Constraints on semileptonic B decays from the measurement of the D^* polarization in $B \to D^* e \bar{\nu}$

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The rates for the decays $B \to De\bar{\nu}$ and $B \to D^* e\bar{\nu}$ are calculated, with emphasis on the consequences of approximate flavor independence of the meson wave functions. We find that one of the form factors is not easily constrained so that the result for the D^* final state is uncertain. However, a recent measurement of the average polarization of the D^* from $B \to D^* e\bar{\nu}$ can be used to constrain this form factor. Implications for the determination of the Kobayashi-Maskawa matrix element V_{cb} are discussed.

I. INTRODUCTION

In the standard model, the quarks couple to the W bosons through the charged weak current

$$J^\mu = \frac{g}{2\sqrt{2}} V_{ij} J^\mu_{ij} , \tag{1.1}$$

where

$$J^\mu_{ij} \equiv \bar{u}_i \gamma^\mu (1-\gamma_5) d_j \tag{1.2}$$

with the u_i (d_i) being the upper (lower) mass eigenstates of the quark doublet of the ith generation. For three generations, the V_{ij} are the elements of the unitary 3×3 Kobayashi-Maskawa matrix.[1] Since the measurement of the B lifetime, many efforts have been made to determine V_{cb} and V_{ub} from studies of B decays.[2] To minimize hadronic complications, it is best to focus on the semileptonic decays

$$B \to X e\bar{\nu} .$$

The study of the lepton spectrum indicates that $|V_{ub}| \ll |V_{cb}|$ with a recent limit[3] being $|V_{ub}/V_{cb}|^2 < 0.03$. Here we concentrate on the determination of $|V_{cb}|$, with particular emphasis on the importance of the measurement[4] of the D^* polarization in the decay $B \to D^* e\bar{\nu}$.

The B mesons are pseudoscalar mesons which consist of a b quark and a light quark ($\bar{u}$ or $\bar{d}$). We are interested in the final states X_c which result from the quark transition $b \to ce\bar{\nu}$. The differential rate can be written as

$$d\Gamma(B \to X_c e\bar{\nu}) = |V_{cb}|^2 d\hat{\Gamma}(B \to X_c e\bar{\nu}) . \tag{1.3}$$

Theory and experiment suggest that the decay is dominated by $X_c = D$ or D^*, the 0^- and 1^- ground states of the D system. Calculations of these exclusive rates have been carried out by Wirbel *et al.*[5] using an infinite-momentum-frame technique, and by Grinstein, Isgur, and Wise[6] (GIW) using the nonrelativistic quark model.

Our approach is similar to that of GIW, with the goal being to get a better understanding of the model dependence of the calculation. The basic idea is that the results of the calculation are highly constrained by the assumption of flavor independence of the quark-binding potential. As a result of flavor independence, except for a small reduced-mass difference effect, the initial B meson and the final D meson have the same spatial wave functions. This result is more important than the specific form of the wave functions used by GIW. We use this flavor independence to calculate the form factors in the limit in which $\mathbf{p}_X$, the recoil momentum of the D or D^*, vanishes. Having fixed the form factors at this one point, we then look at the effect of a reasonable recoil (q^2) dependence for the form factors. Since our calculation indicates that the shape of the electron spectrum is not affected much by changing the form-factor recoil dependence, we concentrate mainly on the total rates for the D and D^* final states and the polarization of the final state D^*.

Unfortunately, one of the form factors (a_+) in the $B \to D^* e\bar{\nu}$ decay cannot be determined in this limit in a reliable fashion. In GIW a_+ is set equal to zero, but our analysis[7,8] has shown that the rate is strongly dependent on a_+. An unreliable estimate of a_+ given by us in Ref. 8, which is in approximate agreement with that of Ref. 5, reduced the rate of $B \to D^* e\bar{\nu}$ by a factor of about 0.6 compared to the results of GIW. In this paper, we show that a_+ affects only the rate for longitudinally polarized D^*. Therefore, a measurement of the average D^* polarization can be used as a constraint on a_+. As we will discuss, the resulting limits on a_+ seem to rule out the previous theoretical estimates. With this constraint on a_+ we return to the main point of determining $|V_{cb}|$.

The general formalism is summarized in Sec. II. The derivation of the form factors in the $\mathbf{p}_X \to 0$ limit is given in Sec. III. In Sec. IV we discuss the decay rates for different form-factor recoil dependence and different values of a_+. In Sec. V we discuss the polarization of the D^* and how the experiment helps to constrain a_+. Finally, with this constraint on a_+ in mind, we discuss the determination of $|V_{cb}|$.

