dx_i\delta\left[\sum x_i-1\right]\ ,$$

$$[d^2k_\perp]=16\pi^3\delta^2\left[\sum\vec{k}_{\perp i}\right]\prod_{i=1}^{3}d^2k_{\perp i}/16\pi^3\ .$$

Here, we follow the notation of Ref. 2(a).

Concluding, in the light-cone approach the validity of the Poincaré algebra allows one to define the frame-independent wave function ψ which therefore has a meaning of the probability amplitude for finding the constituents with momenta $(x_ip^+,\ x_i\vec{p}_\perp+\vec{k}_{\perp i})$ *within the nucleon.*

III. SYMMETRY PROPERTIES OF THE WAVE FUNCTION

Up to now we have studied a general three-quark state with the kinematical constraints following from the proper Poincaré-group invariance. Before we call a three-quark state, the nucleon we must still consider restrictions which follow from the other kinematical symmetries. Those of which we can discuss without specifying the interquark interactions are parity, permutations, isospin, and color. In this section we shall show that the conventional parity, flavor, and color assignments reduce the number of three-quark wave-function components to four independent amplitudes. In addition, for these helicity-momentum amplitudes we derive a convenient spinor basis.

First, we examine parity invariance. Following Soper[10] we consider the operator of reflection in the xz plane,

$$\mathcal{Y}=\exp(-i\pi J_y)\mathcal{P}=R(0,\pi,0)\mathcal{P}\ ,$$

as the light-cone parity. Clearly, $\mathcal{Y}$ commutes with the Lorentz transformation along the z axis. Applying the operator $\mathcal{Y}$ to the single-particle light-cone state $|\mathbf{p}\lambda\rangle$ in Eq. (A1) we get

$$\mathcal{Y}|p^+p^1p^2\lambda\rangle=c_P(-1)^{1/2-\lambda}|p^+p^1-p^2-\lambda\rangle\ .\qquad(7)$$

Here, c_P is the intrinsic parity of the particle. Thus, the action of $\mathcal{Y}$ simply changes the transverse momentum from $\vec{p}_\perp$ to $\hat{p}_\perp=(p^1,-p^2)$ and flips the helicity. To derive the desired transformation law for a general state $|\mathbf{p}\lambda\psi\rangle$ we assume the nucleon and quark intrinsic parities are the same and obtain, from Eq. (6),

$$\psi_\lambda(x_i,\vec{k}_{\perp i},\lambda_i)=(-1)^{3/2+\sum\lambda_i}\psi_{-\lambda}(x_i,\hat{\mathbf{k}}_{\perp i},-\lambda_i)\ .\qquad(8)$$

Thus, with the invariance under the light-cone parity the three-quark state is described by eight quark-helicity amplitudes.

Next we consider restrictions imposed by isospin and color symmetry. Requiring that the nucleon is a flavor doublet and a color singlet one obtains two representations of a proton-p light-cone state with helicity λ, viz.,

$$|p\lambda M_S\rangle=\sum_{\lambda_i}\int\frac{[dx][d^2k_\perp]}{\sqrt{x_1x_2x_3}}\psi_\lambda^{M_S}(\hat{1},\hat{2},\hat{3})\epsilon_{\alpha\beta\gamma}[2u^{\alpha\dagger}(\hat{1})u^{\beta\dagger}(\hat{2})d^{\gamma\dagger}(\hat{3})-d^{\alpha\dagger}(\hat{1})u^{\beta\dagger}(\hat{2})u^{\gamma\dagger}(\hat{3})-u^{\alpha\dagger}(\hat{1})d^{\beta\dagger}(\hat{2})u^{\gamma\dagger}(\hat{3})]|0\rangle\qquad(9a)$$

and

$$|p\lambda M_A\rangle=\sum_{\lambda_i}\int\frac{[dx][d^2k_\perp]}{\sqrt{x_1x_2x_3}}\psi_\lambda^{M_A}(\hat{1},\hat{2},\hat{3})\epsilon_{\alpha\beta\gamma}[u^{\alpha\dagger}(\hat{1})d^{\beta\dagger}(\hat{2})-d^{\alpha\dagger}(\hat{1})u^{\beta\dagger}(\hat{2})]u^{\gamma\dagger}(\hat{3})|0\rangle\ .\qquad(9b)$$

Here, we adopt the shorthand notation

$$u^{\alpha\dagger}(\hat{i})\equiv b^\dagger(\mathbf{p}_i,\lambda_i,t_i=\tfrac{1}{2},c_i=\alpha)\ ,$$

$$d^{\alpha\dagger}(\hat{i})\equiv b^\dagger(\mathbf{p}_i,\lambda_i,t_i=-\tfrac{1}{2},c_i=\alpha)\ ,$$

with $i=1$, 2, and 3. $\hat{1}$, $\hat{2}$, and $\hat{3}$ are collective momentum-helicity indices; $\epsilon_{\alpha\beta\gamma}$ is the totally antisymmetric tensor in color space. The superscripts on the states (M_S,M_A) indicate that the wave functions are symmetric or antisymmetric under interchange of the first two quarks.

For practical purposes, such as the relativistic CQM building, it appears convenient to have a representation of the invariant wave functions (9) in which all of the dynamics is contained in a set of invariant momentum amplitudes. In a sense it would factorize out spin from the problem.

