Realistic pseudoscalar-vector chiral Lagrangian and its soliton excitations

P. Jain and R. Johnson*
Physics Department, Syracuse University, Syracuse, New York 13244-1130

Ulf-G. Meissner
Center for Theoretical Physics, Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

N. W. Park and J. Schechter
Physics Department, Syracuse University, Syracuse, New York 13244-1130

(Received 13 November 1987)

A reasonable low-energy chiral Lagrangian of vectors and pseudoscalars involves three *a priori* unknown coëfficients for terms proportional to the Levi-Civita symbol $\epsilon_{\mu\nu\alpha\beta}$. We argue that these cannot, from a theoretical standpoint, be reliably determined by consideration of electromagnetic processes. Hence we use purely strong-interaction processes which enable us to determine two out of three of these. We examine the nucleon as a soliton excitation in the model and find that adjustment of the third parameter does not greatly change the, in most respects accurate, description of the nucleon obtained in a previous treatment by Meissner, Kaiser, and Weise. The paper includes a new and careful formulation of the addition of electromagnetism to the full Lagrangian.

I. INTRODUCTION

The Skyrme model[1] provides an appealing picture of the nucleon as a solitonic excitation in a chiral-symmetric Lagrangian of π meson fields. The recent point of view about this model is that the underlying theory of QCD should organize itself at low energies to give an effective chiral Lagrangian constructed out of the fields associated with the low-lying meson multiplets. There is little argument about which are the low-lying mesons; experiment and the quark model combine to tell us that they comprise the pseudoscalar and vector multiplets. One may eventually want to include additional particles but it seems very reasonable to proceed one step at a time.

What then is the chiral Lagrangian of pseudoscalars and vectors to proceed with? Although interesting attempts have been made to derive one from QCD, that work still seems to be in a preliminary stage insofar as finding the actual numerical values of the parameters is concerned.[2] At present it seems most reasonable to determine the parameters in a general chiral-symmetric Lagrangian from experiment. This is not easy for those terms proportional to the Levi-Civita symbol $\epsilon_{\mu\nu\alpha\beta}$. We remind the reader that these terms, on one hand, are crucial in stabilizing the soliton (without a need for introducing the "Skyrme" term) and, on the other, naturally occur when one talks about the Wess-Zumino terms and vector-meson dominance for certain electromagnetic interactions of the mesons. The subject of finding these terms and investigating their effect on the solitons is by now a fairly mature one. It is the topic of a very recent comprehensive review[3] (including an announcement of the present paper) by one of us (U.-G.M.) which should be consulted for adequate references to the many interesting articles in the field.

In the present paper we shall start from a general chiral-symmetric Lagrangian of pseudoscalars and vectors written down in Ref. 4 in which there are three unknown constants c_1, c_2, c_3 [see Eq. (2.13)] specifying the ϵ terms. We then apply constraints from strong-interaction meson reactions to determine them. While it is tempting to try to determine the ϵ terms in the strong Lagrangian by assuming some kind of vector-meson dominance for electromagnetic meson reactions, that procedure is not reliable in the sense that, taking into account the non-Abelian anomaly, the low-energy electromagnetic processes can be correctly fit regardless of the values of c_1, c_2, c_3. The relevant strong processes are the decays $\omega \rightarrow 3\pi$, $\phi \rightarrow 3\pi$, and $\phi \rightarrow \pi\rho$. With the standard model for ω-ϕ mixing they enable us to determine the coefficient c_2 in (2.13) of the chiral-invariant term whose leading piece is $\omega\rho\pi$ as well as the linear combination of all three chiral invariants which leads to a contact $\omega 3\pi$ piece. This disentangling is made easy because the ϕ meson is heavy enough to decay into $\pi\rho$ while the ω meson cannot do so. We are left with one undetermined parameter [c_3 in (2.13)] in the strong Lagrangian.

In a similar model to that of Ref. 4, proposed by Fujiwara *et al.*[5] there appears to be even more arbitrariness in the ϵ terms. We show, however, that the two models are identical (this had already been shown for the non ϵ terms[6]) when account is taken of charge-conjugation invariance. We also give a new and perhaps simpler way to introduce electromagnetic interactions into the strong chiral Lagrangian.

Next we study the nucleon as a soliton excitation in this model. The physics of the present model is actually rather similar to that of the "complete model" reviewed in Ref. 7. In that model it was observed that (keeping always the pion decay constant F_π equal to its physical value) the properties of the nucleon would be significantly improved over their values in the original Skyrme model. The main problem that the "complete" model (and other soliton models) have is that the nucleon mass is predicted too high by several hundred MeV. The "complete" model has $\omega\rho\pi$ and $\omega 3\pi$ vertices roughly comparable to the present ones, fixed by a plausible "gauging" approach. The advantage of the present model is that it is chiral invariant without the addition of extra fields. It also contains the undetermined constant c_3 which might be adjusted to try to lower the nucleon mass. We have studied this possibility very carefully and find that there does not seem to be any "magic" value of c_3 which brings the prediction into perfect agreement with experiment. The fit is about the same as the "complete" model. It appears that the "fine details" of the ϵ terms do not make a great deal of difference in the prediction of the nucleon's parameters. On the other hand, we may turn the situation around and use the requirement of an adequate fit to the soliton to find the allowed region for the coefficient c_3.

