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Lattice Hamiltonian in physical gauges

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The form of the lattice Hamiltonian for an arbitrary non-Abelian lattice gauge theory is derived in an arbitrary physical gauge, using the transfer-matrix formalism. The final result is shown to be manifestly Hermitian and polynomial in the link variables.

I. INTRODUCTION

The evolution of lattice gauge theory has involved developments in both the Lagrangian and Hamiltonian formulations of the theory. Although most of the effort in this field has focused on the Monte Carlo simulation of the Lagrangian version, a separate line of development, beginning with the derivation of the lattice gauge Hamiltonian in a temporal gauge,[1,2] has also led to useful insights. The ultimate goal here would be an accurate computation of the spectrum of the lattice Hamiltonian in the weak-coupling regime corresponding to the continuum limit for an asymptotically free theory. Initially, one attempted[3] to extrapolate from spectral calculations at strong coupling; more recently, the emphasis has shifted to the development of techniques suitable for direct application in the continuum regime. In particular, variational[4] and Lanczos[5] techniques have been adapted to lattice field theory, with the result that one is now able to demonstrate, in various asymptotically free two-dimensional theories,[6] the correct perturbative scaling of the mass gap by direct computation of the spectrum of the lattice Hamiltonian.

Convergent results for the spectrum of four-dimensional lattice theory have also been obtained by Lanczos methods in the case of compact U(1) theory.[7] Here, the starting ansatz for the ground-state wave function is basically the Gaussian solution valid in the extreme weak-coupling limit. The evaluation of matrix elements involves incomplete Gaussian integrals which can only be accurately estimated if all zero modes are eliminated, i.e., if a complete gauge fixing is performed. Accordingly, the results of Ref. 7 correspond to a calculation in a physical lattice gauge in which longitudinal photon modes are explicitly eliminated.

In order to proceed to the non-Abelian case, it will be necessary to begin from a physical gauge formulation of the theory—one in which maximal gauge fixing is performed for the spatial links of the lattice. The widely used Kogut-Susskind Hamiltonian[1] corresponds instead to the temporal gauge $A_0=0$. The purpose of this paper is to present a derivation of the form of the lattice Hamiltonian for non-Abelian gauge theories in physical gauges. By "physical gauge," we here mean a formulation in which a maximal tree of spatial links is fixed to unity on each time slice. Thus, these gauges are lattice analogs (and generalizations) of continuum axial gauge $n\cdot A=0$, n_μ spacelike. The derivation is somewhat more complicated than in the temporal case; in particular, there are ordering problems in the kinetic part of the final Hamiltonian (2.28) and (2.29). One important simplifying feature of our final result is the *polynomial* appearance of link variables U in the kinetic part; one might have been confronted with a much more complicated U dependence, making Lanczos calculations difficult, if not impossible. The derivation of the physical gauge Hamiltonian presented in Sec. II is based on transfer matrix techniques first applied by Creutz[2] to the temporal-gauge case.

II. DERIVATION OF THE HAMILTONIAN

The lattice Hamiltonian in the physical gauge will be computed by writing an expression for the transfer matrix $T=e^{-a_0H}$ (a_0 the temporal lattice spacing), and then taking the limit $(1-T)/a_0$, $a_0\to 0$. Thus, we begin with the formulation of the lattice theory on a Euclidean (space-time) lattice with temporal spacing a_0, spatial spacing a, and action

$$S=-\frac{a}{g^2a_0}\sum_{\mathbf{n}i}\mathrm{Re}\,\mathrm{tr}(\Phi_{\mathbf{n}}U'_{\mathbf{n}i}\Phi^\dagger_{\mathbf{n}+i}U^\dagger_{\mathbf{n}i})-\frac{a_0}{g^2a}\sum_{\mathbf{n},i<j}\mathrm{Re}\,\mathrm{tr}(P_{\mathbf{n}ij})\ . \tag{2.1}$$

