Supergravity solitons. I. General framework

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An outline for the construction of an effective theory of interacting solitons in $N=2$ supergravity is presented. The solitons are described by their asymptotic properties, carrying translational and supertranslational degrees of freedom. We discuss briefly the classical and the quantized dynamics for the free soliton. The Lagrangian for the motion of a soliton in a curved supergravity background is exhibited and its implications for an effective supercharge interaction are mentioned.

I. INTRODUCTION

The aim of this paper is to set the framework for and give an outline of a theory of soliton interaction in (ungauged) $N=2$ supergravity. It is the first in a sequence of four papers (Refs. 1–3) whose final goal is an effective quantum field theory of supergravity solitons in an instantaneous-interaction approximation.

Solitons have become an important tool for studying nonperturbative aspects of quantum field theories. It is well known that expanding the elementary fields around a classical "vacuum" (e.g., flat space with all nongravitational fields in the theory set equal to zero) does not in general incorporate all physical degrees of freedom.

The classical field equations may admit solutions with spatially localized energy which are stable against classical and quantum fluctuations. These "solitons" are parametrized by a set of quantities ("collective coordinates") such as the position (center of mass), momentum, and some internal variables. Upon quantization, these additional solitonic degrees of freedom, when added to the elementary field fluctuations, give rise to a richer particle spectrum; i.e., they create the "soliton sector" (or nonperturbative sector) of the full quantized theory. The first field theory where this has been done with great success is the famous "sine-Gordon model" in two dimensions.[4] As a consequence, the perturbative sector may "feel" the existence of solitons, even if they are not excited, and some of its features (such as renormalization properties) may change. Moreover, solitons or bound states of solitons may have a direct physical interpretation.

In this work we are especially interested in solitons arising within supergravity theories. Although supergravity turned out to be nonrenormalizable at the perturbative level, some extended versions arise as the field-theoretical limit of superstring theories. Therefore, it is of interest to see how the structure of supergravity theories is enriched by including nonperturbative degrees of freedom. This investigation is also motivated by the "duality" conjecture whereby one expects that classical solutions play also an important role in the strong-coupling limit.

Gibbons[5] has pointed out that certain extended supergravity theories allow for solitonlike configurations. Generalizing results of Hajicek[6] for the Einstein-Maxwell theory he discussed the general soliton structure of the $N<4$ and $N=8$ theories. We shall consider only the ungauged $N=2$ theory[7] because it is the simplest one admitting solitons. The role of classical solitons therein is played by static extreme black-hole solutions with certain fermionic (gravitino) field excitations. They saturate a Bogomol'nyi-type inequality

$$4\pi G m^2 \geq e^2 + q^2 \ ,$$

which qualifies them of having the least mass (m) for given electric (e) and magnetic (q) charge (cf. Ref. 8). Because of an additional fermionic parameter, the supercharge Q, the solitons to given values of (e,q) form a supermultiplet consisting of all "supertranslated" partners to the purely bosonic black hole.

In a previous paper,[9] we constructed the exact superpartners to the bosonic Einstein-Maxwell multisolitons. The different states of a single soliton are distinguished by position (center of mass), momentum, and a fermionic parameter representing two spinorial (classically anticommuting) degrees of freedom associated with the supercharge of the configuration. Clearly, such a configuration represents a single, noninteracting object. Our aim is to formulate, step by step, an effective dynamics for several *interacting* solitons in the lowest order of approximation that incorporates the effects of the supercharge.

The main idea is to freeze all degrees of freedom in $N=2$ supergravity except the solitonic ones. In other words, we treat the solitons as particles carrying only translational and supertranslational (spinorial) parameters. This description will only be a good approximation if little radiation is set off upon interaction, and the distortions of a soliton configuration by tidal forces is negligible. Since the supergravity field equations admit exact static multisoliton configurations (the Majumdar-Papapetrou solutions), there should be a regime where this truncation of degrees of freedom is justified.

This paper, where we give the program and outline the method, is organized as follows. In Sec. II we review the results of Ref. 9 where the supertranslated solutions were presented, and introduce the point-particle description. In Sec. III the choice of independent Lagrangian variables associated with a soliton leads to the notation of a particle in $N=2$ rigid (also called "flat") superspace. Section IV is devoted to the construction of the free soliton Lagrangian in the slow-motion limit. Attention is focused on the fermionic degrees of freedom associated with the supercharge. Upon quantization, the particle spectrum emerges as the nonrelativistic version of the basic $N=2$ "hypermultiplet." The relativistic generalization of the dynamics is presented in Sec. V. The motion of the free soliton is described by a superspace trajectory. Section VI gives the first steps toward an interaction theory. The soliton is considered as a test particle in a classical supergravity background. We give the Lagrangian and mention briefly how to proceed toward a theory of interacting particles, thereby taking the background as a classical soliton configuration itself. From the Lagrangian, one may then read off the interaction potential between two solitons in an instantaneous approximation. The detailed computations suggested in Secs. V and VI are given in subsequent papers.[1-3]

Once having a classical (and from that a first-quantized) theory of soliton interactions, one may proceed to condense the notion of an arbitrary number of interacting solitons into the language of an effective field theory, exhibiting field operators (and thus creation and annihilation operators) for solitons. This last step, the construction of an effective quantum field theory of solitons in the approximation mentioned above, is also carried out in Ref. 3.

Let us finally remark that in a number of papers[10] localized, static *nongauge* solutions of the $N=2$ supergravity equations have been obtained. However, since it is not clear whether these "superhair" configurations qualify as solitons, we do not include them in our construction.

II. STATIC SOLITONS

In this section we review and generalize results concerning the exact superpartners as derived in Ref. 9. The Majumdar-Papapetrou configuration is considered as the general static classical multisoliton solution of the Einstein-Maxwell theory. In suitable coordinates, the vierbein field and the electromagnetic potential are given by[11]

$$ {}^{(0)}e^0 = V^{-1}dx^0, \quad {}^{(0)}e^i = V\,dx^i \ , \tag{2.1a}$$

$$ k\,{}^{(0)}A = -\zeta V^{-1} dx^0 \ , \tag{2.1b}$$

where

$$ V = 1 + \sum_{J=1}^{n} \frac{Gm_J}{c^2\,|\mathbf{x}-\mathbf{x}_J|} \ , \tag{2.2}$$

and $\zeta = +1$ or -1. (We follow essentially the conventions of Ref. 9 with the exception that the velocity of light c is made explicit in all formulas up to Sec. III, $x^0 = ct$, and that the tangent space indices are now taken from the middle of the alphabet: $m,n,r,\ldots$; see Appendix A.)

