# Two-photon decay of pseudoscalar mesons in hot hadronic matter

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Possible enhancement of coupling of $\pi^0$ to two photons at finite temperature is discussed in connection with second-order or weakly first-order chiral phase transition.

In this Brief Report we discuss the possibility that two-photon decay of pseudoscalar mesons at temperatures close to chiral restoration are enhanced if transition is second or weak first order.

It is well known that the PCAC (partial conservation of axial-vector current) relation with an anomaly,[1]

$$\partial_\mu A^\mu = m_\pi^2 f_\pi \Phi_\pi - (\alpha/4\pi) F_{\mu\nu}\tilde{F}^{\mu\nu} , \quad (1)$$

leads to the formula of $\pi^0 \to 2\gamma$ width as[2]

$$\Gamma(\pi^0 \to 2\gamma) = (64\pi^3)^{-1}(\alpha/f_\pi)^2 m_\pi^3 . \quad (2)$$

As the temperature independence of the anomaly has been proved,[3] the second term of Eq. (1) is not modified at finite temperature. On the other hand, the coupling to axial-vector current, $f_\pi$, would receive finite-temperature correction. If the chiral transition is second order, the pion decay constant $f_\pi$ is expected to vanish at the temperature of chiral restoration $T_\chi$. Therefore formula (2) suggests the enhancement of $\pi^0 \to 2\gamma$ decay at temperature close to $T_\chi$ if the pion mass remains finite.

In order to examine the above heuristic observation we study this process by an effective model which reveals chiral restoration at finite temperature. We also examine the effect of finite current-quark mass in the model.

First we calculate the triangle diagram as depicted in Fig. 1 by using the Matsubara Green's function. Here the internal line means the constituent quark with mass $M$ and meson–constituent-quark coupling is $\gamma_5$ type (coupling constant $G_{\rm CQ}$). The pion is assumed to be fixed to the rest system of thermal medium. Then we have

$$F = \epsilon_{\mu\nu\rho\sigma} e_\mu(k_1) e_\nu(k_2) k_{1\rho} k_{2\sigma} 4e^2 H , \quad (3)$$

where

$$H = G_{\rm CQ} M T \sum_n \int \frac{d\mathbf{p}}{(2\pi)^3} \{(\mathbf{p}^2 + E_n^2 + M^2)[(\mathbf{p}-\mathbf{k})^2 + (E_n - k)^2 + M^2][\mathbf{p}^2 + (E_n - 2k)^2 + M^2]\}^{-1} , \quad (4)$$

here $E_n = 2\pi T(n + \frac{1}{2})$ and $k = |\mathbf{k}| = m_\pi/2$.

These expressions lead to $2\gamma$ decay width as

$$\Gamma_{2\gamma} = (64\pi^3)^{-1} m_\pi^3 (\alpha G_{\rm CQ}/M)^2 I(m_\pi, M, T)^2 , \quad (5)$$

where $I(m_\pi, M, T)$ is given by

$$I(m_\pi, M, T) = \int_M^\infty dE (2M^2/E)(E^2 - m_\pi^2/4)^{-1} \times \ln[(E+p)/M] \tanh(E/T) . \quad (6)$$

The factor $I(m_\pi, M, T)$ represents the explicit temperature dependence of the triangle diagram at finite temperature. If masses $m_\pi$ and $M$ are fixed, $I$ is a decreasing function of temperature and reveals thermal suppression due to interference with induced decay by the medium. However, in Eqs. (4)–(6), we consider that masses $m_\pi$, $M$, and the coupling constant $G_{\rm CQ}$ are temperature dependent.

If the constituent-quark mass decreases near $T_\chi$, the quark loop is enhanced as seen by the factor $(\alpha G_{\rm CQ}/M)^2$ in Eq. (5).