The notation is as follows: $\mathbf{n}$ refers to a site on the lattice (integer four-vector), $\Phi_{\mathbf{n}}$ is the temporal-gauge link variable extending forward in time from site $\mathbf{n}$, $U_{\mathbf{n}i}$ the spatial link variable extending from site $\mathbf{n}$ in direction i ($i=1,2,\ldots,D-1$, where D is the space-time dimension), and $U'_{\mathbf{n}i}$ the link variable corresponding to $U_{\mathbf{n}i}$ at time a_0 later. Finally, $P_{\mathbf{n}ij}$ is a spatial plaquette variable

$$P_{\mathbf{n}ij}\equiv U_{\mathbf{n}i}U_{\mathbf{n}+\hat{\mathbf{x}}_i,j}U^\dagger_{\mathbf{n}+\hat{\mathbf{x}}_j,i}U^\dagger_{\mathbf{n}j}\ . \tag{2.2}$$

We wish to develop the theory in a physical gauge in which as many as possible of the *spatial* links $U_{\mathbf{n}i}$ are specified. For simplicity we shall assume that the gauge freedom is used to set a maximal tree of links on each time slice to unity. A maximal tree is a set of links such that any link added to the set results in a closed loop of links in the set. An example of a maximal tree of spatial links in a $(2+1)$-dimensional theory, corresponding to the continuum gauge choice $A_y=0$, is shown in Fig. 1.

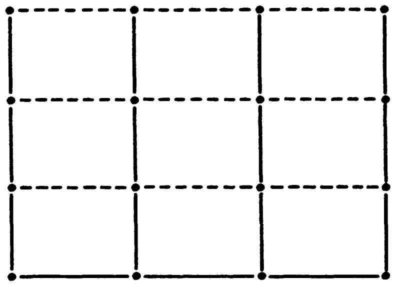

FIG. 1. Maximal tree of links on a two-dimensional lattice (bold lines). Physical links (non-gauge-fixed) are shown as dashed lines.

In addition to all the spacelike links which may be fully specified on each time slice by choice of gauge, a single timelike link $\Phi_{\mathbf{n}_0}$ connecting every pair of adjacent time slices may also be fixed. This will be essential later in removing a zero mode when the $a_0 \to 0$ limit is taken. As regards the spacelike links, our notation will be to write $\sum'_{ni}$ to indicate a restricted sum over links *not* set to unity (i.e., the remaining physical degrees of freedom), and $\sum''_{ni}$ to denote a sum over the links belonging to the maximal gauge-fixed tree on each time slice.

A state of the system at any given time is specified by the physical links $U_{\mathbf{n}i}$ on that time slice

$$|U\rangle \equiv |\{U_{\mathbf{n}i}, \text{ running over nonfixed links}\}\rangle . \tag{2.3}$$

Define gauge operators $\mathbf{Q}_{\mathbf{n}i}$ for each physical link $U_{\mathbf{n}i}$ by

$$e^{i\omega\cdot\mathbf{Q}_{\mathbf{n}i}}|U\rangle = |\{\ldots, e^{i\omega\cdot\mathbf{t}}U_{\mathbf{n}i}, \ldots\}\rangle \tag{2.4}$$

with t_α the generators of the gauge group

$$[t_\alpha, t_\beta] = if_{\alpha\beta\gamma}t_\gamma, \quad \mathrm{tr}(t_\alpha t_\beta) = \delta_{\alpha\beta} . \tag{2.5}$$

From (2.4) it follows directly that, if $\hat{U}_{\mathbf{n}i}$ are the link operators diagonal in representation (2.3),

$$\hat{U}_{\mathbf{n}i}|U\rangle = U_{\mathbf{n}i}|U\rangle , \tag{2.6}$$

the $\mathbf{Q}_{\mathbf{n}i}, \hat{U}_{\mathbf{n}i}$ satisfy the commutation relation

$$[Q^\alpha_{\mathbf{n}i}, \hat{U}_{\mathbf{m}j}] = -\delta_{\mathbf{nm}}\delta_{ij}t^\alpha \hat{U}_{\mathbf{m}j} . \tag{2.7}$$