II. GENERAL FORMALISM

For the decay $B \to X_c e\bar{\nu}$, the T matrix is given by

$$T=\frac{G_F}{\sqrt{2}}V_{cb}\bar{u}_e\gamma_\mu(1-\gamma_5)v_\nu\langle X_c(p_X,S_X)\,|\,J_{cb}^\mu\,|\,B(p_B)\rangle \ . \tag{2.1}$$

The hadronic tensor which arises in the calculation of the decay rate is

$$h_{\mu\nu}=\sum_{S_X}\langle B(p_B)\,|\,J_\nu^\dagger\,|\,X_c(p_X,S_X)\rangle \times\langle X_c(p_X,S_X)\,|\,J_\mu\,|\,B(p_B)\rangle \ . \tag{2.2}$$

By Lorentz invariance, the most general form of this tensor is

$$\begin{aligned}h_{\mu\nu}=&-\alpha g_{\mu\nu}+\beta_{++}(p_B+p_X)_\mu(p_B+p_X)_\nu+\beta_{+-}(p_B+p_X)_\mu(p_B-p_X)_\nu\\&+\beta_{-+}(p_B-p_X)_\mu(p_B+p_X)_\nu+\beta_{--}(p_B-p_X)_\mu(p_B-p_X)_\nu\\&+i\gamma\epsilon_{\mu\nu\rho\sigma}(p_B+p_X)^\rho(p_B-p_X)^\sigma \ .\end{aligned} \tag{2.3}$$

A straightforward calculation shows that in the limit where the electron mass is neglected, the differential decay rate depends only on α, β_{++}, and γ and is given by

$$\frac{d^2\Gamma}{dx\,dy}=|\,V_{cb}\,|^2\frac{G_F^2m_B^5}{32\pi^3}\left\{\alpha\frac{y}{m_B^2}+2\beta_{++}\left[2x\left[1-\frac{m_X^2}{m_B^2}+y\right]-4x^2-y\right]-\gamma y\left[1-\frac{m_X^2}{m_B^2}-4x+y\right]\right\}, \tag{2.4}$$

where $x\equiv(p_e\cdot p_B)/m_B^2$ and $y\equiv t/m_B^2=(p_B-p_X)^2/m_B^2$ are Lorentz scalars. Since we work in the B rest frame, $x=(E_e/m_B)$ and $y=1+m_X^2/m_B^2-2E_X/m_B$.

With $J_\mu=V_\mu-A_\mu$, the matrix elements can be expressed in terms of familiar form factors with only V_μ contributing to the final D state:

$$\langle D(p_X)\,|\,V_\mu\,|\,B(p_B)\rangle=f_+(p_B+p_X)_\mu+f_-(p_B-p_X)_\mu \ , \tag{2.5}$$

$$\langle D^*(p_X,\epsilon)\,|\,V_\mu\,|\,B(p_B)\rangle=ig\epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu}(p_B+p_X)^\rho \times(p_B-p_X)^\sigma \ , \tag{2.6}$$

$$\begin{aligned}\langle D^*(p_X,\epsilon)\,|\,A_\mu\,|\,B(p_B)\rangle=&f\epsilon_\mu^*+a_+(\epsilon^*\cdot p_B)(p_B+p_X)_\mu\\&+a_-(\epsilon^*\cdot p_B)(p_B-p_X)_\mu \ ,\end{aligned} \tag{2.7}$$

where ϵ_μ is the polarization vector of D^* with $(\epsilon\cdot p_X)=0$. Since we will look at the polarization of the D^*, we must separate into longitudinal (L) states (those with $M=0$) and transverse (T) states (those with $M=\pm1$). Of course, the spin quantization axis is the boost direction $\hat{\mathbf{p}}_X$. The crucial point is that in the B rest frame, $(\epsilon^*\cdot p_B)=\epsilon_0^*m_B$. However, only the longitudinal-polarization vector $\epsilon_\mu^{(L)}$ is boosted and gets a zero component, $\epsilon_0^{(L)}=(\,|\,\mathbf{p}_X\,|\,/m_{D^*})$. Therefore, a_+ only affects the longitudinal rate. Note that a_- does not contribute for $m_e=0$. Since the matrix elements depend on the polarization, we will separate the hadronic tensor according to $h_{\mu\nu}=h_{\mu\nu}^{(T)}+h_{\mu\nu}^{(L)}$.

For the $B\to De\bar{\nu}$ decay, the quantities in Eq. (2.4) are related to the form factors by

$$\alpha=\gamma=0,\quad \beta_{++}=f_+^2 \ , \tag{2.8}$$

while for $B\to D^*e\bar{\nu}$

$$\alpha^{(L)}=\gamma^{(L)}=0 \ , \tag{2.9a}$$

$$\alpha^{(T)}=f^2+4m_B^2g^2\mathbf{p}_X^2 \ , \tag{2.9b}$$

$$\gamma^{(T)}=2gf \ , \tag{2.9c}$$

$$\begin{aligned}\beta_{++}^{(L)}=&\frac{1}{16\mathbf{p}_X^2}\frac{m_B^2}{m_{D^*}^2}\left[1-y-\frac{m_{D^*}^2}{m_B^2}\right]^2f^2\\&+\frac{1}{2}\frac{m_B^2}{m_{D^*}^2}\left[1-y-\frac{m_{D^*}^2}{m_B^2}\right]fa_++\frac{m_B^2}{m_{D^*}^2}\mathbf{p}_X^2a_+^2 \ ,\end{aligned} \tag{2.9d}$$

$$\begin{aligned}\beta_{++}^{(T)}=&\left[\frac{1}{4m_{D^*}^2}-\frac{1}{16\mathbf{p}_X^2}\frac{m_B^2}{m_{D^*}^2}\left[1-y-\frac{m_{D^*}^2}{m_B^2}\right]^2\right]f^2\\&-m_B^2g^2y \ .\end{aligned} \tag{2.9e}$$

