Excited-state vertices for type-I superstrings

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Requiring conformal invariance and invariance under the G gauges, we construct Lorentz-covariant vertices for the emission of excited states of the type-I superstring.

I. INTRODUCTION

Soon after the understanding of the group-theoretical basis[1] for dual resonance models, Lorentz-covariant vertices were constructed for excited states of the generalized Veneziano model.[2] The basic requirement of such vertices was covariance under the Virasoro generators:

$$[L_N, V(z)] = z^{-N}\left[z\frac{d}{dz} + NJ_S\right]V(z) . \tag{1.1}$$

Covariance under the SU(1,1) subalgebra, $(N=0,\pm 1)$, with arbitrary SU(1,1) spin J_S, was found to be sufficient to ensure the duality property of dual amplitudes provided that all vertices transformed with the same J_S. The Casimir invariant of SU(1,1) is $J_S(J_S+1)$ with J_S negative for the unitary irreducible representations.[3] However for the elimination of negative-norm states, covariance under the full Virasoro algebra (all integer N) was required with the unique conformal spin $J_S=-1$ for each vertex. This, with (1.1), is then seen to be sufficient for the gauges to commute with the integrated vertices

$$V = \oint \frac{dz}{z} V(z) . \tag{1.2}$$

The Veneziano ground-state vertex

$$V_0(z) = :e^{ik\cdot Q(z)}: , \tag{1.3}$$

normal ordered except for the zeroth mode, is covariant with $J_S=-k^2/2$ requiring, therefore, a negative squared mass $m^2=-k^2=-2$ for ghost elimination. The excited-state vertices were obtained by multiplying by certain functions of the conjugate momentum

$$P_\mu = iz\frac{d}{dz}Q_\mu . \tag{1.4}$$

For the superstring the vertices are similarly invariant under the L_N to guarantee ghost elimination,

$$[L_N, V(z)] = z^{-N}\left[z\frac{d}{dz} - N\right]V(z) , \tag{1.5}$$

but in addition the vertices must be obtained by anticommutation with the fermionic gauge operator $G_{1/2}$ to guarantee decoupling of the tachyon:

$$V(z) = z^{1/2}\{G_{1/2}, \tilde{V}(z)\} . \tag{1.6}$$

In Sec. II we review the construction of excited vertices in the Veneziano model using the ϵ limiting procedure to put the zeroth modes on the same basis as the higher modes. In Sec. III we construct the excited vertices of the superstring. In Sec. IV we discuss zero-norm state and results at low-mass levels.

II. EXCITED-STATE VERTICES IN THE VENEZIANO MODEL

In most string-theory calculations, the zeroth modes are treated on a different basis from the higher modes. One writes the familiar string coordinate and momentum as

$$Q_\mu(z) = q_{0\mu} - ip_{0\mu}\ln z + \sum_{n=1}^{\infty} n^{-1/2}(a_\mu^{n\dagger} z^n + a_\mu^n z^{-n}) ,$$
$$P_\mu(z) = iz\frac{d}{dz}Q_\mu(z) = p_{0\mu} + i\sum_{n=1}^{\infty} n^{+1/2}(a_\mu^{n\dagger} z^n - a_\mu^n z^{-n}) . \tag{2.1}$$

The Virasoro generators are then defined through the contour integrals

$$L_N = \frac{1}{2}\oint \frac{dz}{2\pi i z} z^{-N} :P(z)^2: . \tag{2.2}$$

These satisfy

$$[L_N, Q_\mu(z)] = z^{-N} z\frac{d}{dz}Q_\mu(z) \tag{2.3}$$

and

$$[L_N, P_\mu(z)] = z^{-N}\left[z\frac{d}{dz} - N\right]P_\mu(z) . \tag{2.4}$$

In this scheme, the excited-state vertices require a very cumbersome ordering prescription for zeroth modes. For this and several other purposes, therefore, it is convenient to treat the zeroth modes on the same basis as the other oscillators. For instance, in the finiteness proofs of the loop graphs for arbitrary numbers of external particles, the contribution of the loop momentum integration (trace over zeroth modes) must be combined with the correlations of higher modes to produce correlations that are covariant under the modular group. This can be most conveniently accomplished by unifying the translational and vibrational modes. In the group-theoretical analysis, in fact, this unification arises naturally. One defines Q_μ to transform with an infinitesimally small negative SU(1,1) spin $-\epsilon$:

$$[L_N, Q_\mu(z)] = z^{-N}\left[z\frac{d}{dz} - N\epsilon\right]Q_\mu(z) \ . \qquad (2.5)$$

