Kinematical aspects of nonleptonic multiparticle decays of heavy baryons

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I derive formulas for analyzing in a model-independent way the nonleptonic multiparticle decays of spin-$\frac{1}{2}$ baryons. Two- and three-body decays with up to two vector bosons in the final states are considered as special cases. All information contained in the polarizations of spinors and vector bosons is kept. These formulas may also be used to analyze polarized nucleon and meson scattering.

I. INTRODUCTION

QCD is now widely accepted as *the* theory of strong interaction. With the proof of various factorization theorems,[1] it provides the basis for understanding and improving on the parton model, which is successful in describing deep-inelastic scattering of leptons and hadrons as well as other hadronic processes with characteristic scales much larger than a few hundred MeV. Low-energy processes, on the other hand, reflect the underlying approximate chiral flavor symmetry which is certainly one feature of the QCD Lagrangian, although we are yet unable to calculate the low-energy parameters such as the pion decay constant from first principles. The most challenging aspects of QCD lies in describing processes involving intermediate energy scales of a few GeV where neither perturbative QCD nor approximate chiral symmetry provides reliable estimates. Experiments in hadron collisions as well as heavy-baryon production and decays designed to probe the intermediate energy scales can thus give us valuable information about the nonperturbative aspects of QCD. Several recent experiments in exclusive polarized nucleon-nucleon[2-10] and nucleon-meson[11-14] scattering processes as well as the inclusive production of polarized hyperons[15-27] have suggested a strong "spin dependence" of the interaction mechanism. Hence, to analyze such experiments as well as the decays of hyperons and heavy baryons, it is important to include the polarization of the interacting particles. In this paper we intend to give a complete set of formulas for systematically analyzing multiparticle processes involving a pair of baryons and several mesons in a model-independent way. We shall give the formulas in the context of heavy-baryon nonleptonic decays and indicate the modifications necessary for applications to other processes. Only spin-$\frac{1}{2}$ baryons are considered at present. In the following section we describe our notations and method of calculations. The basic formulas which make explicit the dependence on the baryon spin states are given. Formulas for the trace of a pair of fermion bilinears are given in Appendix A. In Sec. III we consider processes involving one or two mesons in the final states which are special cases of our general formulas. Formulas for two-body decays are, of course, well known; we give them for completeness and for comparison with the general cases. A brief discussion is given in the final section.

II. GENERAL CONSIDERATIONS

Let $M(k_1\sigma_1,k_2\sigma_2)$ be the amplitude for the decay of a baryon of momentum k_1, helicity σ_1 into another baryon of momentum k_2, helicity σ_2 and any number of yet unspecified mesons. We introduce two spacelike unit vectors p and q such that

$$p^2=q^2=-1,\quad p\cdot q=0,\quad p\cdot k_i=q\cdot k_i=0\ ,i=1,2\ . \tag{2.1}$$

A convenient choice of p,q in terms of an arbitrary reference vector P_0 is the following. Let

$$\Delta(abcd)=\epsilon_{\mu\nu\rho\sigma}a^\mu b^\nu c^\rho d^\sigma \tag{2.2}$$

and let $\Delta(\cdot bcd)$ be the four-vector such that

$$a\cdot\Delta(\cdot bcd)=\Delta(abcd)$$

for any four-vector a. We define

$$\Delta(a_1a_2a_3;a_1'a_2'a_3')=\begin{vmatrix} a_1\cdot a_1' & a_1\cdot a_2' & a_1\cdot a_3' \\ a_2\cdot a_1' & a_2\cdot a_2' & a_2\cdot a_3' \\ a_3\cdot a_1' & a_3\cdot a_2' & a_3\cdot a_3' \end{vmatrix}\ . \tag{2.3}$$

This definition can be trivially extended to any number of four-vectors $a_1,\dots,a_n$. We also let $\Delta(\cdot a_2a_3;a_1'a_2'a_3')$ be the four-vector such that

$$a_1\cdot\Delta(\cdot a_2a_3;a_1'a_2'a_3')=\Delta(a_1a_2a_3;a_1'a_2'a_3')$$

for any four-vector a_1. Let

$$\alpha_+=k_1\cdot k_2+m_1m_2,\quad \alpha_-=k_1\cdot k_2-m_1m_2\ , \tag{2.4}$$

and

$$\Delta_r^2=\frac{\Delta(P_0k_1k_2;P_0k_1k_2)}{\alpha_+\alpha_-}\ . \tag{2.5}$$

We assume the reference vector P_0 is linearly independent of k_1,k_2 so that Δ_r does not vanish.

We can choose p,q to be

$$p=\frac{\Delta(\cdot P_0k_1k_2)}{\sqrt{\alpha_+\alpha_-}\,\Delta_r},\quad q=\frac{\Delta(P_0k_1k_2;\cdot k_1k_2)}{\alpha_+\alpha_-\Delta_r}\ . \tag{2.6}$$

Let us note the following useful identities:

$$A\cdot p\,B\cdot p+A\cdot q\,B\cdot q=\frac{1}{\alpha_+\alpha_-}\Delta(Ak_1k_2;Bk_1k_2)\ , \tag{2.7}$$

$$A\cdot p\,B\cdot q-A\cdot q\,B\cdot p=\frac{\Delta(ABk_1k_2)}{\Delta_0}\ , \tag{2.8}$$

for any four-vectors A,B, where

$$\Delta_0=\Delta(pqk_1k_2)=\pm\sqrt{\alpha_+\alpha_-}\ . \tag{2.9}$$

The choice of p,q in Eq. (2.6) is such that the plus sign holds in Eq. (2.9). We shall assume this sign convention in the following.

In terms of p,q, the general invariant amplitude $M(k_1\sigma_1,k_2\sigma_2)$ in four dimensions can be written as

$$M=\bar u_2(A_1+B_1\gamma_5+C_1+C_2\not q+D_1\not p\gamma_5+D_2\not q\gamma_5+E_1\not p\not q+F_1\not p\not q\gamma_5)u_1\ , \tag{2.10}$$

where $u_1=u(k_1,\sigma_1)$, $u_2=u(k_2,\sigma_2)$ are the spinors describing the helicity eigenstates. The convention we choose for helicity eigenstates are such that the spinors $u(k,\sigma)$ and antispinors $v(k,\sigma)$ satisfy the relations[28,29]

$$u(k,\sigma)\bar u(k,\sigma')=\tfrac{1}{2}(m+\not k)(\not e^0\sigma_0+\not e^i\gamma_5\sigma_i)_{\sigma\sigma'}\ , \tag{2.11}$$

$$v(k,\sigma)\bar v(k,\sigma')=\tfrac{1}{2}(m-\not k)(\not e^0\bar\sigma_0+\not e^i\gamma_5\bar\sigma_i)_{\sigma\sigma'}\ , \tag{2.11a}$$

where σ_0 is the 2×2 identity matrix, σ_i the usual Pauli matrices, $\bar\sigma_a=\sigma_2\sigma_a\sigma_2$, and $e^a(k)$ are the four-vectors

$$e^0(k)=\frac{1}{m}k,\quad m^2=k^2\ , \tag{2.12}$$

$$e^1(k)=\frac{1}{|k|(|k|+k_z)}\times(0,-k_z(|k|+k_z)-k_y^2,k_xk_y,k_x(|k|+k_z))\ , \tag{2.13}$$

$$e^2(k)=\frac{1}{|k|(|k|+k_z)}\times(0,-k_xk_y,k_z(|k|+k_z)+k_x^2\ ,\ -k_y(|k|+k_z))\ , \tag{2.14}$$

$$e^3(k)=-\frac{1}{m}\left[|k|,\frac{k^0}{|k|}\mathbf{k}\right]\ . \tag{2.15}$$

In the above, $|k|$ is the absolute value and k_x,k_y,k_z are the components of the three-momentum $\mathbf{k}$. For a vector particle, we also use the four-vectors $e^a(k)$ to describe its polarization states $\epsilon(k,\lambda)$:

$$\epsilon(k,\lambda)=e^\lambda(k)\ . \tag{2.16}$$

$A_1,B_1,\ldots,E_1,F_1$ in Eq. (2.10) are functions of Lorentz invariants that we may construct from the momenta and polarization vectors of the mesons as well as k_1 and k_2. Our task is to compute $M(k\sigma_1,k_2\sigma_2)\bar M(k_1\sigma_1',k_2\sigma_2')$ with

$$\bar M=\bar u_1(\bar A_1-\bar B_1\gamma_5+\bar C_1\not p+\bar C_2\not q-\bar D_1\gamma_5\not p-\bar D_2\gamma_5\not q+\bar E_1\not q\not p-\bar F_1\gamma_5\not q\not p)u_2\ , \tag{2.17}$$

where $\bar A_1$ means taking complex conjugate *and* changing the polarization λ of any vector boson into λ' in the argument of A_1. The various traces involving the fermion bilinears $u_1\bar u_1,u_2\bar u_2$ are given in Appendix A. From Eq. (A14), one can see that only the following combinations of $A_1,B_1,\ldots,E_1,F_1$ appear in $M\bar M$:

$$A=\sqrt{\alpha_+}\,A_1+i\sqrt{\alpha_-}\,F_1,\quad B=\sqrt{\alpha_-}\,B_1+i\sqrt{\alpha_+}\,E_1\ , \tag{2.18}$$

$$C=i\sqrt{\alpha_-}\,C_1+\sqrt{\alpha_+}\,D_2,\quad D=i\sqrt{\alpha_+}\,D_1+\sqrt{\alpha_-}\,C_2\ .$$

