Exclusive decays $B \to K^{(i)}\gamma$ resulting from $b \to s\gamma$

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Recently it has been suggested that the quark transition $b \to s\gamma$ might have a large enhancement. We use the quark model to estimate the rates for exclusive decays $B \to K^{(i)}\gamma$ that result from $b \to s\gamma$. The $K^{(i)}$ are various K-meson excited states. While the ground state $K^*(0.892)$ accounts for only a small fraction of the inclusive $b \to s\gamma$ rate, we estimate that a substantial fraction ($\sim 37\%$) of $b \to s\gamma$ decays will result in $K^{(i)}$ with $m_{K^{(i)}} < m_{D^*}$.

The decay $b \to s\gamma$, which in the standard model is assumed to go through the so-called electromagnetic penguin diagram, is an important place to test loop effects in various models. In particular, a number of authors have discussed the predictions of the standard model[1,2] and extensions such as supersymmetry[3] or a proposed fourth generation.[4] Notably, they have found that the rate for $b \to s\gamma$ may have a large enhancement. If these decays are truly enhanced, one would like to know how they would show up experimentally. Probably the best place to look is in the decays of the pseudoscalar B meson, $B \to K^{(i)}\gamma$, where $K^{(i)}$ is one of the excited states of the K system. Note that the decay to the 0^- ground state, $B \to K\gamma$, is forbidden by angular momentum conservation.

The lowest-mass allowed state is the ${}^3S_1$ radial ground state $K^*(0.892)$. For this decay, an experimental upper bound of $B(B \to K^*\gamma) < 1.8 \times 10^{-3}$ exists.[5] Consequently, several authors[6,7] have attempted to use this bound to get a limit on $b \to s\gamma$. They claim that in the quark model a substantial portion (97% in Ref. 6 and 50–63 % in Ref. 7) of the total $b \to s\gamma$ decays go to the $K^*(0.892)$. However, because of the large recoil of the K^*, this is very unlikely. In particular, we use the quark model and find a large suppression of this mode. However, even if the K^* mode is suppressed, a substantial fraction of the $b \to s\gamma$ decays can show up in excited states $K^{(i)}$.

Since the photon is monochromatic, the total rate for $B \to K^{(i)}\gamma$ for several of the lowest-mass $K^{(i)}$ could be measured by looking for photons with energy above some $E_\gamma^{\min}$. We will look at only those states with $m_K < m_{D^*} = 2.01$ GeV to avoid background from $B \to D^*\gamma$. These states, along with their quark-model quantum-number assignments, are listed in Table I (Ref. 8). Note that since the mixing of the 1^+ states $K_1(1.28)$ and $K_1(1.40)$ is not known, we take the quark-model states K_{1A} and K_{1B} to have an average mass of 1.34 GeV. Also, since the $K^*(1.79)$ can be a mixture of $3\,{}^3S_1$ and $1\,{}^3D_1$, we treat it both ways and assume that the other mixture is near in mass. We will work in the B rest frame, where the photon energy is fixed to be $E_\gamma = (m_B/2)(1 - m_K^2/m_B^2)$ for each $K^{(i)}$. Therefore, to distinguish between a photon from $B \to K^*(1.79) + \gamma$ ($E_\gamma = 2.33$ GeV) and one from $B \to D^* + \gamma$ ($E_\gamma = 2.25$ GeV) would require photon energy resolution of $\sim 3.5\%$.

It is important to point out that we are only estimating the contribution to $B \to K^{(i)}\gamma$ from spectator decays

TABLE I. Results for the $K^{(i)}$ with $m_K < 2$ GeV. The quark-model quantum-number assignments are straightforward, except where possible mixing is involved. The 1^+ states $K_1(1.28)$ and $K_1(1.40)$ are mixtures of the quark-model states K_{1A} and K_{1B}. We assume K_{1A} and K_{1B} have the average mass of 1.34 GeV. Also, $K^*(1.79)$ is included as both $1\,{}^3D_1$ and $3\,{}^3S_1$ states since it is probably a mixture and the other combination should have a similar mass.