At the end of all calculations, ϵ is taken to zero, although in intermediate stages this is a singular limit. For instance, the string coordinate is

$$Q_\mu(z) = F(a^\dagger, z^{-1}) + F(a,z) \ , \qquad (2.6)$$

where

$$F(a,z) \equiv \sum_{n=0}^{\infty} \gamma_n a_\mu^n z^{-n-\epsilon} \ , \qquad (2.7)$$

$$\gamma_n \equiv [\Gamma(n+2\epsilon)/n!]^{1/2} \ , \qquad (2.8)$$

$$[a_\mu^m, a_\nu^{n\dagger}] = \delta^{mn} g_{\mu\nu}, \quad m,n = 0,1,2,\ldots \ . \qquad (2.9)$$

By expanding (2.6) to first order in $\sqrt{\epsilon}$ and comparing with (1.3), one makes the identification

$$q_{0\mu} = \frac{1}{\sqrt{2\epsilon}}(a_\mu^0 + a_\mu^{0\dagger}), \quad p_{0\mu} = i\sqrt{\epsilon/2}(a_\mu^{0\dagger} - a_\mu^0) \ . \qquad (2.10)$$

The ϵ-dependent term in (2.5) depends therefore on $\sqrt{\epsilon}$ and higher powers of ϵ. In general it is safe to neglect terms of order ϵ and higher. The operator a_μ^n annihilates the vacuum for $n=0$ as well as positive values. We will take z on the unit circle so that $Q(z) = F^\dagger + F$. The vertex of (1.3) can then be written

$$V_0(z) = e^{ik\cdot F^\dagger} e^{ik\cdot F} e^{k^2/4\epsilon} \ . \qquad (2.11)$$

F and $F^\dagger$ vary as $\epsilon^{-1/2}$ as $\epsilon \to 0$, but, if the zeroth modes of (2.11) are brought into a single exponential, the $k^2/4\epsilon$ term cancels and all ϵ dependence can be subsumed into $q_{0\mu}$ and $p_{0\mu}$. The commutator with L_N is then, using (2.5),

$$[L_N, V_0(z)] = z^{-N} z\frac{d}{dz} V_0(z) - N\epsilon e^{ik\cdot F^\dagger} ik\cdot Q e^{ik\cdot F} e^{k^2/4\epsilon} \ . \qquad (2.12)$$

After the two exponentials have been brought together, it is safe to neglect terms of order $\sqrt{\epsilon}$ and higher:

$$e^{ik\cdot F^\dagger}\epsilon ik\cdot Q e^{ik\cdot F} = (\epsilon ik\cdot Q + k^2/2) e^{ik\cdot F^\dagger} e^{ik\cdot F}$$
$$= (k^2/2) e^{ik\cdot F^\dagger} e^{ik\cdot F} \ . \qquad (2.13)$$

Thus

$$[L_N, V_0(z)] = z^{-N}\left[z\frac{d}{dz} - Nk^2/2\right]V_0 \ . \qquad (2.14)$$

If we operate with $iz\, d/dz$ on (2.5), we have

$$[L_N, P_\mu(z)] = z^{-N}\left[z\frac{d}{dz} - N\right]\left[z\frac{d}{dz} - N\epsilon\right] iQ_\mu(z)$$
$$= z^{-N}\left[z\frac{d}{dz} - N(1+\epsilon)\right]P_\mu(z)$$
$$+ z^{-N} N^2 i\epsilon Q_\mu(z) \ . \qquad (2.15)$$

Equation (2.13) shows that, when sandwiched between exponentials $e^{ik\cdot F^\dagger}$ and $e^{ik\cdot F}$, $i\epsilon Q_\mu$ is equivalent to $k_\mu/2$. Thus the effective commutator is

$$[L_N, P_\mu(z)] \simeq Z^{-N}\left[z\frac{d}{dz} - N\right]P_\mu(z) + z^{-N}N^2 k_\mu/2 \qquad (2.16)$$

and

$$[L_N, :V_0\zeta\cdot P:] = z^{-N}\left[z\frac{d}{dz} - N(1+k^2/2)\right]:V_0\zeta\cdot P:$$
$$+ V_0 z^{-N} N^2 \zeta\cdot k/2 \ . \qquad (2.17)$$