III. FORM FACTORS IN THE $\mathbf{p}_X\to0$ LIMIT

We now proceed to the calculation of the matrix elements of Eqs. (2.5), (2.6), and (2.7). The normalized meson state vectors are given by

$$|X(p_X,S_X)\rangle=\sqrt{2m_X}\int d^3\mathbf{p}\,\phi_X(\mathbf{p})\left[\sum\chi_{s\bar{s}}^{S_X}\right]\left|q\left[\frac{m_q}{m_X}\mathbf{p}_X+\mathbf{p},s\right]\bar{q}\left[\frac{m_{\bar{q}}}{m_X}\mathbf{p}_X-\mathbf{p},\bar{s}\right]\right\rangle , \tag{3.1}$$

where $\sum\chi_{s\bar{s}}^{S_X}$ is the factor which couples the spins to $S_X=0$ or 1. The same formula holds for the initial spin-zero B meson with the same wave function $\phi_B(\mathbf{p})=\phi_X(\mathbf{p})\equiv\phi(\mathbf{p})$ because of flavor independence. Because we are essentially using a "weak-binding" approximation, some question exists about whether to use the meson mass m_X or a "weak-binding mass" $\tilde{m}_X=m_q+m_{\bar{q}}$ in the normalization factor $\sqrt{2m_X}$. However, using the standard normalization

$$\langle X(p'_X,S'_X)\,|\,X(p_X,S_X)\rangle=2E_X\delta^3(\mathbf{p}'_X-\mathbf{p}_X)\delta_{S'_XS_X}$$

requires the normalization factor $\sqrt{2E_X}$. Therefore, the $\sqrt{2m_X}$ factor, with the real meson mass m_X, arises naturally in the nonrelativistic limit. In any case, it is the only reliably known mass.

The matrix elements may now be written in the B rest frame ($\mathbf{p}_B=0$) as

$$\langle X(p_X,S_X)\,|\,J_{cb}^{\mu}\,|\,B\rangle=\sqrt{4m_Xm_B}\times\int d^3\mathbf{p}\,\phi^*\left[\mathbf{p}+\frac{m_d}{m_X}\mathbf{p}_X\right]\times\phi(\mathbf{p})\langle S_X\,|\,J_{cb}^{\mu}\,|\,S_B\rangle . \tag{3.2}$$

The last factor in Eq. (3.2) is the spinor matrix element. In the spectator model,

$$J_{cb}^{\mu}=\bar{c}(\mathbf{p}+\mathbf{p}_X,\bar{s})\gamma^{\mu}(1-\gamma_5)b(\mathbf{p},s) . \tag{3.3}$$

Using a nonrelativistic approximation, we now calculate the matrix element in the small-$\mathbf{p}_X$ limit. While it is clearly wrong to treat the light quark nonrelativistically, we feel that this is not very important because the light quark is a spectator. We insert the nonrelativistic limit of the spinors $\bar{c}(\mathbf{p}+\mathbf{p}_X,\bar{s})$ and $b(\mathbf{p},s)$ into Eq. (3.3), keeping terms of order (p/m_c), (p/m_b), and (p_X/m_c). In order to do the spin projections, we also treat the light-quark spin nonrelativistically since it is coupled to the $\bar{c}$ in $|S_X\rangle$. For the transition $B\to De\bar{\nu}$, only the vector current V_μ contributes yielding the results

$$\langle S_D\,|\,V_0\,|\,S_B\rangle=1 , \tag{3.4a}$$

$$\langle S_D\,|\,\mathbf{V}\,|\,S_B\rangle=\left[\frac{\mathbf{p}+\mathbf{p}_X}{2m_c}+\frac{\mathbf{p}}{2m_b}\right] . \tag{3.4b}$$

For $B\to D^*e\bar{\nu}$ we get

$$\langle S_{D^*}\,|\,\mathbf{A}\,|\,S_B\rangle=\boldsymbol{\epsilon}^* , \tag{3.5a}$$

$$\langle S_{D^*}\,|\,A_0\,|\,S_B\rangle=\boldsymbol{\epsilon}^*\cdot\left[\frac{\mathbf{p}+\mathbf{p}_X}{2m_c}+\frac{\mathbf{p}}{2m_b}\right] , \tag{3.5b}$$

$$\langle S_{D^*}\,|\,V_0\,|\,S_B\rangle=0 , \tag{3.5c}$$

$$\langle S_{D^*}\,|\,\mathbf{V}\,|\,S_B\rangle=i\boldsymbol{\epsilon}^*\times\left[\frac{\mathbf{p}+\mathbf{p}_X}{2m_c}-\frac{\mathbf{p}}{2m_b}\right] . \tag{3.5d}$$

