Bubbles without cores

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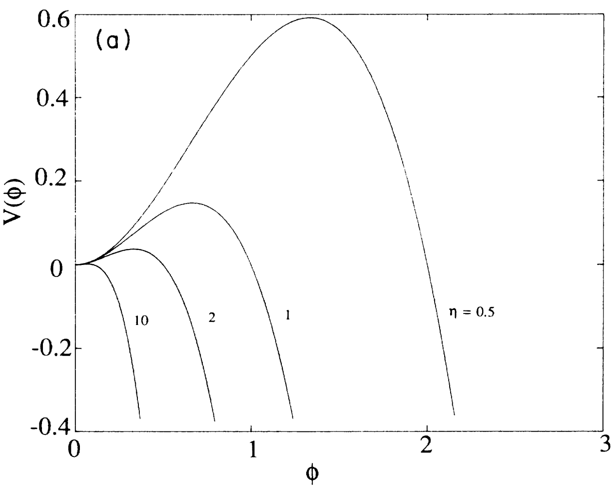
The decay of a false vacuum in a theory without a true vacuum is studied. Using variational arguments, we find that for a class of potentials of the type $a\phi^i + b\phi^j$ ($a > 0, b < 0$ and $i < j$) the necessary condition for the existence of bounce solutions in four dimensions is $2 \leq i < j < 4$. The thin-wall condition breaks down for these solutions, but not necessarily for the semiclassical approximation. A bounce solution associated with potential $m^2\phi^2 - \eta\phi^3 + \lambda\phi^4$ is presented to discuss the transition from a thin-wall bubble to a thick-walled one. The validity conditions of using these solutions to describe the false vacuum decay within the framework of the semiclassical approximation are discussed.

I. INTRODUCTION

Instantons[1] have been widely used to describe false-vacuum decay. For a system of a single scalar field, there are general arguments to show the existence of bounce solutions,[2] if there is more than one local minimum with unequal energies in the classical potential. Furthermore, if the potential satisfies some conditions, the bounce solution of the least Euclidean action is necessarily spherically symmetric in N-dimensional Euclidean space.[3] Recently, potentials without lower bounds have caused some attention in cosmology,[4] higher-dimensional theories,[5] and quantum cosmology.[6] The quantum field theory of these potentials can be defined, but without particle interpretation. A quantum-mechanics example was treated in detail by Barton.[7] Even though the general overshooting-undershooting-type argument concerning the existence of bounce solutions cannot be applied to these potentials with a barrier [see Fig. 1(a)], one may expect that a bounce also exists. Then one can follow the usual procedure to evaluate the Euclidean action of the bounce to obtain the decay rate per unit space volume, at least in the semiclassical limit. However, it has been shown that a massive scalar field ϕ with a quartic interaction of the wrong sign (i.e., $\lambda\phi^4, \lambda < 0$) has no instanton solutions.[8] Approximate Euclidean solutions have been developed to calculate the decay rate of the false vacuum.[9] Mathematically established as it is, one cannot help but wonder what has happened physically to prevent the formation of an instanton in this case. Is it generally true that a potential having a barrier but unbounded from below cannot have a bounce solution?

This paper is an attempt to address these questions directly. Employing the same variational arguments as in Ref. 8, we find a general necessary condition for the existence of bounce solutions for a class of potentials of the form $a\phi^i + b\phi^j$ ($a > 0$, $b < 0$ and $i < j$) [Fig. 1(a)] is $2 \leq i < j < 4$ in four dimensions. Thus, a potential of the form $\phi^2 - \phi^3$ could have a bounce solution, but not for a potential of the form $\phi^2 - \phi^6$. This result seems to imply that the theorem of Ref. 3 holds even for potentials only somewhere positive, if their admissible condition (4) is