$n\,{}^{2s+1}L_J$	J^P	Meson	$I(n,L)$	$\Gamma(B \to K^{(i)}\gamma)/\Gamma(b \to s\gamma)$
$1\,{}^1S_0$	0^-	$K(0.498)$		Forbidden
$1\,{}^3S_1$	1^-	$K^*(0.892)$	0.193	4.5%
$1\,{}^1P_1$	1^+	$K_{1B}(\sim 1.34)$		Forbidden
$1\,{}^3P_1$	1^+	$K_{1A}(\sim 1.34)$	0.339	6.0%
$1\,{}^3P_0$	0^+	$K_0^*(1.35)$		Forbidden
$2\,{}^3S_1$	1^-	$K^*(1.41)$	0.268	7.3%
$1\,{}^3P_2$	2^+	$K_2^*(1.43)$	0.346	6.0%
$2\,{}^1S_0$	0^-	$K(1.46)$		Forbidden
$1\,{}^3D_2$	2^-	$K_2(1.77)$	0.322	4.4%
$1\,{}^3D_3$	3^-	$K_3^*(1.78)$	0.322	3.5%
$1\,{}^3D_1$	1^-	$K^*(1.79)$	0.322	0.88%
$3\,{}^3S_1$	1^-	$K^*(1.79)$	0.216	3.9%
$3\,{}^1S_0$	0^-	$K(1.83)$		Forbidden

$b \to s\gamma$. There are other possible contributions including diagrams in which $b \to s\gamma$ is accompanied by the exchange of a gluon with the spectator. Such contributions, which are not considered in the QCD-corrected quark-level calculations (Refs. 1 and 7), are very hard to calculate in the quark model. Since we cannot rule out their contribution to $B \to K^{(i)}\gamma$, observation of $B \to K^{(i)}\gamma$ does not necessarily give information about the rate for $b \to s\gamma$. However, our purpose here is not to use the measurement of $B \to K^{(i)}\gamma$ to set limits on $b \to s\gamma$. Rather, we merely give an indication of how an enhanced $b \to s\gamma$ rate would show up in low-lying $K^{(i)}$ resonances. Therefore, we will not consider nonspectator decays.

The meson decay $B(p_B) \to K^{(i)}(k) + \gamma(q)$ has a T matrix given by

$$T = \epsilon^{\mu}(q) \langle K^{(i)}(k) \,|\, J_{\mu}^{\text{eff}} \,|\, B(p_B) \rangle \; , \tag{1}$$

where $\epsilon^{\mu}(q)$ is the photon polarization. The Lorentz condition requires $\epsilon \cdot q = 0$.

The spectator model is used here, so that the quark level decay is $b \to s\gamma$ with the spectator ($\bar{u}$ or $\bar{d}$) having no effect on the decay except in the binding into mesons. By Lorentz invariance and gauge invariance, J_{μ}^{eff} must have the form

$$J_{\mu}^{\text{eff}} = a V_{\mu} + b A_{\mu} \; , \tag{2a}$$

where

$$V_{\mu} = i q^{\nu} \bar{s}(\mathbf{k}_s) \sigma_{\mu\nu} b(\mathbf{k}_b) \; , \tag{2b}$$

$$A_{\mu} = i q^{\nu} \bar{s}(\mathbf{k}_s) \gamma_5 \sigma_{\mu\nu} b(\mathbf{k}_b) \; . \tag{2c}$$

The coefficients a and b are calculated in the model of interest (Refs. 1–4 and 7). In the impulse approximation, J_{μ}^{eff} is treated as an effective short-distance operator. Therefore, a and b should include all short-distance (perturbative) QCD corrections (see Refs. 1 and 7). However, the long-distance QCD effects are too slow to affect the quark decay, and they are treated by taking the matrix elements of J_{μ}^{eff} between meson states. If the quarks were free, the resulting decay rate would be

$$\Gamma^{\text{free}}(b \to s\gamma) = \frac{E_{\gamma}^3}{\pi} (\,|a|^2 + |b|^2) \; , \tag{3}$$

where $E_{\gamma} = (m_b/2)(1 - m_s^2/m_b^2)$ is fixed on the quark level. Since the s quark must combine with the spectator to form a meson with unit probability, we will take the free quark rate to be an estimate of the inclusive rate for $b \to s\gamma$. Of course, this rate is uncertain because the quark masses are uncertain.