Substituting the approximate forms of Eq. (3.4) into Eq. (3.2), one finds the following two equations for the $B\to De\bar{\nu}$ form factors defined in Eq. (2.5):

$$(m_B+E_X)f_++(m_B-E_X)f_-=\sqrt{4m_Dm_B}\int d^3\mathbf{p}\,\phi^*\left[\mathbf{p}+\frac{m_d}{m_D}\mathbf{p}_X\right]\phi(\mathbf{p}) , \tag{3.6a}$$

$$(f_+-f_-)\mathbf{p}_X=\sqrt{4m_Dm_B}\int d^3\mathbf{p}\,\phi^*\left[\mathbf{p}+\frac{m_d}{m_D}\mathbf{p}_X\right]\phi(\mathbf{p})\left[\frac{\mathbf{p}}{2m_b}+\frac{\mathbf{p}}{2m_c}+\frac{\mathbf{p}_X}{2m_c}\right] . \tag{3.6b}$$

The $\mathbf{p}_X\to0$ limit of Eq. (3.6a) yields

$$(m_B+m_D)f_++(m_B-m_D)f_-=\sqrt{4m_Dm_B} . \tag{3.7a}$$

In Eq. (3.6b) we take the limit of small $\mathbf{p}_X$ and neglect the term proportional to (m_d/m_D) in the argument of ϕ^*. Note that the linear $\mathbf{p}$ terms integrate to zero by parity. The result is

$$(f_+-f_-)=\frac{\sqrt{m_Dm_B}}{m_c} . \tag{3.7b}$$

Thus in the $\mathbf{p}_X=0$ limit we find

$$f_+=\left[\frac{m_D}{m_B}\right]^{1/2}\left[1+\frac{m_B-m_D}{2m_c}\right] , \tag{3.8a}$$

$$f_-=\left[\frac{m_D}{m_B}\right]^{1/2}\left[1-\frac{m_B+m_D}{2m_c}\right] . \tag{3.8b}$$

Similarly, from Eq. (3.5) we find the following results for the $B\to D^*e\bar{\nu}$ form factors defined in Eqs. (2.6) and (2.7) in the limit $\mathbf{p}_X\to0$:

$$f=\sqrt{4m_B m_{D^*}}\ , \tag{3.9a}$$

$$g=\frac{1}{2m_c}\left[\frac{m_{D^*}}{m_B}\right]^{1/2}, \tag{3.9b}$$

$$a_+(m_B+m_{D^*})+a_-(m_B-m_{D^*})=\left[\frac{m_{D^*}}{m_B}\right]^{1/2}\left[\frac{m_{D^*}}{m_c}-2\right]. \tag{3.9c}$$

The results in Eqs. (3.8) and (3.9) follow essentially from flavor independence. The main corrections are uncertain terms of order (m_d/m_c). In the limit of setting $m_d=0$, they essentially agree with the results of GIW. Relativistic corrections are probably small and QCD corrections seem inappropriate since the results follow from flavor independence in the limit $(m_c/m_b)\rightarrow 1$. To determining a_+ would require a second equation for a_+ and a_- in the small-$\mathbf{p}_X$ limit. This equation would come from the three-vector part of the matrix element of the axial-vector current [see Eq. (2.7)]. However, in the B rest frame,

$$(\epsilon^*\cdot p_B)=\epsilon_0^* m_B=\begin{cases}\dfrac{m_B}{m_{D^*}}\,|\,\mathbf{p}_X\,| & \text{for } \epsilon_\mu^{(L)}\ ,\\ 0 \ \text{ for } \epsilon_\mu^{(T)}\ . \end{cases} \tag{3.10}$$

Therefore, from Eq. (2.7) we see that the coefficients of a_+ and a_- in $\langle D^*\,|\,\mathbf{A}\,|\,B\,\rangle$ are of order $|\,\mathbf{p}_X\,|^2$. When we extended the calculation of $\langle D^*\,|\,\mathbf{A}\,|\,B\,\rangle$ to order $|\,\mathbf{p}_X\,|^2$ in Ref. 8, we found

$$a_+-a_-=\frac{-1}{\sqrt{m_{D^*}m_B}}\ . \tag{3.11}$$

Combining this with Eq. (3.9c) for $\mathbf{p}_X=0$ we find

$$a_+=\frac{-1}{\sqrt{4m_{D^*}m_B}}\left[1+\frac{m_{D^*}}{m_B}\left[1-\frac{m_{D^*}}{m_c}\right]\right], \tag{3.12}$$

$$fa_+=-0.96\ . \tag{3.13}$$

While this is our best guess for a_+, the calculation is not trustworthy because at order (p^2/m^2) there are significant relativistic effects which are not included in our derivation. In Ref. 5, the infinite-momentum-frame analysis gives the value $fa_+\simeq -0.77$ at $\mathbf{p}_X=0$. However, as we will show in Sec. V, these values are probably ruled out by the measurement of the average D^* polarization.