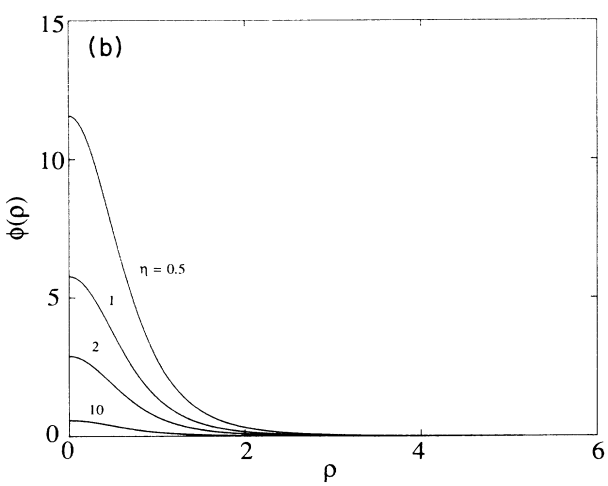


FIG. 1. (a) A typical class of potentials with a barrier but without lower bound. Here we choose $V(\phi) = m^2\phi^2 - \eta\phi^3$ as an example and set $m^2 = 1$. (b) O(4) bounce solutions for the corresponding potentials of (a). ρ is the radial variable of the O(4) coordinate.

satisfied, namely, there exist positive numbers p,q,α,β such that $\alpha<\beta<4$, and $V-p\,|\phi|^{\alpha}+q\,|\phi|^{\beta}\geq 0$, for all ϕ. Furthermore, for those potentials which do have bounce solutions, the "walls" are never thin. The breakdown of the thin-wall condition is hardly surprising if one considers these unbounded potentials as the large-energy-difference limit of a double-well potential. However, by considering these potentials we can demonstrate that the semiclassical formalism for the decay process is not necessarily described by a thin-wall bubble, and a thin-wall bubble may not be reliable under the semiclassical approximation.

The rest of the paper is organized as follows. In Sec. II we review the well-known semiclassical formalism for vacuum decay from the point of view of spontaneous symmetry breaking. In Sec. III we use variational arguments to establish three theorems about bounce solutions and use them to obtain our criterion for unbounded potentials. Examples with numerical solutions are presented and discussed. In Sec. IV we use an example to discuss conditions for the thin-wall solutions and their relation to the semiclassical approximation. Concluding remarks are presented in Sec. V. In this paper we will use the Planck unit, $\hbar=c=1$, except in Sec. II where $\hbar$ is preserved. We choose signatures $(+,-,-,-)$ for Minkowski space and $(+,+,+,+)$ for Euclidean space.

II. REVIEW OF THE FORMALISM

Vacuum decay in quantum field theory can be viewed as a phenomenon of spontaneous symmetry breaking. The unusual part is that both vacua are local minima and there exists an energy barrier between them. Thus, classically both minima are stable, but long-wavelength quantum fluctuations can "detect" the global minimum (true vacuum), and render the local minimum (false vacuum) unstable. The classical analogue of this phenomenon is the first-order phase transition,[10] where statistical fluctuations replace the quantum ones. Since the spontaneous symmetry breaking is triggered by quantum effects, all relevant information should be in the effective action.

Consider a system of a single real scalar field $\phi(x)$ with a self-interaction potential $V(\phi)$ which has two local minima, the false vacuum ϕ_f and the true vacuum ϕ_t, separated by a potential barrier. The action is of the form

$$S=\int d^4x\,[\tfrac{1}{2}(\nabla\phi)^2-V(\phi)]\ . \tag{2.1}$$

The most fundamental quantity in the functional formalism is the vacuum persistence amplitude (VPA), $\langle 0^+|0^-\rangle_J$, in the presence of the external source $J(x)$. VPA is defined by a functional integral of the exponential of the action $S[\phi,J]$ over the function space of ϕ with a proper measure $[d\phi]$,

$$\langle 0^+|0^-\rangle_J=\int[d\phi]e^{iS[\phi,J]/\hbar}\ . \tag{2.2}$$

All the Green's functions can be generated by functional differentiation with respect to J. One can also express the VPA by the generating functional of connected Green's functions $W[J]$ as