In fact, if the Goldberger-Treiman relation for the constituent quark holds at finite temperature, i.e.,

$$g_A m = G_{\rm CQ} f_\pi , \quad (7)$$

Eq. (5) is expressed as

$$\Gamma_{2\gamma} = (64\pi^3)^{-1} m_\pi^3 (\alpha g_A/f_\pi)^2 I(m_\pi, M, T)^2 , \quad (8)$$

where $g_A$ is the axial-vector coupling constant of the con-

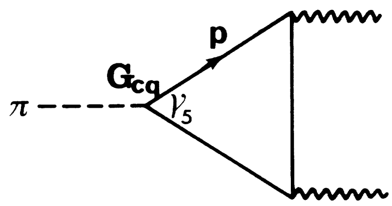


FIG. 1. A triangle diagram for $\pi^0 \to 2\gamma$ decay at finite temperature. Internal lines are the Matsubara Green's functions for the constituent quark.

stituent quark. As $g_A$ has a value around unity, vanishing of $f_\pi$ in the case of second-order chiral restoration seems to enhance $\Gamma_{2\gamma}$. However, the validity of the Goldberger-Treiman relation at finite temperature is not obvious and should be examined. In a successful model of the Nambu–Jona-Lasinio type studied by Hatsuda and Kunihiro,[4] the relation holds in the limit of vanishing current-quark mass. But in the case of finite current-quark mass, the relation is violated and in fact $G_{\rm CQ}$ also vanishes at $T_\chi$. Based on the latter fact, decoupling of the pion from matter near critical temperature has been stressed.[5] Thus a systematic calculation of temperature dependence of relevant masses and couplings to evaluate Eq. (5) is required.

Here we perform a calculation following their model.[5] In the model, the Lagrangian is given by

$$L = \bar{q}(i\gamma^\mu \partial_\mu - M_0)q + g[(\bar{q}q)^2 + (\bar{q}i\gamma_5 \tau q)^2] , \tag{9}$$

where $M_0$ is the current-quark mass. The constituent-quark mass is determined from a gap equation as

$$1 - M_0/M = (3g/\pi^3)T\sum \int_{|\mathbf{p}|<\Lambda} d\mathbf{p}\{\mathbf{p}^2 + [(2n+1)\pi T]^2 + M^2\}^{-1} . \tag{10}$$

Values of coupling constant $g$ and cutoff $\Lambda$ are adjusted to reproduce low-energy data such as $f_\pi$ and $m_\pi$ at $T=0$. The pion mass $m_\pi$ and pion–constituent-quark coupling $G_{\rm CQ}$ are obtained by pole and residue of chain approximation of quark-antiquark scattering amplitude in a pseudoscalar channel of the form

$$\Pi_0^R(\omega, \mathbf{p}, T)/[1 - 2g\Pi_0^R(\omega, \mathbf{p}, T)] , \tag{11}$$

where $\Pi_0^R$ is the lowest-order contribution of the retarded

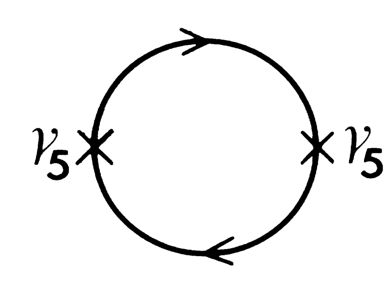


FIG. 2. A diagram for $\Pi_0^R$ (Ref. 4). Lines correspond to the constituent quarks.

correlator for $\bar{q}(i\gamma_5\tau)q$ shown in Fig. 2. After determining $m_\pi$, $M$, and $G_{\rm CQ}$ by these steps we can calculate Eq. (5).

Results of the temperature dependence of relevant parameters are shown in Fig. 3(a). We see an increase of $f_\pi^{-1}$ and $G_{\rm CQ}/M$ and an approximate validity of the Goldberger-Treiman relation except for the region very close to the vanishing point of $f_\pi$ where $m_\pi = 2M$. Following the authors of Ref. 5 we define $T_\chi$ by this point. The coupling-mass ratio $G_{\rm CQ}/M$ quickly turns over when the temperature comes very close to $T_\chi$. This is due to a finite current-quark mass which leads to vanishing of $G_{\rm CQ}$ at the same point.

In Fig. 3(b), the resultant two-photon width by Eq. (5)

