Solvable light-front model of a relativistic bound state in 1+1 dimensions

Mikołaj Sawicki
Institute of Theoretical Physics, Warsaw University, Hoża 69, 00-681 Warsaw, Poland

Lech Mankiewicz
Nicolaus Copernicus Astronomical Center, Bartycka 18, 00-716 Warsaw, Poland

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The bound-state equation at equal light-front time is investigated in the framework of a scalar field model in one space and one time dimension in the limit of an infinitely massive exchanged boson. Analytic expressions for the bound-state mass and form factor are obtained. In a weak-binding limit the results for mass and wave function coincide with those based on the nonrelativistic Schrödinger equation with δ-type potential. For weakly bound systems the form factor reduces to the proper static limit in the whole domain of momentum transfers. In general, the quality of the static approximation is controlled by a dimensionless parameter ($\bar{B}/m$) that characterizes the strength of the binding.

I. INTRODUCTION

The problem of a two-body bound state in a framework of a scalar field model quantized on the light front has been studied in various aspects in papers by Feldman, Fulton, and Townsend,[1] Karmanov,[2] Müller,[3] Brodsky, Ji, and Sawicki,[4] Sawicki,[5] and Celenza, Ji, and Shakin.[6] Also studies in one space and one time dimensions attract nowadays a revigorated interest. The model of fermions interacting by scalar bosons has been studied by Brooks and Frautschi[7] in the usual space-time quantization and by Pauli and Brodsky[8] in light-front quantization. Eller, Pauli, and Brodsky[9] studied quantum electrodynamics in light-front quantization. In this paper we study a two-body bound state in a scalar field model quantized on the light front in one space and one time dimensions, in the limit of an infinitely massive exchanged boson. This setup corresponds to a contact interaction and allows for analytic solution for the mass and wave function of the two-body bound state for all physically admissible values of the coupling constant. A comparison has been made with a similar model by Glöckle, Nogami, and Fukui[10] based on a two-body Dirac equation in one space dimension with contact interaction. For a weakly bound system the results are identical and coincide with nonrelativistic quantum mechanics.

Since our wave function is both Lorentz invariant and explicitly known, we are able to calculate the elastic electromagnetic form factor of the model "deuteron" in a rigorous way. Again, for weakly bound systems, our result for the form factor agrees with the results of Ref. 10 and is equivalent to a static approximation. For strongly bound systems, however, the static approximation fails even for small momentum transfers.

The paper is organized as follows. In Sec. II we present the model. In Sec. III we give solutions for the mass and the wave function of the bound state. In Sec. IV we study the elastic form factor. A short summary is given in Sec. V.

II. THE MODEL

In this paper we study properties of a relativistic two-body bound state in a solvable model in one space and one time dimension. The model is based on the light-front description of the relativistic system of two scalar particles with mass m interacting via exchange of a heavy scalar boson of mass μ. We study the limit $\mu \to \infty$ corresponding to a contact interaction.

The Lagrangian of the system is

$$L = -\tfrac{1}{2}(m^2\psi^2 - \partial_\nu\psi\partial^\nu\psi) - \tfrac{1}{2}(\mu^2\phi^2 - \partial_\nu\phi\partial^\nu\phi) + g\psi^2\phi \ , \tag{2.1}$$

where $x^\nu = (x^0, z)$.

Quantizing on the light front we get

$$\psi(x) = \int_0^\infty \frac{dk^+}{(2\pi)^{1/2}} \frac{1}{\sqrt{2}} b(k^+) e^{-i(\omega_k x^+ + k^+ x^-)/2} + \text{H.c.} \ , \tag{2.2}$$

where $x^{+(-)} = x^0 \pm z$, $k^+ = k^0 + k^z$, $\omega_k = m^2/k^+$, and the nonvanishing commutators are

$$[b(k^+), b^\dagger(l^+)] = \delta(k^+ - l^+) \ . \tag{2.3}$$

We construct the two-body sector of the relativistic bound state with the overall light-front momentum P^+,