$$e^{iW[J]/\hbar}=\langle 0^+|0^-\rangle_J\ . \tag{2.3}$$

For a stable vacuum we would expect that after adiabatically turning on and off the source J, the VPA would only develop a phase, or equivalently that W is a real number. However, for a metastable or unstable vacuum, we would expect that W will yield an imaginary part,[11] even when $J=0$. The false-vacuum-decay probability for the whole spacetime is, therefore,

$$|\langle 0^+|0^-\rangle|^2=e^{-2\,\mathrm{Im}W[0]/\hbar}\ . \tag{2.4}$$

The effective action Γ is obtained from W by a functional Legendre transformation. However, for $J=0$, they can be identified. Thus, we can set out to calculate the imaginary part of the effective action of the system. The classical analogue of the effective action is the free energy, and the free energy is the relevant quantity in the description of the first-order phase transition. Before we start to evaluate the right-hand side (RHS) of Eq. (2.2), two points need to be noted. First, the functional integral as it stands is ill defined. One way to remedy this problem is to insert an $i\epsilon\phi^2$ term in the Lagrangian. However, for treating nonperturbative problems such as vacuum decay, it proves to be convenient to make a Wick rotation $(t\to -i\tau)$ and define the integral in Euclidean space. From now on we will assume that is done and proceed in Euclidean space. It is convenient to define the Euclidean action S_E as

$$S_E=\int d^4x\,[\tfrac{1}{2}(\nabla\phi)^2+V(\phi)]\ , \tag{2.5}$$

then W can be evaluated by

$$W=-\hbar\ln\int[d\phi]e^{-S_E[\phi]/\hbar}\ . \tag{2.6}$$

Second, one should integrate all configurations of ϕ satisfying the boundary condition $\lim_{t\to\pm\infty}\phi(x)=\phi_{\mathrm{vac}}$ where ϕ_{vac} is any vacuum under study. After the Wick rotation, we have to change the boundary of time from Minkowskian infinity to Euclidean infinity. If we restrict ourselves further to finite-action configurations (anticipating a steepest-descent approximation), the boundary condition would be changed to

$$\lim_{|x|\to\infty}\phi(x)=\phi_{\mathrm{vac}}\ . \tag{2.7}$$

We also adjust the classical potential $V(\phi_{\mathrm{vac}})=0$ to ensure the finiteness of the action.

Having defined the functional integral in the Euclidean space, we can now evaluate the RHS of Eq. (2.6). Except for free fields, an approximation scheme is necessary. We shall adopt the loop expansion, because it preserves the full symmetry of the theory; in particular, we shall apply the steepest-descent method to expand the action about its stationary point $\bar{\phi}$ which satisfies the Eucidean field equation

$$\Box\bar{\phi}-\frac{dV}{d\bar{\phi}}=0\ . \tag{2.8}$$

The expansion parameter would be assigned as $\hbar$. The action can then be expanded as

$$S_E[\phi]=S_E[\bar{\phi}]+\frac{1}{2}\int\int d^4x\, d^4y\frac{\delta^2 S_E[\bar{\phi}]}{\delta\bar{\phi}(x)\delta\bar{\phi}(y)}[\phi(x)-\bar{\phi}(x)][\phi(y)-\bar{\phi}(y)]+O(\hbar^3) \ . \tag{2.9}$$

The second term on the RHS of Eq. (2.9) has a Gaussian form, and can be integrated out. Thus, we obtain the well-known one-loop result

$$W=-\hbar\ln\left\{\sum_{\{\bar{\phi}\}} e^{-S_E[\bar{\phi}]/\hbar}\left[\det\left[\frac{\delta^2 S_E}{\delta\bar{\phi}^2}\right]\right]^{-1/2}\right\}+O(\hbar^2) \ . \tag{2.10}$$