The transfer matrix is defined as the operator whose matrix elements directly give the contribution to the exponentiated action arising from two adjacent time slices. In a physical gauge, the temporal links represent dependent field degrees of freedom. Correspondingly, they are integrated over in the definition of the transfer matrix T:

$$\langle U'|T|U\rangle = \int \prod_{\mathbf{n}\neq\mathbf{n}_0} d\Phi_{\mathbf{n}} \exp\left[\frac{a}{g^2 a_0}\sum_{\mathbf{n}i}\mathrm{Re\,tr}(\Phi_{\mathbf{n}}U'_{\mathbf{n}i}\Phi^\dagger_{\mathbf{n}+i}U^\dagger_{\mathbf{n}i})\right]\exp\left[\frac{a_0}{g^2 a}\sum_{\mathbf{n},i<j}\mathrm{Re\,tr}(P_{\mathbf{n}ij})\right] . \tag{2.8}$$

In (2.8), in contrast with (2.1), the sums over sites $\mathbf{n}$ run only over a single time slice. At this point, all links are included, even those belonging to the tree of gauge-fixed links. Define a set of c-number integration link fields,

$$G_{\mathbf{n}i} \equiv e^{i\omega^\alpha_{\mathbf{n}i}t_\alpha} , \tag{2.9}$$

one for each physical link $U_{\mathbf{n}i}$. It is straightforward to verify

$$T = \int \prod_{\mathbf{n}\neq\mathbf{n}_0} d\Phi_{\mathbf{n}} \prod_{\mathbf{n}i}{}' dG_{\mathbf{n}i} \exp\left[i\sum_{\mathbf{n}i}{}'\omega_{\mathbf{n}i}\cdot\mathbf{Q}_{\mathbf{n}i}\right]\exp\left[\frac{a}{g^2 a_0}\sum_{\mathbf{n}i}\mathrm{Re\,tr}(\Phi_{\mathbf{n}}G_{\mathbf{n}i}\hat{U}_{\mathbf{n}i}\Phi^\dagger_{\mathbf{n}+i}\hat{U}^\dagger_{\mathbf{n}i}) + \frac{a_0}{g^2 a}\sum_{\mathbf{n},i<j}\mathrm{Re\,tr}(\hat{P}_{\mathbf{n}ij})\right] . \tag{2.10}$$

It should be emphasized that in (2.10) $\Phi_{\mathbf{n}}, G_{\mathbf{n}i}$ are c-number integration variables, while $\mathbf{Q}_{\mathbf{n}i}$ and $\hat{U}_{\mathbf{n}i}$ (and $\hat{P}_{\mathbf{n}ij}$, constructed from the latter) are noncommuting operators [cf. (2.7)]. Accordingly, the order of the exponential factors in (2.10) is significant. In particular, the Hermiticity (actually, *symmetry*) of T

$$\langle U'|T|U\rangle = \langle U|T|U'\rangle \tag{2.11}$$

manifest in (2.8), is somewhat concealed in (2.10). It will once again be apparent in our final result for H, however.

The evaluation of the $\Phi_{\mathbf{n}}$ and $G_{\mathbf{n}i}$ integrals in (2.10) would appear to be a daunting task, as indeed it would be, in the general case. Fortunately, the derivation of the lattice Hamiltonian requires only the evaluation of (2.10) in the limit $a_0 \to 0$. First, let us note that as the gauge freedom is now completely eliminated, the remaining links represent physical degrees of freedom, and must approach unity to the extent the corresponding lattice spacing is taken to zero (we do not expect infinitely large physical field values to contribute to the functional integral). Thus (2.10) may be evaluated in the limit $a_0 \to 0$ by a saddle-point calculation in which the temporal links are expanded about unity

$$\Phi_{\mathbf{n}} = e^{i\phi^\alpha_{\mathbf{n}}t_\alpha} \equiv e^{i\phi_{\mathbf{n}}\cdot\mathbf{t}} = 1 + i\phi_{\mathbf{n}}\cdot\mathbf{t} - \tfrac{1}{2}(\phi_{\mathbf{n}}\cdot\mathbf{t})^2 + \cdots . \tag{2.12}$$

Of course, it follows from the form of the exponent in (2.10) that the integral is also forced into the region $G_{\mathbf{n}i} \sim 1$ as $a_0 \to 0$. Thus, we also expand in the variables $\omega_{\mathbf{n}i}$ in (2.9).