IV. RATES FOR $B\rightarrow De\bar{\nu}$ AND $B\rightarrow D^*e\bar{\nu}$

To calculate the decay rate, we must now make an assumption about the behavior of the form factors away from $\mathbf{p}_X=0$. We assume that there exists a common $\mathbf{p}_X$ dependence (or y dependence) for all the form factors given by $F(y)$ which satisfies the boundary condition

$$F(y_m)=1\ .$$

The value $y_m\equiv(1-m_X/m_B)^2$, which is the maximum value of y, corresponds to $\mathbf{p}_X=0$ in the B rest frame. The decay rates are then given by

$$\hat{\Gamma}(D_i)\equiv\frac{\Gamma(B\rightarrow D_i e\bar{\nu})}{|\,V_{cb}\,|^2}=\int dy\,|\,F(y)\,|^2\int dx\left[\frac{d\hat{\Gamma}_i^0}{dx\,dy}\right], \tag{4.1}$$

where the $(d\hat{\Gamma}_i^0)$ are obtained by substituting the $\mathbf{p}_X=0$ results from Sec. III into Eqs. (2.8) or (2.9) and then plugging these terms into Eq. (2.4). Here D_i is either D or D^*.

In GIW, a particular form of harmonic-oscillator wave functions is used. This leads to an exponential form-factor recoil dependence

$$F_e(y)=\exp\left[\frac{s}{2}(y-y_m)\right], \tag{4.2}$$

where $s\simeq 1.61$ (Ref. 9). As an alternative, we consider the simplest form suggested by dispersion relations, a pole at the mass m^* of the B_c^* vector meson. This gives

$$F_p(y)=\text{const}\times\frac{1}{m^{*2}-t}=\frac{y_p-y_m}{y_p-y}\ , \tag{4.3}$$

where $y_p\equiv(m^*/m_B)^2$. By making an analogy with other mesons, we assume $[m(B_c^*)-m(B_d^*)]\simeq[m(D_s^+)-m(K)]$ which gives $m(B_c^*)\equiv m^*\simeq 6.8$ GeV. For all calculations, the masses $m_D=1.87$ GeV, $m_{D^*}=2.01$ GeV, $m_B=5.27$ GeV, and $m_c=1.8$ GeV are used.

We first explore the effect of the choice of different form factors. The results for $\hat{\Gamma}(D)$ and $\hat{\Gamma}(D^*)$, the total rates divided by $|\,V_{cb}\,|^2$ as defined in Eq. (4.1), are given in Table I for $a_+=0$. In addition to $F_e(y)$ and $F_p(y)$, we show the results for $F(y)=1$ as well as the results given by GIW. We note that the two different form factors $F_e(y)$ and $F_p(y)$ give very similar results. This is because the first term in the small recoil expansion, given by $F(y)=1+c(y-y_m)+\cdots$, has approximately the same value ($c\simeq 0.8$) for the two forms factors. While the next terms in the expansions are quite different, the similarity in the results shows that the rate is mainly sensitive to the term c which classically corresponds to the charge radius. Our results using $F_e(y)$ are seen to differ from

TABLE I. Results for total rates with $a_+=0$. In this table, we define $\Gamma(B\rightarrow D_i e\bar{\nu})\equiv|\,V_{cb}\,|^2\hat{\Gamma}(D_i)$. All rates are given in units of 10^{13} sec^{-1}.

	$\hat{\Gamma}(D)$	$\hat{\Gamma}(D^*)$	$\hat{\Gamma}(D)+\hat{\Gamma}(D^*)$	$\hat{\Gamma}(D^*)/\hat{\Gamma}(D)$
$F(y)=1$	1.89	5.41	7.30	2.86
$F_p(y)$	1.23	3.92	5.15	3.19
$F_e(y)$	1.17	3.76	4.93	3.21
GIW	1.10	4.12	5.22	3.75

those of GIW by about 10%. The main reason for this difference is that we normalize our meson states with the physical masses m_B, m_D, and m_{D^*}, while GIW use $\tilde{m}_X = m_d + m_q$. While we feel that our normalization is more reasonable, this difference may represent some measure of uncertainty in the calculation.

The most important uncertainty in the $B \to D^* e \bar{\nu}$ rate is the effect of a_+. Using our pole form factors $F_p(y)$, in Fig. 1 we show the value of $\Gamma(D^*)/\Gamma(D)$ as a function of fa_+ for $-3 < fa_+ < 1$. Over this range of fa_+, the ratio $\Gamma(D^*)/\Gamma(D)$ varies from 1.2 to 5.3 and since $\hat{\Gamma}(D)$ is fixed, $\Gamma(D) + \Gamma(D^*)$ varies by a factor of almost 3. This clearly demonstrates the importance of the a_+ term. Furthermore, as we have discussed, the a_+ term affects the rate for longitudinal $D^*, \Gamma^{(L)}$, but it has no effect on the rate for transverse $D^*, \Gamma^{(T)}$. The effect of this is seen in Fig. 2 where $\Gamma^{(L)}/\Gamma^{(T)}$ is plotted as a function of fa_+. In Sec. V, we discuss how a measurement of the D^* polarization can be used to constrain $\Gamma^{(L)}/\Gamma^{(T)}$.