The configuration (2.1) and (2.2) describes an ensemble of n extreme black holes with masses m_J and electric charges

$$ e_J = \zeta k m_J \tag{2.3}$$

in equilibrium.[12] ζ determines the sign of the electric charges. Since our final aim is to describe the dynamics of solitons, both values of ζ must be considered. For our purpose we exclude the existence of magnetic charges which enables us to work with a globally defined electromagnetic potential.

The Arnowitt-Deser-Misner (ADM) four-momentum and the total electric charge of the Majumdar-Papapetrou fields are

$$ P_m^{\mathrm{ADM}} = (mc,0,0,0), \quad m = \sum_{J=1}^{n} m_J \ , \tag{2.4}$$

$$ e = \sum_{J=1}^{n} e_J = k\zeta m \ . \tag{2.5}$$

The configurations (2.1) are also (purely bosonic) solutions of the $N=2$ supergravity field equations.[13] The iterative application of a local supersymmetry transformation to (2.1), with a time-independent gauge spinor field ϵ which approaches a constant spinor ϵ_∞ at infinity, gives rise to the "supertranslated" solution[9]

$$ e^m = {}^{(0)}e^m - \frac{i}{4}[\bar\epsilon\gamma^m\hat D\epsilon - (\hat D\bar\epsilon)\gamma^m\epsilon] + O(\epsilon^4) \ , \tag{2.6a}$$

$$ kA = k\,{}^{(0)}A + \frac{i}{4}[\bar\epsilon\hat D\epsilon - (\hat D\bar\epsilon)\epsilon] + O(\epsilon^4) \ , \tag{2.6b}$$

$$ k\psi = \hat D\epsilon + O(\epsilon^3) \ , \tag{2.6c}$$

where the supercovariant derivative is taken with respect to the bosonic configuration (2.1). ψ is the (complex) Rarita-Schwinger (gravitino) field of $N=2$ supergravity ($\psi = \psi^1 + i\psi^2 = \psi_\mu dx^\mu, \epsilon = \epsilon^1 + i\epsilon^2$ where ψ_μ^j and ϵ^j are Majorana spinors).

The ADM momentum and electric charge of this configuration are still given by (2.2) and (2.5) while the supercharge[9,14,15] of (2.6) is

$$ Q = -\frac{ic^3}{k}\oint_{S^2_\infty} \gamma_5\gamma\wedge\psi = -i\left[P_m^{\mathrm{ADM}}\gamma^m - \frac{c}{k}e\right]\epsilon_\infty \ , \tag{2.7}$$

which implies, together with (2.4) and (2.5),

$$ \left[P_m^{\mathrm{ADM}}\gamma^m + \frac{c}{k}e\right]Q = 0 \tag{2.8}$$

as a constraint for gauge-generated supercharges.

In order to explore the implications of (2.8), it is useful to split up

$$ -\epsilon_\infty = \begin{bmatrix} a+b \\ a-b \end{bmatrix} \ , \tag{2.9}$$

where a and b are complex two-spinors. Then (2.7) and (2.8) become (we use the Weyl representation of the γ matrices[16])

$$ Q = -imc(\gamma_0 - \zeta)\epsilon_\infty \tag{2.10}$$

and

$$(\gamma_0+\zeta)Q=0 \ , \tag{2.11}$$

which implies, for $\zeta=1$ and $\zeta=-1$,

$$Q=-2imc\begin{bmatrix} b \\ -b \end{bmatrix} \tag{2.12a}$$

and

$$Q=2imc\begin{bmatrix} a \\ a \end{bmatrix} \ , \tag{2.12b}$$

respectively. Thus, only half of the components of ϵ_∞ contribute to the supercharge, while the others give rise to short-range gauge transformations.[9] A gauge spinor ϵ_{sc} of the form

$$\epsilon_{sc}=V^{-1/2}\epsilon_\infty \tag{2.13a}$$

with

$$(\gamma_0-\zeta)\epsilon_\infty=0 \tag{2.13b}$$

is supercovarianty constant; i.e., its satisfies

$$\hat{D}\epsilon_{sc}=0 \tag{2.14}$$

and leaves the configuration (2.1) invariant. Any gauge spinor which approaches ϵ_{sc} at infinity induces only a short-range transformation. We identify all configurations which are short-range gauge related. The resulting equivalence classes are then parametrized by only half the components of ϵ_∞ (a for $\zeta=-1$, b for $\zeta=1$).

The configurations (2.6) with $Q\neq 0$ are interpreted as "superpartners" of the bosonic solution (2.1), much the same way as spatially translated solitons are "translational" partners of each other. Just like the center of mass (position), the supercharge becomes a dynamical variable of the soliton.

In Ref. 9, the iteration (2.6) was carried out to all orders in ϵ for $\zeta=1$, and the exact superpartner was displayed. One may fix a certain configuration of every equivalence class by a suitable choice of ϵ for a given value of Q, e.g.,

$$\epsilon=V^{-1/2}\epsilon_\infty \tag{2.15a}$$

with

$$(\gamma_0+\zeta)\epsilon_\infty=0 \tag{2.15b}$$

which implies the gauge condition

$${}^{(0)}\gamma^{\mu}\,{}^{(1)}\psi_\mu=0 \ , \tag{2.16}$$

where ${}^{(1)}\psi$ is the part of ψ which is linear in ϵ. The supercharge is then proportional to ϵ_∞:

$$Q=2imc\zeta\epsilon_\infty \ . \tag{2.17}$$

With this choice, the lowest order in the expansion (2.6) of the superpartners reads

$$\begin{aligned} e^0&=V^{-1}dx^0+V^{-2}V_{,k}\epsilon_{kji}\mathcal{C}_j dx^i+O(\epsilon^4) \ , \\ e^i&=V\,dx^i+V^{-4}V_{,k}\epsilon_{kij}\mathcal{C}_j dx^0+O(\epsilon^4) \ , \end{aligned} \tag{2.18a}$$

$$kA=\zeta(-V^{-1}dx^0+V^{-2}V_{,k}\epsilon_{kji}\mathcal{C}_j dx^i)+O(\epsilon^4) \ , \tag{2.18b}$$

$$k\psi=(\zeta V^{-3}V_{,k}\gamma_k dx^0+V^{-1}V_{,k}\gamma_k\gamma_i dx^i)\epsilon+O(\epsilon^3) \ , \tag{2.18c}$$

where, in the case $\zeta=1$,

$$\mathcal{C}_i=b^\dagger\sigma_i b \ , \tag{2.19}$$

whereas for $\zeta=-1$,

$$\mathcal{C}_i=a^\dagger\sigma_i a \ . \tag{2.20}$$

The ADM four-momentum, the electric charge, and the supercharge of (2.18) are given by (2.4), (2.5), and (2.17), respectively. Moreover, the g_{0i} components of the metric associated with the vierbein field (2.18a) give rise to an angular momentum

$$S_i=mc\mathcal{C}_i \ . \tag{2.21}$$

We refer to it as the classical "spin" (angular momentum in the rest frame) of the object. As stated in Ref. 9, there is also a magnetic moment associated with the spin, giving the correct quantum-mechanical gyromagnetic ratio.