As shown in Appendix B, for the positive-parity state, there are eight independent spinor amplitudes $\bar{u}(\hat{1})\Gamma_{12}C\bar{u}^{T}(\hat{2})\bar{u}(\hat{3})\Gamma_3 u_\lambda$ that are constructed from covariants $\Gamma_{12}C\otimes\Gamma_3$ sandwiched between the light-cone bispinors u and $\bar{u}$ of Appendix A. Four of the amplitudes, I_k, $k=1,\ldots,4$ are mixed symmetric; the others, J_k, $k=1,\ldots,4$ are mixed antisymmetric. The corresponding covariants $\Gamma_{12}\otimes\Gamma_3$ which are composed of the nucleon momentum p^μ and the Dirac γ matrices are listed in Tables I and II. Therefore, quite generally, we may write the wave functions of (9) in the form

$$\psi^{M_S}(\hat{1},\hat{2},\hat{3})=\sum_{k=1}^{4}[\phi_k^{M_S}(1,2,3)I_k+\phi_k^{M_A}(1,2,3)J_k] \quad (10a)$$

and

$$\psi^{M_A}(\hat{1},\hat{2},\hat{3})=\sum_{k=1}^{4}[\phi_k^{M_S}(1,2,3)J_k+\phi_k^{M_A}(1,2,3)I_k]\ , \quad (10b)$$

where $\phi_k^{M_S},\phi_k^{M_A}$ are the momentum wave functions of a given symmetry under interchange of the first two quarks.

Having constructed the convenient spinor basis for the mixed symmetric and the mixed antisymmetric representations (9) one can prove their equivalence. This can be done simply by the method of Ref. 5. Since the quark operators anticommute, and we have antisymmetry in the color indices, we can freely interchange the quark operators (but not the color indices). Using this property we show that both states are equivalent to

$$|p\!\!\!/\lambda\rangle=\sum_{\lambda_i}\int\frac{[dx][d^2k_\perp]}{\sqrt{x_1x_2x_3}}\tilde{\psi}_\lambda(\hat{1},\hat{2},\hat{3})\epsilon_{\alpha\beta\gamma}\times u^{\alpha\dagger}(\hat{1})u^{\beta\dagger}(\hat{2})d^{\gamma\dagger}(\hat{3})\,|\,0\rangle\ , \quad (11)$$

where $\tilde{\psi}$ is given by

$$\tilde{\psi}_\lambda(\hat{1},\hat{2},\hat{3})=[2\psi_\lambda^{M_S}(\hat{1},\hat{2},\hat{3})-\psi_\lambda^{M_S}(\hat{1},\hat{3},\hat{2})-\psi_\lambda^{M_S}(\hat{2},\hat{3},\hat{1})]/3 \quad (12a)$$

TABLE I. Mixed symmetric covariants.

Amplitude	$\Gamma_{12}C\otimes\Gamma_3$
I_1	$\gamma_\mu C\otimes\gamma^\mu\gamma_5$
I_2	$\sigma_{\mu\nu}C\otimes\sigma^{\mu\nu}\gamma_5$
I_3	$\gamma_\mu C\otimes\gamma_5 p^\mu$
I_4	$i\sigma_{\mu\nu}C\otimes p^\nu\gamma^\mu\gamma_5$

TABLE II. Mixed antisymmetric covariants.

Amplitude	$\Gamma_{12}C\otimes\Gamma_3$
J_1	$\gamma_5 C\otimes I$
J_2	$\gamma_5\gamma_\mu C\otimes p^\mu$
J_3	$C\otimes\gamma_5$
J_4	$\gamma_5\gamma_\mu C\otimes\gamma^\mu$

in the mixed symmetric representation and

$$\tilde{\psi}_\lambda(\hat{1},\hat{2},\hat{3})=[\psi_\lambda^{M_A}(\hat{1},\hat{3},\hat{2})-\psi_\lambda^{M_A}(\hat{3},\hat{2},\hat{1})] \quad (12b)$$

in the mixed antisymmetric representation, respectively. Note that the tilde means just the required isospin projection.

The proton light-cone states (11) are now written in the so-called *uds* basis, which was first introduced by Franklin[4] and recently used extensively in CQM calculations by Isgur and collaborators.[11] In the *uds* basis one carries out only a part of the antisymmetrization that would be required by the full S_3 group; the rest is ensured by the anticommutation properties of the quark operators.

Notice that the above-mentioned equivalence means that in a model consideration one can use the helicity-momentum amplitudes which are either symmetric or antisymmetric under interchange of the first two quarks (compare Sec. V).

To make explicit the restriction imposed by the isospin symmetry we use the *uds* basis form of the proton state vector (11). Requiring that the total isospin of three quarks in (11) be equal to $\frac{1}{2}$, one obtains the relation

$$\tilde{\psi}_\lambda(\hat{1},\hat{2},\hat{3})+\tilde{\psi}_\lambda(\hat{1},\hat{3},\hat{2})+\tilde{\psi}_\lambda(\hat{2},\hat{3},\hat{1})=0\ , \quad (13)$$

which is of course satisfied by both representations (12). Now it is easy to check that condition (13) reduces the number of linearly independent quark-helicity amplitudes of the relativistic nucleon state from eight to four. The four amplitudes we select are $\uparrow\uparrow\uparrow$, $\uparrow\downarrow\uparrow$, $\downarrow\uparrow\downarrow$, $\downarrow\downarrow\downarrow$. Let us illustrate this with an example.

For a helicity-up proton, there are three amplitudes with the total quark helicity $+\frac{1}{2}$, viz.,

$$\tilde{\psi}(1,2,3;\uparrow\uparrow\downarrow)\equiv-2T(1,2,3)\ , \quad (14a)$$

$$\tilde{\psi}(1,2,3;\uparrow\downarrow\uparrow)\equiv\varphi(1,2,3)\ , \quad (14b)$$

$$\tilde{\psi}(1,2,3;\downarrow\uparrow\uparrow)\equiv\varphi'(1,2,3)\ , \quad (14c)$$

where 1, 2, and 3 are collective momentum $(x,\vec{k}_\perp)$ variables.