Defining $\mathcal{F}^\alpha$ by $\mathcal{F}^0=A$, $\mathcal{F}^3=B$, $\mathcal{F}^1=C$, and $\mathcal{F}^2=D$ we have, from Eq. (A22),

$$\begin{aligned}M(k_1\sigma_1,k_2\sigma_2)\bar M(k_1\sigma_1',k_2\sigma_2')&=\mathcal{F}^\alpha\bar{\mathcal{F}}^\beta g^{ab}_{\alpha\beta}(\mathcal{S}_{ba})_{\sigma_1\sigma_1',\sigma_2\sigma_2'}\\&=[\mathcal{F}^\alpha g^{0b}_{\alpha\lambda}(\mathcal{S}_1)_b]_{\sigma_1\sigma_1'}[(\bar{\mathcal{S}}_2)_a g^{a0}_{\lambda\beta}\bar{\mathcal{F}}^\beta]_{\sigma_2\sigma_2'}\\&=[\mathcal{F}^\alpha g^{cb}_{\alpha0}(\mathcal{S}_1)_b]_{\sigma_1\sigma_1'}[(\bar{\mathcal{S}}_2)_a g^{ac}_{0\beta}\bar{\mathcal{F}}^\beta]_{\sigma_2\sigma_2'}\ ,\end{aligned} \tag{2.19}$$

where $g^{ab}_{\alpha\beta}$ are constants equal ±1 or $\pm i$ as given in Eq. (A13); $(S_1)_a$, $(\bar{\mathcal{S}}_2)_a$ are 2×2 matrices defined in Eq. (A2) and $\mathcal{S}_{ba}=(\mathcal{S}_1)_b\otimes(\bar{\mathcal{S}}_2)_a$. Note that for given $\alpha\beta$ or ab, $g^{ab}_{\alpha\beta}$ has only four nonzero entries so that the right-hand side of Eq. (2.19) contains no more than 64 terms. If we sum over initial and final spins of the baryons, only terms involving $(\mathcal{S}_1)_0$ and $(\bar{\mathcal{S}}_2)_0$ are nontrivial so that

$$\sum_{\sigma_1,\sigma_2}M(k_1\sigma_1,k_2\sigma_2)\bar M(k_1\sigma_1,k_2\sigma_2)=4\mathcal{F}^\alpha\bar{\mathcal{F}}^\alpha\ . \tag{2.20}$$

This simple structure shows up, for example, in $e^+e^-\to W^+W^-$ or $q\bar q\to W^+W^-$ (Ref. 30), no matter how complicated the underlying interactions one assumed.

If one of the baryons is changed to an antibaryon, then we have to interchange α_+ and α_- in Eq. (2.18) and change $\mathcal{S}_1,\mathcal{S}_2$, and $g_{\alpha\beta}$ according to the rules stated in Appendix A, Eq. (A23) and below. Equation (2.19) remains valid. If both baryons are changed to antibaryons, then the only changes needed are $\mathcal{S}_1\to\bar{\mathcal{S}}_1$, $\bar{\mathcal{S}}_2\to\mathcal{S}_2$ where $\bar{\mathcal{S}}_1,\mathcal{S}_2$ are defined in Eqs. (A23) and (A25).

The polarization density matrix ρ of a spin-$\frac{1}{2}$ particle is specified by $\mathcal{P}^a$ such that $\rho=\frac{1}{2}\mathcal{P}^a\sigma_a$. It is customary to normalize ρ so that $\mathcal{P}^0=1$. In the basis of helicity eigenstates which we are using, $\mathcal{P}^3$ is related to the longitudinal polarization while $\mathcal{P}^1$ and $\mathcal{P}^2$ are related to the transverse polarizations. Let $\rho_1=\frac{1}{2}\mathcal{P}_1^a\sigma_a$ be the spin density matrix of the first baryon. Then we have

$$\sum_{\sigma_1\sigma_1'}\sum_{\sigma_2\sigma_2'}(\rho_1)_{\sigma_1\sigma_1'}M(k_1\sigma_1,k_2\sigma_2)\overline{M}(k_1\sigma_1',k_2\sigma_2') =2\mathcal{F}^\alpha\overline{\mathcal{F}}^\beta g_{\alpha\beta}^{0b}(P_1)_b\ , \tag{2.21}$$

where

$$(P_1)_b=\left\langle 1,\ \frac{m_1}{\Delta_0}k\cdot\tilde{P}_1, p\cdot\tilde{P}_1, iq\cdot\tilde{P}_1\right\rangle_b\ ,\quad b=0,3,1,2\ , \tag{2.22}$$

$$\tilde{P}_1=-\sum_{i=1}^{3}\mathcal{P}_1^i\bar{e}^{\,i}(k_1)\ ,$$
$$\bar{e}^{\,1}=-e^1,\ \ \bar{e}^{\,2}=-e^2,\ \ \bar{e}^{\,3}=-e^3\ . \tag{2.23}$$

The polarization density matrix of the second baryon $\rho_2=\frac{1}{2}\mathcal{P}_2^a\sigma_a$ is given by

$$\tilde{P}_2=-\sum_{i=1}^{3}\mathcal{P}_2^i\bar{e}^{\,i}(k_2)=-c_3\frac{m_2}{\Delta_0}k-c_1p+ic_2q\ , \tag{2.24}$$

where

$$c_i=\frac{\mathcal{F}^\alpha\overline{\mathcal{F}}^\beta g_{\alpha\beta}^{ib}P_{1b}}{\mathcal{F}^\alpha\overline{\mathcal{F}}^\beta g_{\alpha\beta}^{0b}P_{1b}}\ . \tag{2.25}$$

Using Eq. (A12), one can show that $\tilde{P}_2$ as well as the right-hand side of Eq. (2.21) are real. For unpolarized source, we find the polarizations of the final-state baryons to be

$$c_i=\frac{\mathcal{F}^\alpha\overline{\mathcal{F}}^\beta g_{\alpha\beta}^{i0}}{\mathcal{F}^\alpha\overline{\mathcal{F}}^\alpha}\quad \text{for } \mathcal{P}_1^i=0\ . \tag{2.26}$$

The invariant amplitude M is usually not given in the form of Eq. (2.10). Instead, M may arise in the form

$$M=\bar{u}_2[X_0+Y_0\gamma_5+\not{X}_1+\not{Y}_1\gamma_5+\tfrac{1}{2}(\not{U}_1\not{V}_1-\not{V}_1\not{U}_1) +\tfrac{1}{2}(\not{W}_1\not{Z}_1-\not{Z}_1\not{W}_1)\gamma_5]u_1\ , \tag{2.27}$$

where X_0,Y_0 are scalars and X_1,Y_1,U_1,V_1,W_1,Z_1 are vectors. In general, there can be several tensor and pseudotensor terms, but their treatment is exactly the same as the single term we retain.

The above form of M is related to our form factors $\mathcal{F}^\alpha$ through the combinations

$$\Lambda_1=X_1+(k_-\cdot U_1)V_1-(k_-\cdot V_1)U_1+i\Delta(\cdot k_-W_1Z_1) +i\frac{\alpha_+}{\alpha_+-\alpha_-}\Delta(\cdot Y_1k_-k_+)\ , \tag{2.28}$$

$$\Omega_1=Y_1+(k_+\cdot W_1)Z_1-(k_+\cdot Z_1)W_1+i\Delta(\cdot k_+U_1V_1) +i\frac{\alpha_-}{\alpha_+-\alpha_-}\Delta(\cdot X_1k_-k_+)\ , \tag{2.29}$$

where

$$k_+=\frac{1}{\alpha_+}(m_2k_1+m_1k_2),\quad k_-=\frac{1}{\alpha_-}(m_2k_1-m_1k_2)\ . \tag{2.30}$$

Λ_1,Ω_1 satisfy the relations

$$i\sqrt{\alpha_-}p\cdot\Lambda_1=\sqrt{\alpha_+}\,q\cdot\Omega_1\ ,\qquad i\sqrt{\alpha_+}p\cdot\Omega_1=\sqrt{\alpha_-}\,q\cdot\Lambda_1\ . \tag{2.31}$$

Using Λ_1,Ω_1, we have the relations

$$\frac{1}{\sqrt{\alpha_+}}\mathcal{F}^0=X_0+\frac{\alpha_+}{\alpha_+-\alpha_-}k_+\cdot X_1-\frac{\alpha_-}{\alpha_+-\alpha_-}k_+\cdot\Lambda_1\ , \tag{2.32}$$

$$\frac{1}{\sqrt{\alpha_-}}\mathcal{F}^3=Y_0-\frac{\alpha_-}{\alpha_+-\alpha_-}k_-\cdot Y_1+\frac{\alpha_+}{\alpha_+-\alpha_-}k_-\cdot\Omega_1\ , \tag{2.33}$$

$$\frac{1}{\sqrt{\alpha_-}}\mathcal{F}^1=-ip\cdot\Lambda_1,\quad \frac{1}{\sqrt{\alpha_+}}\mathcal{F}^1=-q\cdot\Omega_1\ , \tag{2.34}$$

$$\frac{1}{\sqrt{\alpha_-}}\mathcal{F}^2=-q\cdot\Lambda_1,\quad \frac{1}{\sqrt{\alpha_+}}\mathcal{F}^2=-ip\cdot\Omega_1\ . \tag{2.35}$$

Let us now explain why of the eight form factors $A_1,B_1,\ldots,E_1,F_1$ only four combinations $\mathcal{F}^\alpha$ appear. For on-shell spinors u_1,u_2 we have

$$\bar{u}_2\not{p}\not{q}\gamma_5u_1=-\frac{i}{2}\bar{u}_2\Delta(pq\gamma\gamma)u_1=i\left[\frac{\alpha_-}{\alpha_+}\right]^{1/2}\bar{u}_2u_1\ , \tag{2.36}$$

where we used relations similar to Eq. (2.8), but with the roles of p,q and k_+,k_- interchanged. Similarly, we find

$$\bar{u}_2\not{q}\gamma_5u_1=\frac{i}{6}\bar{u}_2\Delta(q\gamma\gamma\gamma)u_1=-i\left[\frac{\alpha_+}{\alpha_-}\right]^{1/2}\bar{u}_2\not{p}u_1\ , \tag{2.37}$$

$$\bar{u}_2\not{p}\not{q}u_1=i\left[\frac{\alpha_+}{\alpha_-}\right]^{1/2}\bar{u}_2\gamma_5u_1\ , \tag{2.38}$$

$$\bar{u}_2\not{q}u_1=-i\left[\frac{\alpha_-}{\alpha_+}\right]^{1/2}\bar{u}_2\not{p}\gamma_5u_1\ . \tag{2.39}$$

These relations provide useful consistency checks on the formulas given in Appendix A.