At this point the information about the stability of a vacuum begins to be revealed. For a stable vacuum the only solution satisfying both Eqs. (2.7) and (2.8) would be the trivial one, $\bar{\phi}=\phi_t$. All the eigenvalues of the operator $\delta^2 S/\delta\bar{\phi}^2$ will be positive and W is real. For an unstable vacuum, $\bar{\phi}=\phi_f$ may not be the only solution, but already it has negative eigenvalues and indicates the instability of the vacuum.[12] However, for a metastable vacuum, although $\bar{\phi}=\phi_f$ is a solution and gives a real contribution to W, there exists another solution, $\bar{\phi}_{\text{bounce}}$. (Actually, there are an infinite number of them, as long as they are widely separated.) Each one of them will have a negative mode for the operator $\delta^2 S/\delta\bar{\phi}^2$, and will give an imaginary contribution to W. We must sum over all the n-bounce configurations in the steepest-descent approximation. In addition, since each bounce can have its center at any point of the four-dimensional Euclidean space, we have to integrate over the locations of the centers. This procedure gives us a four-volume V_4 on the RHS of Eq. (2.10). Dividing W by V_4, we shall obtain the well-known one-loop decay rate per three-space volume given by Ref. 2:

$$\frac{W}{V_4}=\left[\frac{B}{2\pi\hbar}\right]^2 e^{-B/\hbar}D[1+O(\hbar)] \ , \tag{2.11}$$

where $B=S_E[\bar{\phi}_{\text{bounce}}]$, and D represents the imaginary contribution from the determinant which is of the order unity. This is the leading contribution to the imaginary part of W.

Thus, a primary task in the study of the vacuum decay problem seems to be finding the bounce solution. It turns out that not all potentials with a barrier can have bounce solutions nor can Eq. (2.11) be applied for all bounce solutions to determine a decay rate. The first point is due to the fact that for some potentials the only stationary point of the classical action is at $\bar{\phi}=\phi_f$. The second point is more obvious. When the coupling constants are large, higher-loop effects are no longer negligible, but we still can have bounce solutions. Although bounce solutions are pure classical and indifferent to quantum effects, the value of the action could signal the strength of quantum effects. That leads us into the topics of the next two sections.

III. VARIATIONAL PRINCIPLE AND BOUNCE SOLUTIONS

We shall summarize three simple theorems in N-dimensional Euclidean space about the scaling properties of the bounce solutions and discuss their implications. We list them here not because they are new, but because they are illuminating and useful for our discussion. [Theorem 1 has appeared in Ref. 1, and a special case of Eq. (3.8) has been used by Linde.[13]]

Theorem 1 (a virial theorem). The action of the bounce is positive definite. In addition, the kinetic part and the potential part have the relation

$$\int \tfrac{1}{2}(\nabla\bar{\phi})^2 d^N x=\left[\frac{N}{2-N}\right]\int V(\bar{\phi})d^N x \ . \tag{3.1}$$

The proof of the theorem is parallel to that of the Derrick theorem.[14] Let us imbed $\bar{\phi}(x)$ in a one-parameter family of functions,

$$\phi_\lambda(x)=\bar{\phi}(x\lambda)\equiv\bar{\phi}(y) \ ; \tag{3.2}$$

then the action of ϕ_λ, from Eq. (2.5), becomes

$$S[\phi_\lambda(x)]=\int d^N y\{\tfrac{1}{2}[\nabla_y\bar{\phi}(y)]^2\lambda^{2-N}+\lambda^{-N}U(\bar{\phi})\} \ . \tag{3.3}$$

Because $\bar{\phi}$ is the stationary point of S under general variations, we should have, in particular,

$$\left.\frac{\delta S}{\delta\lambda}\right|_{\lambda=1}=0 \tag{3.4}$$

and the relation (3.1) follows. Thus, the action of a bounce can be written as

$$S(\bar{\phi})=\frac{2}{N}\int \tfrac{1}{2}(\nabla\bar{\phi})^2 d^N x \tag{3.5}$$

which is, of course, positive definite.

Theorem 2. If $\bar{\phi}_1(x)$ is a bounce solution of a potential $U_1(\phi)$, and $\bar{\phi}_2(x)$ is a bounce solution of the potential $\lambda^2 U_1(\phi)$, then $\bar{\phi}_2(x)=\bar{\phi}_1(x\lambda)$ and $S[\bar{\phi}_2]=\lambda^{2-N}S[\bar{\phi}_1]$.

