$^1P_1$ data and the hyperfine interaction in quarkonia

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The recent $1\,^1P_1$ data for $c\bar{c}$ and $b\bar{b}$ systems are discussed for a general spin-dependent potential given by a short-range vector $[V(r)]$ and a long-range confining scalar $[S(r)]$ potential. It is pointed out that for each $L \geq 1$, the difference between the $^1L_{J=L}$ level mass and the mass of the center of gravity of the three ^3L_J levels is given solely by the spin-spin interaction and so is determined only by $V(r)$, for arbitrary $S(r)$. It is argued that it will be extremely hard to reconcile the present $^1P_1$ data with a simple flavor-independent potential model for quarkonium.

Observations of possible candidates for the $^1P_1$ levels of the $c\bar{c}$ and the $b\bar{b}$ quarkonium systems have been recently reported[1,2] by the R704 and CLEO Collaborations. The discovery of these levels provides added support for the potential-model approach to quarkonia spectra, where the $q\bar{q}$ system is taken to interact nonrelativistically through a potential of the form

$$U = U_0 + U_{\text{spin}} \ . \tag{1}$$

The spin-independent potential U_0 (which may be velocity dependent) will essentially yield the center of gravity of levels with given orbital angular momentum L, whereas U_{spin} contains the spin-dependent terms and gives the splitting both of the $^3S_1$ and $^1S_0$ states and of each $L \geq 1$ level into the four states $^3L_{L-1}$, ^3L_L, $^3L_{L+1}$, and $^1L_1$. We use the ordinary spectroscopic notation $n^{2S+1}L_J$ to denote the energy levels.

The $q\bar{q}$ interaction potential U is *a priori* unknown and, consequently, the same is true for the spin-dependent part U_{spin}. However, if U_{spin} results from a nonrelativistic expansion of the Bethe-Salpeter equation, then to order $(v/c)^2$ it will in general have the form (m_q is the common mass of the constituents q and $\bar{q}$)

$$U_{\text{spin}} = \frac{1}{m_q^2}[\lambda_1 F_1(r)\mathbf{L}\cdot\mathbf{S} + \lambda_2 F_2(r) T_{12} + \lambda_3 F_3(r)\mathbf{S}_1\cdot\mathbf{S}_2] \ . \tag{2}$$

Here, $\mathbf{L}$ and $\mathbf{S}=\mathbf{S}_1+\mathbf{S}_2$ are the relative orbital angular momentum and the total spin operators, respectively, and

$$T_{12} = -4\mathbf{S}_1\mathbf{S}_2 + 12(\mathbf{S}_1\cdot\hat{\mathbf{r}})(\mathbf{S}_2\cdot\hat{\mathbf{r}})$$

denotes the tensor operator. The constants λ_i and the radial functions $F_i(r)$ $(i=1,2,3)$ get determined by the nonrelativistic reduction, once the Lorentz character and the analytic form of the static potential $U_{\text{NR}}(r)$ [contained in $U_0(r)$] is given.

There are strong theoretical[3] and phenomenological[4,5] arguments that U_{NR} be of mixed Lorentz character with a "vector" part $V(r)$ and a "scalar" part $S(r)$. The specific forms of $V(r)$ and $S(r)$ are not known, and different models make different choices. Generally, though, one expects $V(r)$ to contain a short-range Coulomb-type part due to one-gluon exchange, while $S(r)$ is assumed to contain a long-range part responsible for quark confinement. Originally, this confining part was chosen as linear in r,[6] a choice also favored by lattice calculations.[7]

In this paper we also assume that U_{NR} consists of a vector and a scalar part, but avoid as much as possible any specification of $V(r)$ and $S(r)$. For a general potential $U_{\text{NR}} = V(r) + S(r)$, the radial functions are

$$F_1(r) = \frac{V'}{r} - \frac{1}{3}\frac{S'}{r} \ , \tag{3a}$$

$$F_2(r) = -V'' + \frac{V'}{r} \ , \tag{3b}$$

$$F_3(r) = \nabla^2 V \ , \tag{3c}$$

whereas the λ_i take the values

$$\lambda_1 = \tfrac{3}{2}, \quad \lambda_2 = \tfrac{1}{12}, \quad \lambda_3 = \tfrac{2}{3} \ . \tag{4}$$

In the following we discuss some consequences of the spin-dependent potential as given by Eqs. (2)–(4). We analyze the splittings of the spin-triplet and spin-singlet states and particularly the question how the new results on the $^1P_1$ states fit into this potential approach.