Let us examine more closely the cases when only (pseudo)scalar mesons are involved. Of the four complex form factors $\mathcal{F}^\alpha$, there are only seven observables since the overall phase is not observable. From the explicit forms of $g_{\alpha\beta}^{ab}$ as given in Appendix A, we see that by measuring the diagonal elements of polarization correlations of the initial and final baryons, i.e., $\mathcal{F}^\alpha\overline{\mathcal{F}}^\beta g_{\alpha\beta}^{aa}$ for $a=1,2,3$, and the total decay rate we can obtain the absolute values $|\mathcal{F}^\alpha|$. The three independent relative phases of $\mathcal{F}^\alpha$ can then be obtained by measuring the decay asymmetry for polarized source or by measuring the polarizations of the final-state baryons with unpolarized source. More details will be given in the next section when we consider three-body decays with no vector mesons in the final state.

Next consider the cases with one vector boson in the final state. The momentum and polarization vector of the vector boson are p_1 and $e_\lambda(p_1)$, respectively, where $e_\lambda(p_1)$ are four-vectors defined in Eqs. (2.12)–(2.15). Instead of $\mathcal{F}^\alpha$, we shall write $\mathcal{F}^\alpha_\lambda$ which makes explicit the dependence on the polarization vector $e_\lambda(p_1)$. $\bar{\mathcal{F}}^\alpha$ now stands for $(\mathcal{F}^\alpha_{\lambda'})^*$. To expand $\mathcal{F}^\alpha_\lambda$, we choose a complete basis of four-vectors as we did in Eq. (2.6), but with p_1,k_1 play the roles of k_1,k_2 and we choose k_2 now as the reference vector P_0. Explicitly, the basis consists of p_1,k_1 and

$$p'=\frac{\Delta(\cdot p_1k_1k_2)}{[\Delta(p_1k_1k_2;p_1k_1k_2)]^{1/2}}\ ,$$
$$q'=\frac{\Delta(p_1k_1k_2;\cdot p_1k_1)}{[-\Delta(p_1k_1;p_1k_1)\Delta(p_1k_1k_2;p_1k_1k_2)]^{1/2}}\ , \tag{2.40}$$

where M_1 is the mass of the vector boson. We can now write

$$\mathcal{F}^\alpha_\lambda=a_0^\alpha\frac{M_1}{[-\Delta(p_1k_1;p_1k_1)]^{1/2}}e_\lambda(p_1)\cdot k_1 +a_1^\alpha e_\lambda(p_1)\cdot p'+a_2^\alpha e_\lambda(p_1)\cdot q'\ . \tag{2.41}$$

a_i^α are now functions of Lorentz invariants constructed from four-momenta of the various particles involved and are independent of the polarization vectors $e_\lambda(p_1)$. The measurable quantities are $\mathcal{F}^\alpha_\lambda\mathcal{F}^{\beta*}_{\lambda'}g^{ab}_{\alpha\beta}$. If we sum over polarizations of the vector boson we obtain

$$\sum_{\lambda=1}^{3}\mathcal{F}^\alpha_\lambda\mathcal{F}^{\beta*}_\lambda g^{ab}_{\alpha\beta}=\sum_{i=0}^{2}a_i^\alpha a_i^{\beta*}g^{ab}_{\alpha\beta}\ . \tag{2.42}$$

Note that the Hermitian matrix $\mathcal{H}^{\alpha\beta}$ defined by

$$\mathcal{H}^{\alpha\beta}=\sum_{i=0}^{2}a_i^\alpha a_i^{\beta*} \tag{2.43}$$

is degenerate. There exists four nonlinear relations among its elements. Instead of the usual 16 real degrees of freedom for 4×4 Hermitian matrices, $\mathcal{H}^{\alpha\beta}$ has only 12 real degrees of freedom. Hence, only half of the information is contained in the polarization summed cross sections. In principle, only 12 measurements out of the 16, i.e., $\mathcal{H}^{\alpha\beta}g^{ab}_{\alpha\beta}$, are needed to determine $\mathcal{H}^{\alpha\beta}$. If the vector boson is a massless gauge boson, then we may set $a_0^\alpha=0$. $\mathcal{H}^{\alpha\beta}$ now has only 8 real degrees of freedom which is again half of the total degrees of freedom contained in a_1^α and a_2^α.

The above considerations for the one-vector-boson cases works only when there are more than two particles in the final state. For two-body decays, there are only two independent four-momenta involved. This is a degenerate case and will be considered in the next section.

Consider now the cases when there are two vector bosons in the final state. The momenta of the vector bosons are p_1,p_2 and their polarization vectors are $e_{\lambda_1}(p_1)$, $e_{\lambda_2}(p_2)$. $\mathcal{F}^\alpha$ now picks up two indices and becomes $\mathcal{F}^\alpha_{\lambda_1\lambda_2}$ while $\bar{\mathcal{F}}^\alpha$ becomes $(\mathcal{F}^\alpha_{\lambda_1'\lambda_2'})^*$. Choose again a basis, say, $p_1,p_2,p'\sim\Delta(\cdot k_1p_1p_2)$, $q'\sim\Delta(k_1p_1p_2;\cdot p_1p_2)$ and normalize p',q' so that $(p')^2=-1,(q')^2=-1$. From p_1,p_2 we construct orthogonal vectors p_+,p_- as we did for k_1,k_2 in Eq. (2.30). Let us define the tensors $\mathcal{T}^{ij}_{\mu\nu}$ so that

$$\mathcal{T}^{00}_{\mu\nu}=\frac{\alpha'_-}{\alpha'_+}p_{-\mu}p_{-\nu}\ ,$$
$$\mathcal{T}^{01}_{\mu\nu}=\left[\frac{\alpha'_-}{\alpha'_+}\right]^{1/2}p_{-\mu}p'_\nu\ , \tag{2.44}$$
$$\mathcal{T}^{02}_{\mu\nu}=\left[\frac{\alpha'_-}{\alpha'_+}\right]^{1/2}p_{-\mu}q'_\nu,\ \text{etc.},$$

where $\alpha'_+=p_1\cdot p_2+M_1M_2$, $\alpha'_-=p_1\cdot p_2-M_1M_2$ with M_1,M_2 the masses of the two vector bosons. We can now expand $\mathcal{F}^\alpha_{\lambda_1\lambda_2}$:

$$\mathcal{F}^\alpha_{\lambda_1\lambda_2}=a_{ij}^\alpha\mathcal{T}^{ij}_{\mu\nu}e_{\lambda_1}(p_1)^\mu e_{\lambda_2}(p_2)^\nu\ . \tag{2.45}$$

a_{ij}^α are functions of kinematical invariants and are independent of the polarizations $e_{\lambda_1},e_{\lambda_2}$. The measurable quantities are now $\mathcal{F}^\alpha_{\lambda_1\lambda_2}\mathcal{F}^{\beta*}_{\lambda_1'\lambda_2'}g^{ab}_{\alpha\beta}$. If we sum over polarizations of one of the vector bosons, say, sum over λ_2, we get

$$\sum_{\lambda_2}\mathcal{F}^\alpha_{\lambda_1\lambda_2}\mathcal{F}^{\beta*}_{\lambda_1\lambda_2}g^{ab}_{\alpha\beta}=\sum_j a_{ij}^\alpha a_{kj}^{\beta*}\mathcal{T}^{ik}_{\mu\nu}e_{\lambda_1}(p_1)^\mu e_{\lambda_1'}(p_1)^\nu g^{ab}_{\alpha\beta}\ . \tag{2.46}$$

This should be compared with the previous case of a single vector boson.

If we sum over polarizations of both vector bosons, we get

$$\sum_{\lambda_1\lambda_2}\mathcal{F}^\alpha_{\lambda_1\lambda_2}\mathcal{F}^{\beta*}_{\lambda_1\lambda_2}g^{ab}_{\alpha\beta}=\sum_{ij}a_{ij}^\alpha a_{ij}^{\beta*}g^{ab}_{\alpha\beta}\ . \tag{2.47}$$

By measuring these 16 quantities we can determine $\mathcal{H}^{\alpha\beta}=\sum_{ij}a_{ij}^\alpha a_{ij}^{\beta*}$ but these account for only a fraction of the total degrees of freedom.

We can proceed to consider cases with more vector bosons. However, it is clear that for each fixed α, the parametrization of $\mathcal{F}^\alpha$ involves only bosons and depends on how many vector bosons are considered. This is a problem of interest in its own right and will not be considered further here.