Our aim is to describe these configurations solely by their long-range properties, thereby assuming that the solitons play the role of fundamental entities of the theory. Gauge transformations which do not tend to unity at infinity will in general change the asymptotic properties of a configuration, and thus the physical state of the system. The generators of these "improper" gauge transformations (ADM four-momentum, ADM angular momentum, and supercharge) are given by surface integrals at spatial infinity which transform according to the flat-space (Poincaré) supersymmetry algebra.[15] [The electric charge, which may be considered as the global U(1) generator, enters the algebra as a central charge.] The asymptotic spatial translations and rotations as well as the previously considered supertranslations maintain the time independence of the configuration. This gauge freedom gives rise to collective coordinates which play the role of dynamical variables in an asymptotic description. For the single soliton ($n=1$) configuration, we associate a position (center-of-mass) variable with the translational and a spinorial variable, the supercharge, with the supertranslational degrees of freedom. The spatial rotations may be absorbed into the supersymmetry transformations, the reason being, that the nonspherically symmetric part of (2.18) comes from the supercharge.

Describing the soliton only by position and supercharge, means that all other degrees of freedom are neglected. For the interacting case, which is our final aim, this implies that we are working with a highly truncated theory which will only be a good approximation to the underlying field theory if conditions are such that the other degrees of freedom are (almost) not excited. For a soliton in an external field (the subject of Sec. VI and Ref. 2), this can be expected if accelerations and tidal forces are small. In an effective soliton-soliton interaction picture (Ref. 3), one can at best expect validity in the slow-motion limit, where radiation effects are negligible. For the quantum version, this implies that the excitations of the elementary quanta (gravitons, gravitinos, and photons) are suppressed.

The single soliton configuration may then be looked upon as a classical point particle in Minkowski space with mass m and electric charge

$$e=\zeta km \ . \tag{2.22}$$

We associate with it (in a relativistic formulation which may be achieved by Lorentz boosting the static configurations) a spacetime variable $x^{\mu}(s)$, a four-momentum $P^{\mu}(s)$, and a (Grassmann-valued, anticommuting) supercharge $Q(s)$ obeying the constraint

$$\left[P_{\mu}\gamma^{\mu}+\frac{c}{k}e\right]Q=0 \ . \tag{2.23}$$

As the basic equations of motion for the single free soliton we postulate

$$\frac{d}{ds}P^{\mu}=0 \tag{2.24a}$$

and

$$\frac{d}{ds}Q=0 \ . \tag{2.24b}$$

However, it will turn out in the next section that Q is not the appropriate independent variable in a Lagrangian formulation (as opposed to x^{μ}). In Secs. IV and V we will construct an effective Lagrangian for a point particle which reproduces the dynamics (2.24) and the constraint (2.23).

III. THE SUPERTRANSLATIONAL FREEDOM

First we have to find suitable independent dynamical variables. For the bosonic part we have already confined ourselves to the position $x^{\mu}(s)$ describing the translational motion of the soliton as a whole. But how to include the fermionic variable Q in a Lagrange formulation? Since the constraint (2.23) depends on P_{μ}, the set of all possible supercharges Q is not defined *a priori*, but depends on the motion of the soliton. Some trial and error reveals that it is rather problemetic to use Q as an independent dynamical variable in the Lagrangian. We therefore propose a different approach which carries over to the interacting case and which will be, at a later stage of our argumentation, in natural agreement with the existence of the local supersymmetry in the underlying field theory.

We assign to the soliton a (classically anticommuting) dynamical fermionic variable $\theta=(\theta^{\alpha})$ [a complex four-spinor which is decomposed into its Majorana parts as $\theta=\theta^1+i\theta^2\equiv(\theta^j)$]. The dynamics of the particle is determined by the set $(x^{\mu}(s),\theta^{\alpha}(s))$, in other words, by a *trajectory in $N=2$ superspace.* The supercharge Q will then arise as the generator of translations in θ, i.e., of global supersymmetry transformations, the same way as the momentum P_{μ} is associated with the space translations [see (5.2) below]. As a by-product, we obtain an additional gauge symmetry acting on θ and rendering two of its components physically superfluous, thus reidentifying Q and θ in a special gauge. Moreover, Q will essentially play the role of the momentum conjugate to θ. Of course the idea to describe a supersymmetric particle by a trajectory through superspace is not new, and several supersymmetric point particle Lagrangians have been studied (cf. Refs. 17 and 18).

Treating the fermoinic superspace variable as the analogue of x^{μ}, two questions naturally arise. (i) What is the physical meaning of the global supertranslations acting on θ? (ii) May one associate a certain value for θ with a given soliton configuration?

To answer the first question, let us first note that the soliton configurations are obtained by long-range supergravity gauge transformations acting on bosonic (black-hole) solutions. According to (2.7), only the value at infinity of the gauge parameter contributes to the supercharge. The remaining local gauge freedom is lost in the asymptotic description.

Acting on a soliton configuration by a further asymptotic gauge transformation just means to *change the value at infinity of the gauge parameter* (i.e., to pass to another soliton configuration):

$$\epsilon_{\infty}\rightarrow\epsilon'_{\infty} \ . \tag{3.1}$$

Let us denote the infinitesimal version of such a variation as

$$\delta\epsilon_{\infty}=-\bar{\epsilon} \ . \tag{3.2}$$

[The minus sign will give rise to the usual signs in the supersymmetry (SUSY) law.] The according variation of the supercharge is read off from (2.7):

$$\delta Q=i\left[P_{\mu}\gamma^{\mu}-\frac{c}{k}e\right]\bar{\epsilon} \ . \tag{3.3}$$

It is now natural to identify these variations with the rigid supertranslations acting on the $N=2$ superspace in terms of which the point-particle dynamics is formulated. In other words, the space of all values for ϵ_{∞} is identified with the space of all θ's. Postulating the correspondence

$$\theta\leftrightarrow-\epsilon_{\infty} \ , \tag{3.4}$$

(3.2) becomes the action of global SUSY in superspace:

$$\delta\theta=\bar{\epsilon} \ . \tag{3.5}$$

The identification (3.4) is in accordance with the expectation that a purely bosonic soliton (the extreme Reissner-Nordström solution) is associated with the value $\theta=0$. Thus, the answer to the first question is the following: The global $N=2$ SUSY reflects the fact that soliton field configurations are related by (long-range) gauge transformations.