Condition (13) leads to the relations

$$2T(1,2,3)=\varphi(1,3,2)+\varphi(2,3,1)\ , \quad (15a)$$

$$\varphi(1,2,3)=V(1,2,3)-A(1,2,3)\ , \quad (15b)$$

$$\varphi'(1,2,3)=V(1,2,3)+A(1,2,3)\ , \quad (15c)$$

where $V\equiv[\varphi(1,2,3)+\varphi(2,1,3)]/2$ and $A\equiv[\varphi(2,1,3)-\varphi(1,2,3)]/2$. Therefore, there is only one independent amplitude with total quark helicity $+\frac{1}{2}$, say $\varphi(1,2,3)$. Moreover, the only independent amplitude $\varphi(1,2,3)$, in general, has no definite symmetry under interchange of the momentum variables of the first two quarks. The actual importance of the remark is exploited in Ref. 12.

The message of this section is that the kinematical requirements of the Poincaré invariance together with the internal symmetries or, in other words, the model-independent methods, do not restrict the three-quark wave function very strongly. We are left with fourfold ambiguity in the relativistic case. With the result of the symmetry consideration now in hand, we cannot proceed without some information about the interactions of the constituent quarks.

IV. NUCLEON GROUND-STATE MODEL

For a complete determination of the transformation properties of the wave function $\tilde{\psi}$, Eq. (11), one must specify unitary representation of the full Poincaré group. It means that, in addition to the invariance under the kinematical Poincaré generators, the bound state $\tilde{\psi}$ is to be an eigenstate of the two interaction-dependent operators, i.e., the mass operator M and the spin $\mathcal{J}^2$. The main difficulty of the program is that it requires a large amount of information on nonperturbative effects such as those of confinement, chiral-symmetry breaking, condensate, etc., implemented into the light-cone scheme.[8] Therefore, it seems more appropriate and simple to start with a relativistic model of the wave function rather than a relativistic model of dynamics itself.

We begin our discussion by commenting on the nonrelativistic description of the nucleon ground state. It is not difficult to see that, apart from some trivial notational modifications, the above discussion of the internal symmetries can be directly applied to any Galileo-invariant quark dynamics yielding identical symmetry restrictions. Therefore, we rederive here the Franklin-Rolnick theorem with the aim of shedding some light on the dynamical assumptions involved.

For the lowest-energy nucleon state, it is customary to assume that the three valence quarks are all in the same s state. Hence, the total orbital angular momentum equals zero and there is only one linearly independent quark-helicity amplitude, viz., $\uparrow\downarrow\uparrow$. Furthermore, because of the s-orbit assumption, the total orbital (momentum) wave function $\phi(1,2,3)$ is completely symmetric. Thus, Eqs. (15) and (14) give

$$\begin{aligned}\tilde{\psi}_\lambda(\hat{1},\hat{2},\hat{3})&=\phi(1,2,3)\chi_\uparrow(\lambda_1,\lambda_2,\lambda_3)\ ,\\ \chi_\uparrow(\lambda_1,\lambda_2,\lambda_3)&=(-2\delta_{\lambda_1\uparrow}\delta_{\lambda_2\uparrow}\delta_{\lambda_3\downarrow}+\delta_{\lambda_1\uparrow}\delta_{\lambda_2\downarrow}\delta_{\lambda_3\uparrow}\\ &\quad+\delta_{\lambda_1\downarrow}\delta_{\lambda_2\uparrow}\delta_{\lambda_3\uparrow})\ .\end{aligned}\tag{16}$$

It is clear from (16) that for the lowest-energy nucleon state there is *the static* spin wave function $\chi_\uparrow(\lambda_1,\lambda_2,\lambda_3)$ which is uniquely determined if we assume valence-quark dominance, use Fermi statistics, and invoke the conventional parity and color assignments. Let us stress, however, that the uniqueness of the nonrelativistic nucleon state follows from the extra weak nevertheless dynamical assumption that the ground-state quarks are all in the same s state.

To implement analogous assumptions for the light-cone description of the ground nucleon state it is convenient to employ the spinor-basis representation (10). In the Fock-state approach there exists much intuition (a wide discussion of which can be found in Refs. 3 and 13) for the following *basic ansatz:* the ground nucleon state is described by a single, totally symmetric momentum-space wave function $\phi(1,2,3)$ which depends only on the light-cone off-shell energy

$$\mathcal{E}=p^+\left[p^- -\sum_{i=1}^{3}p^-{}_i\right]=m_N^2-\sum_{i=1}^{3}(k_{\perp i}^2+m_i^2)/x_i\ ,$$

i.e.,

$$\phi(1,2,3)=\phi(\mathcal{E})\ .\tag{17}$$

There is already quite extensive wave-function phenomenology[2,13-17] which assumes (for simplicity reasons) that in the nonperturbative domain the ϕ falls off exponentially in $\mathcal{E}$, i.e.,

$$\phi(1,2,3)=A\exp(\mathcal{E}/6\alpha^2)\ .\tag{18}$$

Note the presence of a mass-scale parameter α which gives the characteristic size of the composite state.