V. THE POLARIZATION OF THE D^*

The D^* which emerges from the decay $B \to D^* e \bar{\nu}$ is in general polarized. The polarization is described by a vector (the spin direction) and a tensor[10]

$$T_2^0 \equiv \langle 3S_z^2 - S(S+1) \rangle = \frac{\Gamma^{(T)} - 2\Gamma^{(L)}}{\Gamma^{(T)} + \Gamma^{(L)}} \ . \qquad (5.1)$$

In the D^* rest frame, the angular distribution of the decay $D^* \to D\pi$ is given by

$$F(\theta^*) = 1 + A \cos^2\theta^* \ , \qquad A = \frac{2\Gamma^{(L)} - \Gamma^{(T)}}{\Gamma^{(T)}} \ , \qquad (5.2)$$

where θ^* is the angle between the π direction and the boost direction $\hat{\mathbf{p}}_X$. The result does not depend on the vector polarization because parity is conserved in the $D^* \to D\pi$ decay, but does determine T_2^0 by

$$T_2^0 = \frac{-2A}{3+A} \ . \qquad (5.3)$$

A recent CLEO experiment (Ref. 4) measures the D^* decay angular distribution for a decay sample subject to two cuts. These cuts are electron energy $E_e > 1.2$ GeV and $\cos\theta_{\pi e} < -0.7$, where $\theta_{\pi e}$ is the angle between the decay pion and the electron in the B rest frame. Since the pion is moving very slowly in the D^* rest frame, we assume that $\hat{\mathbf{p}}_\pi$ (the π direction in the B rest frame) is a good approximation to the direction $\hat{\mathbf{p}}_X$. With this approximation we can translate these cuts into cuts in the phase-space region (x,y). We then integrate the differential decay rate over the cut phase-space region

$$\hat{\Gamma}_{\text{cut}}^{(I)} = \int_{\text{cut}} dx\, dy \left[\frac{d\hat{\Gamma}^{(I)}}{dx\, dy} \right] , \qquad (5.4)$$

where $I = L$ or T. These rates are then put into Eq. (5.1) to determine T_2^0. The resulting values of T_2^0, which are to be compared with the experiment with the cuts, are plotted in Fig. 3 as a function of fa_+. We have used pole form factors here, but the results are very similar if we use the exponential form factors.

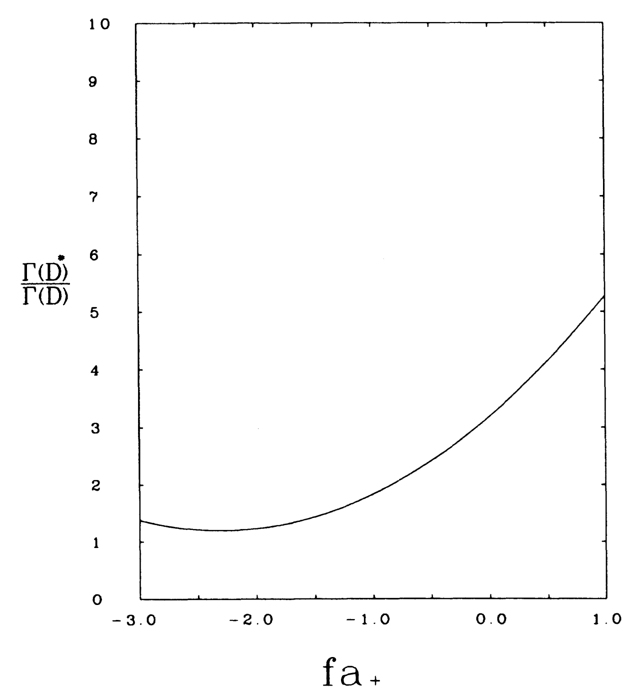

FIG. 1. $\Gamma(D^*)/\Gamma(D)$ as a function of fa_+ with pole form factors $F_p(y)$.