The spatial plaquette contribution (involving $\hat{P}_{\mathbf{n}ij}$) in (2.10) goes directly into the potential energy part H_{pot} of the lattice Hamiltonian in an obvious way. Since $T = e^{-a_0 H}$, we have

$$H_{\mathrm{pot}} = -\frac{1}{g^2 a}\sum_{\mathbf{n},i<j}\mathrm{Re\,tr}(\hat{P}_{\mathbf{n}ij}) . \tag{2.13}$$

Henceforth, we concentrate on the remaining parts of (2.10), which lead to the kinetic part of the Hamiltonian,

$H_{\rm kin}$. Also, for notational simplicity, we henceforth drop the carets over the link variables $U_{\mathbf{n}i}$, understood to be operators obeying (2.7).

The quadratic part of the timelike plaquette piece of (2.10) is found after a short calculation to be

$$\mathrm{Re}\,\mathrm{tr}(\Phi_{\mathbf{n}} G_{\mathbf{n}i} U_{\mathbf{n}i} \Phi^\dagger_{\mathbf{n}+i} U^\dagger_{\mathbf{n}i})_{\rm quad} = -\tfrac{1}{2} \,|\, D_i \boldsymbol{\phi}_{\mathbf{n}} - \boldsymbol{\omega}_{\mathbf{n}i} \,|^2 \,. \tag{2.14}$$

Here D_i is the covariant derivative, $D_i \phi^\alpha_{\mathbf{n}} \equiv \Omega^{\alpha\beta}_{\mathbf{n}i} \phi^\beta_{\mathbf{n}+i} - \phi^\alpha_{\mathbf{n}}$, with Ω the representative of U in the adjoint:

$$\Omega^{\alpha\beta}_{\mathbf{n}i} = \mathrm{tr}(t_\alpha U_{\mathbf{n}i} t_\beta U^\dagger_{\mathbf{n}i}) \,. \tag{2.15}$$

The power counting for $a_0 \to 0$ is facilitated by the rescalings $\phi_{\mathbf{n}} \to a_0^{1/2} \phi_{\mathbf{n}}$, $\omega_{\mathbf{n}i} \to a_0^{1/2} \omega_{\mathbf{n}i}$. After this shift

$$\exp\left[i \textstyle\sum' \omega_{\mathbf{n}i} \mathbf{Q}_{\mathbf{n}i} \right] \to 1 + i a_0^{1/2} \textstyle\sum' \omega_{\mathbf{n}i} \cdot \mathbf{Q}_{\mathbf{n}i} - \tfrac{1}{2} a_0 \left[\textstyle\sum' \omega_{\mathbf{n}i} \cdot \mathbf{Q}_{\mathbf{n}i} \right]^2 + \text{higher-order terms} \,. \tag{2.16}$$

We are only interested in terms up to order a_0, as $T \sim 1 - a_0 H$ for $a_0 \to 0$. Now, the quartic terms in the expansion are already $O(a_0)$, so these terms may be brought down from the exponential and $\mathbf{Q}$ dropped entirely in (2.16). The contribution of any quartic terms to H could then only depend on the $U_{\mathbf{n}i}$ and would take the form

$$\int d\phi_{\mathbf{n}} d\omega_{\mathbf{n}i} J(\phi_{\mathbf{n}}) J(\omega_{\mathbf{n}i})(\text{quartic terms}) \exp\left[-\frac{a}{2g^2} \sum_{\mathbf{n}i} |\, D_i \boldsymbol{\phi}_{\mathbf{n}} - \boldsymbol{\omega}_{\mathbf{n}i} \,|^2 \right] . \tag{2.17}$$

If we now examine the mutilated transfer matrix $\tilde{T}$, obtained by setting $\mathbf{Q}_{\mathbf{n}i} = 0$ from the outset,

$$\tilde{T} = \int d\Phi_{\mathbf{n}} dG_{\mathbf{n}i} \exp\left[\frac{a}{g^2 a_0} \sum_{\mathbf{n}i} \mathrm{Re}\,\mathrm{tr}(\Phi_{\mathbf{n}} G_{\mathbf{n}i} U_{\mathbf{n}i} \Phi^\dagger_{\mathbf{n}+i} U^\dagger_{\mathbf{n}i}) \right] = \int d\Phi_{\mathbf{n}} dG_{\mathbf{n}i} \exp\left[\frac{a}{g^2 a_0} \sum_{\mathbf{n}i} \mathrm{Re}\,\mathrm{tr}(G_{\mathbf{n}i}) \right] = U \text{ independent}$$

