Gravitational field of a global string

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The gravitational field of a straight global string is derived in the linear approximation to general relativity, and the resulting trajectories of test particles are found. Part of the gravitational effect of global strings is a deficit angle which increases logarithmically with the distance to the string core. In this regard, a global string resembles a gauge string. A new feature, not present for gauge strings, is a repulsive gravitational potential outside the core. The lensing properties of the global string, as well as other classical effects, are studied.

I. INTRODUCTION

Phase transitions of quantum fields in the early Universe may produce very thin tubes of false vacuum, known as cosmic strings.[1,2] Depending on whether the symmetry that is broken during the phase transition is local or global, the corresponding topological defects are called gauge or global strings. Gauge strings have their energy concentrated in a very thin tube, the radius of which is of order the symmetry-breaking scale. Global strings, instead, are such that their energy extends to regions far beyond the central core, the energy density being proportional to r^{-2}.

The gravitational effect of an infinite straight gauge string is equivalent to the removal from flat space-time of a wedge of angular size $8\pi\mu$ (Refs. 3–6). Here μ is the energy per unit length of the gauge string, in units of the Planck mass. The linear approximation to the metric, in ordinary cylindrical coordinates $(0\le\theta\le 2\pi)$, is

$$ds^2=-dt^2+dz^2+dr^2+r^2(1-8\mu)d\theta^2 \ . \qquad (1)$$

One important consequence is that the string can act as a gravitational lens, a property which may be observed through the formation of double images of quasars.[3]

In this paper we derive the metric outside a straight global string in the linear approximation to general relativity. We will see that a global string produces a repulsive gravitational field outside the core in addition to an angular deficit. The angular deficit is similar to that of a gauge string but grows logarithmically with the distance to the core.[7,8] In the case of the global string, however, the repulsive gravitational field overcomes the effect of the deficit angle when the velocity of the particle that is being deflected by the string is small enough (typically when $V<\frac{1}{10}c$).

II. THE METRIC AROUND A GLOBAL STRING

The most general static metric with cylindrical symmetry with respect to an axis z and Lorentz invariance in the (t,z) plane reads, in cylindrical coordinates $(0\le\theta\le 2\pi)$,

$$ds^2=A(r)(-dt^2+dz^2)+dr^2+r^2B(r)d\theta^2 \ . \qquad (2)$$

The nonvanishing components of the Ricci tensor for this metric are

$$R_t^t=R_z^z=-\frac{1}{2}\frac{\ddot A}{A}-\frac{1}{2r}\frac{\dot A}{A}-\frac{1}{4}\frac{\dot A\dot B}{AB} \ , \qquad (3a)$$

$$R_r^r=-\frac{1}{2}\frac{\ddot B}{B}-\frac{1}{r}\frac{\dot B}{B}+\frac{1}{4}\left[\frac{\dot B}{B}\right]^2-\frac{\ddot A}{A}+\frac{1}{2}\left[\frac{\dot A}{A}\right]^2 \ , \qquad (3b)$$

$$R_\theta^\theta=-\frac{1}{2}\frac{\ddot B}{B}-\frac{1}{r}\frac{\dot B}{B}+\frac{1}{4}\left[\frac{\dot B}{B}\right]^2-\frac{1}{r}\frac{\dot A}{A}-\frac{1}{2}\frac{\dot A\dot B}{AB} \ . \qquad (3c)$$

We want to solve Einstein equations with a global string as the source. To be definite, let us adopt an explicit model of the string core. Consider a complex scalar field with action density:

$$\mathcal{L}=\tfrac{1}{2}g^{\mu\nu}\partial_\mu\phi^*\partial_\nu\phi-\frac{\lambda}{4}(\phi^*\phi-v^2)^2 \ . \qquad (4)$$

The field configuration

$$\phi=vf(r)e^{i\theta}$$

solves the equation of motion if

$$\ddot f+\left[\frac{1}{r}+\frac{\dot A}{A}+\frac{1}{2}\frac{\dot B}{B}\right]\dot f-\frac{f}{r^2B}=\frac{f}{\delta^2}(f-1)^2 \ . \qquad (5)$$

Here $\delta=(v\sqrt{\lambda})^{-1}$ is the core radius of the string. The function $f(r)$ grows linearly when $r<\delta$ and exponentially approaches unity as soon as $r\gtrsim\delta$. Thus, taking $f=1$ outside the core is a very good approximation to the exact solution. The energy-momentum tensor is then given, outside the core, by

$$T_t^t=T_z^z=T_r^r=-T_\theta^\theta=-\frac{v^2}{2r^2B} \ . \qquad (6)$$

Note that a global string does not have tension along the z direction only. It has equally large tension in the radial direction and an equally large pressure in the θ direction.

