PHYSICAL REVIEW D VOLUME 37, NUMBER 3 1 FEBRUARY 1988

Decay constants, mass formulas, and mixings with radially excited states of the light and heavy ground-state pseudoscalar mesons

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(Received 16 October 1986; revised manuscript received 28 July 1987)

In the absence of mixings with radially excited states and/or no exotic states, asymptotic flavor symmetry predicts, for the ground-state pseudoscalar mesons, $f_K=f_\pi$, $f_+^{K\pi}(0)=1$, and $m_F^2-m_D^2=m_K^2-m_\pi^2$, etc. In this paper, we assume that the observed small deviations from these predictions originate mainly from the mixing between the ground states and the first radially excited states. Then, new sum rules $f_+^{ab}(0)=\cos(\theta_a-\theta_b)$ and $f_b/f_a=\cos(\theta_b-\phi)/\cos(\theta_a-\phi)$ are derived, where θ_a ($a=\pi,K,D,F,\dots$) is the mixing angle between the ground-state meson a and its radially excited counterpart a' and ϕ is a flavor-independent constant with $|\phi|\le\pi/4$. The experimentally indicated and theoretically plausible value $f_+^{K\pi}(0)\simeq0.973$ leads to $\theta_K-\theta_\pi\simeq13°$ (for convenience, $\theta_K>\theta_\pi$ is assumed) and $f_K/f_\pi\simeq1.163$ to $\theta_\pi-\phi\simeq-40°$. This value of $\theta_\pi-\phi$ imposes an interesting restriction on the decay constant f_P, $f_P/f_\pi\lesssim1.4$, for any pseudoscalar meson P ($P=\pi,K,D,F,\dots$). The recently observed (but still probably preliminary) value of $f_+^{DK}(0)\simeq0.73$ yields $\theta_D-\theta_K\simeq43°$ (case I) or $\theta_D-\theta_K\simeq-43°$ (case II) crudely. The observed deviation $\epsilon\equiv(m_K^2-m_\pi^2)-(m_F^2-m_D^2)\simeq-0.165$ GeV^2 from the simple mass formula actually favors case II and predicts tentative values $f_D/f_\pi\simeq0.5$ and $f_F/f_\pi\simeq0.7$, which can be improved with the progress of experiment.

I. INTRODUCTION

Very recently, the Mark III Collaboration[1] has significantly improved the upper limit of the D-meson decay constant f_D as $f_D<340$ MeV ($f_D/f_\pi<2.6$) from their search of $D^+\to\mu^+\nu$ decay. They have also reported the branching ratios[2] of the $D^0\to K^-e^+\nu$ and $D^+\to\bar K^0e^+\nu$ decays, which give information for $f_+^{DK}(0)$. This recent progress on D-decay experiments rekindles our interest in a unified understanding[3] of light- and heavy-pseudoscalar-meson decay constants f_P ($P=\pi,K,D,\dots$) and of the value of their vector-current form factor $f_+^{PP'}(q^2)$ ($PP'=K\pi$, $DK,\dots$) at $q^2=0$.

A powerful method for studying hadron physics which contains heavy as well as light quarks is the method[4] of "asymptotic flavor symmetry" plus "equal-time commutators involving the charges of underlying symmetry groups." It treats broken symmetries without using the language of perturbation theory. Many successful sum rules have been derived from this method.

However, in many of these sum rules, observed values show some deviations. For example, the method leads to the sum rules $f_\pi=f_K=f_D=\cdots$, $f_+^{K\pi}(0)=f_+^{DK}(0)=\cdots=1$, and $m_K^2-m_\pi^2=m_F^2-m_D^2$, if there are no mixings between the ground-state pseudoscalar mesons and their radially excited states and/or exotic states. Since glueballs cannot have isospin and/or strangeness and exotic mesons with $J^P=0^-$ cannot exist in S states, the above observed deviations suggest the presence of some mixings between the ground-state pseudoscalar mesons and their radially excited states.

In this paper we derive new sum rules which include the effect of radially excited states by using a simplifying assumption that those observed deviations arise dominantly from the mixings between the ground states and the first radially excited states. We estimate the mixing angles θ_π, θ_K, θ_D, and θ_F from the observed f_K/f_π, $f_+^{K\pi}(0)$, and $f_+^{DK}(0)$ and predict the ratios f_D/f_π and f_F/f_π.

