Analysis of the ρ mass spectrum in τ decay

K. K. Gan
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305
(Received 12 February 1988)

The ρ mass spectrum in τ decay is calculated from the pion form factor measured in e^+e^- annihilation using the conserved-vector-current hypothesis. The calculation reproduces the mass spectrum observed in the τ data, including the high-mass region near $\rho(1600)$.

The τ lepton has been a subject of extensive study since its discovery in 1975.[1] All measurements indicate that it is a sequential lepton in the standard model of electroweak interactions.[2] However, the sum of the measured branching ratios for exclusive decay modes with one charged particle in the final state is significantly smaller than the measured one-prong topological branching ratio.[3] The one-prong decay $\tau^-\to\rho^-\nu_\tau$ has the largest branching ratio of any exclusive channel, and it is related to the measured cross section for $e^+e^-\to\pi^+\pi^-$ by the conserved-vector-current (CVC) hypothesis.[4] The value of this branching ratio has been measured by many experiments with good precision and all measurements are in good agreement with the CVC prediction. Since the branching ratio is related to the integrated cross section for $e^+e^-\to\pi^+\pi^-$, a more precise test of the CVC hypothesis is to compare the shape of the ρ mass spectrum with the CVC prediction. The high-mass region near the $\rho(1600)$ is of particular interest because the τ is also expected to decay into the $\rho(1600)$, but the experimental data from Mark II (Ref. 5) and Mark III (Ref. 6) show no enhancement in this region. In this paper we calculate the ρ mass spectrum and compare the result with the Mark II data.

The decay $\tau^-\to\pi^-\pi^0\nu_\tau$ proceeds through the strangeness-conserving, hadronic weak vector current. The coupling strength of the weak vector current to $\pi^-\pi^0$ is related to the coupling strength of the electromagnetic vector current to $\pi^+\pi^-$ by CVC. The electromagnetic coupling can be calculated from the measured cross section for $e^+e^-\to\pi^+\pi^-$ and consequently the decay rate[7] is related to the cross section by

$$\Gamma(\tau^-\to\pi^-\pi^0\nu_\tau)=\frac{G_F^2\cos^2\theta_C}{96\pi^3 m_\tau^3}\times\int_0^{m_\tau^2}dQ^2(m_\tau^2-Q^2)^2(m_\tau^2+2Q^2)\times\frac{\sigma^{(1)}_{e^+e^-\to\pi^+\pi^-}(Q^2)}{\sigma_{e^+e^-\to\mu^+\mu^-}(Q^2)},$$

where G_F is the Fermi coupling constant, m_τ is the mass of the τ, $\cos\theta_C$ is the cosine of the Cabibbo angle, $\sigma^{(1)}_{e^+e^-\to\pi^+\pi^-}(Q^2)$ is the cross section for $e^+e^-\to\pi^+\pi^-$ with total isospin of 1 at $E^2_{\mathrm{c.m.}}=Q^2$, and

$$\sigma_{e^+e^-\to\mu^+\mu^-}(Q^2)=(4\pi\alpha^2)/3Q^2$$

is the cross section for $e^+e^-\to\mu^+\mu^-$. The cross section for $e^+e^-\to\pi^+\pi^-$ is related to the pion form factor by

$$\sigma^{(1)}_{e^+e^-\to\pi^+\pi^-}(Q^2)=\frac{\pi\alpha^2}{3Q^2}\left[1-\frac{4m_\pi^2}{Q^2}\right]^{3/2}|F_\pi(Q^2)|^2 .$$

Therefore, the differential decay rate into $\pi^-\pi^0$ with invariant mass Q is

$$d\Gamma(\tau^-\to\pi^-\pi^0(Q)\nu_\tau)=\frac{G_F^2 m_\tau^4\cos^2\theta_C}{192\pi^3}\left[1-\frac{Q^2}{m_\tau^2}\right]^2\times\left[1+\frac{2Q^2}{m_\tau^2}\right]\left[1-\frac{4m_\pi^2}{Q^2}\right]^{3/2}\times\left[\frac{Q}{m_\tau}\right]|F_\pi(Q^2)|^2dQ .$$

Note that the differential rate is suppressed near the two-pion threshold and near the τ mass. The form factor has been measured by many experiments[8–15] as shown in Fig. 1. The solid curve shows the best fit[16] by Barkov *et al.* on the data, taking into account ρ-ω interference, and possible contributions of $\rho(1250)$ and $\rho(1600)$.