The proof of this theorem is a trivial exercise of the variational arguments of those in Theorem 1 and Eq. (2.8). The application of this theorem is that we can explore a class of bounce solutions by scaling the potential which corresponds to scaling coupling constants. We can estimate, for example, when the thin-wall assumption or the semiclassical formula (2.11) fails.

Theorem 3. If the potential V consists of only two terms, $V(\phi)=a_i\phi^i+a_j\phi^j$, then the bounce solution $\bar{\phi}_{a_i,a_j}(x)$ is related to $\bar{\phi}_{1,1}(x)$ by

$$\bar{\phi}_{a_i,a_j}(x)=u\bar{\phi}_{1,1}[x(a_i u^{i-2})^{1/2}] \ , \tag{3.6}$$

where

$$u = \left[\frac{a_j}{a_i} \right]^{1/(i-j)} . \quad (3.7)$$

The action $S(a_i,a_j)$ is linearly related to $S(1,1)$ by

$$S(a_i,a_j) = \left[\frac{a_j^{(2-i)N/2+i}}{a_i^{(2-j)N/2+j}} \right]^{1/(i-j)} S(1,1) . \quad (3.8)$$

Furthermore, in four dimensions, if $i<j$, and $a_i>0$, $a_j<0$, then nontrivial monotone solutions with finite action and vanishing at infinity cannot exist unless $2 \le i < j < 4$.

To prove the first part of the theorem, we can redefine our field ϕ by a dimensionless field ψ as $\phi(x)=u\psi(x)$, followed by a scaling of the coordinate x,

$$x = y \left[\frac{a_j^{2-i}}{a_i^{2-j}} \right]^{1/2(i-j)} . \quad (3.9)$$

Equations (3.6) and (3.8) follow directly after the substitution into the field equation (2.8) and the action (2.5). To prove the second part of the theorem, we can consider a family of functions $\phi_\lambda(x)=\lambda\bar{\phi}(x\lambda)$. By the same procedure as we did for Eqs. (3.3) and (3.4), we have, in four dimensions,

$$\int [(i-4)a_i\bar{\phi}(x)^i + (j-4)a_j\bar{\phi}(x)^j] d^4x = 0 . \quad (3.10)$$

Since $\bar{\phi}$ is required to be monotone, the only possible solution is either $i,j>4$ or $i,j<4$. Now let us study the asymptotic behavior of the bounce solution, assuming they exist. Let $\bar{\phi} \sim \alpha x^s$ and $V(\phi) \sim \phi^i$ near $\phi \sim 0$. Then from Eq. (2.8), we find $s=2/(2-i)$ when x approaches infinity. From Theorem 1 we know the action can be expressed solely by a four-volume integral of the potential V. Therefore, asymptotically the action approaches $\lim_{x\to\infty} x^{4+si}$. From these two relations one can conclude that only when $2<i\le 4$ can we have a finite action of bounce solutions. For the case of $i=2$, the asymptotic solution is of the form e^{-x}, so the action is also convergent, proving the theorem.

It is easy to see from Eq. (3.10) that if $i=4$, any nontrivial solution could occur only when $a_j=0$, and one particular family of solutions has been found by Fubini.[15] Another way to understand this is that it is not possible to have a dimensionless action (in Planck units) formed by a single mass scale a_j, since a_i is dimensionless in this case (unless dimensionful boundary conditions are introduced). Therefore, we would not expect to find a bounce with a finite but nonzero action for any "flat" potential[16] of the form $V=-\phi^i$ except for $i=4$. For different dimensions, e.g., $N=3$, a massive $\lambda\phi^4$ theory could have bounce solutions, because now two dimensionful constants can make up a dimensionless action.