$$|\psi_{P^+}\rangle = \frac{1}{\sqrt{2}} \int_0^\infty dk^+ \int_0^\infty dl^+ \delta(P^+ - k^+ - l^+) b^\dagger(k^+) \times b^\dagger(l^+) |0\rangle \psi_{P^+}(k^+, l^+) \ , \tag{2.4}$$

with the normalization

$$\int_0^\infty dk_1^+ \, |\psi_{P^+}(k_1^+, P^+ - k_1^+)|^2 = P^+ \tag{2.5}$$

so that

$$\langle \psi_{P^+} | \psi_{P'^+} \rangle = P^+ \delta(P^+ - P'^+) \; . \tag{2.6}$$

In lowest order in the coupling constant g the bound-state wave function ψ is given by the integral equation

$$\left[M^2 - \frac{m^2}{x_1 x_2} \right] \psi(x_1, x_2) = \frac{g^2}{2\pi} \int_0^1 dy_1 dy_2 \delta(1 - y_1 - y_2) \frac{1}{\sqrt{x_1 x_2 y_1 y_2}} K(x_i, y_i \,|\, M^2) \psi(y_1, y_2) \; , \tag{2.7}$$

where

$$K(x_i, y_i \,|\, M^2) = \frac{\theta(y_1 - x_1)}{y_1 - x_1} \frac{1}{M^2 - \dfrac{m^2}{x_1} - \dfrac{\mu^2}{y_1 - x_1} - \dfrac{m^2}{y_2}} + (1 \leftrightarrow 2) \; . \tag{2.8}$$

Here M is the mass of the bound system, $x_i \equiv k_i^+ / P^+$ is the fraction of the total light front momentum P^+ carried by the ith constituent, $0 < x_i, y_i < 1$, $x_1 + x_2 = 1 = y_1 + y_2$.

To obtain contact interaction we perform the limits $\mu \to \infty$, $g \to \infty$ keeping $\lambda \equiv g^2 / 2\pi\mu^2 m^2$ constant. Equation (2.7) reduces to

$$\psi(x_1, x_2) = \frac{1}{\sqrt{x_1 x_2}} \frac{1}{M^2 - \dfrac{m^2}{x_1 x_2}} (-m^2 \lambda) \int_0^1 dy_1 dy_2 \delta(1 - y_1 - y_2) \frac{1}{\sqrt{y_1 y_2}} \psi(y_1, y_2) \; . \tag{2.9}$$

III. THE SOLUTION

The solution of Eq. (2.9) is clearly the function of the relative light-front variable $x = x_1 - x_2$ and has the form

$$\psi(x) = N \frac{(1 - x^2)^{1/2}}{a^2 + x^2} \; , \tag{3.1}$$

with the normalization constant N fixed by the condition (2.5)

$$N^2 = \frac{2a^2}{1 + \dfrac{1 - a^2}{a} \arctan \dfrac{1}{a}} \; . \tag{3.2}$$

Here we have defined

$$a^2 \equiv (1 - \eta^2)/\eta^2 \; , \tag{3.3}$$

$$\eta = M / 2m \; . \tag{3.4}$$

Since the light-front variables x_i are invariants of Lorentz boosts, the wave function ψ given by Eq. (3.1) is the Lorentz-invariant object. For future use it is convenient to introduce a parallel parametrization of the bound-state mass as

$$B = 2m - M, \quad \bar{B} = B - B^2/4m \; , \tag{3.5}$$

so that

$$\eta^2 = 1 - \frac{\bar{B}}{m} \; . \tag{3.6}$$

The eigenvalue of Eq. (2.9) is given by the formula

$$\lambda = \eta^2 a \frac{1}{\arctan(1/a)} \; . \tag{3.7}$$

In the weak-binding limit ($\bar{B}/m \ll 1$) one has $\bar{B}/m \approx B/m$ and Eq. (3.7) yields

$$B/m = (\lambda \pi / 2)^2 \; , \tag{3.8}$$

whereas the strong-binding limit ($M/2m = \eta \ll 1$) corresponds to $\lambda = 1$.