A word of caution might be added to our mixing parameters. There is a subtle difference between our mixing parameters which are defined for the creation and annihilation operators of physical (i.e., "in" or "out") particles with infinite momenta and the conventional ones defined among field operators. Our broken-SU$_f(N)$ sum rules are always obtained by realizing the equal-time commutators involving the flavor charge V_α in the infinite-momentum frame. Our mixing parameters then appear in the evaluation of the matrix elements of the charge V_α in this asymptotic limit, i.e., in the four-momentum transfer squared $q^2=0$ limit, where the effect of symmetry breaking will be minimum.

In the usual diagonalization of the mass matrix, the possible q^2 dependence of mixing parameters is never considered. For the study of deviations of f_P and $f_+^{PP'}(0)$ from symmetry-limit predictions, there are many papers in the literature[5] with many different approaches. In this paper, as shown below, we sometimes predict large configuration mixings. However, it should be noted that the configuration mixing angles estimated below are those at the asymptotic limit, so that it may not necessarily imply that there are also such large configuration mixings in the usual mixing scheme defined for the field operators. What should really be compared between the two different approaches are real observables, i.e., physical masses, decay constants f_P, vector-current form factors $f_+^{PP'}(0)$, and so on.

II. PSEUDOSCALAR-MESON DECAY CONSTANTS IN THE PRESENCE OF MIXINGS

Since our main concern is the study of the subtle effect of intermultiplet mixing between different excitations, we reiterate briefly the main point of our version of asymptotic flavor symmetry.[4] It assumes that the creation and annihilation operators of physical ("in" or "out") hadrons do transform, even in broken symmetry, linearly (including, however, the possible particle mixing) under $SU_f(N)$ transformation, but only in the infinite-momentum limit. Let us consider the annihilation operators $a_\alpha(\mathbf{p},\lambda)$ of physical particles with momentum $\mathbf{p}$, helicity λ, and physical $SU_f(N)$ indices α ($\pi,K,\ldots$). The transformation of $a_\alpha(\mathbf{p},\lambda)$ under the $SU_f(N)$ generator V_i can be expressed as $[V_i,a_\alpha(\mathbf{p},\lambda)] = i\sum_\beta u_{i\alpha\beta}a_\beta(\mathbf{p},\lambda)+\delta u_{i\alpha\lambda}(\mathbf{p})$. In exact $SU_f(N)$ symmetry, $\delta u_{i\alpha\lambda}(\mathbf{p})=0$ for *any* value of $\mathbf{p}$ and the indices α and β belong to the *same* $SU_f(N)$ multiplet. However, on the right-hand side of the above equation the first term should pick up, in broken symmetry, *all* the terms *linear* in $a_\beta(\mathbf{p},\lambda)$. In principle, a_β should be taken over *all* possible particles β, which have the same J^{PC} or J^P as the particle α, including those which belong to $SU_f(N)$ multiplets different from the one involving α or glueballs or exotics, etc. All the remaining terms are denoted by $\delta u_{i\alpha\lambda}$. Our asymptotic symmetry requires that $\delta u_{i\alpha\lambda}$ vanish as $1/|\mathbf{p}|^{1+\epsilon}$ ($\epsilon>0$) as $\mathbf{p}\rightarrow\infty$. Therefore, $a_\beta(\mathbf{p},\lambda)$ can be *linearly* related to the (hypothetical) $SU_f(N)$ representation operator $a_j(\mathbf{p},\lambda)$ (but only in the limit $\mathbf{p}\rightarrow\infty$) by $a_\alpha(\mathbf{p},\lambda)=\sum_j C_{\alpha j}(\lambda)a_j(\mathbf{p},\lambda)$ at $\mathbf{p}\rightarrow\infty$. Here $a_j(\mathbf{p},\lambda)$ satisfies the usual $SU_f(N)$ commutation relations with the generator V_i. The orthogonal matrix $C_{\alpha j}$ then involves $SU_f(N)$ particle mixing parameters which are defined in the asymptotic limit. These mixing parameters will be determined, in the process of asymptotic realization of the constraint algebras involving the charges (i.e., vector and axial-vector charges) of underlying symmetry groups of QCD, as will be described below.