Virasoro covariance requires that the polarization ζ_μ be transverse to k_μ. Elimination of negative-norm states requires that $1+k^2/2=1$ or that this vertex describes a massless vector meson. A Lorentz-spin J particle on the leading trajectory is described by the vertex

$$V_{1,J}(z) = :V_0 P^J: \ . \qquad (2.18)$$

Here, and in the following, suppressed Lorentz indices are taken to be transverse. Covariance of the normal-ordered vertex is ensured by contracting with a traceless polarization tensor. Vertices for particles on daughter trajectories are constructed by contracting transverse indices in (2.18) and[2] by using the following $S_\mu^{(n)}$:

$$S_\mu^{(1)} \equiv P_\mu \ , \qquad (2.19)$$

$$S_\mu^{(n+1)} \equiv \left[z\frac{d}{dz} - \frac{2n}{k^2}k\cdot P\right]S_\mu^{(n)} \ . \qquad (2.20)$$

It can be proven by induction that $S^{(n)}$ transforms with conformal spin, $-n$, so that

$$V(z) = V_0 \prod_{n=1}^{\infty} (S^{(n)})^{\lambda_n} \qquad (2.21)$$

corresponds to a state in the Veneziano spectrum with $k^2/2 + \sum n\lambda_n = 1$ providing that the normal ordering does not destroy the Virasoro covariance. The problem of maintaining covariance after normal ordering has not as yet been solved on deep daughter trajectories and, as we will see, the problem recurs in the case of the superstring. One cannot construct longitudinal $S^{(n)}$ that are covariant under the full Visaroro algebra although it is possible if one requires only SU(1,1) covariance $(N=0,\pm 1)$:

$$[L_N, k\cdot S^{(2)}+k\cdot\dot{P}+k^2/2]$$
$$=z^{-N}\left[z\frac{d}{dz}-2N\right](k\cdot S^{(2)}+k\cdot\dot{P}+k^2/2)$$
$$+z^{-N}\frac{k^2}{2}N(1-N^2)\ . \qquad (2.22)$$

Having used this brief review of known results to define our techniques, we may proceed to construct the superstring excited vertices.

III. EXCITED STATE VERTICES FOR TYPE-I SUPERSTRINGS

The realization of the Virasoro generators that satisfy (2.5) *et seq* is

$$L_N=\frac{1}{2}\oint\frac{dz}{2\pi iz}z^{-N}\left[(P_\mu-Ni\epsilon Q_\mu)^2-H_\mu z\frac{d}{dz}H_\mu\right]. \qquad (3.1)$$

The corresponding G gauges, for half odd integer r, are

$$G_r=\oint\frac{dz}{2\pi iz}z^{-r}[P_\mu(z)-2ri\epsilon Q_\mu(z)]H_\mu(z)\ , \qquad (3.2)$$

where $H_\mu(z)=\sum(b_\mu^{s\dagger}z^s+b_\mu^s z^{-s})$, summing s over positive half odd integers. To the requisite order $\sqrt{\epsilon}$, taking Q_μ to behave as $\epsilon^{-1/2}$ near $\epsilon=0$, it is easy to show that these operators satisfy the usual algebra:

$$[L_N,L_M]=(M-N)L_{M+N}+c\delta_{M+N,0}\ , \qquad (3.3)$$

$$\{G_r,G_s\}=2L_{r+s}+b\delta_{r+s,0}\ , \qquad (3.4)$$

$$[L_N,G_s]=\left[s-\frac{N}{2}\right]G_{N+s}\ . \qquad (3.5)$$

For the superstring we require that the physical state vertices V be covariant under the Virasoro algebra with $J_S=-1$ to eliminate ghosts and that they be obtained by anticommuting some $\tilde{V}$ with $G_{1/2}$ to eliminate tachyons:

$$[L_N,V]=z^{-N}\left[z\frac{d}{dz}-N\right]V\ , \qquad (3.6)$$

$$V=z^{1/2}\{G_{1/2},\tilde{V}\}\ . \qquad (3.7)$$

Consider the following lemma.

