Hamiltonian formulation of a gauge-invariant massive spin-$\frac{3}{2}$ theory

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A gauge-invariant theory describing a free massive spin-$\frac{3}{2}$ particle, obtained previously by dimensional reduction of a massless theory, is quantized using Dirac's procedure. The quantum theory is shown to be free from negative-norm states despite the absence of constraints obtained from differentiating the Lagrangian equations of motion (i.e., "secondary constraints"). This is in direct contrast to all other half-integral-spin theories avoiding secondary constraints, which invariably have indefinite metric.

I. INTRODUCTION

The study of theories for higher-spin particles has had a long and interesting history, mainly due to the presence of inconsistencies in the interacting theories for such particles.[1] Of the successful attempts made to solve the problem of inconsistencies, one of considerable interest is that of supergravity, incorporating an interacting theory of spin-$\frac{3}{2}$ and spin-2 particles, later extended to include spin-1 particles as well.[2] Dimensional reduction as a procedure for obtaining a consistent theory of massive higher-spin particles has been suggested recently.[3,4] In particular, causality of classical propagation modes has been shown for spin-1 and spin-$\frac{3}{2}$ particles in interacting theories obtained by dimensional reduction.[5] That such a spin-$\frac{3}{2}$ theory can be quantized in a manner free of the usual inconsistencies anticipated in half-integral higher-spin theories[6] as yet remains to be shown.

The procedure of dimensional reduction from five to four dimensions, when applied to gauge-invariant massless theories, gives rise to gauge-invariant massive theories.[3] This feature of gauge invariance, which in the case of massless supergravity renders the theory consistent, is expected to play a useful role also in the case of massive theories. The concomitant feature of the absence of secondary constraints (i.e., constraints obtained by differentiating the Lagrangian equations of motion), occurring in these massive theories, may be desirable, as it can lead to a causal classical theory.[7] These features are present in the free as well as the interacting theories.[5]

At the same time, theories describing half-integral spin in which secondary constraints are absent have invariably been found to have negative-norm states, even in the absence of interactions,[8] in agreement with a theorem by Johnson and Sudarshan.[6] All these theories, moreover, are multispin theories. Any procedure mainly developed to avoid secondary constraints in the Rarita-Schwinger theory for spin-$\frac{3}{2}$ particles also leads not only to an indefinite metric, but also to multispin equations.[9]

It is therefore of great interest to investigate if the massive spin-$\frac{3}{2}$ theory obtained by dimensional reduction leads to a quantum theory with positive-definite norm, and whether the constraint structure is consistent with a unique spin. Though our ultimate aim is to study the physically more interesting case of interacting spin-$\frac{3}{2}$ particles, it is of sufficient interest to investigate even the free massive theory for the above-mentioned reasons.

In this paper we study the quantization by Dirac's procedure[10] of the free massive spin-$\frac{3}{2}$ theory[3] obtained by dimensional reduction of the massless Rarita-Schwinger theory. Although the Dirac quantization of the massive spin-$\frac{3}{2}$ theory has been discussed in the past,[11] our theory has the new feature of gauge invariance, which in Dirac's formulation leads to first-class constraints. This, together with the absence of secondary constraints (which in our Hamiltonian formulation would be seen as the absence of tertiary constraints), makes the study interesting. We find the significant result that despite the absence of secondary constraints, there are no negative-norm states. Moreover, unlike in the earlier examples[9] which avoided secondary constraints at the cost of having multiple spins, our theory describes a unique spin, viz., $\frac{3}{2}$.

II. CONSTRAINT STRUCTURE OF THE SPIN-$\frac{3}{2}$ THEORY

The free massive spin-$\frac{3}{2}$ particle is described by a Lagrangian which we obtained in Ref. 3 by dimensionally reducing the gauge-invariant Rarita-Schwinger Lagrangian for a massless spin-$\frac{3}{2}$ particle in five dimensions, and retaining only a single massive mode. Without reiterating this procedure here, we start with the Lagrangian obtained in Ref. 3:

$$\mathcal{L} = -i\bar{\psi}_\mu\gamma^{\mu\nu\lambda}\partial_\nu\psi_\lambda - i\bar{\psi}_\mu\gamma^{\mu\nu}\partial_\nu\phi - i\bar{\phi}\gamma^{\mu\nu}\partial_\mu\psi_\nu - m\bar{\psi}_\mu\gamma^{\mu\nu}\psi_\nu \,, \tag{1}$$

where ψ_μ is the vector-spinor representing the spin-$\frac{3}{2}$ field and ϕ is an auxiliary ("Stueckelberg") spinor field necessary for gauge invariance. Our notation is that greek suffixes represent the four-dimensional Lorentz indices, and in what follows, latin suffixes (taking values 1,2,3) will be used for three-dimensional space indices. The metric used is $(+,-,-,-)$. A γ with two or more indices denotes an antisymmetric product of that number of Dirac γ matrices.