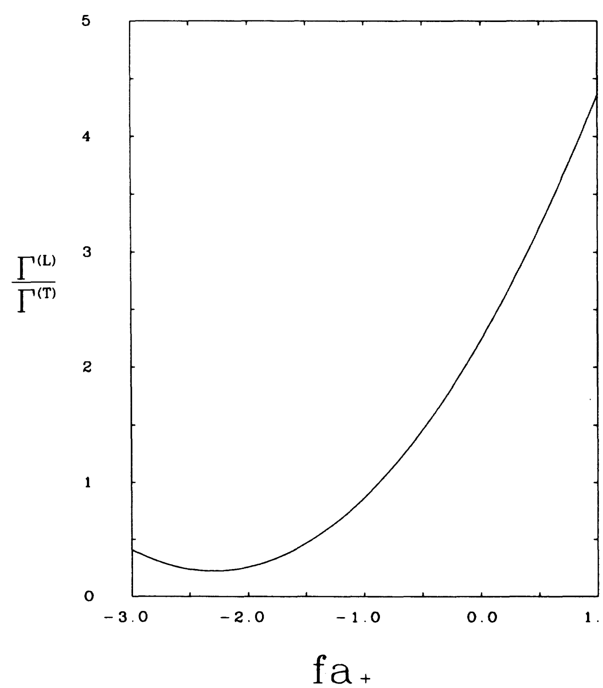

FIG. 2. $\Gamma^{(L)}/\Gamma^{(T)}$ as a function of fa_+ with pole form factors $F_p(y)$.

A result of the CLEO experiment[4] is $A > 3.0$ (99% confidence limit) or, from Eq. (5.3),

$$T_2^0 < -1.0 \ . \tag{5.5}$$

From Fig. 3, we find that this gives the limit

$$fa_+ > 0.35 \ . \tag{5.6}$$

An additional solution of $fa_+ < -4.8$ seems unreasonably large in magnitude. This result seems to rule out both previous theoretical estimates of fa_+; $fa_+ \simeq -0.96$ in Ref. 8 and $fa_+ \simeq -0.77$ in Ref. 5. However, care must be taken in interpreting this result strictly as a limit on a_+. This is illustrated by examining an alternative approach to the calculation of $\hat{\Gamma}(B \to De\bar{\nu})$ and $\hat{\Gamma}(B \to D^*e\bar{\nu})$ which has been given by Suzuki.[11] He uses the free-quark model and projects the final quarks into states of total spin 0 or 1. At $\mathbf{p}_X = 0$, our results, including our theoretical estimate of $fa_+ \simeq -1$, essentially agree with his. Since he is not treating the binding of the quarks into mesons, he does not introduce a recoil dependence based upon meson wave functions or dispersion relations. However, he does find a q^2 (or y) dependence of the form factors from his relativistic analysis of the free-quark transition amplitude. He finds that all the form factors do not behave the same away from $\mathbf{p}_X = 0$. In particular, f increases with respect to a_+ as y decreases from y_m. This partially counteracts the effect that a_+ has on the polarization. Suzuki finds a value of $\Gamma^{(L)}/\Gamma^{(T)} \simeq 1.4$ while from Fig. 2 we find $\Gamma^{(L)}/\Gamma^{(T)} \simeq 0.91$ for $fa_+ \simeq -1$. While both binding effects and relativistic boosting effects probably contribute to the recoil dependence of the form factors, it is not possible at this time to include them both in a completely consistent manner. Therefore, we have only treated the binding form-factor recoil dependence here.

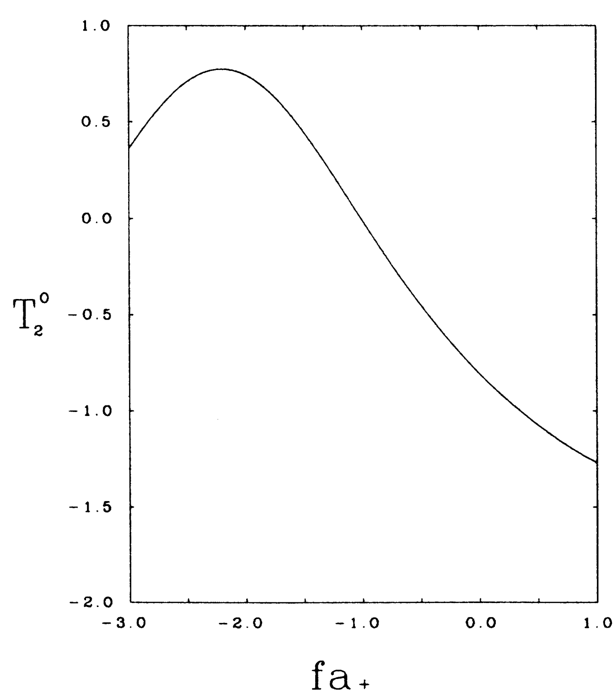

FIG. 3. T_2^0 as a function of fa_+ with pole form factors $F_p(y)$. T_2^0 is determined here by putting the $\hat{\Gamma}_{\text{cut}}^{(I)}$, the rates for the cut phase-space region as defined in Eq. (5.4), into Eq. (5.1).