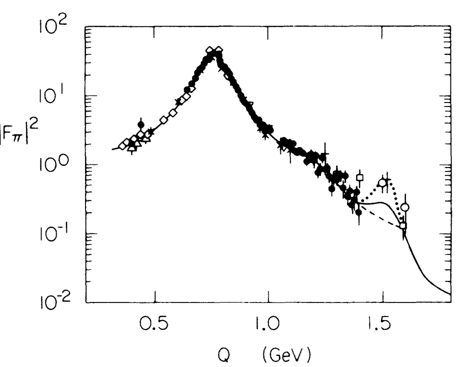

FIG. 1. Measurements of the pion form factor from e^+e^- annihilations as a function of center-of-mass energy Q: Novosibirsk (●, Ref. 8; ◇, Ref. 9; △, Ref. 10), Orsay (▽, Ref. 11; ×, Ref. 12) and Frascati (□, Ref. 13; +, Ref. 14; ○, Ref. 15). See the text for explanations of the curves.

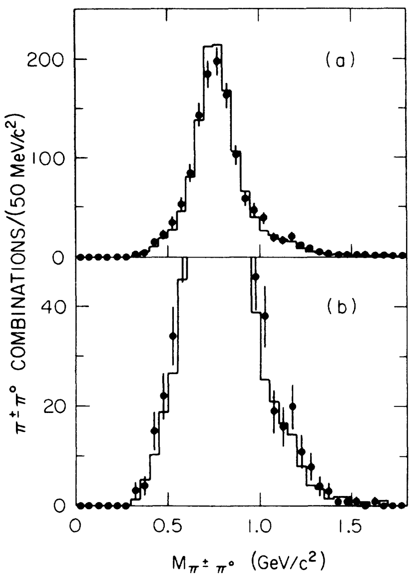

FIG. 2. The $\pi^+\pi^-$ invariant-mass spectrum, as measured by the Mark II Collaboration together with the CVC prediction (histogram), normalized to the same number of events. (b) is an enlarged view of (a).

The CVC prediction for the mass spectrum is compared with the Mark II measurements from Ref. 5. A Monte Carlo simulation program was written to smear the CVC mass spectrum to account for the effects of detector resolution and event-selection criteria. In the program, τ pairs are produced according to the standard electroweak theory with α^3 QED corrections,[17] and then are allowed to decay with the known branching ratios.[18] The charged-particle momentum and the photon energy are smeared according to the known detector resolution. Photons that are too close together are merged into a single photon to account for the granularity of the detector. The $\pi^-\pi^0$ candidates are then selected with the criteria described in Ref. 5. The resulting mass spectrum, normalized to the same number of events as the data, is shown in Fig. 2 using 50-MeV/c^2 bins. The mass resolution is dominated by the π^0 mass resolution, which is about 20 MeV/c^2 (Ref. 19). The CVC prediction is consistent[20] with the Mark II measurement, including the high-mass region near $\rho(1600)$.

There are only a few $\pi^\pm\pi^0$ events in the high-mass region. Data on the form factor in the high-Q region are also sparse. Nevertheless, it is still interesting to compare the CVC predictions for various assumptions on the values of the form factor in this region. The two possible extremes are shown in Fig. 1: The dashed curve corresponds to almost no $\rho(1600)$ contribution. The dotted curve, which passes through the data, corresponds to the maximum probable $\rho(1600)$ contribution, although it should be noted that this may not be a realistic representation of the form factor because the resonance is too narrow compared to that in the Particle Data Group compilation.[21] All three curves predict a few high-mass events as shown in Table I. Within the errors, all three predictions are consistent with the data.

Since the $\rho(1600)$ meson also decays into 4π final states, the number of high-mass events can also be estimated from the branching ratio for $\tau^- \to (4\pi)^- \nu_\tau$. The branching ratio for $\tau^- \to \pi^-\pi^+\pi^-\pi^0\nu_\tau$ is measured[2] to be $(5.0\pm0.5)\%$. The experimental branching ratio for $\tau^- \to \pi^- 3\pi^0 \nu_\tau$ is poorly known. However, there is a prediction,[3] $B(\tau^- \to \pi^- 3\pi^0\nu_\tau) = 1.0\%$, from CVC using the measured cross section for $e^+e^- \to 2\pi^+2\pi^-$. It is not necessary that all the 4π decays proceed through $\rho(1600)$, although the measured cross sections for $e^+e^- \to 2\pi^+2\pi^-$ and $e^+e^- \to \pi^+\pi^-2\pi^0$ have a broad maximum at a center-of-mass energy near 1500 MeV (Ref. 22). Even with only half of the 4π decays[23] proceeding through $\rho(1600)$, $\sim 60\pm20$ events from $\rho(1600)$ are expected in the high-mass region in Fig. 2. Since only a few events are observed and the rate is consistent with the CVC expectation, this indicates that the measured branching ratio for $\rho(1600)\to 2\pi$ is probably too large.[24]

In conclusion, the ρ mass spectrum in τ decay is calculated using CVC from the pion form factor measured in e^+e^- annihilation. The calculation reproduces the mass spectrum observed in the data, including the high-mass region near the $\rho(1600)$. This reaffirms the validity of CVC and hence the validity of the measured branching ratio for $\tau^- \to \pi^-\pi^0\nu_\tau$. Consequently, this analysis does not find any discrepancy that could provide a solution to the problem in the one-charged-particle decay.