The Lagrangian in (1) is invariant under the gauge transformations

$$\psi_\mu(x)\to\psi'_\mu(x)=\psi_\mu(x)+\partial_\mu\epsilon(x)\ ,$$
$$\phi(x)\to\phi(x)'=\phi(x)+im\,\epsilon(x)\ , \tag{2}$$

where $\epsilon(x)$ is an infinitesimal parameter. At the classical level, one can choose a gauge in which $\phi(x)=0$ leading to the standard Lagrangian for a massive spin-$\frac{3}{2}$ particle.

To study the Hamiltonian approach, we use Casalbuoni's modifications[12] of the definitions of the canonical momenta and Poisson brackets, suitable to a theory with anticommuting field variables.

Starting with the Lagrangian (1), we first determine the canonical momenta $\bar\pi_\mu$, π_μ, $\bar\pi$, and π conjugate, respectively, to ψ^μ, $\bar\psi^\mu$, ϕ, and $\bar\phi$:

$$\bar\pi_\mu\equiv\frac{\partial^l\mathcal{L}}{\partial\dot\psi^\mu}=i\bar\psi^\lambda\gamma_{\lambda0\mu}+i\bar\phi\gamma_{0\mu}\ , \tag{3}$$

$$\pi_\mu\equiv\frac{\partial^l\mathcal{L}}{\partial\dot{\bar\psi}^\mu}=0\ , \tag{4}$$

$$\bar\pi\equiv\frac{\partial^l\mathcal{L}}{\partial\dot\phi}=i\bar\psi^\mu\gamma_{\mu0}\ , \tag{5}$$

$$\pi\equiv\frac{\partial^l\mathcal{L}}{\partial\dot{\bar\phi}}=0\ , \tag{6}$$

where ∂^l/∂ denotes the "left" derivative.[12]

As this is a theory with only first-order time derivatives, the conjugate momenta define primary constraints:

$$\bar f_\mu\equiv\bar\pi_\mu-i\bar\psi^\lambda\gamma_{\lambda0\mu}-i\bar\phi\gamma_{0\mu}=0\ , \tag{7}$$

$$f_\mu\equiv\pi_\mu=0\ , \tag{8}$$

$$\bar f\equiv\bar\pi-i\bar\psi^\mu\gamma_{\mu0}=0\ , \tag{9}$$

$$f\equiv\pi=0\ . \tag{10}$$

The Hamiltonian density obtained from (1) using (3)–(6) is

$$\mathcal{H}\equiv-\bar\pi_\mu\dot\psi^\mu+\dot{\bar\psi}^\mu\pi_\mu-\bar\pi\dot\phi+\dot{\bar\phi}\pi-\mathcal{L}$$
$$=i\bar\psi_\mu\gamma^{\mu i\nu}\partial_i\psi_\nu+i\bar\psi_\mu\gamma^{\mu i}\partial_i\phi+i\bar\phi\gamma^{i\mu}\partial_i\psi_\mu+m\bar\psi_\mu\gamma^{\mu\nu}\psi_\nu\ . \tag{11}$$

In view of the constraints (7)–(10), the full Hamiltonian density is given by

$$\mathcal{H}'=\mathcal{H}-\bar f_\mu v^\mu+\bar v^\mu f_\mu-\bar f v+\bar v f\ , \tag{12}$$

where v^μ, $\bar v^\mu$, v, and $\bar v$ are undetermined Lagrange-multiplier spinors. The overbars on the v's are as yet just notation, and do not imply Dirac conjugation, though it will turn out to be so.