III. TWO-BODY AND THREE-BODY DECAYS

A. Two-body decays

The cases of two-body decays are degenerate in the sense that there are only two independent four momenta out of the three external particles. As a result, there is no "natural" way to choose a reference vector P_0 to define p,q without referring to particles or apparatuses outside the three-particle system. Let us first discuss the well-known case when only scalar mesons are present in the final state. In this case, in the amplitude M as given in Eq. (2.27), only X_0, Y_0 are nonzero and they are constants as the only kinematical invariants we can construct in this case are constants. Hence of the four form factors $\mathcal{F}^\alpha$, only $\mathcal{F}^0$ and $\mathcal{F}^3$ are nonvanishing. Instead of the seven independent real quantities to be determined in the general case, we now have only three: namely, $|\mathcal{F}^0|$, $|\mathcal{F}^3|$, and the relative phase of $\mathcal{F}^0$ and $\mathcal{F}^3$. From Eq. (2.21) and the explicit form of $g^{ab}_{\alpha\beta}$, we find that the differential decay rate for a polarized source is

$$d\Gamma=\frac{1}{k_1^0}[\,|\mathcal{F}^0|^2+|\mathcal{F}^3|^2-2\,\mathrm{Re}\mathcal{F}^0\bar{\mathcal{F}}^3(P_1)_3]d\Phi_2 \;, \tag{3.1}$$

where $d\Phi_2$ is the two-particle invariant phase-space element. In the rest frame of the decaying particle, $e^1(k_1)=\langle 0,-1,0,0\rangle$, $e^2(k_1)=\langle 0,0,1,0\rangle$, $e^3(k_1)=\langle 0,0,0,-1\rangle$ so that

$$\tilde{P}_1=\langle 0,\mathcal{P}_1\rangle$$

and we find from Eq. (2.22) that

$$(P_1)_3=\frac{m_1}{\Delta_0}k\cdot\tilde{P}_1=|\mathcal{P}_1|\cos\theta \;, \tag{3.2}$$

where θ is the angle between $\mathcal{P}_1$ and $\mathbf{k}_2$ in the rest frame of k_1. Hence we recovered the well-known formula[31] in this case. For the polarization of the decayed baryon, we find from Eqs. (2.24) and (2.25) that

$$\mathcal{P}_2^3=c_3=\frac{\alpha+(P_1)_3}{1+\alpha(P_1)_3} \tag{3.3}$$

and

$$c_1p-ic_2q=\frac{1}{1+\alpha(P_1)_3}\left\{\gamma\frac{1}{\alpha_+\alpha_-}\Delta(\tilde{P}_1k_1k_2;\cdot k_1k_2)-\beta\frac{1}{\Delta_0}\Delta(\cdot\tilde{P}_1k_1k_2)\right\} , \tag{3.4}$$

where

$$\alpha=\frac{-2\,\mathrm{Re}\mathcal{F}^0\bar{\mathcal{F}}^3}{|\mathcal{F}^0|^2+|\mathcal{F}^3|^2} ,\quad \beta=\frac{-2\,\mathrm{Im}\mathcal{F}^0\bar{\mathcal{F}}^3}{|\mathcal{F}^0|^2+|\mathcal{F}^3|^2} ,\quad \gamma=\frac{|\mathcal{F}^0|^2-|\mathcal{F}^3|^2}{|\mathcal{F}^0|^2+|\mathcal{F}^3|^2} . \tag{3.5}$$

Computing the covariants $\Delta(\tilde{P}_1k_1k_2;\cdot k_1k_2)$ and $\Delta(\cdot\tilde{P}_1k_1k_2)$ in the rest frame of k_1, we get back the usual formulas for the polarizations of the decayed baryon.[31]

Now we turn to the case when the final-state boson is a vector particle. In the amplitude M in Eq. (2.27), only X_0,Y_0,X_1,Y_1 are nonvanishing. Moreover, both X_1,Y_1 must be proportional to $e_\lambda(p_1)$. The on-shell condition of the vector boson implies

$$e_\lambda(p_1)\cdot k_1=e_\lambda(p_1)\cdot k_2 \;.$$

It follows from Eqs. (2.32)–(2.35) that we may write

$$\mathcal{F}^0_\lambda=a^0\frac{M_1}{\Delta_0}e_\lambda(p_1)\cdot k_1,\quad \mathcal{F}^3_\lambda=a^3\frac{M_1}{\Delta_0}e_\lambda(p_1)\cdot k_1 \;,$$
$$\mathcal{F}^1_\lambda=\frac{1}{\sqrt{2}}[a^1e_\lambda(p_1)\cdot p-ia^2e_\lambda(p_1)\cdot q] \;, \tag{3.6}$$
$$\mathcal{F}^2_\lambda=\frac{1}{\sqrt{2}}[a^2e_\lambda(p_1)\cdot p-ia^1e_\lambda(p_1)\cdot q] \;,$$

where p,q are chosen as in Eq. (2.6) with some reference vector P_0. a^α are constants since, as in the previous case, there are no nonconstant Lorentz invariants one can construct from k_1,k_2,p_1.

Let us first sum over the polarizations of the vector boson. We find that

$$\sum_\lambda \mathcal{F}^\alpha_\lambda\bar{\mathcal{F}}^\beta_\lambda=\begin{bmatrix} |a^0|^2 & a^0\bar{a}^3 & 0 & 0\\ \bar{a}^0a^3 & |a^3|^2 & 0 & 0\\ 0 & 0 & \frac{1}{2}(|a^1|^2+|a^2|^2) & \mathrm{Re}a^1\bar{a}^2\\ 0 & 0 & \mathrm{Re}a^1\bar{a}^2 & \frac{1}{2}(|a^1|^2+|a^2|^2)\end{bmatrix}^{\alpha\beta} , \tag{3.7}$$

where $\alpha,\beta=0,3,1,2$.

The differential decay rates follow from Eq. (2.21):

$$d\Gamma=\frac{1}{k_1^0}[a^\alpha\bar{a}^\alpha-2(\mathrm{Re}a^0\bar{a}^3+\mathrm{Re}a^1\bar{a}^2)(P_1)_3]d\Phi_2 \;. \tag{3.8}$$

The polarization of the decayed baryon in this case is given by

$$\mathcal{P}_2^3=c_3=\frac{\alpha_2+\alpha_3(P_1)_3}{1+\alpha_1(P_1)_3} \tag{3.9}$$

and

$$c_1p - ic_2q = \frac{1}{1+\alpha_1(P_1)_3}\left[\alpha_4 \frac{1}{\alpha_+\alpha_-}\Delta(\tilde{P}_1 k_1 k_2; \cdot k_1 k_2) - \alpha_5 \frac{1}{\Delta_0}(\cdot \tilde{P}_1 k_1 k_2)\right], \quad (3.10)$$

where

$$\alpha_1 = \frac{-2\,\mathrm{Re}(a^0\bar{a}^3 + a^1\bar{a}^2)}{a^\alpha \bar{a}^\alpha},$$
$$\alpha_2 = \frac{-2\,\mathrm{Re}(a^0\bar{a}^3 - a^1\bar{a}^2)}{a^\alpha \bar{a}^\alpha},$$
$$\alpha_3 = \frac{|a^0|^2 + |a^3|^2 - |a^1|^2 - |a^2|^2}{a^\alpha \bar{a}^\alpha}, \quad (3.11)$$
$$\alpha_4 = \frac{|a^0|^2 - |a^3|^2}{a^\alpha \bar{a}^\alpha}, \quad \alpha_5 = \frac{-2\,\mathrm{Im}(a^0\bar{a}^3)}{a^\alpha \bar{a}^\alpha},$$

and $(P_1)_3$ is again given by Eq. (3.2). α_i's now satisfy the relation

$$\tfrac{1}{4}(\alpha_1+\alpha_2)^2 + \alpha_4^2 + \alpha_5^2 = \tfrac{1}{4}(1+\alpha_3)^2 . \quad (3.12)$$

If the polarization of the decaying particle is known or is at our disposal, then we can measure all $\alpha_i, i=1,2,\ldots,5$ as well as $a^\alpha \bar{a}^\alpha$. The only quantities we cannot measure from these polarization-summed cross sections are the relative phase between, say, a^0 and a^1 and the difference $|a^1|^2 - |a^2|^2$. On the other hand, suppose we are trying to measure the polarization of the decaying particle. In the case of scalar meson, we have only to measure the decay asymmetry and the longitudinal polarization of the final-state baryon as can be seen from Eqs. (3.1)–(3.3). This is not the case for vector boson in the final state. Measuring the quantities in Eqs. (3.8)–(3.10) as a function of $\cos\theta$ will give us α_2 and $\alpha_i|\mathcal{P}_1|$, $i=1,3,4,5$. We can now use the identity Eq. (3.12) to obtain

$$|\mathcal{P}_1| = \frac{1}{1-\alpha_2^2}\left[\alpha_2\alpha_1|\mathcal{P}_1| - \alpha_3|\mathcal{P}_1| \pm \sqrt{(\alpha_2\alpha_3|\mathcal{P}_1| - \alpha_1|\mathcal{P}_1|)^2 + 4(1-\alpha_2^2)(\alpha_4^2|\mathcal{P}_1|^2 + \alpha_5^2|\mathcal{P}_1|^2)}\right]. \quad (3.13)$$

When $|a^1|, |a^2|$ are small compared to $|a^0|$ and $|a^3|$, we find that we have to take the plus sign in Eq. (3.3). Hence, if we can use a continuity argument, the plus sign should be taken in Eq. (3.3). The sign of $\mathcal{P}_1$ can, of course, be determined by just examining the $\cos\theta$ dependence of various measurable quantities. Thus, for the purpose of determining the polarization of the decaying particle, we need not measure the polarization of the final-state vector boson or its correlation with other physical quantities. However, we do have to measure, in general, the polarization of the decayed baryon.