With this assumption, Eqs. (10) give

$$\psi^{M_S}(\hat{1},\hat{2},\hat{3})=\phi(1,2,3)\sum_{k=1}^{4}a_kI_k(\hat{1},\hat{2},\hat{3})\tag{19a}$$

and

$$\psi^{M_A}(\hat{1},\hat{2},\hat{3})=\phi(1,2,3)\sum_{k=1}^{4}b_kJ_k(\hat{1},\hat{2},\hat{3})\ ,\tag{19b}$$

where a_k,b_k are free parameters of the model (see Ref. 18). Now we use the uds basis, Eq. (11), and employ the isospin projection to the wave functions (19). The details of the calculation are relegated to Appendix B. It turns out that the resulting isospin projections of the spinor amplitudes I_k,J_k are interrelated. Namely, $\tilde{J}_4=\tilde{J}_3-\tilde{J}_1$ and $\tilde{I}_4=2\tilde{I}_3+m_N(2\tilde{I}_1-\tilde{I}_2)/4$ (see Ref. 15). Now that we know these relations, we can define the nucleon ground-state model by the following two (equivalent) wave-function expansions:

$$\tilde{\psi}(\hat{1},\hat{2},\hat{3})=\phi(1,2,3)\sum_{k=1}^{3}a_k\tilde{I}_k\tag{20a}$$

or

$$\tilde{\psi}(\hat{1},\hat{2},\hat{3})=\phi(1,2,3)\sum_{k=1}^{3}b_k\tilde{J}_k\ ,\tag{20b}$$

each given by three linearly independent [see Eqs. (B4)–(B11)] spinor amplitudes $\tilde{I}_k$ or $\tilde{J}_k$, respectively.

The parametrization (20) can serve as an analogue of the Franklin-Rolnick theorem. It shows that the kinematic-symmetry requirements even if supplemented with some plausible assumptions on the momentum wave function (17) are insufficient for the uniqueness of the nucleon ground state. After that we are still left with threefold ambiguity in the relativistic case, represented by the expansion coefficients, either a_k or b_k. They can be taken from the experiment (see Ref. 16) or from another dynamical ansatz[14] (see also Sec. V). The lack of uniqueness can be definitely traced back to the fact that the Poincaré group in the light-cone approach contains as many as three Hamiltonians (i.e., the interaction-dependent generators), as opposed to the Galileo group that contains a single Hamiltonian. It is suggested by the model of Sec. V that the angular condition $J=\frac{1}{2}$ is required to fix the admixture coefficients a_k or b_k.

V. NUCLEON SPIN-WAVE-FUNCTION MODEL

In this section we consider a very elementary model of the spin wave function in order to illustrate the role of the angular condition $J=\frac{1}{2}$ for the determination of the nucleon ground-state wave function (20). It is convenient to start with a system of relativistic but noninteracting quarks. Hence, we follow to some extent the mock-hadron method of Isgur[19] and use the corresponding prescription. The nucleon is a collection of quarks with the wave function (17) of bound quarks in the physical nucleon and the mock-nucleon mass equal to the mean total invariant mass of the free quarks. As is shown below this prescription defines a procedure to deal with the problem of the spin and mass operators.

For noninteracting particles the equal-x^+ and the equal-t (T) descriptions are equivalent. They are related by a unitary transformation which, for a single particle with spin $\frac{1}{2}$, is given in Eq. (A8). Now, using the standard method of the equal-t approach[20] we can construct a state vector $|\vec{p}=0W\frac{1}{2}l=l'=0\rangle_T$ which describes the valence quarks in their center-of-momentum (c.m.) frame ($\vec{p}_1+\vec{p}_2+\vec{p}_3=0$), all in the same s state ($l=l'=0$), being in an eigenstate of the total angular momentum $J=\frac{1}{2}$ and the invariant mass $W_{123}=W$. For the mock nucleon, $|\vec{p}=0W\cdots\rangle_T=|\vec{p}=0m_N\cdots\rangle_T$. Thus, the mock-nucleon state $|\vec{p}=0m_N\cdots\rangle_T$ in the c.m. frame equals the mock-nucleon light-cone state *at rest:*

$$|\vec{p}=0m_N\cdots\rangle_T=|\mathring{p}m_N\cdots\rangle\ ,$$

where $\mathring{p}=(m_N,\vec{0}_\perp)$. Hence, by this prescription, the mock-nucleon state is the eigenstate of the mass operator, the spin, as well as **P**.

To construct a three-particle state with a given total angular momentum $J=\frac{1}{2}$ and a helicity λ in the ls coupling scheme we use the same methods as in Ref. 21. We start with the standard Clebsch-Gordan prescription

$$|\vec{p}_{12}=0\lambda_{12}W_{12}S_{12}l_{12}=0\rangle_T=\sum_{\lambda_1\lambda_2}C^{S_{12}\lambda_{12}}_{\frac{1}{2}\lambda_1\frac{1}{2}\lambda_2}\int\frac{d^3k_1}{\epsilon_1}\frac{d^3k_2}{\epsilon_2}\delta^4(k_1+k_2-\mathring{p}_{12})|\vec{k}_1\lambda_1\rangle_T|\vec{k}_2\lambda_2\rangle_T \tag{21}$$

with $k_1=(\epsilon_1,\vec{k})$, $k_2=(\epsilon_2,-\vec{k})$, $\mathring{p}_{12}=(W_{12},\vec{0})$, and $W_{12}=\epsilon_1+\epsilon_2$ is the invariant mass of the (12) pair. A vector of the form (21) describes the pair (12) in its c.m. frame being in an eigenstate of the angular momentum $J_{12}^2=S_{12}^2$, λ_{12}, and the invariant mass W_{12}. Next, we perform the special Lorentz transformation $l(p_{12}\leftarrow\mathring{p}_{12})=l(p_1\leftarrow k_1)l(p_2\leftarrow k_2)$ of the vectors (21) to the overall c.m. frame in which $\vec{p}_1+\vec{p}_2+\vec{p}_3=0$. After using (A7) we obtain the result

$$|\vec{p}_{12}=-\vec{p}_3\lambda_{12}W_{12}S_{12}l_{12}=0\rangle_T=\sum_{\substack{\lambda_1\lambda_2\\ \lambda_1'\lambda_2'}}C^{S_{12}\lambda_{12}}_{\frac{1}{2}\lambda_1\frac{1}{2}\lambda_2}\int\frac{d^3p_1}{\epsilon_1}\frac{d^3p_2}{\epsilon_2}\delta^4(p_1+p_2-p_{12})|\mathbf{p}_1\lambda_1'\rangle|\mathbf{p}_2\lambda_2'\rangle D^{\frac{1}{2}*}_{\lambda_1\lambda_1'}(U(k_1)U(\mathring{p}_{12}))$$
$$\times D^{\frac{1}{2}*}_{\lambda_2\lambda_2'}(U(k_2)U(\mathring{p}_{12}))\ . \tag{22}$$