Lemma 1: If $\tilde{V}$ is Virasoro covariant with conformal spin J_S, i.e.,

$$[L_N,\tilde{V}]=z^{-N}\left[z\frac{d}{dz}+NJ_S\right]\tilde{V}\ ,$$

and if $\tilde{V}$ is also covariant under the fermionic gauges, i.e.,

$$\{G_r,\tilde{V}\}=z^{-r}V\ , \qquad (3.8)$$

with V being independent of r, then V is also Virasoro covariant with conformal spin $J_S-\frac{1}{2}$.

To prove Lemma 1 we use (3.5) to write

$$[L_N,V]=z^r[L_N,\{G_r,\tilde{V}\}]=z^r\{[L_N,G_r],\tilde{V}\}+z^r\{G_r,[L_N,\tilde{V}]\}$$
$$=z^r(r-N/2)\{G_{N+r},\tilde{V}\}+z^rz^{-N}\left[z\frac{d}{dz}+NJ_S\right]\{G_r,\tilde{V}\}$$
$$=z^{-N}(r-N/2)z^r\{G_r,\tilde{V}\}+z^{-N}\left[z\frac{d}{dz}-r+NJ_S\right]z^r\{G_r,\tilde{V}\}$$
$$=z^{-N}\left[z\frac{d}{dz}+N(J_S-\tfrac{1}{2})\right]V\ . \qquad (3.9)$$

The problem reduces therefore to finding the covariant $\tilde{V}$ vertices with $J_S=-\frac{1}{2}$, whereupon the excited state vertices V are obtained from (3.7). We begin by recording the effective anticommutators (when sandwiched by the ordered plane wave)

$$\{G_r,Q_\mu\}=-iz^{-r}H_\mu\ , \qquad (3.10)$$

$$\{G_r,H_\mu\}=z^{-r}(P_\mu-rk_\mu)\ , \qquad (3.11)$$

$$[L_N,H_\mu]=z^{-N}\left[z\frac{d}{dz}-N/2\right]H_\mu\ , \qquad (3.12)$$

$$z^{1/2}\{G_{1/2},:V_0H:\}=:V_0(P+k\cdot HH):\ . \qquad (3.13)$$

The right-hand side defines the vertex of the massless vector state of the superstring, and one can see again that gauge covariance requires Lorentz transversality. To go up the leading trajectory, it is not sufficient to multiply by powers of P as in the Veneziano model since P by itself is not covariant under the G gauge. Acting on (3.10) with $z\ d/dz$ represented by an overdot, we get

$$\{G_r,P_\mu\}=z^{-r}(\dot{H}_\mu-rH_\mu)\ . \qquad (3.14)$$

To cancel the r-dependent term we form the combination

$$S_\mu^{(1)}=P_\mu-k\cdot HH_\mu/k^2\ . \qquad (3.15)$$

In the limit of vanishing H, $S_\mu^{(1)}$ reduces to the Veneziano model $S_\mu^{(1)}$ of (2.19). The transverse part of $S^{(1)}$ is Virasoro and G gauge covariant:

$$[L_N,S^{(1)}]=z^{-N}\left[z\frac{d}{dz}-N\right]S^{(1)}, \tag{3.16}$$

$$[G_r,S^{(1)}]=z^{-r}\left[\dot{H}-\frac{k\cdot PH}{k^2}+\frac{k\cdot HP}{k^2}\right]. \tag{3.17}$$

The superstring vertex for the spin-J particle on the leading trajectory is, therefore,

$$\begin{aligned}V_J&=z^{1/2}\{G_{1/2},:V_0HS^{(1)(J-1)}:\}\\&=z^{1/2}\{G_{1/2},:V_0H[P^{J-1}-(J-1)k\cdot HHP^{J-2}/k^2]:\}\\&=:V_0[(P+k\cdot HH)P^{J-1}-(J-1)H\dot{H}P^{J-2}]:.\end{aligned} \tag{3.18}$$

Higher than first powers of the anticommuting fields vanish inside the normal-ordered product. One may check explicitly the covariance of this V_J under the Virasoro gauges. These leading trajectory vertices agree with the recent results of Yamamoto.[4]