We would now like to know under what conditions the constraints (7)–(10) are unchanged under time evolution. These conditions are obtained by setting to zero the Poisson brackets of the constraints with the Hamiltonian $H'=\int d^3x\,\mathcal{H}'$. We have to be careful in defining Poisson brackets for our fermionic field variables as mentioned earlier. The appropriate definition is[12]

$$\{f_1,f_2\}=-\sum_i\left[\frac{\partial^v f_1}{\partial P_i}\frac{\partial^l f_2}{\partial q_i}\mp\frac{\partial^v f_2}{\partial P_i}\frac{\partial^l f_1}{\partial q_i}\right]\ , \tag{13}$$

where the lower sign is used if both f_1 and f_2 are fermionic, and the upper sign in all other cases.

We find that $\dot f_i=0$, $\dot{\bar f}_i=0$, $\dot f=0$, and $\dot{\bar f}=0$ lead to the determination of the multipliers v^i, $\bar v^i$, v, $\bar v$, whereas $\dot f_0=0$ and $\dot{\bar f}_0=0$ lead to new constraints. The equation $\dot f_i=0$ gives

$$-i\gamma^0(\gamma^i{}_j v^j+\gamma^i v)=-i\gamma^{ij\mu}\partial_j\psi_\mu-i\gamma^{ij}\partial_j\phi-m\gamma^{i\mu}\psi_\mu\ , \tag{14}$$

whereas $\dot f=0$ leads to

$$i\gamma^0\gamma_i v^i=i\gamma^{i\mu}\partial_i\psi_\mu\ . \tag{15}$$

Equations (14) and (15) serve to determine v^i and v. The condition $\dot f_0=0$ gives the secondary constraint

$$g\equiv-i\gamma^{ij}\partial_i\psi_j-i\gamma^i\partial_i\phi-m\gamma^i\psi_i=0\ . \tag{16}$$

Incidentally, this would have been the primary constraint in a Lagrangian formulation.

Equating the time derivatives of $\bar f_i$ and $\bar f$ to zero leads to the equations obtained by taking the Hermitian conjugate of both sides of (14) and (15), respectively, with the identification

$$v^{i\dagger}=\bar v^i\gamma^0,\quad v^\dagger=\bar v\gamma^0\ . \tag{17}$$

Thus, henceforth, we need not worry about the quantities $\bar v^i$ and $\bar v$; they may be obtained from v^i and v by the standard Dirac conjugation. Finally, equating $\dot{\bar f}_0$ to zero gives a constraint, which we call $\bar g$, and which is obtained by taking the Dirac conjugate of both sides of Eq. (16).

Equations (14) and (15) determine v^i and v to be

$$v^i=\gamma^0\left[-\frac{i}{3}\gamma^i g-\gamma^j\partial_j\psi^i+\partial^i\gamma\cdot\psi+\partial^i\phi-im\psi^i\right]\ , \tag{18}$$

$$v=\gamma^0\left[\frac{i}{3}g-\gamma^j\partial_j\phi+im\gamma\cdot\psi\right]\ . \tag{19}$$

We should now investigate if the constraints $g=0$ and $\bar g=0$ are time independent. Evaluating the Poisson brackets $\{g,H'\}$ and $\{\bar g,H'\}$, and using the above expressions for v^i and v, we find after some algebra that the brackets indeed vanish, imposing no further constraints. There are thus no tertiary constraints (equivalent to the absence of secondary constraints in the Lagrangian formulation). Also, the Lagrange multipliers v^0 and $\bar v^0$ remain undetermined predicting the first-class nature of the constraints f_0 and $\bar f_0$.

The constraints on the system are the primary constraints f_μ, $\bar f_\mu$, f, $\bar f$, and the secondary constraints g and $\bar g$. An examination of the Poisson brackets among these constraints reveals that f_0 and $\bar f_0$ are indeed first class, with vanishing Poisson brackets with each other, as well as with all the remaining constraints. The other Poisson brackets are as follows (of course, only Poisson brackets of barred quantities with unbarred quantities are nonzero):

$$\{f_i^\alpha(\mathbf{x},t),\bar f_j^\beta(\mathbf{y},t)\}=-i(\gamma^0\gamma_{ij})^{\alpha\beta}\delta^{(3)}(x-y)\ , \quad (20a)$$

$$\{f_i^\alpha(\mathbf{x},t),\bar f^\beta(\mathbf{y},t)\}=-i(\gamma^0\gamma_i)^{\alpha\beta}\delta^{(3)}(x-y)\ , \quad (20b)$$