We denote the physical ground state, first, second, ..., excited states as $|a\rangle=|a_1\rangle$, $|a'\rangle=|a_2\rangle$, $|a''\rangle=|a_3\rangle,\ldots$, and their corresponding states in the flavor-symmetry limit as $|\hat{a}_1\rangle$, $|\hat{a}_2\rangle$, $|\hat{a}_3\rangle,\ldots$. Physical state $|a_1\rangle$ with momentum $\mathbf{p}$ can then be expressed in the limit $\mathbf{p}\rightarrow\infty$ by

$$|a_i\rangle=\sum A_{ij}\,|\hat{a}_j\rangle\ , \tag{2.1}$$

where A is an orthogonal matrix:

$$AA^T=A^TA=1\ . \tag{2.2}$$

The matrix element of a flavor charge V_α ($\alpha=1,2,\ldots$) between the (symmetric) states $\langle\hat{a}_i|$ and $|\hat{b}_j\rangle$ is of course given by

$$\lim_{\mathbf{p}\rightarrow\infty}\langle\hat{a}_i|V_\alpha|\hat{b}_j\rangle=\delta_{ij}\ , \tag{2.3}$$

where the $SU_f(N)$ Clebsch-Gordan coefficients have been omitted for convenience. Then, the value of the matrix elements of V_α taken between two *physical* pseudoscalar meson states $\langle a_i|$ and $|b_j\rangle$ with $\mathbf{p}\rightarrow\infty$, which is equal to $f_{+ij}^{ab}(0)$, is given by

$$\lim_{\mathbf{p}\rightarrow\infty}\langle a_i|V_\alpha|b_j\rangle\equiv(F_+^{ab})_{ij}\equiv f_{+ij}^{ab}(0)=(AB^T)_{ij}\ . \tag{2.4}$$

Note that for $\mathbf{p}\rightarrow\infty$, i.e., at the limit of zero four-momentum transfer squared, $q^2=0$, $f_-(0)$ form factors do not contribute. Here and hereafter, we often omit the "limit" symbol, since all the computations are performed in the limit $\mathbf{p}\rightarrow\infty$ to take advantage of asymptotic symmetry.

The physical pseudoscalar-meson decay constants f_{P_i} ($P_i=\pi,K,D,F,\ldots$) are defined by

$$\sqrt{2p_0}\langle 0|A_\mu^\alpha(0)|P_i(\mathbf{p})\rangle\equiv i(F_P)_i p_\mu \equiv if_{P_i}(m_{P_i}^2)p_\mu\ . \tag{2.5}$$

A_μ^α and P_i transform in the same way under $SU_f(N)$. The equal-time commutation relation between vector charge V_a^b and axial-vector current $A_{c\mu}^a$ is given in obvious notation by

$$[A_{c\mu}^a(0),V_a^b]=A_{c\mu}^b(0)\ , \tag{2.6}$$

where a, b, and c denote the $SU_f(N)$ indices and we consider the case with $a\neq b\neq c$. The matrix elements of (2.6) inserted between the vacuum $\langle 0|$ and the physical ground-state pseudoscalar-meson state $|0^-(b\bar{c});\mathbf{p}\rangle$ is given in the limit $\mathbf{p}\rightarrow\infty$ by[6]

$$\sum\langle 0|A_{c\mu}^a|0^-(a\bar{c});\mathbf{p}\rangle\langle 0^-(a\bar{c});\mathbf{p}|V_a^b|0^-(b\bar{c});\mathbf{p}\rangle=\langle 0|A_{c\mu}^b|0^-(b\bar{c});\mathbf{p}\rangle\ . \tag{2.7}$$

In deriving (2.7), annihilation of the vacuum by the charge V_a^b (but only in the asymptotic limit) is used.[4]

We rewrite (2.7) in the form

$$\sum_k\langle 0|A_\mu|a_k\rangle\langle a_k|V|b_i\rangle=\langle 0|A_\mu|b_i\rangle\ , \tag{2.8}$$

where mesons $(a\bar{c})_i$ and $(b\bar{c})_j$ are now denoted as a_i and b_j, respectively. Therefore, using the notations defined by (2.4) and (2.5), we obtain

$$\sum_k f_{ak}(AB^T)_{ki}=f_{bi}\ , \tag{2.9}$$

that is,

$$F_b=F_aAB^T=F_aF_+^{ab}\ , \tag{2.10}$$

where A and B are the mixing matrices for the mesons a_i and b_i, respectively. The relation (2.9) implies

$$f_b/f_a=(AB^T)_{11}+(f_{a'}/f_a)(AB^T)_{21}+(f_{a''}/f_a)(AB^T)_{31}+\cdots \quad (2.11)$$

We now use a simplifying assumption that as far as the mixings of the ground-state mesons are concerned, the mixings with the first radially excited states are most important compared with the mixings with the second and higher radially excited states. It is likely in general that $|A_{ij}|$ with $j=i\pm1$ are sizable, while $|A_{ij}|$ with $j\neq i$ and $j\neq i\pm1$ are relatively small in comparison to $|A_{ij}|$ with $j=i\pm1$. (This corresponds to the simplifying assumption used in the mass matrix approach that mixings are caused only through the nondiagonal elements of mass matrix M_{ij} with $j=i\pm1$.) We use this approximation only when the external states (in realizing the equal-time commutators) are the ground-state mesons.