To construct vertices for the daughter trajectories, we need generalizations of the $H\equiv H^{(1)}$ and $S^{(1)}$ to some $H^{(n)}$ and $S^{(n)}$. It is convenient to introduce the intermediate field

$$R_\mu^{(n)}=z^r\{G_r,H_\mu^{(n)}\} \tag{3.19}$$

in terms of which we will define

$$S_\mu^{(n)}=R_\mu^{(n)}-(2n-1)[k\cdot H,H_\mu^{(n)}]/2k^2 \tag{3.20}$$

with $R^{(1)}=P$. The transverse $R^{(n)}$ are Virasoro covariant by Lemma 1 but not G gauge covariant. The $S^{(n)}$ are constructed to be Virasoro and G gauge covariant. We will show that the required recursion relations are

$$H^{(n+1)}=\dot{H}^{(1)}-\frac{1}{2k^2}\{k\cdot H,R^{(n)}\}-\frac{2n-1}{k^2}k\cdot PH^{(n)}, \tag{3.21}$$

$$R^{(n+1)}=\dot{R}^{(n)}-\frac{2n}{k^2}k\cdot PR^{(n)}+\frac{1}{2k^2}[k\cdot H,\dot{H}^{(n)}]-\frac{2n-1}{2k^2}[k\cdot\dot{H},H^{(n)}]. \tag{3.22}$$

We assume that for some n, $H^{(n)}$ and $R^{(n)}$ satisfy

$$\{G_r,H^{(n)}\}=z^{-r}R^{(n)}, \tag{3.23}$$

$$[L_N,H^{(n)}]=z^{-N}\left[z\frac{d}{dz}-N(n-\tfrac{1}{2})\right]H^{(n)}, \tag{3.24}$$

$$[L_N,R^{(n)}]=z^{-N}\left[z\frac{d}{dz}-Nn\right]R^{(n)}. \tag{3.25}$$

We have shown that these equations are valid for $n=1$. Taking the $z\,d/dz$ derivative of both sides of Eqs. (3.24) and (3.25), one has

$$[L_N,\dot{H}^{(n)}]=z^{-N}\left[z\frac{d}{dz}-N(n+\tfrac{1}{2})\right]\dot{H}^{(n)}+z^{-N}N^2(n-\tfrac{1}{2})H^{(n)}, \tag{3.26}$$

$$[L_N,\dot{R}^{(n)}]=z^{-N}\left[z\frac{d}{dz}-N(n+1)\right]\dot{R}^{(n)}+z^{-N}N^2nR^{(n)}. \tag{3.27}$$

From (2.16), (3.24), and (3.25) one has

$$\left[L_N,\frac{k\cdot P}{k^2}H^{(n)}\right]=z^{-N}\left[z\frac{d}{dz}-N(n+\tfrac{1}{2})\right]\frac{k\cdot P}{k^2}H^{(n)}+z^{-N}\frac{N^2}{2}H^{(n)} \tag{3.28}$$

and

$$\left[L_N,\frac{k\cdot P}{k^2}R^{(n)}\right]=z^{-N}\left[z\frac{d}{dz}-N(n+1)\right]\frac{k\cdot P}{k^2}R^{(n)}+z^{-N}\frac{N^2}{2}R^{(n)}. \tag{3.29}$$

From (3.12), (3.25), and (3.26) one has

$$\left[L_N,\left[\frac{k\cdot H}{2k^2},\dot{H}^{(n)}\right]\right]=z^{-N}\left[z\frac{d}{dz}-N(n+1)\right]\left[\frac{k\cdot H}{2k^2},\dot{H}^{(n)}\right]+\frac{z^{-N}N^2(n-\tfrac{1}{2})}{2k^2}[k\cdot H,H^n] \tag{3.30}$$

and

$$\left[L_N,\left\{\frac{k\cdot H}{2k^2},R^{(n)}\right\}\right]=z^{-N}\left[z\frac{d}{dz}-N(n+\tfrac{1}{2})\right]+\left\{\frac{k\cdot H}{2k^2},R^{(n)}\right\}. \tag{3.31}$$