(by a shift of the $G_{\mathbf{n}i}$ integration). Consequently, the quartic terms contribute at most an irrelevant constant to the Hamiltonian. It may similarly be shown that the Jacobians $J(\phi_{\mathbf{n}})$, $J(\omega_{\mathbf{n}i})$ in the Haar measure [see (2.17)] are also irrelevant in the limit $a_0 \to 0$, and may be set to unity.

The cubic terms in the expansion of the timelike plaquettes are not, however, excluded by the above argument. In fact, they turn out to be crucial in restoring the Hermiticity of the Hamiltonian [note that at the moment all $\mathbf{Q}$ factors remain to the *left* of all U factors in (2.10)]. By the antisymmetry of $\mathrm{Re}\,\mathrm{tr}(it_\alpha t_\beta t_\gamma) = -\tfrac{1}{2} f_{\alpha\beta\gamma}$, it follows that the only surviving cubic term is obtained by taking the linear term from each of $\Phi_{\mathbf{n}}$, $G_{\mathbf{n}i}$, and $U_{\mathbf{n}i} \Phi^\dagger_{\mathbf{n}+i} U^\dagger_{\mathbf{n}i}$. This yields

$$\mathrm{Re}\,\mathrm{tr}(\Phi_{\mathbf{n}} G_{\mathbf{n}i} U_{\mathbf{n}i} \Phi^\dagger_{\mathbf{n}+i} U^\dagger_{\mathbf{n}i})_{\rm cubic} = -\tfrac{1}{2} f_{\alpha\beta\gamma} \phi^\alpha_{\mathbf{n}} \omega^\beta_{\mathbf{n}i} \Omega^{\gamma\delta}_{\mathbf{n}i} \phi^\delta_{\mathbf{n}+i} \,. \tag{2.18}$$

After rescaling we find the following expression, correct to $O(a_0)$ for $T_{\rm kin} \sim 1 - a_0 H_{\rm kin}$:

$$T_{\rm kin} \sim \int \prod_{\mathbf{n} \neq \mathbf{n}_0} d\phi_{\mathbf{n}} \prod_{\mathbf{n}i}{}' d\omega_{\mathbf{n}i} \left[1 - \frac{ia}{2g^2} a_0 \sum_{\mathbf{m}i,\mathbf{n}j}{}' \omega_{\mathbf{m}i} \cdot \mathbf{Q}_{\mathbf{m}i} \cdot f_{\alpha\beta\gamma} \phi^\alpha_{\mathbf{n}} \omega^\beta_{\mathbf{n}j} \phi^\delta_{\mathbf{n}+j} \Omega^{\gamma\delta}_{\mathbf{n}j} - \frac{1}{2} a_0 \left[\textstyle\sum' \omega_{\mathbf{m}i} \cdot \mathbf{Q}_{\mathbf{m}i} \right]^2 \right] \exp\left[-\frac{a}{2g^2} \sum_{\mathbf{n}i} |\, D_i \boldsymbol{\phi}_{\mathbf{n}} - \boldsymbol{\omega}_{\mathbf{n}i} \,|^2 \right] . \tag{2.19}$$

Note that

$$\sum_{\mathbf{n}i} |\, D_i \boldsymbol{\phi}_{\mathbf{n}} - \boldsymbol{\omega}_{\mathbf{n}i} \,|^2 = \sum_{\mathbf{n}i}{}' |\, D_i \boldsymbol{\phi}_{\mathbf{n}} - \boldsymbol{\omega}_{\mathbf{n}i} \,|^2 + \sum_{\mathbf{n}i}{}'' |\, \Delta_i \boldsymbol{\phi}_{\mathbf{n}} \,|^2 \,, \tag{2.20}$$