As an application of these theorems, we consider a four-dimensional system with potentials $m^2\phi^2-\eta\phi^3$ where m^2, $\eta>0$ [Fig. 1(a)].[17] Bounce solutions with O(4) symmetry are obtained numerically as illustrated in Fig. 1(b). From Eq. (3.8), the values of the action associated with various coupling constants turn out to be

$$S(m,\eta) = \left[\frac{m}{\eta} \right]^2 S(1,1) . \quad (3.11)$$

Indeed, numerical calculation confirms this relation and $S_{1,1}=45.43$. One observes immediately that the action approaches zero as m does, as was discussed in the last paragraph. When η approaches zero the action diverges, because the vacuum $\phi=0$ becomes more stable, the decay rate vanishes. Moreover, from Eqs. (3.6) and (3.7) we find the bounce solution for (m,η) relates to that for (1,1) by

$$\bar{\phi}_{m,\eta}(x) = \frac{m^2}{\eta} \bar{\phi}_{1,1}(xm) , \quad (3.12)$$

which also agrees with Fig. 1(b). One may notice that these solutions have no region of constant ϕ no matter how we change the coupling constants. Therefore, the usual pictures of thin-wall bubbles cannot apply. This leads to the question of when the thin-wall condition fails, and the next section.

IV. FROM THE THIN WALL

Since the field equation (2.8) is nonlinear, it is difficult to find exact solutions to evaluate the classical action, let alone loop corrections. A useful analytic method has been developed by Coleman[2] to approximate the classical action; namely, the thin-wall approximation. The primary condition for the use of the thin-wall formula is to have a potential with two wells whose energy difference is very small. A dimensionless parameter to quantify the last statement can be the ratio of the barrier to $V(\phi_f)-V(\phi_t)$, or the dimensionless coupling constant deviation from the degenerated double-well potential. Under this assumption we can compute a zeroth-order approximation of the bounce action by separating it into two readily evaluated parts, $S=S_{\text{core}}+S_{\text{wall}}$. S_{core} is approximated by a four-dimensional constant field configuration ϕ_t of size R (four-dimensional bubble radius), and S_{wall} is approximated by a one-dimensional kink configuration which can be integrated exactly. (Now we can see why our previous examples have no cores. For potentials with huge energy difference between the vacua, bounce solutions do not start near the true vacuum. Hence the acceleration is not zero, and ϕ cannot stay constant.) Here we have assumed that the potential is admissible by the definition of Ref. 3, and considered only bounce solutions with O(4) symmetry. We can fix the value of R by minimizing S with respect to R. The size of the wall r, can be estimated from the kink solution. From Sec. III we know that $m^2\phi^2$, if present, will dictate the behavior of ϕ near the false vacuum at large x as $\phi-\phi_t \sim e^{-mx}$, so we can estimate the thickness of the wall as $1/m$. Therefore, we can write down the condition for the thin-wall approximation in terms of the intrinsic parameters of the theory as[2]

$$\frac{R}{r} = \frac{3S_1 m}{-V(\phi_t)} \gg 1 , \quad (4.1)$$

where $S_{\text{wall}}=2\pi^2R^3S_1$ and S_1 the one-dimensional kink action.