I. *S*-WAVE MASS SPLITTING

This is given by the $\mathbf{S}_1\cdot\mathbf{S}_2$ part in Eq. (2) and one has, to first order,

$$M(^3S_1) - M(^1S_0) = \frac{2}{3m_q^2}\langle\nabla^2 V\rangle_0 \ , \tag{5}$$

where the subscript indicates that the expectation value is taken for the (unperturbed) $L=0$ state wave function ψ_s. The empirical regularity[8]

$$[M(^3S_1)]^2 - [M(^1S_0)]^2 \simeq \text{const} \ , \tag{6}$$

which works for practically all (light and heavy) $I\neq 0$ mesons has been investigated recently[9,10] and suggests that

$$\langle\nabla^2 V\rangle_0 = \text{const}\times\mu \ , \tag{7}$$

where μ is the reduced mass of the system. For our purpose we note that $\nabla^2 V$ alone is responsible for these mass splittings. We will come back to the consequences of (7) later on.

II. $L \geq$ MASS SPLITTINGS

For a given $L \geq 1$ we denote the mass of the three ^3L_J levels by M_J ($J = L-1, L, L+1$) and the mass of the $^1L_{J=L}$ level by M'_L. Then, treating U_{spin} to first order, one obtains, for a completely general (not only vector and scalar) potential U_{NR},

$$M_{L-1} = m_L - (L+1)\lambda_1 A_{L1} - \frac{2(L+1)}{2L-1}\lambda_2 A_{L2} + \tfrac{1}{4}\lambda_3 A_{L3} , \tag{8a}$$

$$M_L = m_L - \lambda_1 A_{L1} + 2\lambda_2 A_{L2} + \tfrac{1}{4}\lambda_3 A_{L3} , \tag{8b}$$

$$M_{L+1} = m_L + L\lambda_1 A_{L1} - \frac{2L}{2L+3}\lambda_2 A_{L2} + \tfrac{1}{4}\lambda_3 A_{L3} , \tag{8c}$$

$$M'_L = m_L - \tfrac{3}{4}\lambda_3 A_{L3} , \tag{8d}$$

where m_L is the common mass given by U_0, and the constants A_{Li} denote the expectation values

$$A_{Li} \equiv \frac{1}{m_q^2}\langle F_i(r)\rangle_L , \quad i = 1,2,3 . \tag{9}$$

In the case of pure vector + scalar potential these constants together with the λ_i are given by Eqs. (3) and (4).

In previous papers,[11] Eq. (8) was used to derive sum rules connecting the four masses for some cases of interest by noting that the three A_{Li} are not independent. The key observation was that

$$\langle \nabla^2 V(r)\rangle_L = \left\langle V'' + \frac{2V'}{r}\right\rangle_L , \tag{10}$$

if V is no more singular than $1/r$ near $r=0$. [Note that the $\delta^3(\mathbf{r})$ term, if present in $\nabla^2 V$, does not contribute to the expectation value for states with $L \geq 1$.]

The new observation we want to make here concerns a relation between the mass $\bar{M}_L$ of the center of gravity (COG) of the three ^3L_J states and the mass M'_L of the corresponding spin-singlet state. In fact, one gets from Eq. (8), again for a completely general potential,

$$\bar{M}_L \equiv \frac{(2L-1)M_{L-1} + (2L+1)M_L + (2L+3)M_{L+1}}{3(2L+1)}$$

$$= m_L + \tfrac{1}{4}\lambda_3 A_{L3} . \tag{8e}$$

This together with Eq. (8d) gives

$$\bar{M}_L - M'_L = \lambda_3 A_{L3} . \tag{11}$$

That is to say, the center of gravity of the three triplet states ^3L_J differs from the singlet ^1L_L state solely due to the spin-spin interaction. That is true for all values of $L \geq 1$ and for the arbitrary potential U_{NR}. This remarkable result is due to the fact that the spin orbit and the tensor separately cancel in the center of gravity for arbitrary L. The particular case for $L=1$ has been noted by many authors earlier.[12]