Therefore, in the estimates of physical parameters of ground-state mesons, we may approximately write

$$A=\begin{bmatrix}\cos\theta_a & \sin\theta_a\\ -\sin\theta_a & \cos\theta_a\end{bmatrix},\quad B=\begin{bmatrix}\cos\theta_b & \sin\theta_b\\ -\sin\theta_b & \cos\theta_b\end{bmatrix}, \quad (2.12)$$

so that

$$AB^T=\begin{bmatrix}\cos(\theta_a-\theta_b) & \sin(\theta_a-\theta_b)\\ -\sin(\theta_a-\theta_b) & \cos(\theta_a-\theta_b)\end{bmatrix}. \quad (2.13)$$

Then we obtain, from (2.4),

$$\begin{aligned}f_+^{ab}(0)&=\cos(\theta_a-\theta_b)\,,\\ f_+^{ab'}(0)&=-f_+^{a'b}(0)=\sin(\theta_a-\theta_b)\,.\end{aligned} \quad (2.14)$$

From (2.11) we then obtain

$$f_b/f_a=\cos(\theta_a-\theta_b)-(f_{a'}/f_a)\sin(\theta_a-\theta_b) \quad (2.15)$$

and

$$f_a/f_b=\cos(\theta_a-\theta_b)+(f_{b'}/f_b)\sin(\theta_a-\theta_b)\,. \quad (2.16)$$

Eliminating f_a/f_b from (2.15) and (2.16), we get

$$\tan(\theta_a-\theta_b)=-\frac{(f_{a'}/f_a)-(f_{b'}/f_b)}{1+(f_{a'}/f_a)(f_{b'}/f_b)}\,, \quad (2.17)$$

which leads to

$$\begin{aligned}\tan(\theta_a-\phi)&=-f_{a'}/f_a\,,\\ \tan(\theta_b-\phi)&=-f_{b'}/f_b\,,\end{aligned} \quad (2.18)$$

where the flavor-independent parameter ϕ can be identified with [considering the symmetry limit in (2.18)]

$$\tan\phi=f_{a2}/f_{a1}=f_{b2}/f_{b1}=\cdots\,. \quad (2.19)$$

Since it is likely that $|f_{P2}|\leq|f_{P1}|$, the angle ϕ may be restricted to $|\phi|\leq\pi/4$. From (2.15), (2.16), and (2.18) we can readily obtain

$$\frac{f_b}{f_a}=\frac{\cos(\theta_b-\phi)}{\cos(\theta_a-\phi)}\,, \quad (2.20)$$

$$\tan(\theta_a-\phi)=-\frac{f_b/f_a-f_+^{ab}(0)}{\pm\{1-[f_+^{ab}(0)]^2\}^{1/2}}\,, \quad (2.21)$$

$$\tan(\theta_b-\phi)=\frac{f_a/f_b-f_+^{ab}(0)}{\pm\{1-[f_+^{ab}(0)]^2\}^{1/2}}\,, \quad (2.22)$$

where $\pm$ signs correspond to $\theta_a<\theta_b$ and $\theta_a>\theta_b$, respectively.

Using the fairly well-known observed value[7,8]

$$f_+^{\pi K}(0)=0.973\pm0.027 \quad (2.23)$$

$[\,|f_+(0)\sin\theta_C|=0.216\pm0.003$ from the K_{e3} decay and $V_{su}\equiv\sin\theta_C=0.222\pm0.003]$ into formula (2.14), we get

$$\theta_K-\theta_\pi=(13.3^{-13.3}_{+5.6})^\circ\,, \quad (2.24)$$

where, for convenience, we have chosen $\theta_K>\theta_\pi$, and errors -13.3° and $+5.6^\circ$ have come from errors $+0.027$ and -0.027 in (2.23), respectively. From the relations (2.21) and (2.22) with the usually accepted value (see Ref. 9)