$$\{f^\alpha(\mathbf{x},t),\bar f_j^\beta(\mathbf{y},t)\}=i(\gamma^0\gamma_j)^{\alpha\beta}\delta^{(3)}(x-y)\ , \quad (20c)$$

$$\{f^\alpha(\mathbf{x},t),\bar f^\beta(\mathbf{y},t)\}=0\ , \quad (20d)$$

$$\{g^\alpha(\mathbf{x},t),\bar f_i^\beta(\mathbf{y},t)\}=(i\gamma^j{}_i\partial_j+m\gamma_i)^{\alpha\beta}\delta^{(3)}(x-y)\ , \quad (20e)$$

$$\{g^\alpha(\mathbf{x},t),\bar f^\beta(\mathbf{y},t)\}=i(\gamma^i)^{\alpha\beta}\partial_i\delta^{(3)}(x-y)\ , \quad (20f)$$

$$\{g^\alpha(\mathbf{x},t),\bar g^\beta(\mathbf{y},t)\}=0\ . \quad (20g)$$

In these equations, α,β denote Dirac indices. The remaining Poisson brackets are obtained by taking complex conjugates of Eqs. (20). Even though g and $\bar g$ do not have nonvanishing Poisson brackets with the other constraints, we notice that the combinations

$$h=\gamma^0 g+\partial^i f_i+imf \quad (21)$$

and

$$\bar h=\bar g\gamma^0-\partial^i\bar f_i+im\bar f \quad (22)$$

have zero Poisson brackets with all the other constraints, and with each other. Hence $h=0$ and $\bar h=0$ are first-class constraints.

We have therefore the first-class constraints f_0, $\bar f_0$, h, and $\bar h$, and the second-class constraints f_i, $\bar f_i$, f, and $\bar f$. The first-class constraints generate gauge transformations:

$$\delta_\epsilon\psi^\mu(x)\equiv\left\{\int d^3y\,\bar h(y)\epsilon(y),\psi^\mu(x)\right\}$$
$$=-\partial^i\epsilon(x)\delta_i^\mu\ ,$$
$$\delta_\epsilon\phi(x)\equiv\left\{\int d^3y\,\bar h(y)\epsilon(y),\phi(x)\right\}$$
$$=-im\epsilon(x)\ , \quad (23)$$
$$\delta_{\epsilon'}\psi^\mu(x)\equiv\left\{\int d^3y\,\bar f_0(y)\epsilon'(y),\psi^\mu(x)\right\}$$
$$=\epsilon'(x)\delta_0^\mu\ ,$$
$$\delta_{\epsilon'}\phi(x)\equiv\left\{\int d^3y\,\bar f_0(y)\epsilon'(y),\phi(x)\right\}=0\ ,$$

and similar relations obtained by considering Poisson brackets of h and f_0 with $\bar\psi^\mu$ and $\bar\phi$. The above correspond to the transformations (2), under which the Lagrangian is invariant, only for $\epsilon'(x)=-\partial_0\epsilon(x)$. This is quite analogous to the situation in the case of electrodynamics, for example.

At this stage, it is good to pause and take stock of the number of independent degrees of freedom. We started with the five spinors ψ_μ ($\mu=0,1,2,3$) and ϕ, and their five canonical momenta. With the constraints f_μ, f, h, the number of independent spinors is reduced to $10-6=4$. We have to choose a gauge corresponding to the first-class constraints f_0 and h. This will constrain two spinors, leaving two free. The resulting phase space thus has 8 degrees of freedom, 4 corresponding to each of these two spinors. This is correct for a particle of spin $\frac{3}{2}$. More concretely, the constraints f_μ and f serve to fix the conjugate momenta, the gauge condition corresponding to f_0 should fix ψ_0 in terms of other fields, the constraint h together with its gauge condition reduces ψ_i, ϕ, and their conjugate momenta to two independent spinors.

Equipped with a detailed classification of the constraints, we will now quantize the theory after choosing a gauge.

III. DIRAC BRACKETS IN THE GAUGE $\psi_0=\phi=0$

To enable us to construct the anticommutation relations, we must (i) choose a gauge and then (ii) determine the Dirac brackets corresponding to the augmented set of second-class constraints χ_i, viz., the original set of second-class constraints together with the first-class constraints and the gauge conditions.