The average values of light-front momenta are easily calculable. One has clearly

$$\langle x_1 \rangle = \langle x_2 \rangle = \tfrac{1}{2} \tag{3.9}$$

and

$$\langle x_1^2 \rangle = \langle x_2^2 \rangle = \tfrac{1}{4} + \frac{N^2}{8} \left[-3 + \frac{1}{a}(1 + 3a^2) \arctan \frac{1}{a} \right] \; . \tag{3.10}$$

Thus in the weak-binding limit ($\bar{B}/m \ll 1$) we have

$$\langle x_i^2 \rangle \approx \tfrac{1}{4} \; , \tag{3.11}$$

whereas for the strong-binding limit ($M/2m \ll 1$) we get

$$\langle x_i^2 \rangle \approx (\tfrac{3}{10})^2 \; , \tag{3.12}$$

i.e.,

$$\langle k_i^{+2} \rangle \approx \tfrac{3}{10} (P^+)^2 \; . \tag{3.13}$$

In the rest frame ($P^+ = M$) this reduces to $\langle k_i^{+2} \rangle = 0.3 M^2$.

For the purpose of comparison to nonrelativistic quantum mechanics we cast Eq. (2.9) into another form. To this end we introduce the relativistic relative momentum p as the new variable, defining

$$x_{1,2} = \frac{1}{2} \left[1 \pm \frac{p}{\epsilon(p)} \right] \quad (-\infty < p < +\infty) \; , \tag{3.14}$$

where

$$\epsilon(p)=(m^2+p^2)^{1/2} . \tag{3.15}$$

Using definitions (3.14), (3.15), and (3.5) we transform Eq. (2.9) into the form

$$\left[\frac{p^2}{m}+\bar{B}\right]\phi(p)=\frac{\lambda}{2}\int_{-\infty}^{+\infty}dp'\frac{m}{\epsilon(p')}\phi(p') , \tag{3.16}$$

where the new wave function is defined by

$$\phi(p)=(1-x^2)^{1/2}\psi(x) . \tag{3.17}$$

Using (3.1) and (3.14) we write (3.17) explicitly:

$$\phi(p)=N\frac{m^2}{1+a^2}\frac{1}{p^2+m^2a^2/(1+a^2)} . \tag{3.18}$$

The structure of Eq. (3.16) is analogous to that of the Lippmann-Schwinger equation

$$\left[\frac{p^2}{2m^*}+B\right]\tilde{\phi}(p)=\frac{\lambda}{2}\int_{-\infty}^{+\infty}dp'\tilde{\phi}(p') , \tag{3.19}$$

describing a one-dimensional nonrelativistic two-body system with the reduced mass $m^*=m/2$, bound by the contact potential that in the position space has the form ($z=z_1-z_2$):

$$V(z)=-\pi\lambda\delta(z) . \tag{3.20}$$

The solution of Eq. (3.19) is

$$B/m=\left[\frac{\lambda\pi}{2}\right]^2 , \tag{3.21}$$

the wave function reads

$$\tilde{\phi}(p)=\frac{2\kappa^3}{\pi}\frac{1}{p^2+\kappa^2} , \tag{3.22}$$

and its Fourier transform to the position space is

$$\tilde{\phi}(z)=\sqrt{\kappa}e^{-\kappa|z|} , \tag{3.23}$$

where we defined in the standard way $\kappa^2\equiv mB$.

Indeed, in the low-momentum region of the phase space (i.e., for $p'/m\ll 1$) and for weakly bound systems (i.e., for $\bar{B}/m\approx B/m\ll 1$) the kernel of Eq. (3.16) coincides with that of Eq. (3.19) and we recognize the approximate solution (3.8) as an exact solution (3.21) of the corresponding nonrelativistic equation. Likewise, for a weakly bound system the wave function (3.18) reduces to the expression (3.22).