As a consequence of this interpretation, the answer to the second question turns out to be negative. Because of the constraint (2.23), only half of the components of ϵ_∞ (and thus of θ) contribute to the supercharge. Varying θ by an $\tilde{\epsilon}$ which satisfies

$$\left[P_\mu \gamma^\mu - \frac{c}{k} e\right]\tilde{\epsilon}=0 \tag{3.6}$$

has no effect on Q and thus on the physical state. At this stage it should not be surprising that, as will be worked out in Ref. 1, $\tilde{\epsilon}$ in (3.6) may also depend on the time variable s, giving rise to an additional *local* gauge symmetry of the particle dynamics. This symmetry may be used to impose a gauge condition on θ. At the level of soliton field configurations, this corresponds to a choice of ϵ_∞ for each value of Q. One possible choice has been made in (2.15b) and (2.17). For notational ease, from now on $\tilde{\epsilon}$ is simply denoted by ϵ.

In the rest frame (and also in the low-velocity limit) the decomposition of θ in gauge and nongauge contribution follows from (2.9):

$$\theta = \begin{bmatrix} a+b \\ a-b \end{bmatrix} . \tag{3.7}$$

The gauge condition (2.15b) imposed on ϵ_∞ now reads

$$a=0 \tag{3.8a}$$

for positive e and

$$b=0 \tag{3.8b}$$

for negative e. The remaining components of θ are then in one-to-one correspondence with Q and constitute the "true" degrees of freedom [cf. Eq. (2.17)]. The nonrelativistic Lagrangian as given in the next section will be formulated entirely in terms of these. From the superspace point of view, we have now exhibited the reason why this is possible: The gauge components of θ will not enter the nonrelativistic Lagrangian and are therefore completely arbitrary. Getting rid of them is most naturally done by setting them equal to zero, as in (3.8).

The decomposition (3.7) will be used to evaluate the nonrelativistic limit of the full Lagrangian in Ref. 1. It carries over to the case of interacting solitons.[2,3] Its relativistic generalization is also worked out in Refs. 1–3.

Moreover, by suitably eliminating the redundant degrees of freedom in the relativistic version, the relation between four-momentum and four-velocity will become

$$P^\mu = m\frac{dx^\mu}{ds} \tag{3.9}$$

if s is the proper time.

IV. NONRELATIVISTIC DYNAMICS OF THE FREE SOLITON

Let us first consider the slow-motion (nonrelativistic) limit. The lowest-order term of the constraint (2.23) is just (2.11) and therefore independent of the velocity. From Eqs. (2.12a) and (2.12b) we infer that, to this order, the possible supercharges are parametrized by b or a for $\zeta=1$ to $\zeta=-1$, respectively. These quantities we take as the independent dynamical variables.

The problem in the slow-motion limit is posed as follows. Can we find a point-particle Lagrangian with $\mathbf{x}(t)$ and $a(t)$ or $b(t)$ as dynamical variables, which describes the free slowly moving soliton and which reproduces the correct supercharge and spin?

We restrict ourselves to the case $\zeta=-1$ (the other case is completely analogous). Assuming that the Lagrangian L depends only on $\mathbf{x}(t)$, $\dot{\mathbf{x}}(t)$, $a(t)$, and $\dot{a}(t)$ (the overdot denoting the derivative with respect to t and the components $a_A, \dot{a}_A$, $A=1,2$, anticommute with each other), we impose the following conditions: (i) The resulting equations of motion should read $\ddot{\mathbf{x}}=0$ and $\dot{a}=0$; (ii) the bosonic part of L (i.e., setting $a=\dot{a}=0$) should be that of a free nonrelativistic particle with mass m; (iii) the Lagrangian should be translational and rotational invariant [a_A transforming as an SU(2) spinor]; (iv) since we want to suppress all (self-) interactions at this level, L is required to be quadratic in a and $\dot{a}$.

Conditions (i)–(iv) lead to the ansatz

$$L=\tfrac{1}{2}m\dot{\mathbf{x}}^2 + mc[a^\dagger f(\mathbf{x},\dot{\mathbf{x}})\dot{a} + \text{H.c.}] , \tag{4.1}$$

where f is a dimensionless 2×2 matrix function which, when expanded in powers of $\dot{\mathbf{x}}/c$, reads

$$f(\mathbf{x},\dot{\mathbf{x}})=\alpha \mathbb{1} + \beta\frac{\dot{x}^i}{c}\sigma_i + O(\dot{\mathbf{x}}^2/c^2) \tag{4.2}$$

with α and β complex constants. This follows essentially from the requirement (iii). We only keep the lowest-order term and determine α by a method that proved useful in the theory of scalar field solitons in two dimensions (see Ref. 19). Replace a by $a(t)$ in the explicit superpartner solution (2.18) and insert into the field-theoretical Lagrangian of $N=2$ supergravity.[20] From the result

$$\int d^3x\, \mathcal{L} = imc(a^\dagger \dot{a} - \dot{a}^\dagger a) + O(a^4) \tag{4.3}$$

we read off $\alpha=i$.

At this point we should critically remark that this method is problematic in curved space: Applying it to the position variable $\mathbf{x}(t)$ of a black hole gives, apart from a constant and a total time derivative,

$$\int d^3x\, \mathcal{L} = \tfrac{1}{2}M\dot{\mathbf{x}}^2 \tag{4.4}$$

but M is coordinate dependent and may even be infinite.

However, further support for the value of α is obtained by computing the conserved quantities associated with spatial rotations. Applying

$$\delta x^i = \epsilon_{ijk}\lambda^j x^k, \quad \delta a = -\frac{i}{2}\lambda^i \sigma_i a \ , \tag{4.5}$$

to L, one finds the generator of rotations to be

$$L_i = \epsilon_{ijk}x^j p_k - i\alpha mc\,\mathcal{C}_i \ , \tag{4.6a}$$

$$\mathbf{p}=m\dot{\mathbf{x}} \ . \tag{4.6b}$$

The first part of L_i is the usual orbital angular momentum, the second part, the classical spin, coincides with (2.21) only for the above choice of α.

Thus, the nonrelativistic limit of the free point-particle Lagrangian is

$$L=\tfrac{1}{2}m\dot{\mathbf{x}}^2+imc(a^\dagger\dot{a}-\dot{a}^\dagger a) \tag{4.7}$$

for $\zeta=-1$, and

$$L=\tfrac{1}{2}m\dot{\mathbf{x}}^2+imc(b^\dagger\dot{b}-\dot{b}^\dagger b) \tag{4.8}$$

in the $\zeta=1$ case. A similar Lagrangian was considered in a different context (in $N=1$ supersymmetry) by Casalbuoni.[21] Note that the combinations $\mathbf{x}, c^{1/2}a$, respectively, $c^{1/2}b$, play a role of "nonrelativistic" quantities.