We choose the gauge conditions

$$\psi^0=0,\quad \bar\psi^0=0\ , \quad (24a)$$

$$\phi=0,\quad \bar\phi=0\ . \quad (24b)$$

To ensure that Eqs. (24) are permissible gauge conditions, and that no gauge freedom is left, we must verify that for the augmented set of constraints χ_i, the determinant $\det(\{\chi_i(x),\chi_j(y)\})$ is nonzero. We have done this by explicitly inverting the matrix C of Poisson brackets: $C_{ij}(x,y)=\{\chi_i(x),\chi_j(y)\}$.

To complete the matrix C, we need the Poisson brackets of ψ_0 and $\bar\psi_0$,

$$\{\psi_0^\alpha(\mathbf{x},t),\bar f_0^\beta(\mathbf{y},t)\}=-\delta^{\alpha\beta}\delta^{(3)}(x-y)\ , \quad (25a)$$

$$\{f_0^\alpha(\mathbf{x},t),\bar\psi_0^\beta(\mathbf{y},t)\}=-\delta^{\alpha\beta}\delta^{(3)}(x-y)\ , \quad (25b)$$

the rest being zero, and the Poisson brackets of ϕ and $\bar\phi$,

$$\{\phi^\alpha(\mathbf{x},t),\bar f^\beta(\mathbf{y},t)\}=-\delta^{\alpha\beta}\delta^{(3)}(x-y)$$
$$=\{f^\alpha(\mathbf{x},t),\bar\phi^\beta(\mathbf{y},t)\}\ , \quad (26a)$$

$$\{\phi^\alpha(\mathbf{x},t),\bar h^\beta(\mathbf{y},t)\}=-im\delta^{\alpha\beta}\delta^{(3)}(x-y)$$
$$=\{h^\alpha(\mathbf{x},t),\bar\phi^\beta(\mathbf{y},t)\}\ , \quad (26b)$$

with the remaining ones vanishing. We do not give here any further details of the inversion of the matrix C, which is straightforward, but only quote the results for the Dirac brackets defined by

$$\{X(x),Y(y)\}^*=\{X(x),Y(y)\}-\sum_{ij}\int d^3z\,d^3z'\{X(x),\chi_i(z)\}\times C_{ij}^{-1}(z,z')\{\chi_j(z'),Y(y)\}\ , \quad (27)$$

where $C_{ij}^{-1}(z,z')$ satisfies

$$\int d^3z'' C_{ik}^{-1}(z,z'')\{\chi_k(z''),\chi_j(z)\} = \delta_{ij}\delta^{(3)}(z-z') . \quad (28)$$

Note that all the above quantities are defined at a common instant of time. In actual practice C and C^{-1} also have Dirac indices α and β, which have been suppressed.

It is sufficient to state the Dirac brackets of ψ_i with ψ_j and $\psi_j^\dagger$, since the constraints and gauge conditions can be used inside Dirac brackets, and they serve to define all other quantities in terms of ψ_i and $\bar{\psi}_j$.

We get

$$\{\psi_i^\alpha(\mathbf{x},t),\psi_j^\beta(\mathbf{y},t)\}^* = 0 = \{\psi_i^{\alpha *}(\mathbf{x},t),\psi_j^{\beta *}(\mathbf{y},t)\}^* , \quad (29a)$$

$$\{\psi_i^\alpha(\mathbf{x},t),\psi_j^{\beta *}(\mathbf{y},t)\}^* = \left[\frac{2i}{3m^2}\partial_i\partial_j - \frac{1}{3m}(\gamma_i\partial_j - \gamma_j\partial_i) + i(g_{ij} - \tfrac{1}{3}\gamma_i\gamma_j)\right]^{\alpha\beta}\delta^{(3)}(x-y) . \quad (29b)$$

We can now make a transition to the quantum theory by replacing all the Dirac brackets by $1/i$ times the anticommutators. From (29) we get the anticommutators

$$\{\psi_i^\alpha(\mathbf{x},t),\psi_j^\beta(\mathbf{y},t)\}_{\rm ac} = 0 = \{\psi_i^{\alpha\dagger}(\mathbf{x},t),\psi_j^\beta(\mathbf{y},t)\}_{\rm ac} , \quad (30a)$$

$$\{\psi_i^\alpha(\mathbf{x},t),\psi_j^{\beta\dagger}(\mathbf{y},t)\}_{\rm ac} = \left[-\frac{2}{3m^2}\partial_i\partial_j - \frac{i}{3m}(\gamma_i\partial_j - \partial_i\gamma_j) - (g_{ij} - \tfrac{1}{3}\gamma_i - \gamma_j)\right]^{\alpha\beta}\delta^{(3)}(x-y) . \quad (30b)$$