Outside the core, Einstein's equations are

$$R_t^t=R_z^z=R_r^r=0 \ , \qquad (7a)$$

$$R_\theta^\theta=\frac{8\pi Gv^2}{Br^2} \ . \qquad (7b)$$

These equations are readily solved in the linear approximation, i.e., assuming that both A and B are very close to unity. The result is

$$ds^2=\left[1-4\mu\ln\frac{r}{\delta}\right](-dt^2+dz^2)+dr^2+r^2\left[1-8\mu\left[\ln\frac{r}{\delta}+c\right]\right]d\theta^2 \ . \tag{8}$$

Here

$$\mu\equiv\pi G v^2 \ .$$

In terms of μ, the energy per unit length that is outside the core of the string up to a distance r is, in units of the Planck mass,

$$\lambda=G\int_\delta^r 2\pi r T_{00}(r)dr=\mu\ln\frac{r}{\delta} \ .$$

The linear approximation to the metric makes sense as long as $\mu\ln r/\delta \ll 1$. For distances r up to the present horizon that condition requires the symmetry-breaking scale v to be smaller than 10^{17} GeV. Recall that δ is the core radius. We denote as μc the energy per unit length contained in the core itself. Since that quantity is model dependent, we wish to keep track of its effects separately from those of the rest of the string. Inside the core, at least for the specific model given by Eq. (4), both $A(r)$ and $B(r)$ differ from unity by terms quadratic in r. But we shall not be concerned with the interior metric here, nor with a careful matching of the interior and exterior metrics at $r=\delta$, since Eq. (8) is sufficient for our purposes.

Comparison of Eqs. (1) and (8) shows that the effective deficit angle produced by a global string increases logarithmically with the distance to the core,[7,8] whereas the gauge string deficit angle is constant. However, there is also a more qualitative difference: a global string has a repulsive gravitational potential.[9] A freely moving particle near the string experiences an outward proper acceleration

$$\ddot{r}=\frac{2\mu}{r} \ . \tag{9}$$

Two parallel global strings separated by a distance d repel each other with a gravitational force per unit length $(\pi v^2/d)2\mu\ln d/\delta$. This gravitational force differs by a factor $2\mu\ln d/\delta$ from the force due to the Goldstone-boson field itself. The latter force is attractive or repulsive according to the relative orientation of the two strings.

III. THE GEODESICS

Let us now write down the equations for the geodesics in a plane perpendicular to the z axis in the metric (2). From $(D/dp)(dx^\mu/dp)=0$ we have

$$\left[\frac{dr}{dp}\right]^2=\frac{1}{A(r)}-\frac{J^2}{r^2B(r)}-M^2 \ , \tag{10a}$$

$$r^2\frac{d\theta}{dp}B(r)=J \ , \tag{10b}$$

$$\frac{dt}{dp}=\frac{1}{A(r)} \ . \tag{10c}$$

Here J and M are integration constants. J represents the angular momentum of the trajectory and M is the ratio between the proper time along the trajectory and the affine parameter p, i.e., $ds^2=-M^2dp^2$.

From Eqs. (10) the shape of the paths, $r=r(\theta)$, can be found. It is given by

$$\left|\frac{d\theta}{dr}\right|=\Delta(r)\equiv\frac{r_0}{r^2}B^{-1/2}(r)\left[\frac{B(r)}{B(r_0)}\frac{A(r_0)}{A(r)}\left[\frac{1-M^2A(r)}{1-M^2A(r_0)}\right]-\left[\frac{r_0}{r}\right]^2\right]^{-1/2} \ . \tag{11}$$

Here we have chosen to parametrize the trajectories in terms of the distance of closest approach to the core, r_0, instead of the angular momentum J.