From (3.24) and (3.26) one has

$$\left[L_N,\frac{k\cdot\dot{H}}{k^2}H^{(n)}\right]=z^{-N}\left[z\frac{d}{dz}-N(N+1)\right]\frac{k\cdot\dot{H}}{k^2}H^{(n)}+z^{-N}N^2(n-\tfrac{1}{2})\frac{k\cdot H}{k^2}H^{(n)}. \tag{3.32}$$

Combining (3.26)–(3.32) we see that, having assumed Virasoro covariance at the nth level, we have it also at the $(n+1)$th level:

$$[L_N,H^{(n+1)}]=z^{-N}\left[z\frac{d}{dz}-N(n+\tfrac{1}{2})\right]H^{(n+1)}, \tag{3.33}$$

$$[L_N,R^{(n+1)}]=z^{-N}\left[z\frac{d}{dz}-N(n+1)\right]R^{(n+1)}. \tag{3.34}$$

The proof of (3.23) for all n is similarly proven inductively. One can then prove the G gauge covariance of $S^{(n)}$ for all n:

$$[G_r, S^{(n)}] = z^{-r}\left\{\dot{H}^{(n)} - (2n-1)\frac{k\cdot P}{k^2}H^{(n)} + (2n-1)\left\{\frac{k\cdot H}{2k^2}, R^{(n)}\right\}\right\}$$
$$= z^{-r}\left\{H^{(n+1)} + \frac{n}{k^2}\{k\cdot H, R^{(n)}\}\right\} . \quad (3.35)$$

Modulo further constraints that arise from normal ordering that are as yet unsolved in general, the excited bosonic state vertex for emission from a bosonic line in the type-I superstring is therefore, with non-negative integer λ_n and λ'_n:

$$V(z, \{\lambda_n, \lambda'_m\}) = z^{1/2}\left\{G_{1/2}, :V_0 \prod_n (H^{(n)})^{\lambda_n} \prod_m (S^{(m)})^{\lambda'_m}:\right\} . \quad (3.36)$$

The fermionic nature of $H^{(n)}$ requires that λ_n be less than or equal to 9 with multiple H's antisymmetrized. The even G parity of the physical states requires that $\sum \lambda_n$ be odd. We refer to the Lorentz spin J of the state as the number of symmetrized transverse indices as would be the case in $D=4$,

$$J = \sum (\lambda_n + \lambda'_n) \quad (3.37)$$

and the squared mass is

$$m^2/2 = -\tfrac{1}{2} + \sum_{n=1}^{\infty} [(n-\tfrac{1}{2})\lambda_n + n\lambda'_n] . \quad (3.38)$$

To emit the excited states from fermion lines, one replaces H_μ by $\Gamma_\mu/(i\sqrt{2})$. On the second trajectory one has, *a priori*, a twofold degeneracy:

$$V = z^{1/2}\{G_{1/2}, :V_0 H^{(1)} S^{(2)} (S^{(1)})^{J-2}:\}, \quad J \geq 2 , \quad (3.39)$$

$$V' = z^{1/2}\{G_{1/2}, :V_0 H^{(2)} S^{(1)(J-1)}:\}, \quad J \geq 1 . \quad (3.40)$$

However, V' is a perfect derivative and therefore decouples:

$$V' = -\frac{1}{k^2} z \frac{d}{dz} :V_0 (2P + k\cdot HH) P^{J-1}: . \quad (3.41)$$

The same phenomenon occurs in the Veneziano model, the second trajectory being, in that case, empty. For the superstring, however, the vertex of (3.39) provides a physical state on the first daughter trajectory for $J \geq 2$. The decoupling of excited states will be further discussed in the next section. On the second daughter trajectory one has vertices

$$V'' = z^{1/2}\{G_{1/2}, :V_0 (H^{(1)})^3 (S^{(1)})^J:\}, \quad J \geq 0 . \quad (3.42)$$

The lowest state of this type corresponds to spin $J=0$ in the four-dimensional sense: a product of three antisymmetrized, transverse four-vectors. The vertices of (3.18) and (3.42) agree with those of Ref. 4, although we would identify them with states on the parent and second daughter trajectory, the first daughter trajectory states being given by (3.39).