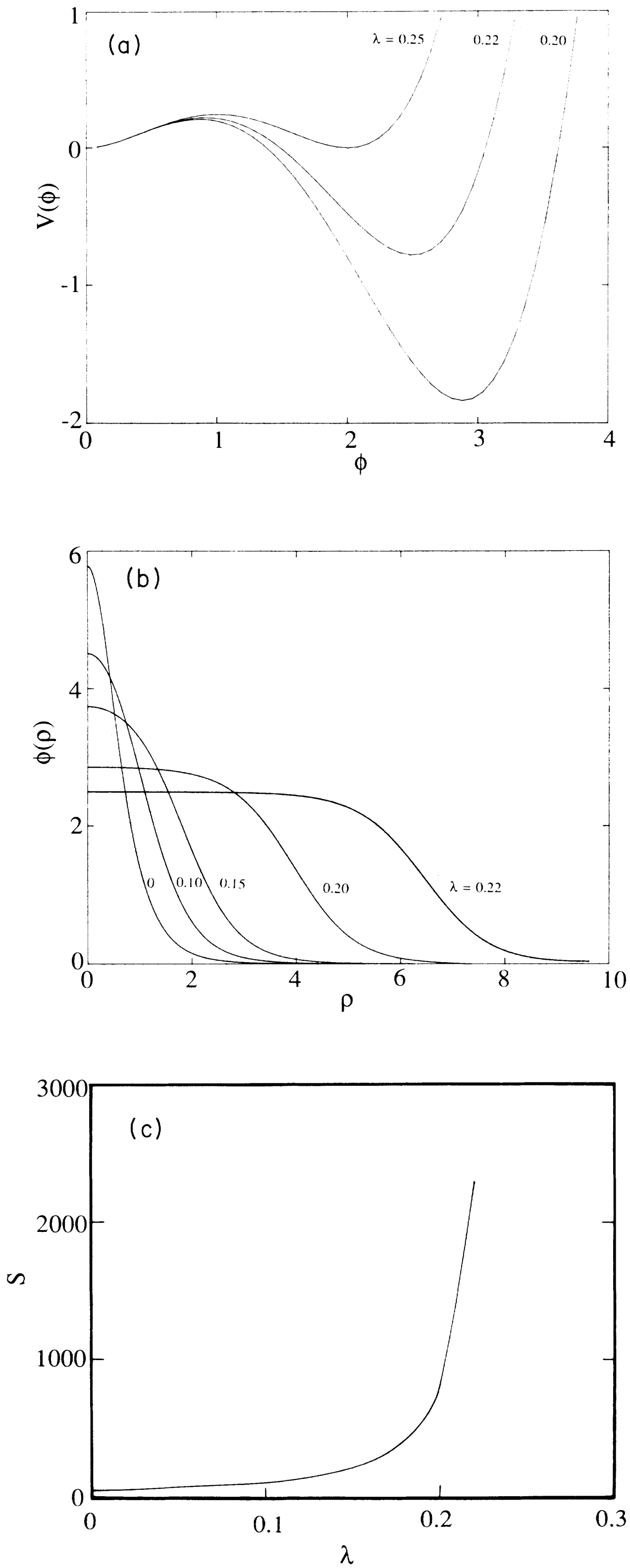


FIG. 2. (a) A class of double-well potentials of the form $V(\phi)=m^2\phi^2-\eta\phi^3+\lambda\phi^4$. Here we set $m^2=1$ and $\eta=-1$. (b) O(4) bounce solutions for the corresponding potentials of (a). ρ is the radial variable of the O(4) coordinate. (c) The Euclidean action S evaluated at the bounce solutions (b) for various coupling constants λ.

To illustrate this point, let us consider a class of potentials[17] of the form

$$V(\phi)=m^2\phi^2-\eta\phi^3+\lambda\phi^4, \quad m^2,\eta,\lambda>0 . \tag{4.2}$$

[See Fig. 2(a).] It is not difficult to see that these potentials satisfy all permissible conditions of Ref. 3. Therefore, there should exist at least one nontrivial monotone spherically symmetric solution vanishing at infinity with the least Euclidean action. Examples of numerical solutions and their corresponding actions are presented in Figs. 2(b) and 2(c), respectively. The case of degenerated vacua occurs when $\lambda\equiv\lambda_0=\eta^2/4m^2$. The one-dimensional kink solution is the usual one with a displacement

$$\bar{\phi}_1(\rho)=\frac{m^2}{\eta}\left[1-\tanh\frac{m\rho}{\sqrt{2}}\right] ,$$

where ρ is the O(4) radial variable. Thus,

$$S_1=\frac{2\sqrt{2}m^5}{3\eta^2} . \tag{4.3}$$

Let δ measure the deviation of our potential from the degenerated one:

$$\lambda=\frac{\eta^2}{4m^2}(1+\delta), \quad -1\leq\delta<0 . \tag{4.4}$$

Elementary algebra reveals that

$$V(\phi_t)=\frac{4m^6}{\eta^2}\delta[1-4\delta+O(\delta^2)] . \tag{4.5}$$

From Eqs. (4.1), (4.3), and (4.5), we find that the thin-wall condition for this type of theory is