The latter equation follows from the simple transformation law of the light-cone state under the special Lorentz transformation (A5). Combining the vectors (22) in a similar manner with the vectors $|\vec{p}_3\lambda_3\rangle_T$ we obtain

$$|\vec{p}=0\lambda WSW_{12}l=l'=0\rangle_T=\sum_{\lambda_{12}\lambda_3\lambda_3'}C^{S\lambda}_{S_{12}\lambda_{12}\frac{1}{2}\lambda_3}\int\frac{d^3p_{12}}{\epsilon_{12}}\frac{d^3p_3}{\epsilon_3}\delta^4(p_{12}+p_3-\mathring{p})|\vec{p}_{12}=-\vec{p}_3\lambda_{12}W_{12}S_{12}l_{12}=0\rangle_T|\mathbf{p}_3\lambda_3'\rangle$$
$$\times D^{\frac{1}{2}*}_{\lambda_3\lambda_3'}(U(p_3))\ . \tag{23}$$

To actually perform calculations with these mock-nucleon wave functions we make the identification $U(k_i)U(p_{12})=U(p_1)$, $i=1,2$. Some motivations for doing this are given in Ref. 22.

From (23) and (22), after a simple change of variables, we find an expansion of the mock-nucleon state $|\mathring{p}m_N\cdots\rangle$ into one-particle states of the light-cone dynamics

$$|\mathring{\mathbf{p}}\lambda m_N S l=l'=0\rangle=\sum_{\lambda_i}\int\frac{[dx][d^2k_\perp]}{\sqrt{x_1x_2x_3}}\chi^S_\lambda(\hat{1},\hat{2},\hat{3})\times\prod_{i=1}^{3}|\mathbf{p}_i\lambda_i\rangle\ ,$$

where

$$\chi^S_\lambda(\hat{1},\hat{2},\hat{3})=\sum_{\lambda_{12}\lambda'_i}C^{S\lambda}_{S_{12}\lambda_{12}\frac{1}{2}\lambda'_3}C^{S_{12}\lambda_{12}}_{\frac{1}{2}\lambda'_1\frac{1}{2}\lambda'_2}\prod_{i=1}^{3}D^{\frac{1}{2}*}_{\lambda'_i\lambda_i}(U(p_i))\ .$$

Finally, based on the mock-nucleon prescription, we get the following model for the Lorentz-invariant light-cone wave function (6):

$$\psi_\lambda(\hat{1},\hat{2},\hat{3})=\phi(\mathcal{E})\chi_\lambda(\hat{1},\hat{2},\hat{3})/\sqrt{x_1x_2x_3}\ , \tag{24}$$

where the relativistic spin wave function χ is given by

$$\chi_\lambda(\hat{1},\hat{2},\hat{3})=\sum_{\lambda_{12}\lambda'_i}C^{\frac{1}{2}\lambda}_{S_{12}\lambda_{12}\frac{1}{2}\lambda'_3}C^{S_{12}\lambda_{12}}_{\frac{1}{2}\lambda'_1\frac{1}{2}\lambda'_2}\prod_{i=1}^{3}D^{\frac{1}{2}*}_{\lambda'_i\lambda_i}(U(p_i))\ . \tag{25}$$

Note, in Eq. (25), the coupling between the relative momenta and the quark helicity. Furthermore, the wave function (24) depends on two parameters: viz., the constituent-quark mass and the bound-state scale factor α [see e.g., Eq. (18)], which is to be of the order of the universal hadronic scale.

It is convenient to start with the subsystem (12) with angular momentum $S_{12}=0$ (i.e., with the mixed antisymmetric spin wave function) then using the spinor basis (see Table I) and the isospin projection Eq. (12b) we get the compact and useful expression

$$\tilde{\chi}_\uparrow(\hat{1},\hat{2},\hat{3})=J_\uparrow(\hat{1},\hat{3},\hat{2})-J_\uparrow(\hat{3},\hat{2},\hat{1})\ , \tag{26}$$

where

$$J_\uparrow(\hat{1},\hat{2},\hat{3})\equiv m_NJ_1-J_2 =\bar{u}(\hat{1})(m_N+p_\mu\gamma^\mu)\gamma_5 v(\hat{2})\bar{u}(\hat{3})u_\uparrow\ .$$

Here, u and v are the light-cone spinors. The explicit form of $\tilde{\chi}_\uparrow(\hat{1},\hat{2},\hat{3})$, for four independent quark-helicity components, can be found in Table III. The other components are readily generated with the use of (8) and (15).

The light-cone spin wave function can also be expressed in the mixed symmetric spinor basis. Using Eqs. (B4)–(B11), we obtain

$$\tilde{\chi}_\uparrow(\hat{1},\hat{2},\hat{3})=-3m_N(\tilde{I}_1-\tilde{I}_2/6+2\tilde{I}_3/m_N)/2\ , \tag{27}$$

where $\tilde{I}_k$, $k=1,2,3$, are the isospin projection of the amplitudes of Table I.

This completes the specification of our model and leaves us ready to check its validity. Certainly, the QCD picture of hadrons is not this simple. Although such a weak-binding assumption is very crude and presumably untenable, it is regarded as the first approximation given by pure relativistic kinematics and fundamentals of the quark model (spin-parity assignments, the constituent-quark mass, etc.). Deviations from the model predictions are then regarded as caused by a dynamical agent.