where

$$\Delta_i \boldsymbol{\phi}_{\mathbf{n}} = \boldsymbol{\phi}_{\mathbf{n}+i} - \boldsymbol{\phi}_{\mathbf{n}} \tag{2.21}$$

so that the integral over the $\boldsymbol{\omega}_{\mathbf{n}i}$ is manifestly convergent. The quantity $\sum'' |\, \Delta_i \boldsymbol{\phi}_{\mathbf{n}} \,|^2$ is also positive definite, as setting $\Delta_i \boldsymbol{\phi}_{\mathbf{n}} = 0$ for all links on a maximal tree clearly sets all $\boldsymbol{\phi}_{\mathbf{n}}$ equal. However [see discussion following (2.2)], as one of the $\boldsymbol{\phi}_{\mathbf{n}}$ is gauge fixed to zero, this requires all $\boldsymbol{\phi}_{\mathbf{n}}$ to vanish. Thus, the integral over the $\boldsymbol{\phi}_{\mathbf{n}}$ is also convergent. This granted, and including an obvious normalization factor in the definition of the integrals, one finds

$$\int \prod{}' d\omega_{\mathbf{n}i} \omega^\alpha_{\mathbf{m}i} \omega^\beta_{\mathbf{n}j} \exp\left[-\frac{a}{2g^2} \sum_{\mathbf{n}i}{}' |\, D_i \boldsymbol{\phi}_{\mathbf{n}} - \boldsymbol{\omega}_{\mathbf{n}i} \,|^2 \right] = \frac{g^2}{a} \delta_{\alpha\beta} \delta_{\mathbf{mn}} \delta_{ij} + (D_i \boldsymbol{\phi}_{\mathbf{m}})^\alpha (D_j \boldsymbol{\phi}_{\mathbf{n}})^\beta \,. \tag{2.22}$$

Define a purely kinematic (i.e., U-independent) propagator $K_{\mathbf{mn}}$ by

$$\int \prod_{\mathbf{n} \neq \mathbf{n}_0} d\phi_{\mathbf{n}} \phi^\alpha_{\mathbf{m}} \phi^\beta_{\mathbf{n}} \exp\left[-\frac{a}{2g^2} \sum_{\mathbf{n}i}{}'' |\, \Delta_i \boldsymbol{\phi}_{\mathbf{n}} \,|^2 \right] \equiv \frac{g^2}{a} \delta^{\alpha\beta} K_{\mathbf{mn}} \,. \tag{2.23}$$

Inserting these results in (2.19) one finds

$$H_{\mathrm{kin}}=\frac{g^2}{2a}\left[\sum_{\mathrm{m}i}{}' \mathbf{Q}_{\mathrm{m}i}^2+\sum_{\mathrm{m}i\alpha,\mathrm{n}j\beta}{}' Q_{\mathrm{m}i}^{\alpha}Q_{\mathrm{n}j}^{\beta}\mathbb{L}_{\mathrm{m}i\alpha,\mathrm{n}j\beta}\right]$$
$$+\frac{ia}{2g^2}\int d\boldsymbol{\phi}_{\mathrm{n}}d\boldsymbol{\omega}_{\mathrm{n}i}Q_{\mathrm{m}i}^{\alpha}\omega_{\mathrm{m}i}^{\alpha}f_{\alpha'\beta\gamma}\phi_{\mathrm{n}}^{\alpha'}\omega_{\mathrm{n}j}^{\beta}\phi_{\mathrm{n}+j}^{\delta}\Omega_{\mathrm{n}j}^{\gamma\delta}\exp\left[-\frac{a}{2g^2}\sum{}' \,|D_i\phi_n-\omega_{ni}|^2-\frac{a}{2g^2}\sum{}'' \,|\Delta_i\phi_n|^2\right], \tag{2.24}$$

where $\mathbb{L}$ is the symmetric matrix

$$\mathbb{L}_{\mathrm{m}i\alpha,\mathrm{n}j\beta}\equiv\sum_{\mathrm{m'n'}\gamma}(D_i)_{\alpha\mathrm{m},\gamma\mathrm{m'}}K_{\mathrm{m'n'}}(D_j)_{\beta\mathrm{n},\gamma\mathrm{n'}}, \tag{2.25}$$

$$(D_i)_{\alpha\mathrm{m},\gamma\mathrm{m'}}\equiv\delta_{\mathrm{m'},\mathrm{m}+i}\Omega_{\mathrm{m}i}^{\alpha\gamma}-\delta^{\alpha\gamma}\delta_{\mathrm{mm'}}. \tag{2.26}$$