Now we take into account the polarization of the vector boson. For this purpose we need explicit expressions for $\mathcal{F}_\lambda^\alpha \bar{\mathcal{F}}_{\lambda'}^\beta$. It is convenient to choose $\tilde{P}_1$ as the reference vector for p,q. In evaluating the Lorentz invariants in Eq. (3.6), we choose the direction of the three-momentum of the vector boson as the positive z axis and choose $\mathcal{P}_1$ to lie in the x-z plane with positive x components. With these conventions we find

$$\mathcal{F}_\lambda^0 = -a^0\delta_{\lambda 3}, \quad \mathcal{F}_\lambda^3 = -a^3\delta_{\lambda 3},$$
$$\mathcal{F}_\lambda^1 = -\frac{1}{\sqrt{2}}(a^1\delta_{\lambda 2} + ia^2\delta_{\lambda 1}), \quad (3.14)$$
$$\mathcal{F}_\lambda^2 = -\frac{1}{\sqrt{2}}(a^2\delta_{\lambda 2} + ia^1\delta_{\lambda 1}).$$

It follows that we have

$$\mathcal{F}_1^\alpha \bar{\mathcal{F}}_1^\beta = \frac{1}{2}\begin{bmatrix} |a^2|^2 & \bar{a}^1 a^2 \\ a^1\bar{a}^2 & |a^1|^2 \end{bmatrix}^{\alpha\beta},$$
$$\mathcal{F}_2^\alpha \bar{\mathcal{F}}_2^\beta = \frac{1}{2}\begin{bmatrix} |a^1|^2 & \bar{a}^2 a^1 \\ a^2\bar{a}^1 & |a^2|^2 \end{bmatrix}^{\alpha\beta}, \quad (3.15)$$
$$\mathcal{F}_1^\alpha \bar{\mathcal{F}}_2^\beta = \frac{1}{2}\begin{bmatrix} \bar{a}^1 a^2 & |a^2|^2 \\ |a^1|^2 & a^1\bar{a}^2 \end{bmatrix}^{\alpha\beta}, \quad \alpha,\beta = 1,2$$

and

$$\mathcal{F}_3^\alpha \bar{\mathcal{F}}_1^\beta = -\frac{i}{\sqrt{2}}\begin{bmatrix} a^0\bar{a}^2 & a^0\bar{a}^1 \\ a^3\bar{a}^2 & a^3\bar{a}^1 \end{bmatrix}^{\alpha\beta},$$
$$\mathcal{F}_3^\alpha \bar{\mathcal{F}}_2^\beta = \frac{1}{\sqrt{2}}\begin{bmatrix} a^0\bar{a}^1 & a^0\bar{a}^2 \\ a^3\bar{a}^1 & a^3\bar{a}^2 \end{bmatrix}^{\alpha\beta}, \quad \alpha = 0,3, \beta = 1,2, \quad (3.16)$$

where we have written only the nonzero elements in $\mathcal{F}_\lambda^\alpha \bar{\mathcal{F}}_{\lambda'}^\beta$ for various λ, λ'. To obtain $|a^1|^2 - |a^2|^2$ and the relative phase of, say, a^0 and $a^1 + a^2$, we have only to measure $\mathcal{F}_1^\alpha \bar{\mathcal{F}}_1^\alpha - \mathcal{F}_2^\alpha \bar{\mathcal{F}}_2^\alpha$ and $\mathcal{F}_3^\alpha(\bar{\mathcal{F}}_1^\alpha - i\bar{\mathcal{F}}_2^\alpha)$ which requires observing the correlations of the decay products of the vector particle.

It is interesting to look at the case when the vector boson is a massless gauge particle. Then we have $a^0 = a^3 = 0$. In Eq. (3.11), we find now

$$-\alpha_1 = \alpha_2 = \frac{2\,\mathrm{Re}\,a^1\bar{a}^2}{|a^1|^2 + |a^2|^2}, \quad \alpha_3 = -1, \quad \alpha_4 = \alpha_5 = 0. \quad (3.17)$$

Hence, the decayed baryon is longitudinally polarized if we do not observe the polarization of the vector boson. On the other hand, assume the vector boson is linear polarized with $\lambda=1$. Using the explicit form of $\mathcal{F}_1^{\alpha}\bar{\mathcal{F}}_1^{\beta}$ in Eq. (3.15) we find now

$$c_1 p - i c_2 q = \frac{1}{1+\alpha_1(P_1)_3}\left[\alpha_6 \frac{1}{\alpha_+\alpha_-}\Delta(\tilde{P}_1 k_1 k_2; \cdot k_1 k_2) - \alpha_7 \frac{1}{\Delta_0}\Delta(\cdot \tilde{P}_1 k_1 k_2)\right], \quad (3.18)$$

where

$$\alpha_6 = \frac{-|a^1|^2+|a^2|^2}{|a^1|^2+|a^2|^2}, \quad \alpha_7 = \frac{-2\,\mathrm{Im} a^1 \bar{a}^2}{|a^1|^2+|a^2|^2}. \quad (3.19)$$

Hence, the transverse polarization of the decayed baryon need not vanish in general.

B. Three-body decays

Three-body decays will illustrate most features of the general multibody processes. With three independent four momenta of the four external particles, we can construct two independent Lorentz invariants which are not constants. We may choose them, for example, to be the energy of the decayed baryon and one of the bosons in the rest frame of the decaying particle. Our form factors will depend on these two variables.

Let us begin with the case when no vector bosons are in the final state. We choose the reference vector to be p_1, the four-momentum of one of the mesons observed, in constructing p and q in Eq. (2.6). From Eq. (2.21), we can write down the differential decay rate

$$d\Gamma = \frac{1}{k_1^0}\Bigg[\mathcal{F}^{\alpha}\bar{\mathcal{F}}^{\alpha} - 2\,\mathrm{Re}(\mathcal{F}^0\bar{\mathcal{F}}^3+\mathcal{F}^1\bar{\mathcal{F}}^2)\,|\mathcal{P}_1|\cos\theta - 2\,\mathrm{Im}(\mathcal{F}^0\bar{\mathcal{F}}^2+\mathcal{F}^1\bar{\mathcal{F}}^3)\frac{\Delta(\tilde{P}_1 p_1 k_1 k_2)}{\sqrt{\alpha_+\alpha_-\Delta_r}} + 2\,\mathrm{Re}(\mathcal{F}^0\bar{\mathcal{F}}^1-\mathcal{F}^3\bar{\mathcal{F}}^2) \times \frac{\Delta(p_1 k_1 k_2; \tilde{P}_1 k_1 k_2)}{\alpha_+\alpha_-\Delta_r}\Bigg] d\Phi_3, \quad (3.20)$$

where θ is the angle between $\mathcal{P}_1$ and the three-momentum of the decayed baryon and $d\Phi_3$ is the invariant phase-space elements of the final-state particles. We evaluate the phase-space volume elements and the Lorantz invariants in the large parentheses in Eq. (3.20) in the rest frame of the decaying baryon and find

$$d\Phi_3 = \frac{1}{128\pi^4} dk_2^0 dp_1^0 d(\cos\theta) d\phi, \quad (3.21)$$

$$\frac{1}{\sqrt{\alpha_+\alpha_-\Delta_r}}\Delta(\tilde{P}_1 p_1 k_1 k_2) = -|\mathcal{P}_1|\sin\theta\sin\phi, \quad (3.22)$$

$$\frac{1}{\alpha_+\alpha_-\Delta_r}\Delta(p_1 k_1 k_2; \tilde{P}_1 k_1 k_2) = -|\mathcal{P}_1|\sin\theta\cos\phi, \quad (3.23)$$

where θ is the angle between $\mathcal{P}_1$ and $\mathbf{k}_2$ and ϕ is the angle between the plane of the decayed products and the plane containing $\mathcal{P}_1$ and $\mathbf{k}_2$. More precisely, if we choose the direction of $\mathbf{k}_2$ to be the positive z axis and choose $\mathcal{P}_1$ to lie in the x-z plane with positive x component, then ϕ is the azimuthal angle of $\mathbf{p}_1$. In the phase-space element of Eq. (3.21), we have integrated out an irrelevant angle. Using the following shorthand notation

$$\alpha_{\mu\nu} = \frac{-2\,\mathrm{Re}\mathcal{F}^{\mu}\bar{\mathcal{F}}^{\nu}}{\mathcal{F}^{\alpha}\bar{\mathcal{F}}^{\alpha}}, \quad \beta_{\mu\nu} = \frac{-2\,\mathrm{Im}\mathcal{F}^{\mu}\bar{\mathcal{F}}^{\nu}}{\mathcal{F}^{\alpha}\bar{\mathcal{F}}^{\alpha}}, \quad \mu,\nu = 0,3,1,2;\ \mu\neq\nu, \quad (3.24)$$

$$\gamma_3 = \frac{|\mathcal{F}^0|^2+|\mathcal{F}^3|^2-|\mathcal{F}^1|^2-|\mathcal{F}^2|^2}{\mathcal{F}^{\alpha}\bar{\mathcal{F}}^{\alpha}},$$

$$\gamma_1 = \frac{|\mathcal{F}^0|^2-|\mathcal{F}^3|^2-|\mathcal{F}^1|^2+|\mathcal{F}^2|^2}{\mathcal{F}^{\alpha}\bar{\mathcal{F}}^{\alpha}}, \quad (3.25)$$

$$\gamma_2 = \frac{|\mathcal{F}^0|^2-|\mathcal{F}^3|^2+\mathcal{F}^1|^2-|\mathcal{F}^2|^2}{\mathcal{F}^{\alpha}\bar{\mathcal{F}}^{\alpha}}$$

we can write the differential decay rate as

$$d\Gamma = \frac{1}{k_1^0}\mathcal{F}^{\alpha}\bar{\mathcal{F}}^{\alpha}\{1+|\mathcal{P}_1|[(\alpha_{03}+\alpha_{12})\cos\theta-(\beta_{02}-\beta_{31})\sin\theta\sin\phi+(\alpha_{01}-\alpha_{32})\sin\theta\cos\phi]\}\frac{1}{128\pi^4}dk_2^0 dp_1^0 d(\cos\theta)d\phi. \quad (3.26)$$