TABLE III. The nucleon light-cone spin wave function $\tilde{\chi}_\uparrow(\hat{1},\hat{2},\hat{3})$; $a_i=x_im_N+m$, $p^{L,R}=p^1\mp ip^2$.

$\lambda_1\lambda_2\lambda_3$	$\tilde{\chi}_\uparrow(\hat{1},\hat{2},\hat{3})\sqrt{x_1x_2x_3}$
$\uparrow\uparrow\uparrow$	$-2a_1a_2p^L_3+a_3(a_1p^L_2+a_2p^L_1)$
$\downarrow\downarrow\downarrow$	$-2a_3p^R_1p^R_2+p^R_3(a_1p^R_2+a_2p^R_1)$
$\uparrow\downarrow\uparrow$	$a_1a_2a_3+p^R_2(2a_1p^L_3-a_3p^L_1)$
$\downarrow\uparrow\downarrow$	$p^R_1p^L_2p^R_3+a_2(2a_3p^R_1-a_1p^R_3)$

It is an interesting feature of the light-cone approach that a large number of observables have an exact expression in terms of the wave function[2] (e.g., form factors, charge radii, magnetic moments, structure functions, distribution amplitudes, etc.). Thus, if the wave functions are known, then a large number of phenomena can be interrelated. Some interesting consequences of the relation between low- and high-momentum-transfer nucleon properties are given in the following paper.

VI. CONCLUSION

In this paper we have shown, even without explicit information about details of QCD dynamics, we can make a number of basic statements concerning the form of the valence-quark wave function of the nucleon. The nucleon is described by Poincaré-invariant quantum mechanics. In the weak-binding limit, we have found the relativistic spin structure uniquely determined. Comparison of our model predictions with data on some low- and high-momentum-transfer properties of the nucleon may be found in Refs. 12 and 23–26. These give further indication that fundamentals of the quark model, together with relativistic kinematics, provide a consistent starting point of complete description of the nucleon-valence structure (and of the other light-quark hadrons).

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APPENDIX A: SINGLE-PARTICLE STATE ON THE LIGHT CONE

In this appendix we make all of our wave-function conventions explicit and give a compilation of some notations, definitions, and formulas which are used in the text of the paper. Details can be found in Refs. 9, 10, and 21.

The state of a single particle $|\mathbf{p}\lambda\rangle$ with mass m, spin s, and an arbitrary momentum $p^\mu=(\mathbf{p},p^-)=(p^+,\vec{p}_\perp(\vec{p}^{\,2}_\perp+m^2)/p^+)$ is determined, in the Wigner construction,[27] in terms of the state vectors of a particle at rest $\mathring{p}^{\,\mu}=(m,\vec{0}_\perp,m)$:

$$|\mathbf{p}\lambda\rangle = U(l(p\leftarrow\mathring{p}))\,|\mathring{\mathbf{p}}\lambda\rangle \ . \tag{A1}$$

In the light-cone approach, the helicity state λ is specified by the following standard Lorentz transformation $l(p\leftarrow\mathring{p})$ which carries the rest momentum $\mathring{p}$ into p, viz.,

$$l(p\leftarrow\mathring{p}) = \exp(-i\vec{v}_\perp\cdot\vec{E}_\perp)\exp(-i\omega K^3) \ , \tag{A2}$$

where

$$\vec{v}_\perp = \vec{p}_\perp/p^+, \quad \omega = \ln(p^+/m) \ ,$$

and $\vec{E}_\perp$ are the transverse-Galilean-boost generators defined by

$$E^1 = K^1 - J^2, \quad E^2 = K^2 - J^1 \ ,$$

$\vec{J}$ and $\vec{K}$ being the usual Lorentz generators. Now, it is easy to find how states (A1) transform under all proper Lorentz transformations Λ. We get

$$U(\Lambda)\,|\mathbf{p}\lambda\rangle = \sum_{\lambda'} |\mathbf{p}'\lambda'\rangle D^s_{\lambda'\lambda}(R(\Lambda,p)) \ . \tag{A3}$$

Here, $p' = \Lambda p$ and $R(\Lambda,p)$ is the Wigner rotation in accordance with our choice of the light-cone helicity basis, viz.,

$$R(\Lambda,p) = l^{-1}(p'\leftarrow\mathring{p})\,l(p\leftarrow\mathring{p}) \ . \tag{A4}$$

Note that the Lorentz transformations $l(p'\leftarrow\mathring{p})$ are transitive on the mass shell $p^2 = m^2$:

$$l(\mathring{p}\leftarrow p')\,l(p'\leftarrow p)\,l(p\leftarrow\mathring{p}) = 1 \ .$$

Thus, the states $|\mathbf{p}\lambda\rangle$ transform under l,

$$U(l(p'\leftarrow p))\,|\mathbf{p}\lambda\rangle = |\mathbf{p}'\lambda\rangle \ , \tag{A5}$$

without the Wigner rotation (A4).