We now return to the determination of $|V_{cb}|$. The decay rate for $B \to Xe\bar{\nu}$, summed over all final states X, can be written

$$\Gamma(B \to Xe\bar{\nu}) = |V_{cb}|^2 \left[\hat{\Gamma}(D) + \hat{\Gamma}^{(T)} \left[1 + \frac{\Gamma^{(L)}}{\Gamma^{(T)}} \right] \right] \times (1+\chi)(1+U) \ . \tag{5.7}$$

Here $\hat{\Gamma}^{(T)}$ is the transverse part of $\hat{\Gamma}(D^*)$ defined in Eq. (4.1). Both $\hat{\Gamma}(D)$ and $\hat{\Gamma}^{(T)}$ are independent of a_+, and using pole form factors we find $\hat{\Gamma}(D) = 1.23 \times 10^{13}$ sec^{-1} and $\hat{\Gamma}^{(T)} = 1.21 \times 10^{13}$ sec^{-1}. χ is the ratio of the number of $b \to ce\bar{\nu}$ decays which go to excited states X_c compared to the number which give D or D^*. We accept the GIW estimate of $\chi \simeq 0.11$, but we recognize this as an important uncertainty. U is the ratio of $b \to ue\bar{\nu}$ to $b \to ce\bar{\nu}$ transitions. From the report by Schmidt-Parzefall (Ref. 2), this is clearly between 1% and 10%, so we set $U = 0.05$. Because of the cuts involved, the experimental results for the D^* polarization do not directly determine the ratio of total rates, $\Gamma^{(L)}/\Gamma^{(T)}$. However, by putting the lower limit $fa_+ > 0.35$ into our calculation for the total rates, we can get a lower limit on $\Gamma^{(L)}/\Gamma^{(T)}$. From Fig. 2, we see that this gives

$$\frac{\Gamma^{(L)}}{\Gamma^{(T)}} > 2.9 \ . \tag{5.8}$$

However, this limit is slightly model dependent.

To illustrate the determination of $|V_{cb}|$, we set $\Gamma^{(L)}/\Gamma^{(T)} = 2.9$. Additional data on the D^* polarization would certainly be helpful. We also use the value $\tau_B = 1.17 \times 10^{-12}$ sec and a semileptonic branching ratio of 11.8% so that $\Gamma(B \to Xe\bar{\nu}) = 1.01 \times 10^{11}$ sec^{-1} (see Ref. 2). Using pole form factors, we then find

$$|V_{cb}| = 0.038 \ . \tag{5.9}$$

A larger value of $\Gamma^{(L)}/\Gamma^{(T)}$ would further reduce the value of $|V_{cb}|$. Given $\Gamma^{(L)}/\Gamma^{(T)}$, the major theoretical uncertainties are the form-factor recoil dependence and the excited-state ratio χ. To some extent, these two compensate for each other because an increase in the form-factor suppression away from zero recoil should increase the probability of going to excited states. An extreme case, with $F(y) = 1$ and $\chi = 0$, leads to the lower limit of 0.033 for $|V_{cb}|$.

An alternative approach is to use the measured branching ratio (Ref. 2) of $(7 \pm 1.2 \pm 1.9)\%$ for the decay $B \to D^*e\bar{\nu}$. This gives the exclusive rate $\Gamma(B \to D^*e\bar{\nu}) = (6 \pm 2) \times 10^{10}$ sec^{-1}. We can then write

$$\Gamma(B \to D^*e\bar{\nu}) = |V_{cb}|^2 \left[\hat{\Gamma}^{(T)} \left[1 + \frac{\Gamma^{(L)}}{\Gamma^{(T)}} \right] \right] \ .$$

For $\Gamma^{(L)}/\Gamma^{(T)} = 2.9$, this gives $|V_{cb}| = 0.036 \pm 0.006$ where only the experimental error is included.

VI. CONCLUSION

In this paper we have studied the rates of the most important semileptonic B decays: $B \to De\bar{\nu}$ and $B \to D^* e\bar{\nu}$. For the D^* final state, we have looked at the rates $\Gamma^{(L)}$ (longitudinal) and $\Gamma^{(T)}$ (transverse) which determine the tensor polarization T_2^0. For the rates $\Gamma(B \to De\bar{\nu})$ and $\Gamma^{(T)}$, the assumption of flavor independence determines the necessary form factors (f_+ and f) in the zero-recoil limit. We believe that a good estimate of these rates can be made using reasonable form factors. However, the rate $\Gamma^{(L)}$ depends critically on the form factor a_+ which cannot be reliably determined in the same way.

An experimental measurement of T_2^0 provides a measurement of $\Gamma^{(L)}/\Gamma^{(T)}$ which can be used in place of a theoretical estimate of a_+. Given $\Gamma^{(L)}/\Gamma^{(T)}$, one can attempt to determine $|V_{cb}|$ from the measured semileptonic decay rate. The major uncertainties are the assumed form factors and the correction for decays other than $B \to De\bar{\nu}$ and $B \to D^* e\bar{\nu}$. Using $\Gamma^{(L)}/\Gamma^{(T)}=2.9$ (which at present is only a lower limit), we find a value of $|V_{cb}|=0.038$. Further data on the D^* polarization and on transitions to excited states are needed to obtain a better value and to estimate the uncertainty.

While preparing this paper, we received a similar paper by Grinstein and Wise.[12]

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