The spinor which transforms under spatial rotations as a spin-$\frac{3}{2}$ object is the combination

$$\psi_i^{(3/2)} = \psi_i - \tfrac{1}{3}\gamma_i\gamma^j\psi_j , \quad (31)$$

and we determine its anticommutation relation to be

$$\{\psi_i^{(3/2)\alpha}(\mathbf{x},t),\psi_j^{(3/2)\beta^\dagger}(\mathbf{y},t)\}_{\rm ac} = \left[P_i^k\left[-\frac{2}{3m^2}\partial_k\partial_l - g_{kl}\right]P_j^l\right]^{\alpha\beta}\delta^{(3)}(x-y) , \quad (32)$$

where P_i^k is the projection matrix for spin $\frac{3}{2}$:

$$P_i^k = \delta_i^k - \tfrac{1}{3}\gamma_i\gamma^k . \quad (33)$$

Note that despite the fact that the constraint (16) implies a nonlocal relation between $\gamma^i\psi_i$ and $\psi_i^{(3/2)}$, Eq. (30b) implies that anticommutators of *all* components of ψ_i are local.

We can check that the anticommutators (30b) and (32) are positive definite, as required for consistency. For example, folding (32) with test-function spinors $\phi_i^*(x)$ and $\phi_j(y)$ we get, for

$$M^\dagger M + MM^\dagger \equiv \int d^3x\, d^3y \times \phi^{i\alpha^*}(x)\{\psi_i^{(3/2)\alpha}(x),\psi_j^{(3/2)\beta^\dagger}(y)\}_{\rm ac} \times \phi^{j\beta}(y) , \quad (34)$$

where

$$M = \int d^3x\, \psi_j^{(3/2)\beta^\dagger}(x)\phi^{j\beta}(x) , \quad (35)$$

the expression

$$M^\dagger M + MM^\dagger = \int d^3x \left[\frac{2}{3m^2}|\partial_k\tilde{\phi}^k|^2 - \tilde{\phi}^{k*}\tilde{\phi}_k\right] , \quad (36)$$

where $\tilde{\phi}_k$ is the spin-$\frac{3}{2}$ projection $\tilde{\phi}_k = P_k^l\phi_l$. Because of our metric ($g_{ij} = -\delta_{ij}$), the right-hand side of (36) is positive definite as required. We can similarly check that even the general anticommutator (30b) leads to positive-definite norm. This absence of negative-norm states, in spite of the absence of secondary constraints (i.e., tertiary constraints in the Hamiltonian formulation) is a new feature of our theory.

IV. CONCLUSIONS

We have performed the quantization of the massive gauge-invariant spin-$\frac{3}{2}$ theory. An important result is the positive-definitive nature of the anticommutator between ψ_i and $\psi_j^\dagger$, necessary for a consistent theory. This is especially significant in view of the fact that secondary constraints are absent in the theory, since this feature was generally found to lead to the presence of negative-norm states. We presume that this is due to the redeeming feature of gauge invariance. For example, the theorem of Johnson and Sudarshan[6] is not directly applicable when the theory has gauge invariance, and hence there is no contradiction of our result with the theorem.

In addition, our theory describes a unique spin, unlike other constructions, which avoid secondary constraints at the expense of negative-norm states and multiple spins. In these cases, primary constraints are not enough to restrict the spectrum to a unique spin. Our theory, in which extra ("Stueckelberg") fields are used for gauge invariance, the primary constraints, together with the gauge-fixing conditions are enough to restrict the spectrum only to spin $\frac{3}{2}$.

It would be of great interest to extend this study to the interacting theory. Since the interacting theory obtained by Kaluza-Klein reduction, which has been shown to be classically consistent,[5] also has the properties of gauge invariance and the absence of secondary constraints, we conjecture that negative-norm states will be absent, resulting in a quantum theory which is consistent. Work on this is in progress.

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