We must now calculate the matrix elements of J_{μ}^{eff} between initial and final meson states. Unfortunately, there is at present no reliable way to do this type of calculation. However, in order to get some estimate of the matrix elements, we will use the quark model. We first need normalized meson states. In the quark model, these states take the form

$$|X(p_X, J_X)\rangle = \sqrt{2E_X} \int d^3\mathbf{p}\, \phi_X(\mathbf{p}) \left[\sum \langle J_X M_J \,|\, L M_L S M_S \rangle \chi_{s\bar{s}}^{S M_S} \right] \left| q \left[\frac{m_q}{m_q + m_{\bar{q}}} \mathbf{p}_X + \mathbf{p}, s \right] \bar{q} \left[\frac{m_{\bar{q}}}{m_q + m_{\bar{q}}} \mathbf{p}_X - \mathbf{p}, \bar{s} \right] \right\rangle \; , \tag{4}$$

where $\phi_X(\mathbf{p})$ is the relative momentum wave function and we assume that the angular momenta are coupled to J_X by the Clebsch-Gordan coefficients. The problem of relativistic effects on spin will be addressed later. These states are normalized according to

$$\langle X(\mathbf{p}'_X, J'_X) \,|\, X(\mathbf{p}_X, J_X) \rangle = 2E_X \delta^3(\mathbf{p}'_X - \mathbf{p}_X) \delta_{J'_X J_X} \; .$$

This normalization requires that the spinors in the effective current J_{μ}^{eff} must be normalized to

$$\bar{u}(\mathbf{p}, s') u(\mathbf{p}, s) = \frac{m}{E} \delta_{s's} \; .$$

In the quark model, the B meson is the ${}^1S_0$ ground state and the $K^{(i)}$ states have L and S coupled to total J and radial excitation number $n = 1, 2, 3, \ldots$. The possible $K^{(i)}$ are listed in Table I. Photon transversality requires $M_J = \pm 1$, thus eliminating the $J = 0$ states. In the B rest frame, the matrix elements are

$$\langle K^{(i)}\{n,(L,S)J;\mathbf{k}\} \,|\, J_{\mu}^{\text{eff}} \,|\, B \rangle = \sqrt{4 m_B E_K} \sum_{M_L M_S} \int d^3\mathbf{k}_b \phi_K^*(\mathbf{k}_b + \mathbf{k}') \phi_B(\mathbf{k}_b) \langle J \pm 1 \,|\, L M_L S M_S \rangle \langle S \,|\, J_{\mu}^{\text{eff}} \,|\, S_B \rangle \; , \tag{5}$$

where $\mathbf{k}' \equiv [m_d/(m_d + m_s)]\mathbf{k}$ is the "momentum-mismatch" term which arises because the B rest frame is not the center-of-mass frame of the K. Because $m_B \gg m_K$, this term is large and consequently very important. It introduces an effective operator which connects the ground state B to otherwise orthogonal excited states ($n \neq 1$, $L \neq 0$). This is exactly what we expect since the large relative momentum of the recoiling s quark and the spectator would make it unlikely that they would be in their ground state.

Calculation of the spin matrix elements $\langle S \,|\, J_{\mu}^{\text{eff}} \,|\, S_B = 0 \rangle$ is very easy in the nonrelativistic limit.[9] Unfortunately, the s quark is very relativistic. The approach we take is to treat the b quark extremely nonrelativistically in the current $[J_\mu^{\rm eff}(\mathbf{k}_b \simeq 0)]$ while fixing the s-quark momentum by the two-body quark level decay $b(\mathbf{k}_b \simeq 0) \to s(\mathbf{k}_s) + \gamma(\mathbf{q})$. Thus we set $\mathbf{k}_s \simeq -\mathbf{q}$ and $E_s \simeq m_b - E_\gamma$ in the spinor $\bar{s}(\mathbf{k}_s)$. However, we naturally assume the photon momentum $\mathbf{q}$ is fixed by meson level momentum conservation and ignore the unavoidable inconsistency. We could think of the s quark as having some effective mass m_s^*.

The quantization axis is then chosen along the $\mathbf{q}$ direction so that all particles (except the spectator) have good S_z. We will ignore the problem associated with the relativistic effects on the spin of the spectator. We feel that this is acceptable because the spectator should not affect the decay very much.