We recall that the fermionic parts of these Lagrangians are obtained by taking the superpartner configurations in a specific gauge. The gauge freedom of choosing a different gauge spinor ϵ' than in (2.15), leading to the same supercharge, is reflected by an additional term to (4.7) of the form

$$L_{\text{gauge}}=-2imc(a^\dagger\dot{f}-\dot{f}^\dagger a)+O(f^2)\ , \tag{4.9}$$

where

$$\epsilon-\epsilon'=V^{1/2}\begin{bmatrix} f\\ f\end{bmatrix}$$

and

$$\lim_{|\mathbf{x}|\to\infty} f(\mathbf{x})=0\ .$$

The form of the gauge Lagrangian (4.9) will be justified in Ref. 3. In the present context it can be obtained by the following heuristic argument. First let $f(t,\mathbf{x})$ and find, using the field-theoretical Lagrangian of $N=2$ supergravity.

$$L_{\text{gauge}}=-2imc\left[a^\dagger\frac{d}{dt}[f(t,0)]-\text{H.c.}\right]+O(f^2)\ .$$

Substituting the position vector into the second argument and omitting the explicit time dependence of f, one arrives at (4.9). The freedom described by this part of the whole Lagrangian corresponds to the local supersymmetry of the underlying field theory. In the quantum version, it may be compensated by an adequate local gauge transformation on the physical states, as we shall see below.

The canonical momenta associated with the fermionic variables (a and $a^\dagger$ treated as independent variables) are

$$\pi=\frac{\vec{\partial}}{\partial\dot{a}^\dagger}L=-imca\ , \tag{4.10a}$$

$$\pi^\dagger=L\frac{\overleftarrow{\partial}}{\partial\dot{a}}=imca^\dagger\ . \tag{4.10b}$$

Calculating the Hamiltonian, we notice that the fermionic part does not contribute to the total energy:

$$H=\frac{\mathbf{p}^2}{2m}\ . \tag{4.11}$$

This is in agreement with the superpartner configurations where the fermionic part in the vierbein does not contribute to the ADM energy integral. Let us now quantize the system (4.7). The relations (4.10) are second-class constraints.[22] Introducing graded Dirac brackets $[\ ,\ \}_*$ to eliminate the π's and going over to the graded commutator, we arrive at the canonical (anti)commutation relations

$$[x^i,p_j]=i\hbar\delta^i{}_j\ , \tag{4.12a}$$

$$\{a_A,a_B^\dagger\}=\frac{\hbar}{2mc}\delta_{AB}\ , \tag{4.12b}$$

all others vanish. The supercharge as operator on the quantum states is defined by (2.12b). One easily checks that the anticommutation relations among the a's and $a^\dagger$'s are identical with

$$\begin{aligned}&\{Q^\alpha,Q^\beta\}=\{\bar{Q}_\alpha,\bar{Q}_\beta\}=0\ ,\\ &\{Q^\alpha,\bar{Q}_\beta\}=2\hbar(P_\mu\gamma^\mu+mc)^\alpha{}_\beta\ ,\end{aligned} \tag{4.13}$$

where

$$P_\mu=(mc,0,0,0)\ .$$

This algebra is exactly the $N=2$ SUSY algebra with central electric charge $e=-km$ acting on states in the rest frame. (For a review on the global SUSY algebra see, e.g., Ref. 23.) This also justifies the value of α.

The operators a and $a^\dagger$ (or b and $b^\dagger$, for $\zeta=1$) constitute a four-dimensional Clifford algebra. They play the role of the (nonrelativistic) supercharge and change the spin by $\pm\hbar/2$, respectively. Together with the standard $L^2(\mathbb{R}^3)$ representation for (4.12a), the algebra may be realized on the space of superfields

$$\begin{aligned}\Psi(\mathbf{x},t,a^\dagger)&=\phi(\mathbf{x},t)+\left[\frac{mc}{\hbar}\right]^{1/2}a_A^\dagger\chi^A(\mathbf{x},t)\\ &\quad+\frac{1}{2}\frac{mc}{\hbar}\epsilon^{AB}a_A^\dagger a_B^\dagger\lambda(\mathbf{x},t)\\ &\equiv\begin{bmatrix}\phi\\ \chi\\ \lambda\end{bmatrix}\end{aligned} \tag{4.14}$$

with $a_A^\dagger$ acting by left multiplication and

$$a_A=\frac{\hbar}{2mc}\frac{\vec{\partial}}{\partial a_A^\dagger}\ . \tag{4.15}$$

The spin operator

$$S_i=mca^\dagger\sigma_i a$$

obeys the SO(3) algebra, and its action on Ψ is

$$\phi\to 0,\quad \chi\to\frac{\hbar}{2}\sigma_i\chi,\quad \lambda\to 0\ . \tag{4.16}$$

Thus the quantum states for the free soliton form a multiplet with spins $(0,\tfrac{1}{2},0)$.

The global SUSY transformation induced by a on the superfield is

$$\delta\Psi=(mc/\hbar)^{1/2}(a^{\dagger}\nu-\nu^{\dagger}a)\Psi \tag{4.17}$$

which for the components reads

$$\delta\phi=-\nu_A^{\dagger}\chi_A \ , \tag{4.18a}$$

$$\delta\chi_A=-\nu_A\phi+\epsilon_{AB}\nu_B^{\dagger}\lambda \ , \tag{4.18b}$$

$$\delta\lambda=-\epsilon_{AB}\nu_A\chi_B \ , \tag{4.18c}$$

where $\nu=(\nu_A)$ is a constant anticommuting spinor. This may be considered as the nonrelativistic limit of the transformation law for the basic $N=2$ "hypermultiplet" (which has extreme central charges[24]).

Varying the multiplet according to (4.18) by a *local* $\nu(\mathbf{x})$ compensates the effect of the gauge Lagrangian (4.9) if we identify

$$\nu(\mathbf{x})=2(mc/\hbar)^{1/2}f(\mathbf{x}) \ . \tag{4.19}$$

(This is in analogy to the local phase-factor transformation of the Dirac field to compensate a gauge transformation of the electromagnetic potential.) The possibility of altering L by such a term corresponds to the freedom of generating a gravitino field $\psi=d\epsilon$ from flat space. Equation (4.18) with local ν therefore reflects the existence of the supergauge freedom in $N=2$ supergravity.

The time evolution for the superfield is given by the Hamiltonian (4.11)

$$i\hbar\frac{\partial}{\partial t}\Psi=-\frac{\hbar^2}{2m}\Delta\Psi \tag{4.20}$$

which consists of two free Schrödinger equations for the scalar fields and a free Pauli equation for the spinor field.

The multiplet structure coincides with results derived by Gibbons[5] who has discussed the possibility of solitons in extended supergravity theories. Relying on the gauge condition (2.16), he argued that only the extreme charged black holes have superpartners and may be considered as solitons [for remarks on (2.16) see Ref. 9].