The vectors $|\mathbf{p}\lambda\rangle$ are assumed to be normalized in a Lorentz-invariant manner:

$$\langle\mathbf{p}'\lambda'\,|\,\mathbf{p}\lambda\rangle = \delta_{\lambda'\lambda}16\pi^3 p^+\delta^3(\mathbf{p}-\mathbf{p}') \ . \tag{A6}$$

Using, once again, the general formula (A3), one can find that the light-cone helicity basis $|\mathbf{p}\lambda\rangle$ and a conventional helicity basis of the equal-t approach $|\vec{p}\lambda\rangle_T$ are related by a rotation

$$|\vec{p}\lambda\rangle_T = \sum_{\lambda'} |\mathbf{p}\lambda'\rangle D^{s*}_{\lambda\lambda'}(U(p)) \ . \tag{A7}$$

For $s=\frac{1}{2}$, the relation (A7) is given by the Melosh transformation[28]

$$|\vec{p}\uparrow\rangle_T = \frac{(p^+ + m)\,|\mathbf{p}\uparrow\rangle - p^R\,|\mathbf{p}\downarrow\rangle}{\sqrt{2p^+(p^0+m)}} \ ,$$
$$|\vec{p}\downarrow\rangle_T = \frac{(p^+ + m)\,|\mathbf{p}\downarrow\rangle + p^L\,|\mathbf{p}\uparrow\rangle}{\sqrt{2p^+(p^0+m)}} \ , \tag{A8}$$

where $p^{L,R} = p^1 \mp ip^2$.

To construct the Dirac bispinor representation of the nucleon wave function, we reformulate the unitary noncovariant transformation law (A3) in terms of a covariant nonunitary representation. Let us consider a particle with spin $s=\frac{1}{2}$. By the Weinberg construction,[29] the corresponding covariant representation S coincides with the standard Dirac representation $D(\frac{1}{2},0)\oplus D(0,\frac{1}{2})$ of SL(2,C). A four-component covariant field is then defined as

$$\psi_\alpha(x) = \int_{p^+>0}\frac{dp^+d^2p_\perp}{16\pi^3p^+}\sum_\lambda\{u_\alpha(\mathbf{p},\lambda)e^{-ipx}b(\mathbf{p},\lambda) + v_\alpha(\mathbf{p},\lambda)e^{ipx}d^\dagger(\mathbf{p},\lambda)\} \ ,$$

where the spinors $u(\mathbf{p},\lambda)$ are chosen such that the field $\psi(x)$ transforms covariantly according to the Dirac spinor representation of SL(2,C):

$$U^{-1}(\Lambda)\psi_\alpha(x)U(\Lambda) = S(\Lambda)_\alpha{}^\beta\psi_\beta(\Lambda^{-1}x) \ .$$

Knowing that we can obtain explicit expressions for the Dirac spinors $u(\mathbf{p},\lambda)$ and the charge-conjugate spinors $v(\mathbf{p},\lambda)$, the spinors $u(\mathbf{p},\lambda)$ are given by

$$u(\mathbf{p},\lambda) = \sqrt{m}\begin{bmatrix}\hat{l}(p\leftarrow\mathring{p})\\ \hat{l}(\mathring{p}\leftarrow p)\end{bmatrix} = S(l(p\leftarrow\mathring{p}))u(\mathring{\mathbf{p}},\lambda) \ , \tag{A9}$$

where the matrix $\hat{l}(p\leftarrow\mathring{p})$ of SL(2,C) represents the standard Lorentz transformation (A2). We recall that the generators of rotations in SL(2,C) are the Pauli matrices, $\vec{J}=\frac{1}{2}\vec{\tau}$, and that the generators of Lorentz boosts are $\vec{K}=\frac{1}{2}i\vec{\tau}$. Then, the exponentials in (A2) can be easily worked out, giving

$$u(\mathbf{p},\uparrow) = \frac{1}{\sqrt{p^+}}\begin{bmatrix}p^+\\ p^R\\ m\\ 0\end{bmatrix}, \quad u(\mathbf{p},\downarrow) = \frac{1}{\sqrt{p^+}}\begin{bmatrix}0\\ m\\ -p^L\\ p^+\end{bmatrix} \ . \tag{A10}$$

The spinors are normalized to $\bar{u}u = 2m$. It is useful to notice the following relation between $S(\Lambda)u(\mathbf{p},\lambda)$ and $u(\mathbf{p}',\lambda')$:

$$S(\Lambda)u(\mathbf{p},\lambda) = \sum_{\lambda'} u(\mathbf{p}',\lambda')D^{1/2}_{\lambda'\lambda}(R(\Lambda,p)) \tag{A11}$$

with $p' = \Lambda p$.

The spinors $v(\mathbf{p},\lambda)$ are related to $u(\mathbf{p},\lambda)$ by charge conjugation $C = i\gamma^0\gamma^2$:

$$v(\mathbf{p},\uparrow) = \frac{1}{\sqrt{p^+}}\begin{bmatrix}0\\ -m\\ -p^L\\ p^+\end{bmatrix}, \quad v(\mathbf{p},\downarrow) = \frac{1}{\sqrt{p^+}}\begin{bmatrix}p^+\\ p^R\\ -m\\ 0\end{bmatrix} \ . \tag{A12}$$

The light-cone spinors (A10) and (A12) correspond to the Weyl realization of the γ matrices:

$$\gamma^0 = \begin{bmatrix}0 & I\\ I & 0\end{bmatrix}, \quad \gamma^i = \begin{bmatrix}0 & -\tau^i\\ \tau^i & 0\end{bmatrix} \ ,$$

$$\gamma_5 = \begin{bmatrix}I & 0\\ 0 & -I\end{bmatrix} \ .$$

Finally, notice that the spinors u and v are identical to the ones used in Ref. 3(a).

APPENDIX B: SPINOR BASIS FOR NUCLEON WAVE FUNCTION

We consider the three-quark system described by the light-cone wave function $\Psi_{p\lambda}(\mathbf{p}_i,\lambda_i)$ which is invariant under all kinematical Poincaré transformations (we keep here color and isospin indices implicit). To construct the spinor representation of Ψ we embed the kinematical Lorentz group in the full Lorentz group of three noninteracting particles. In that case, the relativistic invariance under the full Lorentz group is given by

$$\Psi_{p\lambda}(\mathbf{p}_i,\lambda_i)=\sum_{\lambda_i'\lambda_i} D^{\frac{1}{2}*}_{\lambda_1\lambda_1'}(R_1)\cdots D^{\frac{1}{2}*}_{\lambda_3\lambda_3'}(R_3) \times D^{\frac{1}{2}}_{\lambda'\lambda}(R)\Psi_{p'\lambda}(\mathbf{p}_i',\lambda_i') , \quad \text{(B1)}$$

where $p'=\Lambda p$ and R_i is the Wigner rotations.