$$f_K/f_\pi=153.3\ \text{MeV}/131.8\ \text{MeV}=1.163\,, \quad (2.25)$$

we can obtain

$$\begin{aligned}f_{\pi'}/f_\pi&=\tan(\phi-\theta_\pi)=0.82^{+\infty}_{-0.15}\,,\\ \theta_\pi-\phi&=-(39.5^{+50.5}_{-5.7})^\circ\,,\\ f_{K'}/f_K&=\tan(\phi-\theta_K)=0.49^{+\infty}_{-0.22}\,,\\ \theta_K-\phi&=-(26.1^{+63.9}_{-11.2})^\circ\,.\end{aligned} \quad (2.26)$$

The upper value of $f_+^{\pi K}(0)$ in (2.23), $0.973+0.027=1$, leads to $\theta_K-\theta_\pi=0$, $\theta_\pi-\phi=\theta_K-\phi=-90^\circ$, and $f_K/f_\pi=1$. Therefore, errors following from the $+0.027$ value in (2.23) are not consistent with our f_K/f_π input within our model. It will perhaps be more practical to regard errors in (2.24) as $\theta_K-\theta_\pi=(13.3\pm5.6)^\circ$.

Although these numerical values should not be taken too seriously since they are sensitive to the input value[7,8] of $f_+^{\pi K}(0)$, it is, at least, likely that the the value $\theta_\pi-\phi$ lies in the range from -45° to -35°. Then, from (2.20) we obtain an interesting bound on the pseudoscalar-meson decay constants f_P ($P=K,D,F,B,\ldots$):

$$f_P/f_\pi\simeq\cos(\theta_P-\phi)/\cos(-40^\circ)\lesssim1.4\,. \quad (2.27)$$

Similarly, using the recently observed value[2,8]

$$f_+^{KD}(0)=0.73\pm0.05\,, \quad (2.28)$$

obtained from the $D^0\to K^-e^+\nu$ and $D^+\to K^0e^+\nu$ decays $[\,|f_+^{KD}(0)|^2|V_{c\pi}|^2=0.51\pm0.07$ and $V_{cs}=0.974\pm0.001]$ we get

$$\theta_D-\theta_K=\pm(43.1^{-4.4}_{+4.1})^\circ\,, \quad (2.29)$$

where errors -4.4° and $+4.1^\circ$ come from errors $+0.05$ and -0.05 in (2.21), respectively.

This value (2.29) in turn yields a sizable value $|f_+^{DK'}(0)|=|\sin(\theta_D-\theta_K)|\simeq0.26$ for the possible decay $D\to K'e\nu$. However, the rate of the decay becomes very small $[\Gamma(D\to K'e\nu)/\Gamma(D\to Ke\nu)\simeq5.1\times10^{-3}]$ because of the phase volume factor and this $D \to K'e\nu$ decay is probably difficult to observe. Here, we have tentatively assigned[8] $K(1460)$ to K' in the above computation.

According to the choice $\theta_D > \theta_K$ or $\theta_D < \theta_K$, together with the values given in (2.26), we can predict

$$\text{Case I: } \theta_D - \phi = (17.0^{-4.4-63.9}_{+4.1+11.2})^\circ , \tag{2.30}$$

$$f_D/f_\pi = 1.24 \pm 0.03^{+\infty}_{-0.09} , \tag{2.31}$$

or

$$\text{Case II: } \theta_D - \phi = -(69.2^{-4.4+20.8}_{+4.1-11.2})^\circ , \tag{2.32}$$

$$f_D/f_\pi = 0.46 \pm 0.09^{-0.46}_{+0.18} . \tag{2.33}$$

Here the errors $(-4.4/+4.1)$ and $(-63.9/+11.2)$ in (2.30) come from ± 0.05 in (2.28) and ± 0.027 in (2.23), respectively.

Although it may be tempting to imagine $\theta_D > \theta_K$, the observed deviation from the mass sum rule $m_K^2 - m_\pi^2 = m_F^2 - m_D^2$ curiously favors case II as will be discussed in the next section.