Since $J_\mu^{\rm eff}(\mathbf{k}_b \simeq 0)$ is independent of $\mathbf{k}_b$ in this approximation, we can factor the spin matrix element out of the integral. Therefore

$$\begin{aligned}\langle K^{(i)}\{n,(L,S)J;\mathbf{k}\} \mid J_\mu^{\rm eff} \mid B\rangle &= \sqrt{4m_B E_K} \sum_{M_L M_S} \langle J\pm 1 \mid L M_L S M_S\rangle \\ &\quad \times \langle S \mid J_\mu^{\rm eff} \mid S_B\rangle I(n,L,M_L)\,, \end{aligned} \tag{6}$$

where the wave-function overlap is given by

$$I(n,L,M_L) \equiv \int d^3\mathbf{k}_b \phi^*_{n,L,M_L}(\mathbf{k}_b + \mathbf{k}')\phi_B(\mathbf{k}_b)\,. \tag{7}$$

To calculate the wave-function overlap, we use harmonic-oscillator wave functions which have the form

$$\phi_{n,L,M_L}(\mathbf{p}) = F_{n,L}(\,|\mathbf{p}|\,) Y_{LM_L}(\Omega_{\mathbf{p}}) \exp\left[\frac{-\mathbf{p}^2}{2\beta^2}\right] \tag{8}$$

with $\beta_B = 0.41$ GeV and $\beta_K = 0.34$ GeV (Ref. 10). For the states we are considering, we only need the following functions:

$$F_{1,0}(\mathbf{p}) = 2\pi^{-1/4}\beta^{-3/2}\,, \tag{9a}$$

$$F_{2,0}(\mathbf{p}) = (\tfrac{8}{3})^{1/2}\pi^{-1/4}\beta^{-7/2}(\mathbf{p}^2 - \tfrac{3}{2}\beta^2)\,, \tag{9b}$$

$$F_{3,0}(\mathbf{p}) = (\tfrac{1}{30})^{1/2}\pi^{-1/4}\beta^{-11/2}(4\mathbf{p}^4 - 20\beta^2\mathbf{p}^2 + 15\beta^4)\,, \tag{9c}$$

$$F_{1,1}(\mathbf{p}) = (\tfrac{8}{3})^{1/2}\pi^{-1/4}\beta^{-5/2}\,|\mathbf{p}|\,, \tag{9d}$$

$$F_{1,2}(\mathbf{p}) = (\tfrac{16}{15})^{1/2}\pi^{-1/4}\beta^{-7/2}\mathbf{p}^2\,. \tag{9e}$$

Because of orthogonality, if there were no momentum mismatch term $\mathbf{k}'$, there would be no contribution from $L \neq 0$ $K^{(i)}$ and only a very small ($\sim 1\%$) contribution from $L = 0$ radial excitations because $\beta_B \neq \beta_K$. Thus, the ground state $K^*(0.892)$ would dominate. However, the large $\mathbf{k}'$ term creates a large overlap with $L \neq 0$ and $n \neq 1$ excited states. On the other hand, it is easy to show that while $\Delta L \neq 0$ is possible, the only contribution is for $\Delta M_L = 0$ because the mismatch term $\mathbf{k}'$ is along the quantization axis. Therefore, to get $M_J = \pm 1$, we must have a $\Delta M_S = \pm 1$ spin flip. As a result, only triplet ($S = 1$) states are allowed and the spin coupling is trivial. The numerical results for the overlap integrals $I(n,L)$, where only the $M_L = 0$ integrals are nonzero, are included in Table I. Note that the $I(n,L)$ depend on m_K because $\mathbf{k}'$ does.