V. RELATIVISTIC DYNAMICS OF THE FREE SOLITON

Having set up the multiplet structure for the quantized free soliton in the nonrelativistic limit, we proceed in our task to find a method for describing interacting solitons by generalizing the previous results to the relativistic regime. We have in mind a Lorentz-invariant particle dynamics in rigid (flat) $N=2$ superspace. The dynamics of the soliton is to be described by a Lagrangian $L(x^{\mu},\dot{x}^{\mu},\theta^{\alpha},\dot{\theta}^{\alpha})$ (where the overdot means d/ds, and from now on we set $c=1$) which should have the following properties.

(i) Setting $\theta=\dot{\theta}=0$, L reduces to the usual, reparametrization-invariant Lagrangian

$$L=-m\sqrt{\dot{x}^{\mu}\dot{x}_{\mu}} \ . \tag{5.1}$$

(ii) L is invariant under global supersymmetry transformations

$$\delta x^{\mu}=\frac{i}{2}\bar{\epsilon}^{j}\gamma^{\mu}\theta^{j}\equiv\frac{i}{4}(\bar{\epsilon}\gamma^{\mu}\theta-\bar{\theta}\gamma^{\mu}\epsilon), \quad \delta\theta=\epsilon \ . \tag{5.2}$$

(iii) The conserved quantity associated with this symmetry (the supercharge) belongs to the $N=2$ SUSY algebra (represented by Poisson brackets or their quantum version) with extreme central electric charge $e=\zeta km$.

(iv) The condition (2.23) on the supercharge emerges as a constraint of the theory.

(v) The nonrelativistic limit coincides with the model of the previous section.

The following arguments lead to a Lagrangian compatible with the requirements (i)–(v). First replace $\dot{x}^{\mu}$ by (cf. Ref. 21)

$$\dot{x}^{\mu}-\frac{i}{2}\bar{\theta}^{j}\gamma^{\mu}\dot{\theta}^{j} \tag{5.3}$$

in (5.1) to ensure $N=2$ supersymmetry, but with zero central charges. To generate the correct central charges, the supercharge, as the generator of supertranslations acting on superfields,

$$Q^{\alpha}=2i\hbar\frac{\vec{\partial}}{\partial\bar{\theta}_{\alpha}}-\frac{\hbar}{2}(\gamma^{\mu}\theta)^{\alpha}\partial_{\mu} \ , \tag{5.4}$$

has to be modified by adding the term

$$-\frac{i}{2}\zeta m\theta^{\alpha} \ . \tag{5.5}$$

One then checks that the term $\frac{1}{2}\zeta m\epsilon^{jk}\bar{\theta}^{j}\dot{\theta}^{k}$ varies under (5.2) into a total derivative and when added to L, produces the above superspace representation of Q. We thus end up with

$$L=-m\sqrt{\mathcal{A}^{\mu}\mathcal{A}_{\mu}}+\tfrac{1}{2}\mathcal{G} \ , \tag{5.6}$$

where

$$\mathcal{A}^{\mu}=\dot{x}^{\mu}-\frac{i}{2}\bar{\theta}^{j}\gamma^{\mu}\dot{\theta}^{j} \ , \tag{5.7}$$

$$\mathcal{G}=\frac{e}{k}\epsilon^{jk}\bar{\theta}^{j}\dot{\theta}^{k}, \quad e^2=k^2m^2 \ . \tag{5.8}$$

This Lagrangian has been studied previously in a different context by Azcarraga, Lukierski, Frydryszak, and Lusanna.[17] We have not proven that the conditions (i)–(v) lead uniquely to (5.6). We also remark that one may attach a magnetic charge to the particle as well. For the soliton configuration this is achieved by a duality transformation[9,25] which carries over to the point particle model.

In the forthcoming paper,[1] we shall study the implications of this Lagrangian in detail, showing that the requirements are indeed satisfied. Upon (first) quantization, one arrives at the basic $N=2$ hypermultiplet.[24]

VI. SOLITONS IN CURVED SUPERSPACE

Having obtained a relativistically invariant and globally supersymmetric point-particle Lagrangian for the free soliton, we would like to incorporate interactions. We start by studying the dynamics of the point particle in an external field. Since our goal is to derive an effective soliton-soliton interaction, we take the external fields to be those of $N=2$ supergravity. In other words, we seek a locally supersymmetric extension of the Lagrangian constructed so far. This leads naturally to the formulation of the theory in (curved) superspace of $N=2$ supergravity.[20] The dynamics of the system is then described by the motion of a "test" particle in this superspace. The motion of such a "small" soliton should, in principle, follow from the supergravity field equations in a suitable approximation. Since this has not been achieved in a consistent way even for pure gravity, we proceed heuristically and try to anticipate the outcome of such an approximation.

Once the Lagrangian has been found, the background is taken to be a *soliton configuration,* thus leading to an effective "force" between two solitons in an instantaneous interaction approximation. This idea was applied by Gibbons and Ruback[26] to study the dynamics of extreme black holes within the Einstein-Maxwell theory.

Taking into account the foregoing, we require L to meet the following criteria: (i) L is $N=2$ supersymmetric in the sense that a local supersymmetry transformation of the background induces a corresponding transformation of the particle variables; (ii) by choosing the background to be flat, L reduces to (5.6)–(5.8); (iii) the purely bosonic part of L corresponds to the dynamics of a charged particle in the Einstein-Maxwell theory:

$$L(\theta=\dot{\theta}=0)=-m\sqrt{\dot{x}^\mu \dot{x}^\nu g_{\mu\nu}}-e\dot{x}^\mu A_\mu \,, \qquad e^2=k^2m^2 \,. \tag{6.1}$$

We proceed to construct L in terms of the simplest supercovariant objects. Again, we have no proof that the resulting Lagrangian is unique. $N=2$ supergravity in superspace is usually formulated in terms of two superfields, the supervielbein $V_M{}^A$ and the U(1) gauge field B_M (see Appendix B). These fields are defined on superspace with coordinates $z^M=(x^\mu,\theta^{\alpha j})$. A straightforward generalization of $g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu$ is

$$\dot{z}^M V_M{}^A \dot{z}^N V_N^B \eta_{BA} \,, \tag{6.2}$$

where

$$\eta_{AB}=\begin{bmatrix} \eta_{mn} & 0 \\ 0 & \beta C_{ab} \end{bmatrix} \tag{6.3}$$

and β an arbitrary constant. One checks that the case $\beta\neq 0$ disagrees with the property (ii). Thus, we take for the "gravitational" parts of L the term (6.2) with $\beta=0$ in (6.3). A suitable generalization of the second term in (6.1) is obtained by replacing $\dot{x}^\mu$ by $\dot{z}^M$ and eA_μ by $(e/k)B_M$. The resulting Lagrangian reads

$$L=-m\sqrt{\mathcal{A}^m\mathcal{A}^n\eta_{mn}}-\frac{e}{k}\dot{z}^M B_M \,, \tag{6.4}$$

where

$$\mathcal{A}^m=\dot{z}^M V_M{}^m, \quad e^2=k^2m^2 \,. \tag{6.5}$$

Equation (6.4) is at the same time the supersymmetric extension of the Einstein-Maxwell-dynamic (6.1) as well as the extension of the flat superspace Lagrangian (5.6)–(5.8) to curved space. Note that $\mathcal{G}$ of (5.6) which was essential for producing the correct central charge in the algebra, is seen to be the leftover of the U(1) gauge potential B_M in the flat-space limit of (6.4), see also Eq. (B4f) in Appendix B.