If we restrict ourselves to pure scalar and vector potentials, Eq. (11) takes the specific form

$$\bar{M}_L - M'_L = \frac{2}{3m_q^2}\langle \nabla^2 V\rangle_L ; \tag{11a}$$

i.e., only the vector part is responsible for this mass difference. In case one would also allow for a pseudoscalar potential $P(r)$, the expectation value in Eq. (11) would be changed to $\langle \nabla^2(V + \frac{1}{2}P)\rangle_L$.

Equation (11a) will play an important role in our discussion. It has the immediate consequence that if V is pure r^{-1}, then, for *all* $L \geq 1$, $M'_L = \bar{M}_L$ for an *arbitrary* scalar potential S.

III. ANALYSIS OF P LEVELS ($L=1$)

From Eq. (8) one can solve for A_{Li} ($i=1,2,3$) and for m_L in terms of the four masses. Thereby the parameters A_{L1} and A_{L2} can be determined in terms of the ^3L_J masses alone. Only A_{L3} given by Eq. (11) requires the additional knowledge of M'_L. Note that the expressions for A_{L2} and A_{L3} would only contain the vector potential and therefore could tell us more about it.

Masses are presently known for $L=1$ states, namely, for the $1P$ levels of the $c\bar{c}$ and $b\bar{b}$ systems and for the $2\,^3P_J$ levels of the $b\bar{b}$ system. So we restrict ourselves in the following to $L=1$. In this case we get, from (8),

$$18A_{11} = 5M_2 - 3M_1 - 2M_0 , \tag{12a}$$

$$-6A_{12} = 5M_2 - 15M_1 + 10M_0 , \tag{12b}$$

$$2A_{13} = 3\bar{M}_1 - 3M'_1 . \tag{12c}$$

Using the available data[1,2,13] the values of A_{11}, A_{12}, and A_{13} are listed in Table I. Knowledge of A_{11}, A_{12}, A_{13} allows us to separate the expectation values $\langle V'/r\rangle_1$, $\langle V''\rangle_1$ (providing information on the vector potential) and $\langle S'/r\rangle_1$. The corresponding numerical values are given in the lower part of Table I. We note the following.

(a) A_{11}, A_{12}, $\langle V'/r\rangle$, $-\langle V''\rangle_1$, and $\langle S'/r\rangle_1$ are all positive and decrease with increasing constituent mass m_q. The signs of these expectation values suggest that V may be a concave function, which is the choice usually made for detailed fits of the spectra.[6] As to the sign of $\langle \nabla^2 V\rangle_1$ (or, equivalently of A_{13}) no decisive calculation can be drawn from the data at the moment, since this quantity is practically zero for $c\bar{c}$, whereas it is positive for $b\bar{b}$. From simple scaling arguments (see later) one would expect it to have same sign in both cases. This will definitely be so, if V is a pure power in r:

$$V = -Cr^{-\epsilon} \quad (\epsilon > 0, C > 0) . \tag{13}$$

It is interesting to note that in this case the positivity of $\langle \nabla^2 V\rangle_1$ (as required by the $b\bar{b}$ data) implies the bound $\epsilon < 1$.

(b) Comparison of the magnitudes of $\langle V'/r\rangle_1$, $\langle V''\rangle_1$, $\langle S'/r\rangle_1$ for the $1P$ levels of $c\bar{c}$ system with the corresponding values for the $b\bar{b}$ system suggests that all these expectation values scale (as a function of the reduced

TABLE I. Numerical values (in MeV) of the various expectation values which determine spin-splitting calculated using the known $1P$ level masses. Input-mass values used for $c\bar{c}$ ($b\bar{b}$) are $M(^3P_0)=3414.9\pm1.1$ (9859.8±1.3), $M(^3P_1)=3510.7\pm0.5$ (9891.9±0.7), $M(^3P_2)=3556.3\pm0.4$ (9913.3±0.6), and $M(^1P_1)=3525.4\pm0.8$ (9894.8±1.5).