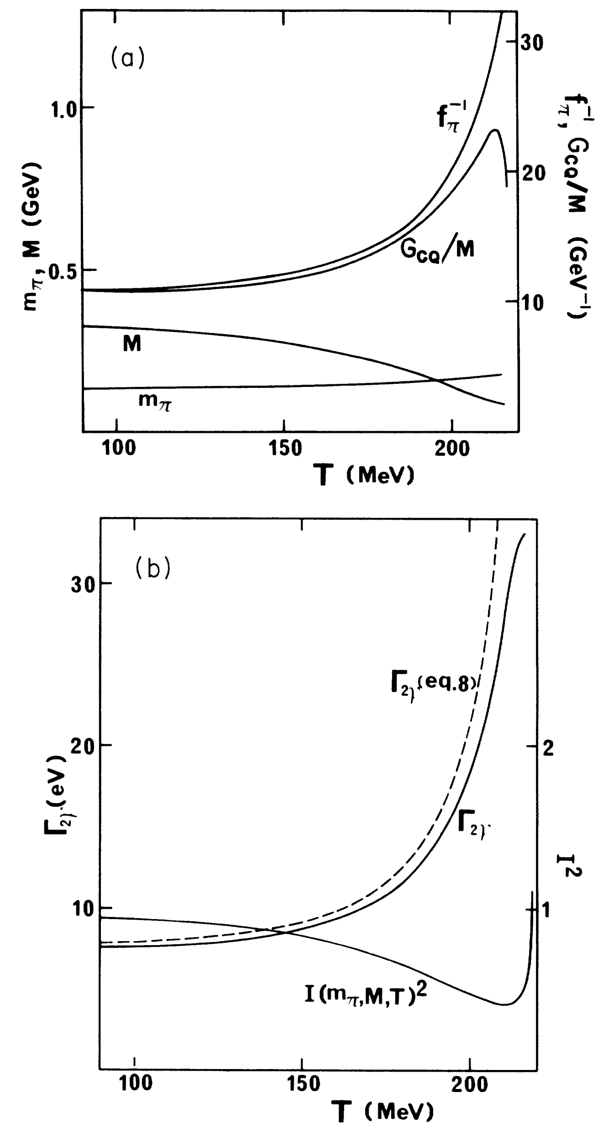


FIG. 3. (a) Pion mass $m_\pi$, constituent-quark mass $M$, coupling-to-mass ratio $G_{\rm CQ}/M$, and inverse of pion decay constant $f_\pi^{-1}$ at finite temperature in the model of Refs. 4 and 5. Values of four-fermion coupling constant $g$, bare quark mass, and cutoff are 0.214 fm$^2$, 5.5 MeV, and 631 MeV, respectively. (b) Two-photon decay width at finite temperature. $\Gamma_{2\gamma}$ [Eq. (8)] is also presented for comparison.

is presented. We also show the temperature dependence of $I(m_\pi,M,T)^2$ to see the effect of thermal suppression and the two-photon decay width by Eq. (8) for a case in which the Goldberger-Treiman relation holds ($g_A$ is fixed to be unity).

The width grows up near $T_\chi$ in spite of the decreasing behavior of $I(m_\pi,M,T)^2$. In the chiral limit this diverges at the critical point as indicated by the calculation of Eq. (8). In the case of finite current-quark mass, it reaches a maximum. [It is noted that a zero of $G_{\mathrm{CQ}}$ is canceled by a singularity of $I(m_\pi,M,T)$ at $T_\chi$.] Thus we observe the enhancement of coupling of $\pi^0$ to photons near $T_\chi$ in contrast with decoupling from matter in this model.

The following conclusion is obtained for the enhancement of the two-photon decay of $\pi^0$ near critical temperature of chiral restoration. A decrease in the mass of the constituent quark leads to an enhancement of the quark loop effect. This feature is expected if chiral restoration is a second-order or weakly first-order transition. In an example of the calculation of the Nambu–Jona-Lasinio–type model, the enhancement overwhelms thermal suppression and the decrease of the pion–constituent-quark coupling. On this point, it is remarked that the model does not simulate a deconfining transition. However, lattice Monte Carlo results indicate the simultaneous occurrence of a chiral and deconfining transition.[6] Thus decoupling of the pion from the quark might be affected by dynamical singularities of liberated quarks. In this regard, it is interesting that recent Monte Carlo results for shielding the mass at finite temperature has shown a clear pionic pole in the neighborhood of the transition point.[7]

Finally we mention phenomenological implications. It is an interesting challenge to observe hadron spectroscopy near the critical temperature in a high-energy collision experiment. In order to extract information of hot hadrons in a fireball, use of a penetrating probe such as the lepton or photon is appropriate.[8,9] The photon pair is especially adequate to observe the mass shift of $\eta'$ and $\eta$ which is expected from effective $\mathrm{U}_1$ recovery at high temperature.[10] A problem in this case might be that the signal is very weak because of the shortness of the hot era of the hadronic fireball in an actual collision.[11] However the enhancement of two-photon decay near critical temperature discussed in this paper intensifies the signal when it also occurs for $\eta'$ and $\eta$.

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