The same Lagrangian has been used by Lusanna and Milewski[18] to derive part of the superfield constraints. For our purpose, (6.4) gives the dynamics of a "small" soliton, classically described by the trajectory $(x^\mu(s),\theta^{\alpha j}(s))$ is curved superspace. The supergravity field configuration (e^m,A,ψ) which enters L through the superfields $V_M{}^m$ and B_M are assumed to satisfy the field equations[13] (in contrast with Ref. 18).

We will study this dynamics in detail in Ref. 2. To quote some of the results we remark that the equations of motion (in an approximation where third-order fermion terms are neglected) are of the type

$$\left[m\gamma_\mu \frac{u^\mu}{\sqrt{u^\rho u_\rho}}-\frac{e}{k}\right]\left[\frac{\hat{D}\theta}{ds}+ku^\mu\psi_\mu\right]=0 \,, \tag{6.6}$$

$$\frac{D}{ds}\left[\frac{mu_\mu}{\sqrt{u^\rho u_\rho}}\right]=eF_{\mu\nu}u^\nu + \bar{Q}^j(k\hat{\psi}_{\mu\nu}{}^j+\tfrac{1}{2}\hat{\Omega}_{\mu\nu}{}^{jk}\theta^k)u^\nu+\cdots \,, \tag{6.7}$$

where

$$u^\mu=\frac{dx^\mu}{ds} \,.$$

$\hat{\psi}$ and $\hat{\Omega}$ are the gravitino field strength and "supercurvature," respectively. The local gauge freedom of (6.6) as well as the terms denoted by dots are exhibited in Ref. 2. The last term of (6.7) contains the expression

$$\tfrac{1}{2}R_{\mu\nu}{}^{mn}S_{mn}u^\nu \,, \tag{6.8}$$

where S_{mn} is the spin tensor, and therefore constitutes a supersymmetric generalization of the Mathisson-Papapetrou equation of motion for a spinning particle.[27]

As mentioned above, a further step is to choose a soliton configuration for the background and derive from L the force between two solitons in the low-velocity limit.

As an example of the results derived in Ref. 3, we display the interaction potential between two solitons, both with $\zeta=-1$, and masses m and m'. Their respective position variables are $\mathbf{x}$ and $\mathbf{x}'$, the (nonrelativistic) supercharge parameters a and a'. Up to some gauge freedom (which will be omitted here), the interaction Lagrangian (which is the negative of the interaction potential) in the instantaneous approximation and for large distances is

$$L_{\text{int}}=\frac{3Gmm'}{c\,|\mathbf{x}-\mathbf{x}'|^3}(\dot{x}^i-\dot{x}'^i)(x^j-x'^j)\epsilon_{ijk} \times(a-a')^\dagger\sigma_k(a-a') \,. \tag{6.9}$$

This is a sort of generalized spin-orbit coupling, where the well-known purely bosonic interaction terms (cf. Ref. 26), which are of the same order, are not included.

The last step in our program is to interpret the instantaneous two-soliton dynamics as the two-particle sector of a (second quantized) field theory. The fields ϕ, χ, and λ which form the multiplet (4.14) will then become field operators in an effective quantum field theory of solitons. This is accomplished in Ref. 3.

VII. CONCLUSIONS

In this paper we have presented the framework for a theory of soliton interaction in $N=2$ supergravity. The solitons are described by their asymptotic properties in terms of translational and supertranslational degrees of freedom, the latter being associated with the supercharge. By looking at the algebra of asymptotic gauge transformations, we argued that the motion of a soliton can be described by a trajectory in $N=2$ superspace. Interactions are implemented by considering curved superspace. Taking the superspace fields to be soliton solutions of $N=2$ supergravity, gives rise to an effective interaction Lagrangian for several solitons in the slow-motion limit.

It is not surprising that in order to set up the dynamics, we often had to use heuristic arguments instead of strictly deducing it from the underlying field theory. Nevertheless, we feel that the results that follow from the presented Lagrangian seem to be reasonable generalizations of what is known in general relativity, e.g., for the motion of a spinning particle in an external gravitational field. We have condensed the line of argumentation in the first of the papers; the forthcoming ones will proceed in a more deductive way making use of the assumptions made here.

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APPENDIX A: CONVENTIONS AND NOTATIONS

Our conventions follow essentially those of Refs. 9, 13, and 16. Especially the metric signature is $(+---)$, and $k^2=4\pi G$. In contrast with Ref. 9, the velocity of light is kept explicitly in Secs. II–IV. The tangent space indices are now taken from the middle of the alphabet $(m,n,r,s,\ldots)$. Four-spinor indices are denoted by $\alpha,\beta,\gamma,\ldots$ while in Secs. IV, A and B are two-spinor indices.

For the γ matrices we use the Weyl representation,[16] especially

$$\gamma_0=\begin{bmatrix} 0 & \underline{1} \\ \underline{1} & 0 \end{bmatrix}$$

which is the reason for the decomposition (2.9). All complex four-spinors may be decomposed into their Majorana parts by $\chi=\chi^1+i\chi^2\equiv(\chi^j)$. The global SO(2) then acts on the index j. For simplicity we write the index j sometimes downstairs (e.g., $\chi_{\alpha j}$).

The index convention in superspace is taken from Ref. 20:

$$M,N=(\mu,\nu,\ldots;\alpha j,\beta j,\ldots) \quad \text{natural indices} ,$$

$$A,B=(m,n,\ldots;aj,bj,\ldots) \quad \text{tangent space indices} .$$

The summation is always of the form

(left upstairs index)$\times$(right downstairs index) ,

e.g., $\dot{z}^M V_M{}^A$. By $(-)^M$ we mean $+1$ for $M=\mu$ and -1 for $M=\alpha j$, analogously $(-)^A$.