I am greatly indebted to F. J. Gilman for stimulating discussions. I would like to thank J. M. Dorfan, K. Hayes, and M. L. Perl for useful discussions. This work was supported by the Department of Energy, Contract DE-AC03-76SF00515.

TABLE I. The number of high-mass events as observed by the Mark II Collaboration and as predicted by CVC for the three assumptions (curves) on the form factor shown in Fig. 1. All errors are statistical only.

$m_{\pi^\pm\pi^0}>$ (GeV/c^2)	Data	CVC (dashed curve)	CVC (solid curve)	CVC (dotted curve)
1.30	11±3.3	10.5±1.8	11.5±1.9	12.9±2.0
1.40	4±2.0	5.6±1.3	6.5±1.4	7.9±1.5
1.50	2±1.4	2.6±0.9	2.8±0.9	3.1±1.0

[1]M. L. Perl *et al.,* Phys. Rev. Lett. **35**, 1489 (1975).
[2]For a recent review see, for example, K. K. Gan, in Proceedings of the SLAC Summer Institute on Particle Physics, Stanford, California, 1987, edited by E. C. Brennan (SLAC, Stanford, CA, to be published); K. K. Gan and M. L. Perl, Int. J. Mod. Phys. A (to be published).
[3]F. J. Gilman and S. H. Rhie, Phys. Rev. D **31**, 1066 (1985).
[4]R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).
[5]J. M. Yelton *et al.,* Phys. Rev. Lett. **56**, 812 (1986).
[6]J. Adler *et al.,* Phys. Rev. Lett. **59**, 1527 (1987).
[7]Y. S. Tsai, Phys. Rev. D **4**, 2821 (1971).
[8]I. B. Vasserman *et al.,* Yad. Fiz. **30**, 999 (1979) [Sov. J. Nucl. Phys. **30**, 519 (1979)]; L. M. Kurdadze *et al., ibid.* **40**, 451 (1984) [**40**, 286 (1984)].
[9]G. V. Anikin *et al.,* Novosibirsk Report No. IYF-83-85, 1983 (unpublished).
[10]I. B. Vasserman *et al.,* Yad. Fiz. **33**, 709 (1981) [Sov. J. Nucl. Phys. **33**, 368 (1981)].
[11]G. Cosme *et al.,* Orsay Report No. LAL-1287, 1976 (unpublished).
[12]A. Quenzer *et al.,* Phys. Lett. **76B**, 512 (1978).
[13]D. Bollini *et al.,* Lett. Nuovo Cimento **14**, 418 (1975).
[14]G. Barbiellini *et al.,* Lett. Nuovo Cimento **6**, 557 (1973).
[15]B. Esposito *et al.,* Phys. Lett. **67B**, 239 (1977); Lett. Nuovo Cimento **28**, 337 (1980).
[16]L. M. Barkov *et al.,* Nucl. Phys. **B256**, 365 (1985).
[17]F. A. Berends and R. Kleiss, Nucl. Phys. **B177**, 237 (1981); F. A. Berends, R. Kleiss, and S. Jadach, *ibid.* **B202**, 63 (1982).
[18]The branching ratios compiled by Gan and Perl (Ref. 2) are used. The results are relatively insensitive to the uncertainties in the multiple-neutral-meson branching ratios as expected.
[19]See, for example, Fig. 1 in K. K. Gan *et al.,* Phys. Lett. B **197**, 561 (1987).
[20]Because of the simplicity of the Monte Carlo model, the estimated width is probably slightly narrower than that in the data.
[21]Particle Data Group, M. Aguilar-Benitez *et al.,* Phys. Lett. **170B**, 1 (1986).
[22]See, for example, Figs. 1 and 2 in Ref. 3.
[23]The measured branching ratio (Ref. 21) of $B[\rho(1600), \rightarrow 2\pi]=(23\pm 7)\%$ is used.
[24]As part of the original analysis of the Mark II data presented here, J. M. Yelton studied the high-mass region for an indication of $\rho(1600)$. His conclusions, that either the $\rho(1600)$ is not often produced in τ decays or more likely that the measured branching ratios for $\rho(1600)\rightarrow 2\pi$ is incorrect, are in agreement with those presented in this paper (Mark II Memo, private communication).