In Fig. 1 we present the relation between the mass of the bound system and the coupling constant λ as given by the formula (3.7) and compare it with the result given by the solution (3.21) of the Lippmann-Schwinger solution. One clearly sees the onset of the difference between relativistic and nonrelativistic solutions as the binding of the system increases. This feature appears to be qualitatively independent from details of interaction and therefore of rather kinematical origin—indeed, in Ref. 5 we studied another extreme case, setting the mass of the exchange boson to zero ($\mu=0$) and found a similar relation between the exact light-front solution and its nonrelativistic, positroniumlike approximation.

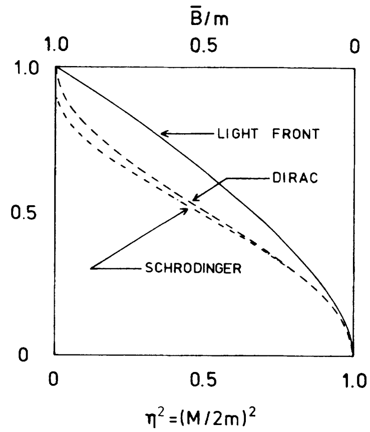

FIG. 1. Coupling constant λ vs $\eta^2=(M/2m)^2$. The solid line represents the solution (3.7) of the light-front equation, the dot-dashed curve represents the solution (3.21) of the Lippmann-Schwinger equation, and the dashed line corresponds to the results of Ref. 10 based on the two-body Dirac equation in one dimension with δ-type contact interaction.

In addition, we draw also the results of Glöckle, Nogami, and Fukui,[10] based on the two-body Dirac equation in one space dimension with a δ-type interaction. In our notation their results reads

$$\lambda=\frac{2}{\pi}\arctan a .$$

IV. FORM FACTORS

Let us treat our composite system as a prototype of a deuteron and probe its structure by means of elastic electron scattering. Let us consider a situation when the deuteron with momentum P absorbs a virtual photon carrying the momentum $q^\nu=(q^0,q^z)$ and remains intact. This means that the photon is absorbed by the proton (constituent 1) which acquires the photon's momentum and then both constituents could be found in the state representing the deuteron with the final momentum P'.

The probability of such a sequence is measured by the elastic electromagnetic form factor $F(Q^2)$, where

$$q^2=(q^0)^2-(q^z)^2\equiv -Q^2 ,$$

so that $Q^2>0$.

In nonrelativistic quantum mechanics one works in a static approximation neglecting a motion of the target and calculating the form factor as the Fourier transform of the charge-density distribution in the target. In this case $Q\equiv q^z$ and we have

$$F_{\rm st}(Q^2)=\int_{-\infty}^{+\infty}dz_1e^{iQz_1}\phi^*(z_1)\phi(z_1) . \tag{4.1}$$

Here z_1 is the distance of the first particle (proton) from the center of the mass of the target (deuteron). The wave function ϕ is taken as the Fourier transform of the exact wave function (3.18). Keeping in mind that $z=2z_1$ ($z=2z_2$), where z is the relative position appearing as the argument of the Fourier transform, one easily obtains the form factor in the static approximation

$$F_{st}(Q^2)=\frac{1}{1+Q^2/(16m\bar{B})} . \tag{4.2}$$

In the case of the weakly bound system one has $m\bar{B}\to mB\equiv\kappa^2$ and the form factor (4.2) becomes identical to what one obtains upon using the nonrelativistic wave function (3.23) from the very beginning

$$F_{st}(Q^2)=\frac{1}{1+Q^2/(16\kappa^2)} \quad \text{(weak binding)} . \tag{4.2'}$$

We note here that the result (4.2) is identical to the static form factor calculated by Glöckle, Nogami, and Fukui[10] for all values of the bound-state mass M.