	$1P(c\bar{c})$	$1P(b\bar{b})$
$A_{11}=\frac{1}{m_q^2}\left\langle\frac{V'}{r}-\frac{1}{3}\frac{S'}{r}\right\rangle_1$	23.3±0.19	9.5±0.25
$A_{12}=\frac{1}{m_q^2}\left\langle -V''+\frac{V'}{r}\right\rangle_1$	121.6±2.24	35.6±2.83
$A_{13}=\frac{1}{m_q^2}\left\langle V''+\frac{2V'}{r}\right\rangle_1$	0±1.28	8.1±2.30
$\frac{1}{m_q^2}\left\langle\frac{V'}{r}\right\rangle_1$	40.6±0.84	14.6±1.21
$\frac{1}{m_q^2}\langle V''\rangle_1$	−81.1±1.6	−21.1±2.05
$\frac{1}{m_q^2}\left\langle\frac{S'}{r}\right\rangle_1$	51.7±2.50	15.3±3.68

mass $\mu_q\equiv m_q/2$) as μ_q. Consequently, one would expect that $\langle\nabla^2V\rangle_1$ has the same scaling behavior. Suppose again now that the vector potential $V(r)$ is pure power as in Eq. (13). Then $\langle\nabla^2V\rangle_1$ would scale as

$$\langle\nabla^2V\rangle_1\sim\mu_q^{(\epsilon+2)/(\nu+2)}\ . \tag{14}$$

Here, we have assumed that there exists an "effective" scaling power ν, determined by the full potential $U_{\rm NR}(r)$, such that a length scales as $\mu_q^{-1/(2+\nu)}$. Clearly, the μ_q scaling suggested by the $1P$ data implies

$$\epsilon\simeq\nu\ . \tag{15}$$

ν will in general depend on the relative magnitude of the parameters of $V(r)$ and $S(r)$, and it could as well be the power of the scalar confining potential $S(r)=\text{const}\times r^\nu$ ($\nu>0$). There are, in fact, some indications from the J/ψ and Υ families[14] that the scalar confining potential solely determines the scaling behavior. In particular, a value $\nu=1$ is possible and leads, via Eq. (15), to the canonical form of the charmonium potential[6] $U_{\rm NR}=-\frac{4}{3}\alpha_s 1/r+\lambda r$. It is interesting to note, that the same conclusion as in Eq. (15) has been arrived at[9] by considering the mass splittings of the $L=0$ quarkonia levels and clearly ties up neatly with Eq. (7).

(c) The new information[1,2] on the masses of the $^1P_1$ states, or equivalently on $\langle\nabla^2V\rangle_1$ does not fall in line with the above points, since A_{13} apparently shows a mass dependence completely different from A_{11} and A_{12}: One would naively expect that $\overline{M}_1$-M_1' should decrease with m_q, while the opposite seems to be the case. Furthermore, the scaling behavior $\langle\nabla^2V\rangle_1\sim\mu_q$, induced in (b) above, would suggest that

$$(\overline{M}_1-M_1')_{c\bar{c}}\simeq\frac{m_b}{m_c}(\overline{M}_1-M_1')_{b\bar{b}}\simeq 16\ \text{MeV}\ ,$$

in manifest contradiction to the experimental finding that the left-hand side is zero. Even on reducing the value of $(\overline{M}_1-M_1')_{b\bar{b}}$ by 2 standard deviations (2σ) one would expect $(\overline{M}_1-M_1')_{c\bar{c}}$ to be about 6 MeV, still more than 4σ off from the experimental value. So, it seems that it will be extremely hard to reconcile the present $^1P_1$ data with a simple potential model for quarkonia.