Finally, we show that the horrible integral in (2.24), arising from the cubic term, is precisely the symmetrizing factor for the terms that precede it. Note that

$$f_{\alpha'\beta\gamma}\phi_{\mathrm{n}}^{\alpha'}(D_j\phi_{\mathrm{n}})^{\beta}\Omega_{\mathrm{n}j}^{\gamma\delta}\phi_{\mathrm{n}+j}^{\delta}=f_{\alpha'\beta\gamma}\phi_{\mathrm{n}}^{\alpha'}(D_j\phi_{\mathrm{n}})^{\beta}[(D_j\phi_{\mathrm{n}})^{\gamma}+\phi_{\mathrm{n}}^{\gamma}]=0 \text{ by antisymmetry}.$$

Thus, the last term in H may be written

$$\frac{ia}{2g^2}\int d\boldsymbol{\phi}_{\mathrm{n}}d\boldsymbol{\omega}_{\mathrm{n}i}Q_{\mathrm{m}i}^{\alpha}\omega_{\mathrm{m}i}^{\alpha}f_{\alpha'\beta\gamma}\phi_{\mathrm{n}}^{\alpha'}[\omega_{\mathrm{n}j}^{\beta}-(D_j\phi_{\mathrm{n}})^{\beta}]\Omega_{\mathrm{n}j}^{\gamma\delta}\phi_{\mathrm{n}+j}^{\delta}e^{-\cdots}$$
$$=\frac{ia}{2g^2}\int d\boldsymbol{\phi}_{\mathrm{n}}d\boldsymbol{\omega}_{\mathrm{n}i}Q_{\mathrm{m}i}^{\alpha}(\omega_{\mathrm{m}i}^{\alpha}+D_i\phi_{\mathrm{m}}^{\alpha})f_{\alpha'\beta\gamma}\phi_{\mathrm{n}}^{\alpha'}\omega_{\mathrm{n}j}^{\beta}\Omega_{\mathrm{n}j}^{\gamma\delta}\phi_{\mathrm{n}+j}^{\delta}\exp\left[-\frac{a}{2g^2}\sum{}'\omega_{ni}^2\right]\exp\left[-\frac{a}{2g^2}\sum{}''\,|\Delta_i\phi_n|^2\right]$$
$$=\frac{ig^2}{2a}Q_{\mathrm{m}i}^{\alpha}f_{\delta\alpha\gamma}K_{\mathrm{m}+i,\mathrm{m}}\Omega_{\mathrm{m}i}^{\gamma\delta}=-\frac{g^2}{2a}Q_{\mathrm{m}i}^{\alpha}[Q_{\mathrm{n}j}^{\beta},\mathbb{L}_{\mathrm{m}i\alpha,\mathrm{n}j\beta}]. \tag{2.27}$$

Substituting this result in (2.24) we obtain the manifestly Hermitian result for the kinetic part of the lattice Hamiltonian:

$$aH_{\mathrm{kin}}=\frac{g^2}{2}\left[\sum_{ni}{}'\mathbf{Q}_{ni}^2+\sum_{\mathrm{m}i\alpha,\mathrm{n}j\beta}{}' Q_{\mathrm{m}i}^{\alpha}\mathbb{L}_{\mathrm{m}i\alpha,\mathrm{n}j\beta}Q_{\mathrm{n}j}^{\beta}\right]. \tag{2.28}$$

Together with (2.13),

$$aH_{\mathrm{pot}}=-\frac{1}{g^2}\sum_{\mathrm{n},i<j}\mathrm{Re}\,\mathrm{tr}(P_{\mathrm{n}ij}) \tag{2.29}$$

we have the complete expression for the lattice Hamiltonian $H=H_{\mathrm{kin}}+H_{\mathrm{pot}}$ in a physical lattice gauge.

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