Consider a trajectory starting from a distance $L\gg r_0$ and with velocity V such that $V^2\gg 2\mu\ln L/r_0$. Most of the deflection imprinted upon the trajectory by the gravitational field will occur in the region $r\approx r_0$, where

$$\Delta(r)\approx\frac{r_0}{r^2}\frac{1}{\left[1-\left[\frac{r_0}{r}\right]^2\right]^{1/2}}\left\{1+4\mu\left[\ln\frac{r}{\delta}+c+\left[1-\frac{1}{2V^2}\right]\frac{\ln r/r_0}{1-(r_0/r)^2}\right]\right\} \ . \tag{12}$$

to first order in μ and μ/V^2. Also $M^2=1-V^2$ to this order. The deflection angle is

$$\epsilon=\pi-\Delta\theta \ ,$$

where

$$\Delta\theta=2\int_{r_0}^\infty\Delta(r)dr \ . \tag{13}$$

Equations (13) and (12) yield

$$\epsilon=-4\pi\mu\left[\ln\frac{r_0}{\delta}+c+\ln 2+1-\frac{1}{2V^2}\right] \ . \tag{14}$$

In our order of approximation, r_0 coincides with the impact parameter of the trajectory. The analogous result to Eq. (14) for a gauge string is $\epsilon = -4\pi\mu$ with μ the string energy per unit length. The global string case differs from that of a gauge string by the dependence of ϵ upon both the impact parameter and the velocity of the trajectory.

In the case of ultrarelativistic particles ($V \approx 1$) the dominant term in Eq. (14) is the term $\ln r_0/\delta$, since it is typically of order 10^2 in any astronomically relevant situation. In that limit the deflection effected by a global string is similar to that by a gauge string whose energy per unit length equals that contained within a radius r_0 of the core of the global string.

There is a threshold velocity V_0 above which the string acts as a convergent lens and below which the string acts as a divergent lens. By demanding $\epsilon = 0$ in Eq. (14) one obtains

$$V_0^2 = \frac{1}{2\left[\ln\dfrac{r_0}{\delta} + c + \ln 2 + 1\right]} \tag{15}$$

If r_0 is an astronomical scale (say the size of a galaxy) and δ is of the order of the grand unification scale, then V_0 is about $\frac{1}{10}$ the speed of light. When a particle moves at speed V_0 there is no net deflection.

Finally, if the particle moves so slowly that $V^2 \ll 2\mu \ln L/r_0$, the repulsive gravitational force is the only relevant effect. The deficit angle in the spacelike metric is negligible compared to the total deflection angle, which is of order π.

IV. CONCLUSIONS

We have seen that the gravitational field of a global string has similar effects upon ultrarelativistic particles than the gravitational field of a gauge string of comparable energy per unit length. Those effects can be summarized in terms of a deflection angle $\epsilon \approx -4\pi\mu \ln r_0/\delta$. Only the logarithmic dependence on the impact parameter gives rise to slight differences. Acting as a gravitational lens, a gauge string will produce an angular separation between double images of a quasar about 1.2 times larger than that produced by a global string with equal energy per unit length at the era of galaxy formation, and there will be a very small magnification of the images in the global string case.[7]

Strings are expected to move usually at speeds close to that of light. However, in a situation where the relative velocity of the string and the surrounding matter is of order V_0, as given by Eq. (15), or smaller, the gravitational effects of gauge and global strings will differ substantially from one another due to the repulsive gravitational field the latter produce.

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[8] In Ref. 7 the energy-momentum-tensor components T_t^t and T_z^z are the same as in our Eq. (4), but the other components are zero. With the source of Ref. 7, the $A(r)$ term in the metric [Eq. (2)] is constant, and thus the repulsive effect is absent.

[9] Notice that if the energy-momentum tensor of the string were to vanish beyond a long-distance cutoff Λ then the metric at $r > \Lambda$ would be that of flat space with a deficit angle $8\pi\mu(\ln\Lambda/\delta + c)$; i.e., there would be no Newtonian gravitational potential. That is the case, for instance, around a gauge string. Within the energy-density distribution of the global string there is, however, a repulsive gravitational potential. It is a non-Newtonian effect.