(d) Of course, it is possible that the short-range vector potential $V(r)$ is such that $\langle\nabla^2V\rangle_1$ be zero for $c\bar{c}$ and not for $b\bar{b}$, or even change sign in going from $c\bar{c}$ to $b\bar{b}$. But this would require fine-tuning of the parameters of V and seems unlikely for a flavor-independent potential. It is clearly impossible for a simple power-law potential of the form Eq. (13). Explicit model calculations[15] using a running coupling constant with $V(r)\sim\alpha_s(r)/r$ also support the expectation that $(\overline{M}_1-M_1')_{c\bar{c}}$ should be much larger than $(\overline{M}_1-M_1')_{b\bar{b}}$.

(e) Since $\overline{M}_1-M_1'\mid_{b\bar{b}}=5.4\pm1.5$ MeV is quite small, it is possible that some corrections not considered so far may account for the different values of $\overline{M}_1-M_1'$ for $c\bar{c}$ and $b\bar{b}$ systems. Two possibilities come to mind.

(1) Higher-order relativistic corrections. It is known from charmonium calculations (e.g., third paper in Ref. 6) that the magnitude of $O((v/c)^2)$ relativistic corrections is about 5–6 % of the nonrelativistic binding energy. Assuming that the next order is of the same relative size we expect the $(v/c)^4$ corrections to be less than 1.5 MeV for $c\bar{c}$ and negligible for $b\bar{b}$ for a given level. The contribution to the difference $\overline{M}_1-M_1'$ would be even smaller.

(2) Unitarity corrections. Coupling to inelastic channels[16,17] ($D\bar{D}$, $D^*\bar{D}$, ...; $B\bar{B}$, ...) can yield rather large corrections to the mass of an individual level. Model calculations[17] find corrections of about 180 MeV (44 MeV) for the $1P$ states of $c\bar{c}$ ($b\bar{b}$) system. However, since the contribution to each level is nearly the same, the correction to the *difference* $\overline{M}_1-M_1'$ is very small, in fact it is found to be less than 0.5 MeV in each case.

So to us it seems that there are only two reasonable possibilities.

(i) $\langle\nabla^2V\rangle_1=0$ for both $c\bar{c}$ and $b\bar{b}$ suggesting that $\nabla^2V=0$ and $V(r)$ is practically a pure Coulomb poten-

tial. As pointed out earlier, Eq. (11) then implies $M'_L = \overline{M}_L$ for *all* $L \geq 1$, and for *arbitrary* scalar potential. This would imply that the $^1P_1(b\bar{b})$ level should lie at 9900.2±1.7 MeV rather than at 9894.8±1.5 MeV. Using the $2\,^3P_J$ masses[13] it would further predict the $2\,^1P_1(b\bar{b})$ mass to be 10261.6±1.7 MeV.

(ii) $\langle \nabla^2 V \rangle_1$ = finite for both $c\bar{c}$ and $b\bar{b}$ systems. This would mean V cannot be a pure Coulomb potential. Further it would be extremely hard to understand in a simple and natural way the increase of $\overline{M}_1 - M'_1$ with μ_q as implied by the data. From Eq. (11a), such an increase would require $\langle \nabla^2 V \rangle_1$ to scale at μ_q^α with $\alpha > 2$. Using the scaling arguments given in (b), for a simple power potential, Eq. (14) would give $\epsilon + 2 = \alpha(\nu + 2)$ with $\alpha > 2$. It is practically impossible to satisfy this condition with reasonable values of ϵ and ν. For positive ν one must have $\epsilon > 2$ which yields a too strong singularity of $V(r)$ near the origin. On the other hand, for $0 < \epsilon < 2$, one needs

$$-\frac{2(\alpha-1)}{\alpha} < \nu < \frac{-2(\alpha-2)}{\alpha},$$

so that $\nu < 0$ for $\alpha > 2$. Thus an increase of $\overline{M}_1 - M'_1$ would go against the rest of the evidence.

Among the above-mentioned two alternatives our preference is for (i) as it offers the possibility of having a simple potential to understand quarkonia spectra.

In summary, we have argued that the presently available data for the 1^1P_1 level of the $c\bar{c}$ and the $b\bar{b}$ systems are difficult to understand theoretically as they stand. The values given for the masses of these states have been obtained from a very difficult experiment and we realize that these values are still tentative.[1,2] The remarks above are to point out the need for further experimentation in this regard.

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