In the actual scattering process, however, the deuteron not only does not remain at rest, but for large momentum transfer suffers a huge recoil. The natural question arises of how good is the static approximation to true form factor. To this end we calculate the form factor from first principles.

We begin with the electromagnetic current for the boson field,

$$j^\nu(x)=i:\psi(\partial^\nu\psi)-(\partial^\nu\psi)\psi: \tag{4.3}$$

and define the form factor in a standard way:

$$\langle\psi_{P'^+}|j^\nu(0)|\psi_{P^+}\rangle=\frac{1}{2}\frac{1}{2\pi}(P+P')^\nu F(q^2) , \tag{4.4}$$

where the state vectors of the initial and final deuterons are given by Eq. (2.4). We extract the form factor upon calculating the longitudinal component of Eq. (4.4). Defining $q=P'-P$ and using (2.2) we obtain

$$\langle\psi_{P'^+}|j^+(0)|\psi_{P^+}\rangle=\frac{1}{2}\frac{1}{2\pi}\int_0^\infty dk_1^+\int_0^\infty dl_1^+\frac{k_1^++l_1^+}{(k_1^+l_1^+)^{1/2}}\delta(l_1^+-k_1^+-q^+)\psi_{P'^+}(l_1^+,P^+-k_1^+)\psi_{P^+}(k_1^+,P^+-k_1^+) . \tag{4.5}$$

For elastic scattering the target in initial and final state is on shell, i.e.,

$$P^2=(P^0)^2-(P^z)^2\equiv P^+P^-=M^2$$

and likewise for P'. Thus

$$q^2=(P'-P)^2=-M^2\frac{(q^+)^2}{P^+P'^+} \tag{4.6}$$

and

$$Q^2=M^2\frac{\alpha^2}{1+\alpha} , \tag{4.7}$$

where we have defined the key parameter

$$\alpha=q^+/P^+ . \tag{4.8}$$

Solving Eq. (4.7) for α we get

$$\alpha=\frac{(Q^2/M^2)+[(Q^2/M^2)^2+4(Q^2/M^2)]^{1/2}}{2} . \tag{4.9}$$

This yields

$$\alpha\approx\begin{cases}Q/M & \text{for } Q/M\ll 1 ,\\ Q^2/M^2 & \text{for } Q/M\gg 1 .\end{cases} \tag{4.10}$$

Relation (4.7) constitutes an essential point—we see that, due to a lack of transversal degrees of freedom in our model, any nonzero momentum transfer in elastic scattering must have a nonvanishing longitudinal component. For this reason we cannot reduce our result to the standard Drell-Yan formula[11] for the latter holds in the particular frame of reference where $q^+=0$, $Q^2=q_\perp{}^2$. This leads in effect to a change in fractional distribution of longitudinal momentum of the bound state among both constituents. Clearly, we have

$$\begin{aligned}P'^+&=P^++q^+=P^+(1+\alpha) ,\\ l_1^+&=k_1^++q^+=P^+(x_1+\alpha) ,\\ l_2^+&=k_2^+=P^+x_2 .\end{aligned} \tag{4.11}$$

Thus the new distribution reads

$$\begin{aligned}y_1&\equiv l_1^+/P'^+=\frac{x_1+\alpha}{1+\alpha} ,\\ y_2&\equiv l_2^+P'^+=\frac{x_2}{1+\alpha} .\end{aligned} \tag{4.12}$$

Combining together Eqs. (4.4), (4.5), (4.8), (4.11), and (4.12) we obtain the form factor as a function of parameter α rather than Q^2.

$$F(\alpha)=\frac{1}{2+\alpha}\int_{-1}^{+1}dx\frac{1+x+\alpha}{\sqrt{(1+x)(1+x+2\alpha)}}\psi(x)\psi(y) . \tag{4.13}$$

Here the wave function is given by Eq. (3.1), $x\equiv x_1-x_2$ and $y\equiv y_1-y_2=(x+\alpha)/(1+\alpha)$. Performing the integration we arrive at the general analytical result for the form factor $F(\alpha)$ that is given in the Appendix. Here we discuss only special cases.