III. POSSIBLE MODIFICATION OF THE SUM RULE $m_F^2 - m_D^2 = m_K^2 - m_\pi^2$

By inserting the exotic charge commutation relation $[\dot{V}_{\bar{K}^0} = (d/dt)V_{\bar{K}^0}$ and $V_{\bar{K}^0} \equiv V_3^2 \equiv V_6 - iV_7$, etc.]

$$[\dot{V}_{\bar{K}^0}, V_{\bar{D}^0}] = 0 \tag{3.1}$$

between the asymptotic states $\langle \pi_i^+ |$ and $| F_j^+ \rangle$, we obtain using asymptotic flavor symmetry, a sum rule

$$\sum_k \langle \pi_i^+ | \dot{V}_{\bar{K}^0} | K_k^+ \rangle \langle K_k^+ | V_{\bar{D}^0} | F_j^+ \rangle = \sum_k \langle \pi_i^+ | V_{\bar{D}^0} | D_k^+ \rangle \langle D_k^+ | \dot{V}_{\bar{K}^0} | F_j^+ \rangle , \tag{3.2}$$

which yields

$$\sum_k (m_{\pi i}^2 - m_{Kk}^2)[U(\pi)U(K)^T]_{ik}[U(K)U(F)^T]_{kj} = \sum_k (m_{Dk}^2 - m_{Fj}^2)[U(\pi)U(D)^T]_{ik}[U(D)U(F)^T]_{kj} \tag{3.3}$$

or

$$U(\pi)^T M_\pi^2 U(\pi) - U(K)^T M_K^2 U(K) = U(D)^T M_D^2 U(D) - U(F)^T M_F^2 U(F) , \tag{3.4}$$

where $U(P)$ denotes the mixing matrix among the mesons P_i and $(M_P^2)_{ij} = \delta_{ij} m_{P_i}^2$.

Of course, when all of the $U(P)$ are unit matrices (i.e., there are no mixings) we recover the simple mass-squared spacing[4] $m_F^2 - m_D^2 = m_K^2 - m_\pi^2$.

However, the observed values of these masses show a slight deviation from the equal-spacing rule:

$$\epsilon \equiv (m_{K^+}^2 - m_{\pi^+}^2) - (m_{F^+}^2 - m_{D^+}^2) = -(0.165 \pm 0.012) \text{ GeV}^2 . \tag{3.5}$$

In the present model the deviation is given by

$$\epsilon = -[\cos(\theta_F - \theta_\pi)]^{-1}[(m_{K'}^2 - m_K^2)\sin(\theta_K - \theta_\pi)\sin(\theta_K - \theta_F) + (m_{D'}^2 - m_D^2)\sin(\theta_D - \theta_\pi)\sin(\theta_D - \theta_F)] , \tag{3.6}$$

which is readily derived from (3.3) via

$$m_{\pi 1}^2 + m_{F1}^2 = [U(\pi)U(F)^T]_{11}^{-1} \sum_k \{m_{Kk}^2[U(K)U(\pi)^T]_{k1}[U(K)U(F)^T]_{k1} + m_{Dk}^2[U(D)U(\pi)^T]_{k1}[U(D)U(F)^T]_{k1}\} . \tag{3.7}$$

Since $m_{K'}^2 - m_K^2 > 0$ and $m_{D'}^2 - m_D^2 > 0$, the observed value of ϵ requires $(\theta_K - \theta_\pi)(\theta_F - \theta_K) < 0$ and/or $(\theta_D - \theta_\pi)(\theta_F - \theta_D) < 0$ (we restrict θ to the range $|\theta| \le \pi/4$). This suggests that in the choice of the values of $\theta_D - \theta_\pi$ discussed in the previous section, case II ($\theta_D - \theta_\pi \simeq -40^\circ$) is favored for the explanation of the observed value (3.5).

For the numerical study of ϵ, it is useful to eliminate $\theta_F - \theta_D$ and $m_{F'}^2 - m_F^2$ from our sum rules, since experimental information is not available for these quantities at present.

For the case of 2×2 mixing matrices, relation (3.4) leads to

$$2\epsilon = (d - c) - (b - a) , \tag{3.8}$$

$$d \sin\delta - c \sin\gamma = b \sin\beta - a \sin\alpha , \tag{3.9}$$

$$d \cos\delta - c \cos\gamma = b \cos\beta - a \cos\alpha , \tag{3.10}$$

where $d = m_{F'}^2 - m_F^2$, $c = m_{D'}^2 - m_D^2$, $b = m_{K'}^2 - m_K^2$, $a = m_{\pi'}^2 - m_\pi^2$, $\delta = 2\theta_F$, $\gamma = 2\theta_D$, $\beta = 2\theta_K$, and $\alpha = 2\theta_\pi$. The first relation (3.8) has been derived from the trace of (3.4), the second relation (3.9) from the 12 (or 21) component of (3.4), and the last relation (3.10) from the difference between the 11 and 22 components of (3.4). The relations (3.9) and (3.10) imply a relation $\mathbf{d} - \mathbf{c} = \mathbf{b} - \mathbf{a}$ among the two-dimensional vectors $\mathbf{a}$ ($|\mathbf{a}| = a$, $\arg \mathbf{a} = \alpha$), and so on. Therefore, in (3.9) and (3.10), we can replace each angle θ_P by $\theta_P - \theta_\pi$. Then, we get