The polarization of the decayed baryon follows from Eq. (2.25). We have, for the longitudinal polarization,

$$\mathcal{P}_2^3 = c_3 = \frac{\alpha_{03}-\alpha_{12}+|\mathcal{P}_1|[\gamma_3\cos\theta-(\beta_{01}-\beta_{32})\sin\theta\sin\phi+(\alpha_{02}-\alpha_{31})\sin\theta\cos\phi]}{1+|\mathcal{P}_1|[(\alpha_{03}+\alpha_{12})\cos\theta-(\beta_{02}-\beta_{31})\sin\theta\sin\phi+(\alpha_{01}-\alpha_{32})\sin\theta\cos\phi]} \quad (3.27)$$

and, for the transverse polarization,

$$c_1=\frac{\beta_{02}+\beta_{31}+|\mathcal{P}_1|[-(\beta_{01}+\beta_{32})\cos\theta-\gamma_1\sin\theta\sin\phi+(\beta_{03}-\beta_{12})\sin\theta\cos\phi]}{1+|\mathcal{P}_1|[(\alpha_{03}+\alpha_{12})\cos\theta-(\beta_{02}-\beta_{31})\sin\theta\sin\phi+(\alpha_{01}-\alpha_{32})\sin\theta\cos\phi]}\ , \tag{3.28}$$

$$-ic_2=\frac{-(\alpha_{01}+\alpha_{32})+|\mathcal{P}_1|[(\alpha_{02}+\alpha_{31})\cos\theta-(\beta_{03}+\beta_{12})\sin\theta\sin\phi-\gamma_2\sin\theta\cos\phi]}{1+|\mathcal{P}_1|[(\alpha_{03}+\alpha_{12})\cos\theta-(\beta_{02}-\beta_{31})\sin\theta\sin\phi+(\alpha_{01}-\alpha_{32})\sin\theta\cos\phi]}\ . \tag{3.29}$$

Note that c_1 gives the transverse polarization perpendicular to the decay plane and c_2 gives the transverse polarization in the decay plane. Conversely, $\mathcal{P}_2^1$ and $\mathcal{P}_2^2$ give the transverse polarization perpendicular to and in the plane of $\mathcal{P}_1$ and $\mathbf{k}_2$, respectively.

If the angle ϕ is not observed, we have to integrate over ϕ in the numerator and denominator in Eqs. (3.27)–(3.29) separately. The resulting equations are similar to the cases of two-body decays. If $|\mathcal{P}_1|$ is not known, by measuring the decay asymmetry and the longitudinal polarization of the decayed baryon, we can obtain, in principle,

$$(\alpha_{03}+\alpha_{12})|\mathcal{P}_1|\ ,\quad \gamma_3|\mathcal{P}_1|\ ,\quad \alpha_{03}-\alpha_{12}\ .$$

Contrary to the two-body case where α_{12} vanishes, in general, we cannot extract $|\mathcal{P}_1|$ from these measurements alone. It is necessary to measure the transverse polarization of the decayed baryon or to observe the ϕ dependence of the decay rate and the longitudinal polarizations.

Note that the formulas in Eqs. (3.26)–(3.29) are general in the sense that they can be applied no matter how many scalar mesons are in the final state as long as we observe only the decayed baryon and one of the decayed mesons while we integrate out all other dynamical variables.

Next we consider the case when there is one vector boson in the final state. The form factors $\mathcal{F}^\alpha$ are parametrized by a_i^α as in Eq. (2.41). a_i^α depends only on two independent kinematical invariants which we may choose to be the energy of the decayed baryon and the vector boson in the rest frame of the decaying baryon. Calculating the scalar products of four-vectors in Eq. (2.41) we obtain

$$\mathcal{F}_3^\alpha=-a_0^\alpha,\quad \mathcal{F}_1^\alpha=a_1^\alpha\sin\omega-a_2^\alpha\cos\omega,\quad \mathcal{F}_2^\alpha=a_1^\alpha\cos\omega+a_2^\alpha\sin\omega\ , \tag{3.30}$$

where ω is the angle between the plane of the decayed products and the plane containing $\mathcal{P}_1$ and the vector boson. It is related to θ and ϕ, which appear in Eqs. (3.21)–(3.23), by

$$\begin{aligned}\sin\omega&=\frac{-\sin\theta\sin\phi}{[\sin^2\theta\sin^2\phi+(\sin\lambda\cos\theta-\cos\lambda\sin\theta\cos\phi)^2]^{1/2}}\ ,\\ \cos\omega&=\frac{\sin\lambda\cos\theta-\cos\lambda\sin\theta\cos\phi}{[\sin^2\theta\sin^2\phi+(\sin\lambda\cos\theta-\cos\lambda\sin\theta\cos\phi)^2]^{1/2}}\ ,\end{aligned} \tag{3.31}$$

where λ is the angle between the decayed baryon and the vector boson and is constrained kinematically to be

$$\cos\lambda=\frac{|p_2|^2-|p_1|^2-|k_2|^2}{2|p_1|\,|k_2|}\ . \tag{3.32}$$

It is convenient to use the helicity eigenstates of the vector boson. Defining

$$\mathcal{F}_\pm^\alpha=\frac{1}{\sqrt{2}}(\mathcal{F}_1^\alpha\pm\mathcal{F}_2^\alpha),\quad a_\pm^\alpha=\frac{1}{\sqrt{2}}(a_1^\alpha\pm a_2^\alpha),\quad \mathcal{F}_0^\alpha=\mathcal{F}_3^\alpha\ ,$$

we have

$$\mathcal{F}_\lambda^\alpha=-e^{-i\lambda(\omega+\pi/2)}a_\lambda^\alpha,\quad \lambda=+,-,0\ . \tag{3.33}$$

For given helicity state λ of the vector boson, the differential decay rate and the polarization of the decayed baryon is given by formulas almost identical to Eqs. (3.26)–(3.29). We have, for example,

$$\begin{aligned}(d\Gamma)_\lambda=&\frac{1}{k_1^0}a_\lambda^\alpha a_\lambda^{\alpha *}\{1+|\mathcal{P}_1|[(\alpha_{03}^\lambda+\alpha_{12}^\lambda)\cos\theta-(\beta_{02}^\lambda-\beta_{31}^\lambda)\sin\theta\sin\phi+(\alpha_{01}^\lambda-\alpha_{32}^\lambda)\sin\theta\cos\phi]\}\\ &\times\frac{1}{128\pi^4}dk_2^0dp_1^0d(\cos\theta)d\phi\ ,\end{aligned} \tag{3.34}$$

where no summation over λ is implied and

$$\alpha_{\mu\nu}^\lambda=\frac{-2\,\mathrm{Re}(a_\lambda^\mu a_\lambda^{\nu *})}{a_\lambda^\alpha a_\lambda^{\alpha *}},\quad \beta_{\mu\nu}^\lambda=\frac{-2\,\mathrm{Im}(a_\lambda^\mu a_\lambda^{\nu *})}{a_\lambda^\alpha a_\lambda^{\alpha *}}\quad (\text{no sum over }\lambda)\ . \tag{3.35}$$

Similarly, γ_i^λ is defined by replacing $\mathcal{F}^\alpha$ by a_λ^α in Eq. (3.25). The polarization of the decayed baryon is then obtained from Eqs. (3.27)–(3.29) by replacing $\alpha_{\mu\nu},\beta_{\mu\nu},\gamma_i$ with $\alpha_{\mu\nu}^\lambda,\beta_{\mu\nu}^\lambda,\gamma_i^\lambda$.

For massless gauge vector bosons, we have to set $a_0^\alpha=0$ and restrict λ to take only the values 1,2 in Eq. (2.41). The differential decay rate and the polarization of the decayed baryon for given helicity $\lambda=\pm$ of the vector boson are given by the same formulas as in the massive case. We can also define polarization for the massless vector boson in the same way as we did for the spin-$\frac{1}{2}$ baryons. It is straightforward to write down the polarization of the vector boson from Eq. (2.21) or more generally from Eq. (2.19) if we do not sum over the spins of the final-state baryon. We shall not give the explicit formulas here.

Finally, let us discuss briefly the case when both final-state bosons are vector bosons. It should be clear by now that for given helicity eigenstates of the vector bosons, formulas in Eqs. (3.27)–(3.29) still apply. We have only to modify the definitions of $\alpha_{\mu\nu},\beta_{\mu\nu},\gamma_i$ properly. For the differential decay rate, we have to modify the factor $\mathcal{F}^\alpha\overline{\mathcal{F}}^\alpha$ in Eq. (3.26) as well, as we did in Eq. (3.34). To see the correct modifications, we have to work out $\mathcal{T}^{ij}_{\mu\nu}e_{\lambda_1}(p_1)^\mu e_{\lambda_3}(p_2)^\nu$ as defined in Eqs. (2.44) and (2.45). This is best done in the center-of-mass (c.m.) frame of the two vector bosons. Defining

$$\begin{aligned}&\mathcal{F}^\alpha_{\pm,\lambda}=\frac{1}{\sqrt{2}}(\mathcal{F}^\alpha_{1,\lambda}\pm i\mathcal{F}^\alpha_{2\lambda}),\quad \mathcal{F}^\alpha_{\lambda,\pm}=\frac{1}{\sqrt{2}}(\mathcal{F}^\alpha_{\lambda 1}\pm i\mathcal{F}^\alpha_{\lambda 2}),\mathcal{F}^\alpha_{0\lambda}=\mathcal{F}^\alpha_{3\lambda}\ ,\\ &a^\alpha_{\pm,j}=\frac{1}{\sqrt{2}}(a^\alpha_{ij}\mp ia^\alpha_{2j}),\quad a^\alpha_{j,\pm}=\frac{1}{\sqrt{2}}(a^\alpha_{j1}\mp ia^\alpha_{j2})\ ,\end{aligned} \tag{3.36}$$

we find

$$\mathcal{F}^\alpha_{\lambda_1\lambda_2}=-\exp\left[i(\lambda_1+\lambda_2)\left[\frac{\pi}{2}-\xi\right]\right]a^\alpha_{\lambda_1\lambda_2},\quad \lambda_1,\lambda_2=+,0,-\ , \tag{3.37}$$

where ξ is the angle between the decay plane and the plane containing the vector bosons and the polarization $\mathcal{P}_1$ of the decaying baryon in the c.m. frame of the two vector bosons. Given the helicities λ_1 and λ_2 of the two vector bosons, only the combinations