IV. EXPLICIT EXCITED STATE VERTICES

It is useful to explicitly compare the covariant vertices at low levels with the spectrum of states as given in the light-cone analysis.[5] To get the correct counting, one must eliminate zero-norm states from the covariant vertices constructed in Sec. III. Suppose a superstring vertex V is related to a vertex $\tilde{V}$ by (3.7) and $\tilde{V}$ itself is given by

$$\tilde{V} = z^{1/2}[G_{1/2}, \tilde{\tilde{V}}] . \quad (4.1)$$

If $\tilde{\tilde{V}}$ transforms with $J_S = 0$, then by Lemma 1, $\tilde{V}$ transforms with $J_S = -\frac{1}{2}$, and V transforms with $J = -1$ as required for ghost elimination. However, in this case

$$z^{1/2}\{G_{1/2}, \tilde{V}\} = z[(G_{1/2})^2, \tilde{\tilde{V}}] = z[L_{+1}, \tilde{\tilde{V}}] = z\frac{d}{dz}\tilde{\tilde{V}} . \quad (4.2)$$

The corresponding physical state is therefore a zero-norm state and decouples. The rule for constructing the zero-norm states of the superstring is to drop back $\frac{1}{2}$ unit in conformal weight and commute with $G_{1/2}$ to find the decoupling $\tilde{V}$.

At the first excited level ($m^2=2$), the covariant vertices together with their SO(9) content are as follows.

Physical states, $\tilde{V}$:

$V_0 H^{(2)}$	9
$V_0 H^{(1)} S^{(1)}$	$9^2 - 1 = 36 + 44$
$V_0 H^{(1)} H^{(1)} H^{(1)}$	$\frac{9!}{3!6!} = 84$

Zero-norm states, $\tilde{\tilde{V}}$:

$V_0 S^{(1)}$	9
$V_0 H^{(1)} H^{(1)}$	$\frac{9!}{2!7!} = 36$

Multiplication by V_0 is intended to mean multiplication from the left by $\exp(ik\cdot F^\dagger)$ and from the right by $\exp(ik\cdot F + k^2/4\epsilon)$. The above covariant vertices with the appropriate trace subtractions are then equal to themselves normal ordered. Products of $H^{(1)}$ are of course antisymmetrized to survive normal ordering. At this level, the normal-ordered vertices are separately covariant after trace subtraction. The spectrum at the first level agrees with the light-cone analysis ($128 = 44 + 84$). Each of the above vertices generates vertices for more massive particles through multiplication by products of $S^{(1)}$.

At the second level, ($m^2=4$), one has the following.

Physical states, $\tilde{V}$:

$V_0 H^{(3)}$	9
$V_0 \{H^{(2)}, S^{(1)}\}$	$9^2 - 1 = 36 + 44$
$V_0 H^{(2)} H^{(1)} H^{(1)}$	$9\times 36 = 9 + 84 + 231$
$V_0 H^{(1)} S^{(2)}$	$9^2 - 1 = 36 + 44$
$V_0 H^{(1)} S^{(1)} S^{(1)}$	$9\times 44 = 9 + 156 + 231$
$V_0 H^{(1)} H^{(1)} H^{(1)} S^{(1)}$	$9\times 84 - 36 = 126 + 594$
$V_0 (H^{(1)})^5$	$\frac{9!}{5!4!} = 126$

Zero-norm states, $\tilde{\tilde{V}}$:

$V_0S^{(2)}$	9
$V_0S^{(1)}S^{(1)}$	$\frac{9\times 10}{2}-1=44$
$V_0H^{(1)}H^{(1)}S^{(1)}$	$9\times 36=9+84+231$
$V_0(H^{(1)})^4$	$\frac{9!}{4!5!}=126$
$V_0H^{(2)}H^{(1)}$	$9^2-1=36+44$

If we subtract the zero-norm-state representations from the physical state representations, we obtain the SO(9) representations found in the light-cone gauge:[5] $9+36+126+156+231+594$. However, the recombinations of these vertices and trace constraints necessary to maintain Virasoro covariance after normal ordering are not at present understood and further work is needed to present a general prescription here and at higher levels.

The mass shift results of Ref. 4 depend on the fact that the vertices considered there involved only $S^{(1)}$ and $H^{(1)}$. We suspect, therefore, that the speculation of unbroken mass degeneracy to one-loop order will not be confirmed on lower daughter trajectories.

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