$$\tan(\delta - \alpha) = \frac{b \sin(\beta - \alpha) + c \sin(\gamma - \alpha)}{b \cos(\beta - \alpha) + c \cos(\gamma - \alpha) - a} , \tag{3.11}$$

$$d=\{[b\sin(\beta-\alpha)+c\sin(\gamma-\alpha)]^2 +[b\cos(\beta-\alpha)+c\cos(\gamma-\alpha)-a]^2\}^{1/2} . \quad (3.12)$$

From (3.8) and (3.12) with input values a, b, c, $\beta-\alpha$, and $\gamma-\alpha$, we can estimate the value of ϵ.

Present data show[8]

$$a=m_{\pi'}^2-m_\pi^2=1.67\pm0.26\ \text{GeV}^2 , \quad b=m_{K'}^2-m_K^2\simeq1.89\ \text{GeV}^2 , \quad (3.13)$$

where we have used $m_{\pi'}=1.3\pm0.1$ GeV and $m_{K'}\simeq1.46$ GeV. On the other hand, there are no data for c. We take very tentatively

$$c=m_{D'}^2-m_D^2\simeq1.55\ \text{GeV}^2 , \quad (3.14)$$

by assuming experimentally well-satisfied equal spacing rule $m_V^2-m_P^2\simeq$const and its plausible counterpart $m_{V'}^2-m_{P'}^2\simeq$const (which can, in fact, be derived in the present formalism), i.e., $m_{D'}^2-m_D^2\simeq(m_{D^{*\prime}}^2-m_{D^*}^2)+(m_{\pi'}^2-m_\pi^2)-(m_{\rho'}^2-m_\rho^2)$, and by tentative identifying $D^*(2420)$ with $D^{*\prime}$. (The predictions of θ_F and ϵ are not very sensitive to the value of c.) Then, case II, $\theta_D-\theta_K\simeq-43.1°$, yields

$$\theta_{Fi}-\theta_\pi\simeq-15.6° \quad (\theta_F-\theta_D\simeq14.2°) , \quad (3.15)$$

$$d=m_{F'}^2-m_F^2\simeq0.939\ \text{GeV}^2 , \quad (3.16)$$

$$\epsilon\simeq-0.414\ \text{GeV}^2 . \quad (3.17)$$

The predicted value of $|\epsilon|$ is somewhat larger in comparison with the observed value (3.5). We, however, consider that the value (3.17) is not so unreasonable in view of our crude estimate based on the still preliminary experimental data. (In contrast, if we take case I, $\theta_D-\theta_K\simeq43.1°$, we obtain $\theta_F-\theta_\pi\simeq52.2°$, $\theta_F-\theta_D\simeq-4.2°$, and $\epsilon\simeq+0.291\ \text{GeV}^2$, and we cannot obtain a negative value of ϵ.)

From relation (2.20) and the value (3.15) of θ_F, we predict crudely

$$f_F/f_\pi\simeq0.74 . \quad (3.18)$$

IV. CONCLUSIONS

In conclusion, under the approximation we have used for the model of mixings we have presented arguments which prefer the choice $\theta_D<\theta_F<\theta_\pi<\theta_K$:

$$\theta_K-\theta_\pi\simeq13°, \quad \theta_\pi-\theta_F\simeq16°, \quad \theta_F-\theta_D\simeq14° , \quad (4.1)$$

and

$$f_D/f_\pi\simeq0.5, \quad f_F/f_\pi\simeq0.7 . \quad (4.2)$$

Here, in the estimates of (4.1) and (4.2), $f_+^{\pi K}(0)\simeq0.973$ and $f_+^{KD}(0)\simeq0.73$ have been used. For the errors, see (2.23) and (2.28). Moreover, in the estimates of θ_F and f_F, $m_{D'}^2-m_D^2\simeq1.55\ \text{GeV}^2$ has tentatively been assumed. Therefore, these numerical values should not be taken too seriously at present.