$$\mathcal{F}^\alpha_{\lambda_1\lambda_2}\mathcal{F}^{\beta*}_{\lambda_1\lambda_2}=a^\alpha_{\lambda_1\lambda_2}a^{\beta*}_{\lambda_1\lambda_2}\quad(\text{no sum over }\lambda_1\lambda_2)$$

appear in the decay rate and the polarization of the decayed baryon. For example, we have

$$\begin{aligned}(d\Gamma)_{\lambda_1\lambda_2}=\frac{1}{k_1^0}a^\alpha_{\lambda_1\lambda_2}a^{\beta*}_{\lambda_1\lambda_2}\{\,|+|\mathcal{P}_1|[(\alpha_{03}^{\lambda_1\lambda_2}+\alpha_{12}^{\lambda_1\lambda_2})\cos\theta-(\beta_{02}^{\lambda_1\lambda_2}-\beta_{31}^{\lambda_1\lambda_2})\sin\theta\sin\phi\\ +(\alpha_{01}^{\lambda_1\lambda_2}-\alpha_{32}^{\lambda_1\lambda_2})\sin\theta\cos\phi]\}\frac{1}{128\pi^4}dk_2^0dp_1^0d(\cos\theta)d\phi\ ,\end{aligned} \tag{3.38}$$

where

$$\alpha_{\mu\nu}^{\lambda_1\lambda_2}=\frac{-2\,\mathrm{Re}(a^\mu_{\lambda_1\lambda_2}a^{\nu*}_{\lambda_1\lambda_2})}{a^\alpha_{\lambda_1\lambda_2}a^{\alpha*}_{\lambda_1\lambda_2}},\beta_{\mu\nu}^{\lambda_1\lambda_2}=\frac{-2\,\mathrm{Im}(a^\mu_{\lambda_1\lambda_2}a^{\nu*}_{\lambda_1\lambda_2})}{a^\alpha_{\lambda_1\lambda_2}a^{\alpha*}_{\lambda_1\lambda_2}}\ (\text{no sum over }\lambda_1\lambda_2)\ . \tag{3.39}$$

To conclude this section we remark, without giving explicit formulas, that the angle ξ appearing in Eq. (3.36) is a function of k_2^0, p_1^0, and ϕ appearing in Eq. (3.26). To derive this relation we have only to relate Lorentz invariants involving the various momenta and the polarization four-vector $\tilde{P}_1$ [see Eq. (2.33)] in different reference frames.

IV. DISCUSSIONS

We have derived formulas for analyzing, in a model-independent way, multiparticle scattering or decay processes involving a pair of polarized spin-$\frac{1}{2}$ baryons, any number of scalar mesons and up to two polarized vector bosons. We use helicity eigenstates of the baryons so that our formulas are Lorentz covariant. The formulas are applied specifically to the multiparticle decay of baryons which will be relevant for charm and bottom baryon decays.

The two-body decays are degenerate cases of our general formulas. In the case when the final-state boson is a vector particle, we find that measuring the decay asymmetry and the longitudinal polarization of the decayed baryon alone is not enough to determine the polarization of the decaying baryon in contrast with the case when the final-state boson is a scalar particle.

The three-body decays are typical of the more general situation. Besides the angle θ between decayed baryon and the polarization $\mathcal{P}_1$ of the decaying baryon, the differential decay rate and the polarization of the decayed baryon depend on another angle ϕ which is the angle between the decay plane and the plane containing $\mathcal{P}_1$ and the decayed baryon. If ϕ is not measured, then we may define asymmetry of the decayed baryon with respect to the polarization $\mathcal{P}_1$ as usual. The formulas for decay asymmetry and polarization are similar to the two-body case. We cannot, however, determine the polarization of the decaying baryon by just measuring the decay asymmetry and the longitudinal polarization of the decayed baryon.

For three-body decays with one or two vector bosons in the final state, we find that for given helicities of the final-state vector boson the formulas are almost exactly the same as the case involving only scalar mesons.

If there are more than two vector bosons involved in the final state, then we may lose the simple relations as in Eqs. (3.33) and (3.37). In these cases, a straightforward generalization of Eqs. (2.41) and (2.44) in parametrizing the form factors need not be the most convenient. A detailed consideration is outside the scope of the present work.

Although we have illustrated our formulas by considering the decay processes only, it is quite clear that they may be applied to scattering and to production processes as well. In particular we have indicated how to modify the basic formula Eq. (2.14) for these processes. They can also be applied to processes involving leptons or partons such as the production of W pair by polarized electron-positron annihilation.

As indicated above, the simplicity of our formulas results from the use of helicity eigenstates. This is, of course, a well-known technique[32] and earlier developments are summarized nicely in Refs. 33 and 34. In particular, meson-baryon and baryon-baryon scattering as well as two- and three-body decays of a spin J particle were discussed in Ref. 33. There the helicity amplitudes are expanded in "multipole parameters." This has the advantage that certain angular dependences of kinematical origins can be easily factored out. In contrast we expanded the helicity amplitudes in terms of the possible Lorentz structures. This has the advantage of maintaining manifest Lorentz covariance so that switching between different reference frames is easy. Moreover, if the amplitude of a process involving a pair of spin-$\frac{1}{2}$ fermions and any number of scalar and vector bosons can be calculated, say, in terms of Feynman diagrams of some model Lagrangian, then the amplitude can be put in the form we wrote down, i.e., Eq. (2.10) or Eq. (2.27). Hence, we believe our results are more convenient to use at least for processes involving only particles with spin $J \leq 1$. Processes involving particles of higher spins can be considered along the same line. We hope to return to this in later works.

The helicity-amplitude calculation of physical processes have received renewed interests recently.[35] Since measurable quantities are usually quadratic in helicity amplitudes, for complicated processes, it is easier to compute helicity amplitudes first and then to obtain the measurable quantities numerically. It is worth emphasizing that by computing all the needed traces of fermion bilinears we have effectively carried out the "squaring" of helicity amplitudes analytically. Moreover, the results can be summarized in a single equation, i.e., Eq. (A22). The specific application of this formula to decay processes in the previous section demonstrated the usefulness of this approach.

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APPENDIX A: TRACES OF FERMION BILINEARS

We want to compute

$$\mathrm{Tr}\Gamma u(k_1\sigma_1)\bar{u}(k_1\sigma_1)\Gamma' u(k_2\sigma_2')\bar{u}(k_2\sigma_2) = G^{ab}(\Gamma,\Gamma')(\sigma_b)_{\sigma_1\sigma_1'}(\bar{\sigma}_a)_{\sigma_2\sigma_2'} , \quad (A1)$$

where σ_0 is the 2×2 identity matrix, σ_i the Pauli matrices, and $\bar{\sigma}_a=\sigma_2\sigma_a\sigma_2$. Γ,Γ' are constructed from $\not{p}$, $\not{q}$, and γ_5. We shall omit the polarization indices on the right-hand side of Eq. (A1) and simply write it as $G^{ab}\,\sigma_b\otimes\bar{\sigma}_a$. It is convenient to define the 2×2 matrices

$$\begin{aligned}&(\mathcal{S}_1)_0=\sigma_0,\ \ (\mathcal{S}_1)_3=\frac{m_1}{\Delta_0}k\cdot e_1^i\sigma_i,\ \ (\mathcal{S}_1)_1=p\cdot e_1^i\sigma_i,\ \ (\mathcal{S}_1)_2=iq\cdot e_1^i\sigma_i\ ,\\ &(\bar{\mathcal{S}}_2)_0=\bar{\sigma}_0,\ \ (\bar{\mathcal{S}}_2)_3=-\frac{m_2}{\Delta_0}k\cdot e_2^i\bar{\sigma}_i,\ \ (\bar{\mathcal{S}}_2)_1=-p\cdot e_2^i\bar{\sigma}_i,\ \ (\bar{\mathcal{S}}_2)_2=iq\cdot e_2^i\bar{\sigma}_1\ ,\end{aligned} \quad (A2)$$

where

$$k=k_2-k_1,\ \ e_1^i=e^i(k_1),\ \ e_2^i=e^i(k_2)\ , \quad (A3)$$

$$\alpha_+=k_1\cdot k_2+m_1m_2,\ \ \alpha_-=k_1\cdot k_2-m_1m_2,\ \ \Delta_0=\Delta(pqk_1k_2)=\pm\sqrt{\alpha_+\alpha_-}\ . \quad (A4)$$

The four-vectors $e^i(k)$ are defined in Eqs. (2.13)–(2.15). We shall choose p,q so that the plus sign in Eq. (A4) holds in the following.

Instead of $\sigma_a\otimes\bar{\sigma}_b$, we shall use

$$\mathcal{S}_{ab}=(\mathcal{S}_1)_a\otimes(\bar{\mathcal{S}}_2)_b \quad (A5)$$

to expand the trace in Eq. (A1) so that the right-hand side be written as $g^{ab}(\Gamma,\Gamma')\mathcal{S}_{ba}$. Our results will be given in terms of the 4×4 matrices $g^{ab}(\Gamma,\Gamma')$ for various Γ,Γ'. The ordering of columns and rows of g^{ab} is

$$g^{ab}=\begin{bmatrix} g^{00} & g^{03} & g^{01} & g^{02}\\ g^{30} & g^{33} & g^{31} & g^{32}\\ g^{10} & g^{13} & g^{11} & g^{12}\\ g^{20} & g^{23} & g^{21} & g^{22}\end{bmatrix} . \quad (A6)$$

As an example we have

$$T_r u_1\bar{u}_1 u_2\bar{u}_2=\alpha^+\left[\sigma_0\otimes\bar{\sigma}_0-\frac{\Delta(e_1^i k_1 k_2;e_2^j k_1 k_2)}{\alpha_+\alpha_-}\sigma_i\otimes\bar{\sigma}_j-\frac{m_1 m_2}{\alpha_+\alpha_-}k\cdot e_1^i k\cdot e_2^j\sigma_i\otimes\bar{\sigma}_j\right]$$

which will be given by

$$g^{ab}(1,1)=\alpha_+\begin{bmatrix}1&&&\\&1&&\\&&1&\\&&&1\end{bmatrix}=\alpha_+\begin{bmatrix}\sigma_0&\\&\sigma_0\end{bmatrix}, \tag{A7}$$

where we have used Eq. (2.7).