This rather complicated momentum-dependent transformation law can be simplified by adopting an informal version of the procedure[30] of replacement of the S matrix by the "M function." For this aim, we display the helicity dependence by factorizing out of the light-cone spinor wave function (A9) of each particle, the remaining M function

$$\Psi_{p\lambda}(\hat{1},\hat{2},\hat{3})=\bar{u}(\hat{1})^{\alpha}[C\bar{u}(\hat{2})]_{\gamma}\bar{u}(\hat{3})^{\beta}u(\mathbf{p},\lambda)_{\delta}M_{\alpha}{}^{\gamma}{}_{\beta}{}^{\delta}$$

then transforming simply under the Lorentz group (dropping the four-momentum delta function)

$$M(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3)=S_1(\Lambda^{-1})S_N(\Lambda^{-1})M(\mathbf{p}_1',\mathbf{p}_2',\mathbf{p}_3') \times S_2(\Lambda)S_3(\Lambda) .$$

As such M is a Lorentz invariant which is a direct product of 4×4 matrices, i.e., $\Gamma_{12}\otimes\Gamma_3$, composed of particle momenta and γ matrices. With the parity invariance, there is $8=(2\times\frac{1}{2}+1)^4/2$ independent M functions each multiplied by an invariant function of momentum $\phi(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3)$.

When the interactions between quarks are turned on, the above spinor representation is still valid, since the invariant wave functions (B1) remain invariant under the kinematical subgroup of the Lorentz group of the interacting system. Hence, we have the following spinor representation of the wave functions in (9):

$$\psi_\lambda(\hat{1},\hat{2},\hat{3})=\sum_{k=1}^{8}\phi_k(1,2,3)\chi_\lambda^{(k)}(\hat{1},\hat{2},\hat{3}) , \quad \text{(B2)}$$

where

$$\chi_\lambda^{(k)}(\hat{1},\hat{2},\hat{3})=\bar{u}(\hat{1})\Gamma_{12}^{(k)}C\bar{u}^{T}(\hat{2})\bar{u}(\hat{3})\Gamma_3^{(k)}u(\mathbf{p},\lambda) . \quad \text{(B3)}$$

In order to construct eight independent invariant spinor amplitudes χ_λ we must realize that for any interacting system there is only one four-momentum [viz., the nucleon momentum $p^{\mu}=(p^+,\vec{p}_\perp,(\vec{p}_\perp^{\,2}+m^2)/p^+)$] at our disposal for the contractions with the γ matrices. Because there is no single-particle p_i^-, four-momenta of individual quarks do not exist. It is worthwhile to stress that in the invariant amplitudes χ_λ the whole information about the total momentum p is only retained in the invariant mass m_N.

Now, notice that among the complete set of 4×4 matrices, ten are symmetric (i.e., $\gamma_\mu C$, $\sigma_{\mu\nu}C$) and six are antisymmetric (i.e., C, $\gamma_4 C$, $\gamma_5\gamma_\mu C$) where $\sigma_{\mu\nu}=i[\gamma_\mu,\gamma_\nu]/2$. Taking this into account, for the three-quark state of positive parity one gets four invariant spinor amplitudes $\chi^{M_S}\equiv I_k$, $k=1,\ldots,4$ which are symmetric, and four amplitudes $\chi^{M_A}\equiv J_k$, $k=1,\ldots,4$ which are antisymmetric under interchange of the first two quarks ($1\leftrightarrow 2$). They are given in Tables I and II, respectively.

Now if we write $\phi\equiv\phi^{M_S}+\phi^{M_A}$, then we obtain the spinor representation of the three-quark light-cone states given in (10).

We complete this discussion by noting that by a number of Fierz transformations one can get the isospin projections[31] [see Eq. (12)] of the spinor amplitudes I_k and J_k, viz.,

$$\tilde{I}_1=I_1 , \quad \text{(B4)}$$

$$\tilde{I}_2=I_2 , \quad \text{(B5)}$$

$$\tilde{I}_3=(I_3+I_4)/3+m_N(-2I_1+I_2)/12 , \quad \text{(B6)}$$

$$\tilde{I}_4=2(I_3+I_4)/3+m_N(2I_1-I_2)/12 , \quad \text{(B7)}$$

$$\tilde{J}_1=(-2I_1+I_2)/4 , \quad \text{(B8)}$$

$$\tilde{J}_2=I_3+I_4+m_N(2I_1+I_2)/4 , \quad \text{(B9)}$$

$$\tilde{J}_3=(2I_1+I_2)/4 , \quad \text{(B10)}$$

$$\tilde{J}_4=I_1 . \quad \text{(B11)}$$

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[22] There are several reasons for not using the exact product of the Melosh transformations. The first, technical, reason is that the approximate form makes the model calculations tractable since it enables to perform analytically all the $[dk_\perp]$ multiple integrations. The second, pragmatic, reason is that the approximate, more symmetric form appears to provide the basis for a successful phenomenology. A plausible rationale for this fact is provided by the arguments recently elaborated by Isgur and collaborators in Ref. 1(b) that the CQM regime with $\langle p\rangle\approx m$ is not too far from the nonrelativistic limit. Note that the identification $U(k_i)U(p_{12})=U(p_i)$ is strictly valid in the nonrelativistic limit.

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