For small values of momentum transfer α we expand our result into the power series in α. The terms linear in α cancel exactly and in the particular case of a weakly bound system ($a^2\approx B/m\ll 1$) the result simplifies to

$$\begin{aligned}F(\alpha)&=\frac{1}{1+\dfrac{\alpha^2}{4B/m}}+O(\alpha^4)\\ &=\frac{1}{1+\dfrac{Q^2}{16\kappa^2}}+O\left[\frac{Q^4}{M^4}\right] \quad (\alpha\ll 1,\ \text{weak binding}) ,\end{aligned} \tag{4.14}$$

where the second equality follows from Eq. (4.10). This agrees with our result for static approximation, cf. Eq. (4.2′).

On the other hand, for large momentum transfers ($\alpha \gg 1$) and again for weakly bound systems ($\bar{B}/m \ll 1$) we obtain

$$F(\alpha)=\frac{4B/m}{\alpha}+O(1/\alpha^2) \quad (\alpha \gg 1, \text{ weak binding}) . \tag{4.15}$$

However, for large values of α relation (4.10) yields $\alpha \approx Q^2/M^2$. This again yields the Q^{-2} falloff. Explicitly, for weak binding ($M \approx 2m$), Eq. (4.15) reads

$$F(Q^2)=\frac{16\kappa^2}{Q^2}+O\left[\frac{M^4}{Q^4}\right] \quad (Q/M^2 \gg 1, \text{ weak binding}) . \tag{4.16}$$

Again, for weakly bound system the large-Q^2 behavior of the full form factor agrees exactly with the predictions of the static approximation.

V. SUMMARY

In the framework of a scalar field theory quantized at equal light-front time we investigated the model in one space and one time dimensions with interaction Lagrangian $L = g\psi^2\phi$. When the exchanged boson becomes infinitely massive the model is exactly solvable by means of simple quadratures. The mass and the wave function of the bound state are calculated and shown to reduce to nonrelativistic results for the case of weakly bound systems. The elastic electromagnetic form factor is also obtained in an analytical form. For weakly bound systems the true form factor reduces exactly to the results based on the static approximation both for small and large momentum transfers. This is not the case for systems with considerable binding—even for small momentum transfers the exact form factor yields the behavior (A7) that differs from the static approximation (4.2) as the control parameter $\bar{B}/m$ increases.

Finally, we point out that in the classical Drell-Yan treatment[11] of the elastic form factor one works in the infinite momentum frame, where not only $q^+=0$ and $\mathbf{q}_\perp \neq 0$, but in addition one always satisfies the condition $\mathbf{q}_\perp \ll P^z$. The last condition allows for an approximate treatment of individual momenta of constituents in the final-state wave function. On the contrary, due to a lack of transversal dimensions in our model we are not only forced to work with $q^+ \neq 0$, but in fact in the light-front approach one deals with finite[8] momenta and an exact treatment of the redistribution of momenta in the final state is found vital to ensure proper results simultaneously for small and large momentum transfers.

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APPENDIX

In this appendix we calculate the form factor defined by Eq. (4.13). Using Eq. (3.1) we obtain

$$F(\alpha)=\frac{N^2}{2+\alpha}[(1+\alpha)^2 I_0(\alpha)-\alpha(1+\alpha)I_1(\alpha) -(1+\alpha)I_2(\alpha)] , \tag{A1}$$

where N^2 is given by Eq. (3.2) and

$$I_n(\alpha)=\int_{-1}^{+1}\frac{x^n}{a^2(1+\alpha)^2+(x+\alpha)^2}\frac{dx}{x^2+a^2} , \quad n=0,1,2 . \tag{A2}$$