$$\frac{R}{r}=\frac{-1}{2\delta}[1+4\delta+O(\delta^2)]\gg 1 , \tag{4.6}$$

and the action is

$$S=\frac{-\pi^2m^2}{6\eta^2\delta^3}[1+12\delta+O(\delta^2)] . \tag{4.7}$$

If we examine Fig. 2(b), we can observe how small the validity range of the thin-wall approximation is. In that example $\lambda_0=0.25$, and for $\delta=0.3$, we see that $R/r\sim 1$. If we compare Fig. 2(c) with Eq. (4.7), we see at $\delta=0.12$ the leading term of Eq. (4.7) gives 951.9, but the numerical evaluation yields 2319. Therefore we must remember that the thin-wall approximation is a zeroth-order approximation, and one can use it only when the double well is nearly degenerate. Figure 2(c) also reveals that the classical action decreases sharply as the true-vacuum energy is lowered. In this particular case it will not vanish as $\lambda\rightarrow 0$, because bounce solutions exist even when the true vacuum disappears as was discussed in Sec. III. Hence, the decay rate is greater as the energy difference between the two vacua is enlarged. The factor D of (2.11) can no longer be neglected when S is of the order 1. To determine whether higher-loop effects are important, we have to consider coupling constants, because the n-loop correction contains coupling constants up to order n. From Theorem 2 we know that if all the coupling constants are multiplied by a factor κ, then the corresponding bounce action is divided by a factor κ. Therefore a small classical action does imply strong quantum effects. There is no direct relation between the semiclassical approximation and the thin-wall approximation. As we have seen from our example, the thin-wall condition is determined by a dimensionless parameter, which is independent of the absolute value of coupling constants. However, we do see from Eq. (4.7) and Fig. 2 that when the energy difference between the two vacua increases, the bounce action decreases, and the quantum effects are enhanced.

V. CONCLUSION

We find that it is possible to use the Euclidean semiclassical formalism to describe the decay of a false vacuum without a true vacuum. The necessary conditions are (1) small coupling constants to ensure that the higher-loop effects are negligible, (2) the classical potential behaves as $-\phi^j$, $j \leq 4$ for large ϕ to ensure that ϕ_f can be reached at infinity, and (3) near ϕ_f the classical potential must approach ϕ^i, $2 \leq i \leq 4$ to ensure that ϕ_f is a classical solution and the derivative of $\bar{\phi}$ vanishes as $\bar{\phi} \rightarrow \phi_f$. Although the theorem we have shown is only for polynomial potentials, it seems that nonpolynomial potentials can also be applied. For example, from our conclusion a potential of the form[18]

$$V = \frac{m^2}{2}\phi^2 \left[1 - \ln \frac{\phi^2}{\phi_0^2} \right]$$

should have a bounce solution. Indeed, $\phi(\rho) = \phi_0 e^{-m^2\rho^2/2}$ is the one.

We also notice that if we approach the unbounded potential from a double-well potential, then the action decreases rapidly as the energy of the true vacuum is lowered. This indicates the decay rate increases rapidly as the quantum instability tries to overcome the classical stability. It is interesting to compare this phenomenom with the tunneling rate of particle quantum mechanics which can be thought of as a one-dimensional (time dimension) quantum field theory. Since it is one dimensional, the Euclidean field equation (2.8) in spherical coordinates has no damping term (which in general has the form $[(N-1)/\rho]d\phi/d\rho$). Therefore the value of the field at the center of the bubble is fixed by the value where $V(\phi_{\text{center}}) = V(\phi_f)$ and is independent of the position of the true vacuum. Equivalently, the tunneling rate for a quantum-mechanical particle is independent of the potential function beyond the barrier in the semiclassical limit. This difference, as in spontaneous symmetry breaking, is attributed to the infinite degrees of freedom of the field theory.

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