APPENDIX B: $N=2$ SUPERGRAVITY SUPERSPACE

The coordinates of superspace are $z^M=(x^\mu,\theta^{\alpha j})$. The fundamental superfields are the supervielbein $V_M{}^A$ and the U(1) gauge potential B_M (Refs. 20 and 28), defined by their transformation properties under local supertranslations $\Xi^M(z)$, Lorentz rotations $L^{mn}(z)=-L^{nm}(z)$, and U(1) gauge transformations $\Lambda(z)$:

$$\delta V_M{}^A=(\partial_M\Xi^N)V_N{}^A+\Xi^N\partial_N V_M{}^A+\tfrac{1}{2}L^{mn}(X_{mn})^A{}_B V_M{}^B , \tag{B1a}$$

$$\delta B_M=(\partial_M\Xi^N)B_N+\Xi^N\partial_N B_M+\partial_M\Lambda , \tag{B1b}$$

where

$$\tfrac{1}{2}L^{mn}(X_{mn})^A{}_B=\begin{bmatrix} L^m{}_n & 0 \\ 0 & \tfrac{1}{2}L^{mn}(\sigma_{mn})^a{}_b\delta^{jk} \end{bmatrix} .$$

To lowest order in θ one requires

$$\Xi^M=(\xi^\mu,\epsilon^{\alpha j})+O(\theta) ,$$

$$\Lambda=k\lambda+O(\theta) ,$$

$$L^{mn}=l^{mn}+O(\theta) ,$$

$$V_M{}^A=\begin{bmatrix} e^m{}_\mu & k\psi_\mu{}^{aj} \\ 0 & \delta^a{}_\alpha\delta^k{}_j \end{bmatrix}+O(\theta) ,$$

$$B_M=(kA_\mu,0)+O(\theta) ,$$

where $(e^m{}_\mu,\psi_\mu{}^{aj},A_\mu)$ are the elementary $N=2$ supergravity fields and $(\xi^\mu,\epsilon^{\alpha j},l^{mn},\lambda)$ are the parameters of local space-time translations, supergauge transformations, local Lorentz rotations, and U(1) gauge transformations, respectively. On the level of the (space-time) supergravity fields,[20]

$$\delta e^m{}_\mu=-ik\bar{\epsilon}^j\gamma^m\psi_\mu{}^j+\mathcal{L}_\xi e^m{}_\mu+l^m{}_n e^n{}_\mu , \tag{B2a}$$

$$\delta\psi_\mu{}^j=\frac{1}{k}\hat{D}_\mu\epsilon^j+\mathcal{L}_\xi\psi_\mu{}^j+\tfrac{1}{2}l^{mn}\sigma_{mn}\psi_\mu{}^j , \tag{B2b}$$

$$\delta A_\mu=-\epsilon^{jk}\bar{\epsilon}^j\psi_\mu{}^k+\mathcal{L}_\xi A_\mu+\partial_\mu\lambda . \tag{B2c}$$

$\mathcal{L}_\xi$ is the Lie derivative acting only on the world indices, e.g.,

$$\mathcal{L}_\xi e^m{}_\mu=(\partial_\mu \xi^\nu)e^m{}_\nu+\xi^\nu\partial_\nu e^m{}_\mu \ .$$

Since we assume that (e^m, ψ, A) satisfy the supergravity field equations,[13] the variation of the connection $\omega_\mu{}^{mn}$ and other derived quantities follows from (B2).

It is now required that the superspace transformation law (B1) just reproduces (B2). This determines the expansion of Ξ^M, L^{mn}, Λ, and the fields $V_M{}^A$, B_M in terms of $\theta^{\alpha j}$ and the space-time supergravity fields (and their derivatives). Making use of the local gauge algebra, one arrives at the results

$$\Xi^\mu=\xi^\mu-\frac{i}{2}\bar\theta^j\gamma^\mu\epsilon^j+\frac{k}{2}(\bar\theta^j\gamma^\mu\psi_\rho{}^j)(\bar\theta^k\gamma^\rho\epsilon^k)+O(\theta^3)\ , \tag{B3a}$$

$$\Xi^{\alpha j}=\epsilon^{\alpha j}+\frac{ik}{2}(\bar\theta^k\gamma^\mu\epsilon^k)\psi_\mu{}^{\alpha j}-\tfrac{1}{2}l^{mn}(\sigma_{mn}\theta^j)^\alpha+O(\theta^2)\ , \tag{B3b}$$

$$L^{mn}=l^{mn}-\frac{i}{2}(\bar\theta^j\gamma^\mu\epsilon^j)\omega_\mu{}^{mn} -\frac{k}{2}\epsilon^{jk}\bar\theta^j(\hat F^{mn}+\tfrac{1}{2}\epsilon^{mnpq}\hat F_{pq}\gamma_5)\epsilon^k+O(\theta^2)\ , \tag{B3c}$$

$$\Lambda=k\lambda-\frac{ik}{2}(\bar\theta^j\gamma^\mu\epsilon^j)A_\mu-\tfrac{1}{2}\epsilon^{jk}\bar\theta^j\epsilon^k+O(\theta^2)\ , \tag{B3d}$$

$$V_\mu{}^m=e^m{}_\mu-ik(\bar\theta^j\gamma^m\psi_\mu)-\frac{i}{2}\bar\theta^j\gamma^m\mathcal{D}_\mu{}^{jk}\theta^k+O(\theta^3)\ , \tag{B4a}$$

$$V_{\alpha j}{}^m=\frac{i}{2}(\bar\theta^j\gamma^m)_\alpha\ , \tag{B4b}$$

$$V_\mu{}^{aj}=k\psi_\mu{}^{aj}+(\mathcal{D}_\mu{}^{jk}\theta^k)^a+O(\theta^2)\ , \tag{B4c}$$

$$V_{\alpha j}{}^{ak}=\delta^a{}_\alpha\delta^k{}_j\ , \tag{B4d}$$

$$B_\mu=kA_\mu-k\epsilon^{jk}(\bar\theta^j\psi_\mu{}^k)-\tfrac{1}{2}\epsilon^{jk}(\bar\theta^j\mathcal{D}_\mu{}^{kl}\theta^l)+O(\epsilon^3)\ , \tag{B4e}$$

$$B_{\alpha j}=-\tfrac{1}{2}\epsilon^{jk}\bar\theta_{\alpha k}\ , \tag{B4f}$$

where

$$\mathcal{D}_\mu{}^{jk}=\tfrac{1}{2}\omega_\mu{}^{rs}\sigma_{rs}\delta^{jk}-\frac{ik}{2}\epsilon_{jk}\hat F^{rs}\sigma_{rs}\gamma_\mu\ . \tag{B5}$$

This combination also appears in the supercovariant derivative

$$\hat D_\mu\chi^j=\partial_\mu\chi^j+\mathcal{D}_\mu{}^{jk}\chi^k\ . \tag{B6}$$

Note that $V_{\alpha j}{}^m$ and $B_{\alpha j}$ do not vanish even in the case of a flat background configuration. This fact is responsible for (6.4) to attain the correct flat-space limit (5.6).

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