The right-hand side of Eq. (A7) can be further shortened by introducing a tensor product so that

$$\begin{bmatrix}\sigma_0&\\&\sigma_0\end{bmatrix}=\sigma_0\otimes\sigma_0,\quad\begin{bmatrix}&\sigma_0\\\sigma_0&\end{bmatrix}=\sigma_0\otimes\sigma_1,\quad\begin{bmatrix}&-i\sigma_0\\i\sigma_0&\end{bmatrix}=\sigma_0\otimes\sigma_2,\quad\text{etc.} \tag{A8}$$

The extension of this tensor product to the general cases is self-evident. It is only a short-hand notation and should not be confused with the tensor product in Eq. (A5).

In deriving our results, we used the identity

$$e_1^i\cdot e_2^j=-\frac{\alpha_++\alpha_-}{2\alpha_+\alpha_-}k\cdot e_1^i k\cdot e_2^j-\frac{1}{\alpha_+\alpha_-}\Delta(e_1^i k_1 k_2;e_2^j k_1 k_2)\ . \tag{A9}$$

Γ,Γ' appearing in Eq. (A1) are of the following eight choices:

$$\Gamma_\alpha:\ \Gamma_0=\frac{1}{\sqrt{\alpha_+}}1,\quad\Gamma_3=\frac{1}{\sqrt{\alpha_-}}\gamma_5,\quad\Gamma_1=\frac{-i}{\sqrt{\alpha_-}}\not{p},\quad\Gamma_2=\frac{-i}{\sqrt{\alpha_+}}\not{p}\gamma_5\ , \tag{A10}$$

$$\Gamma_{\bar{\alpha}}:\ \Gamma_{\bar{0}}=\frac{-1}{\sqrt{\alpha_-}}\not{p}\not{q}\gamma_5,\quad\Gamma_{\bar{3}}=\frac{-1}{\sqrt{\alpha_+}}\not{p}\not{q},\quad\Gamma_{\bar{1}}=\frac{-i}{\sqrt{\alpha_+}}\not{q}\gamma_5,\quad\Gamma_{\bar{2}}=\frac{-i}{\sqrt{\alpha_-}}\not{q}\ . \tag{A11}$$

We define

$$g^{ab}_{\alpha\beta}=g^{ab}(\Gamma_\alpha,\Gamma_\beta),\quad g^{ab}_{\alpha\bar{\beta}}=g^{ab}(\Gamma_\alpha,\bar{\Gamma}_{\bar{\beta}}),\quad\text{etc.}, \tag{A12}$$

where, as usual, $\bar{\Gamma}=\gamma^0\Gamma^+\gamma^0$, Γ^+ being the Hermitian conjugate of Γ. Omitting the indices a,b, we give $g_{\alpha\beta}$ as

$$g_{\alpha\beta}=\begin{bmatrix}\sigma_0\otimes\sigma_0&-\sigma_1\otimes\sigma_0&-\sigma_2\otimes\sigma_1&-i\sigma_3\otimes\sigma_1\\-\sigma_1\otimes\sigma_3&\sigma_0\otimes\sigma_3&-\sigma_3\otimes\sigma_2&i\sigma_2\otimes\sigma_2\\-\sigma_1\otimes\sigma_2&\sigma_0\otimes\sigma_2&\sigma_3\otimes\sigma_3&-i\sigma_2\otimes\sigma_3\\i\sigma_0\otimes\sigma_1&-i\sigma_1\otimes\sigma_1&-i\sigma_2\otimes\sigma_0&\sigma_3\otimes\sigma_0\end{bmatrix}_{\alpha\beta}, \tag{A13}$$

where $\alpha,\beta=0,3,1,2$, i.e., the ordering of columns and rows are as in Eq. (A6). $g_{\alpha\bar{\beta}}, g_{\bar{\alpha}\beta}, g_{\bar{\alpha}\bar{\beta}}$ are given by the relations

$$g_{\alpha\bar{\beta}}=ig_{\alpha\beta},\quad g_{\bar{\alpha}\beta}=-ig_{\alpha\beta},\quad g_{\bar{\alpha}\bar{\beta}}=g_{\alpha\beta}\ . \tag{A14}$$

We regard $g_{\alpha\beta}$ as a 4×4 matrix whose elements are themselves 4×4 matrices, thus, $g_{00}=\sigma_0\otimes\sigma_0$, $g_{03}=-\sigma_1\otimes\sigma_0$, etc. With this remark we note that

$$g_{\alpha\beta}=g_{\alpha0}\cdot g_{0\beta}=g_{0\beta}\cdot g_{\alpha0}\ , \tag{A15}$$

where the dot on the right-hand side stands for multiplication of 4×4 matrices. Explicitly

$$g^{ab}_{\alpha\beta}=g^{ac}_{\alpha0}g^{cb}_{0\beta}=g^{ac}_{0\beta}g^{ac}_{\alpha0}\ , \tag{A16}$$

where summation over c is understood.

We can also show that

$$g^{ab}=g^{a0}\cdot g^{0b}=g^{0b}\cdot g^{a0}\ , \tag{A17}$$

where the asterisk stands for the complex conjugate, and

$$g^{ab}_{\alpha\beta}=(-1)^{\alpha+\beta}\epsilon_b(g^{ab}_{s(\alpha)s(\beta)})^*\ , \tag{A19}$$

where

$$\epsilon_0=\epsilon_1=1,\quad\epsilon_2=\epsilon_3=-1\ ,$$
$$s(0)=2,\quad s(2)=0\ , \tag{A20}$$
$$s(1)=3,\quad s(3)=1\ .$$

Moreover, one can easily see from Eq. (A13) that

$$g^{ab}_{\alpha\beta}=\epsilon(a,b)(g^{ab}_{\beta\alpha})^*=\epsilon(\alpha,\beta)(g^{ab}_{\alpha\beta})^*\ , \tag{A21}$$

where $\epsilon(x,y)=\pm1$ and the minus sign holds if and only if x or y but not both equals 2.

Summarizing our results we have

$$\mathrm{Tr}\Gamma_{*}u(k_1\sigma_1)\bar{u}(k_1\sigma_1')\bar{\Gamma}_{\#}u(k_2\sigma_2')\bar{u}(k_2\sigma_2) = g^{ab}_{*\#}[(\mathcal{S}_1)_b]_{\sigma_1\sigma_1'}[(\bar{\mathcal{S}}_2)_a]_{\sigma_2\sigma_2'} , \quad \text{(A22)}$$

where the asterisk and # stand for α or $\bar{\alpha}$, $\alpha=0,1,2,3$.

Finally, we comment on the changes that have to be made if we replace one or both of the baryons by antibaryons. Suppose $u(k_1\sigma_1)$ is changed to $v(k_1\sigma_1)$ in Eq. (A22). Then we have to change $\mathcal{S}_{ab}$, Γ_α, $\Gamma_{\bar{\alpha}}$, and $g_{\alpha\beta}$ in the following way.

(i) Change $\mathcal{S}_1$ to $\bar{\mathcal{S}}_1$:

$$(\bar{\mathcal{S}}_1)_0=\bar{\sigma}_0,\quad (\bar{\mathcal{S}}_1)_3=-\frac{m_1}{\Delta_0}k\cdot e_1^i\bar{\sigma}_i ,$$
$$(\bar{\mathcal{S}}_1)_1=-p\cdot e_1^i\bar{\sigma}_i,\quad (\bar{\mathcal{S}}_1)_2=iq\cdot e_1^i\bar{\sigma}_1 . \quad \text{(A23)}$$

(ii) Redefine $\Gamma_\alpha,\Gamma_{\bar{\alpha}}$ by interchanging α_+ and α_- while leaving Δ_0 unchanged in Eqs. (A10) and (A11).

(iii) Change $g_{\alpha\beta}$ to $\bar{g}_{\alpha\beta}$ which is obtained by multiplying $g_{\alpha\beta}$ for fixed α,β on the right by $\sigma_3\otimes\sigma_0$. Thus, for example, $\bar{g}_{00}=\sigma_3\otimes\sigma_0$, $\bar{g}_{03}=i\sigma_2\otimes\sigma_0$, etc. In general we have

$$\bar{g}^{\,ab}_{\alpha\beta}=\epsilon_b g^{ab}_{\alpha\beta}=(-1)^{\alpha+\beta}g^{s(\alpha)s(\beta)}_{ab} , \quad \text{(A24)}$$

where ϵ_b and $s(\alpha)$ are defined in Eq. (A20).

If $u(k_2\sigma_2')$ is changed to $v(k_2\sigma_2')$ in Eq. (A22), we apply the same procedures as above except in step (i) we shall change $\bar{\mathcal{S}}_2$ to $\mathcal{S}_2$ instead of changing $\mathcal{S}_1$ to $\bar{\mathcal{S}}_1$. $\mathcal{S}_2$ is defined, as it should be, as

$$(\mathcal{S}_2)_0=\sigma_0,\quad (\mathcal{S}_2)_3=\frac{m_2}{\Delta_0}k\cdot e_2^i\sigma_i ,$$
$$(\mathcal{S}_2)_1=p\cdot e_2^i\sigma_i,\quad (\mathcal{S}_2)_2=iq\cdot e_2^i\sigma_i . \quad \text{(A25)}$$

If both u_1,u_2 are changed to v_1,v_2, then we apply both changes as described above so that $\Gamma_\alpha,\Gamma_{\bar{\alpha}}$ and $g_{\alpha\beta}$ will remain unchanged while $\mathcal{S}_1\to\bar{\mathcal{S}}_1,\bar{\mathcal{S}}_2\to\mathcal{S}_2$.

This completes our computation for traces of fermion bilinears.

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