Since a spin flip is required, the spin matrix elements are easily calculated to be

$$\langle S \mid A_0 \mid S_B\rangle = \langle S \mid V_0 \mid S_B\rangle = 0\,, \tag{10a}$$

$$\langle S \mid A_i \mid S_B\rangle = E_\gamma \left[\frac{m_b + m_s - E_\gamma}{2(m_b - E_\gamma)}\right]^{1/2} \times \left[\frac{m_b + m_s}{m_b + m_s - E_\gamma}\right] \tilde{\epsilon}_i^*\,, \tag{10b}$$

$$\langle S \mid V_i \mid S_B\rangle = -i \left[\frac{m_b + m_s - E_\gamma}{2(m_b - E_\gamma)}\right]^{1/2} \times \left[\frac{m_b + m_s}{m_b + m_s - E_\gamma}\right] \epsilon_{ijk} q_j \tilde{\epsilon}_k^*\,, \tag{10c}$$

where $\tilde{\epsilon}_\mu$ is the transverse "spin polarization" which is the polarization vector for $J = 1$ final states. For $J > 1$, there is some complicated relation between $\tilde{\epsilon}_\mu$ and the rank-J spin tensor, but since the coupling is trivial ($\Delta M_S = \pm 1$) this is not important for the rate. Summing over the two photon polarizations, we find that the total rate is given by

$$\Gamma(B \to K^{(i)}\gamma) = g^{(i)} \mid \langle J1 \mid L011\rangle \mid^2 \times \mid I(n,L) \mid^2 \frac{E_\gamma^3}{\pi}(\,|a|^2 + |b|^2)\,, \tag{11a}$$

where

$$g^{(i)} = \left[1 - \frac{E_\gamma}{m_B}\right]\left[\frac{m_b + m_s - E_\gamma}{2(m_b - E_\gamma)}\right] \times \left[\frac{m_b + m_s}{m_b + m_s - E_\gamma}\right]^2\,. \tag{11b}$$

For all calculations, we used the following numerical values: $m_B = 5.27$ GeV, $m_b = 5.0$ GeV, $m_s = 0.55$ GeV, $m_u = m_d = 0.33$ GeV, and the $K^{(i)}$ masses from Table I. We find that the $g^{(i)}$ only range from 1.0 to 1.1, so that the important factor is the overlap integral. Note the similarity with Eq. (3) for the free quarks, keeping in mind that E_γ is slightly different. The fraction of the free quark rate for each channel is shown in the last column of Table I. Notice that the $K^*(0.892)$ is not dominant, contributing only about 4.5% of the inclusive rate for $b \to s\gamma$. Actually, we see that no single state dominates the inclusive rate. This is not surprising since it is merely the result of the very large momentum transferred to the light s quark in this two-body decay. However, we see that the total contribution from all the

$K^{(i)}$ with $m_K < 2$ GeV is $\sim 37\%$ of the inclusive rate. This is a substantial fraction of the inclusive rate, and when combined with the predictions for the inclusive rate (Refs. 1–4 and 7), it indicates that it may be possible to observe the decay $b \to s\gamma$ by looking for all photons with $E_\gamma \gtrsim 2.33$ GeV. However, to distinguish these photons from those coming from $B \to D^*\gamma$ would require a photon energy resolution of a few percent.

Gluon-exchange diagrams, which we have not included, could also contribute to $B \to K^{(i)}\gamma$. Since recoil momentum is shared with the spectator, the effect of momentum mismatch suppression would be somewhat lessened. Without an estimate of the relative importance of these nonspectator contributions, observation of $B \to K^{(i)}\gamma$ cannot be used as a measure of the $b \to s\gamma$ rate. However, if $b \to s\gamma$ is large, our calculation indicates that it is likely to show up in these low-lying $K^{(i)}$ resonances.

Also, it is important to note that due to the large uncertainties in this quark-model calculation, the results in Table I are merely an estimate of the contribution from these decays. In particular, the results depend rather strongly on the explicit wave functions used. This is because the $K^{(i)}$ is so far from zero recoil, where flavor independence helps to constrain the wave-function overlap. For example, changing the values of β_B and β_K, the "characteristic momentum" parameters in the wave functions, will change the amount of suppression of the various modes. However, this change primarily tends to redistribute the $b \to s\gamma$ decays among these $K^{(i)}$ states. Such changes would not alter our basic conclusion that a large fraction of the $b \to s\gamma$ rate is likely to show up in these low-lying $K^{(i)}$ resonances.

Upon the completion of this work, we received a revised version of Ref. 7 in which their previous result of 50–63 % of $b \to s\gamma$ going to the ground state $K^*(0.892)$ was revised down to about 7%.

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