The argument can be extended to B mesons in a straightforward way. In this connection, the recent result of Suzuki[10] that f_B is not larger than f_π in his model is interesting to us.

A recent experiment[1] has made remarkable progress toward an upper bound of the value of f_D, and we hope that more data soon become available to check the validity of our sum rules. Similar experiments in B-meson decays are also eagerly awaited.

In this paper we have estimated the values of $\theta_P-\phi$. Finally, let us speculate on θ_P themselves, although we cannot predict the values of θ_P from asymptotic symmetry alone. The experimental values[8] yield, for the relation

$$m_{\pi'}^2-m_\pi^2\simeq(m_{K'}^2-m_K^2)\cos2(\theta_K-\theta_\pi) , \quad (4.3)$$

$1.67\pm0.27\ \text{GeV}^2$ for the left-handed side and $\simeq1.69\ \text{GeV}^2$ for the right-handed side. In the formulation of asymptotic flavor symmetry, the masses of flavor multiplets in the symmetry limit never play a role, since we always deal only with physical "in" or "out" fields. However, if we nevertheless assume that physical masses and symmetry masses are related by the simple relations inferred from the usual diagonalization of mass matrix,

$$m_{\pi'}^2-m_\pi^2=(m_{\hat\pi2}^2-m_{\hat\pi1}^2)\cos2\theta_\pi , \quad m_{K'}^2-m_K^2=(m_{\hat K2}^2-m_{\hat K1}^2)\cos2\theta_K , \quad (4.4)$$

the empirical relation (4.3) suggests

$$m_{\hat\pi2}^2-m_{\hat\pi1}^2\simeq m_{\hat K2}^2-m_{\hat K1}^2 \quad (4.5)$$

and

$$\theta_\pi\simeq0 . \quad (4.6)$$

This is probably a very natural result since $SU_I(2)$ is a good symmetry in $SU_f(N)$. Then, we may deduce $\phi\simeq39.5°$ from (2.26) and obtain

$$\tan\phi\equiv f_{\hat P2}/f_{\hat P1}\simeq0.82 , \quad (4.7)$$

which also seems reasonable.

ACKNOWLEDGMENTS

The research of one of the authors (Y.K.) was supported in part by the Ministry of Education in Japan. One of the authors (S.O.) thanks Dr. K. Terasaki for calling his attention to Ref. 10.

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$$(4E_1E_2)^{1/2}\langle 1^+(\mathbf{p}_1) | V_\mu(0) | 0^-(\mathbf{p}_0)\rangle \equiv \epsilon_\mu(\mathbf{p}_1) f_1(q^2) + [p_0 \epsilon(\mathbf{p}_1)] \times [(p_0+p_1)_\mu f_2(q^2) + (p_0-p_1)_\mu f_3(q^2)] ,$$

we obtain, in the limit of $\mathbf{p}_1 = \mathbf{p}_0 \to \infty$,

$$\langle 1^+(\mathbf{p}_1) | V | 0^-(\mathbf{p}_0)\rangle = (2\pi)^3 \delta^3(\mathbf{p}_0 - \mathbf{p}_1)[f_1(0) + (m_0^2 - m_1^2) f_2(0)]/2m_1 ,$$

where we have used the 1^+ polarization vector $\epsilon_\mu(\mathbf{p}_1)_{\lambda=0} = (0,0,E_1/m_1, |\mathbf{p}_1|/m_1)$. In unbroken symmetry [such as $SU(2)_I$], the relation

$$[f_1(0) + (m_0^2 - m_1^2) f_2(0)]/2m_1 \equiv f^{0\to 1}(0) = 0$$

is of course required. However, in the framework of "asymptotic symmetry" plus constraint algebra $[V^a_{c\mu}(0), V^b_a] = V^b_{c\mu}(0)$, the asymptotic matrix elements of $SU_f(3)$ current $V^K_\mu(0)$ $(V_{K^0} \equiv V^3_2 \equiv V_6 + iV_7$, etc.) can be related linearly to those of $SU_I(2)$ current $V^\pi_\mu(0)$ $(V_{\pi^+} \equiv V^2_1 \equiv V_1 + iV_2$, etc.). Therefore, $f^{0\to 1}(0)$ remains zero *independently* of the physical multiplet masses involved. However, at finite value of q^2, 1^+ state can, of course, contribute.

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