The integrals are calculated in a standard way, yielding

$$I_k(\alpha)=\frac{P_k(\alpha)}{2}\ln\frac{(1+a^2)(1+\alpha)^2}{(1+a^2)(1+\alpha)^2-4\alpha} +\frac{R_k(\alpha)-\alpha P_k(\alpha)}{a(1+\alpha)} \times\left[\arctan\frac{1}{a}+\arctan\frac{1-\alpha}{a(1+\alpha)}\right] +2Q_k(\alpha)\frac{1}{a}\arctan\frac{1}{a}, \quad k=0,1 , \tag{A3}$$

where

$$P_0(\alpha)=\frac{2\alpha}{D(\alpha)}, \quad P_1(\alpha)=-Q_0(\alpha) ,$$
$$R_0(\alpha)=\frac{4\alpha^2-w(\alpha)}{D(\alpha)}, \quad R_1(\alpha)=-2\alpha\frac{w(\alpha)+a^2}{D(\alpha)} , \tag{A4}$$
$$Q_0(\alpha)=\frac{w(\alpha)}{D(\alpha)}, \quad Q_1(\alpha)=a^2P_0(\alpha) ,$$

and

$$w(\alpha)=2\alpha a^2+\alpha^2(1+a^2) ,$$
$$D(\alpha)=w^2(\alpha)+4\alpha^2a^2 . \tag{A5}$$

The last integral yields

$$I_2(\alpha)=\frac{1}{a(1+\alpha)}\left[\arctan\frac{1}{a}+\arctan\frac{1-\alpha}{a(1+\alpha)}\right] -a^2I_0(\alpha) \tag{A6}$$

completing the calculation of the form factors.

(i) For small values of α ($\alpha \ll 1$) we expand expressions (A3)–(A6) into the power series and collect the terms up to order α^2. To check the result we expanded the integrand of Eq. (4.13) into a Taylor series in α and integrated term by term, in any case arriving at

$$F(\alpha)=1-\frac{\alpha^2}{2}+\alpha^2\frac{N^2}{4}\left\{-\frac{3}{2(1+a^2)}-\frac{1}{3a^2(1+a^2)^2}-\frac{1}{2a^4(1+a^2)}-\frac{5}{3(1+a^2)^2}+\frac{1}{3a^2(1+a^2)}+\left[\left[\frac{3}{2a}+\frac{1}{a^3}-\frac{1}{2a^5}\right]\arctan\frac{1}{a}\right]\right\}. \tag{A7}$$

For the special case of the weakly bound system ($a^2\approx B/m \ll 1$) this result reduces to the formula (4.14).

(ii) For large values of α ($\alpha \gg 1$) we use the expansions

$$I_0(\alpha)\sim\frac{1}{\alpha^2}\frac{2}{a(1+a^2)}\arctan\frac{1}{a}+O(\alpha^{-3}),$$
$$I_1(\alpha)\sim\frac{1}{\alpha^3}\frac{4}{(1+a^2)^2}\left[-1+a\arctan\frac{1}{a}\right]+O(\alpha^{-4}), \tag{A8}$$

and

$$\left[\arctan\frac{1}{a}+\arctan\frac{1-\alpha}{a(1+\alpha)}\right]\sim\frac{2a}{\alpha(1+a^2)}. \tag{A9}$$

Combining (A8) and (A9), with (A1) and using Eq. (3.2) we observe that the leading behavior is provided by the term (A8):

$$F(\alpha)=\frac{1}{\alpha}\frac{4a}{1+\frac{1-a^2}{a}\arctan\frac{1}{a}}\frac{1}{1+a^2}\arctan\frac{1}{a}+O\left[\frac{1}{\alpha^2}\right]. \tag{A10}$$

In particular, for weak binding ($a^2\approx B/m \ll 1$) one arrives at Eq. (4.15).

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