Section II contains a brief review of the strong Lagrangian of Ref. 4 with some additional remarks. In Sec. III we add electromagnetism in a new way, show why electromagnetic processes cannot be used to determine c_1, c_2, c_3 and compare with the literature. Section IV discusses the partial determination of c_1, c_2, c_3 from the strong decays of the ω and ϕ mesons. A related Appendix briefly discusses the ω-ϕ mixing angle. In Sec. V we give the $\mathrm{U}(2)\times\mathrm{U}(2)$ reduction of the ϵ terms, obtained by dropping the third flavor. This is in preparation for discussing the classical soliton, whose equations are formulated in Sec. VI. Section VII contains a detailed discussion of the numerical results for the soliton properties and comparison with the "complete" model. The outlook is surveyed in Sec. VIII.

II. STRONG-INTERACTION LAGRANGIAN OF PSEUDOSCALARS AND VECTORS

A chiral-invariant Lagrangian constructed out of fields belonging to the pseudoscalar and vector-meson nonets can encode two crucial features of low-energy strong-interaction physics: (i) the spontaneous breakdown of chiral symmetry and (ii) the experimental fact that the pseudoscalars and vectors are the lowest-lying multiplets (S-wave bound states in the quark model). Many different approaches can be used to construct such a Lagrangian. In this section we shall briefly review the Lagrangian presented in Ref. 4 and make some further remarks.

In this model the 0^- nonet ϕ transforms nonlinearly under chiral $\mathrm{U}(3)\times\mathrm{U}(3)$. The matrix with a linear transformation property is [note that in the usual isospin notation $\phi=(1/\sqrt{2})\tau\cdot\pi+\cdots$]

$$U=\exp\left[\frac{2i\phi}{F_\pi}\right], \tag{2.1}$$

where $F_\pi \simeq 132$ MeV. It is convenient to define

$$\xi=U^{1/2}=\exp\left[\frac{i\phi}{F_\pi}\right]. \tag{2.2}$$

The vector-meson nonet matrix ρ_μ, which also transforms nonlinearly, is related to auxiliary linearly transforming "gauge fields" A_μ^L and A_μ^R by

$$A_\mu^L=\xi\rho_\mu\xi^\dagger+\frac{i}{g}\xi\partial_\mu\xi^\dagger , \tag{2.3a}$$

$$A_\mu^R=\xi^\dagger\rho_\mu\xi+\frac{i}{g}\xi^\dagger\partial_\mu\xi , \tag{2.3b}$$

where g is a gaugelike coupling constant. From the above we see that A_μ^L and A_μ^R are related by

$$A_\mu^L=UA_\mu^R U^\dagger+\frac{i}{g}U\partial_\mu U^\dagger . \tag{2.4}$$

There are three main pieces in the action

$$\Gamma=\int(\mathcal{L}_1+\mathcal{L}_2)d^4x+\Gamma_3 . \tag{2.5}$$

The first piece is a gauge-invariant kinetic term for the vectors:

$$\begin{aligned}&\mathcal{L}_1=-\tfrac{1}{4}\mathrm{Tr}[F_{\mu\nu}(\rho)F_{\mu\nu}(\rho)] ,\\ &F_{\mu\nu}(\rho)=\partial_\mu\rho_\nu-\partial_\nu\rho_\mu-ig[\rho_\mu,\rho_\nu] .\end{aligned} \tag{2.6}$$

[Note that we are using the "$x_4=ict$" metric convention, the vectors are normalized so that $\rho=(1/\sqrt{2})\tau\cdot\rho+(1/\sqrt{2})1\omega+\cdots$.] The second piece breaks the strong gauge invariance; written in terms of the auxiliary fields it is simply

$$\mathcal{L}_2=-m_0^2\mathrm{Tr}(A_\mu^L A_\mu^L+A_\mu^R A_\mu^R)+B\,\mathrm{Tr}(A_\mu^L U A_\mu^R U^\dagger) , \tag{2.7}$$

where m_0 and B are two constants. It is convenient to express g, m_0^2, and B in terms of three "physical" quantities, $g_{\rho\pi\pi}$, m_V^2, and k:

$$g=\frac{g_{\rho\pi\pi}}{k},\quad m_0^2=\frac{m_V^2(1+k)}{8k},\quad B=\frac{m_V^2(1-k)}{4k} , \tag{2.8}$$

$$k=\frac{g_{\rho\pi\pi}^2F_\pi^2}{m_V^2} . \tag{2.9}$$

Using (2.8) and (2.9) as well as (2.3a) and (2.3b) we reexpress (2.7) as

$$\begin{aligned}\mathcal{L}_2=&-\tfrac{1}{2}m_V^2\mathrm{Tr}(\rho_\mu\rho_\mu)-i\frac{F_\pi^2 g_{\rho\pi\pi}}{2}\mathrm{Tr}[\rho_\mu(\partial_\mu\xi\xi^\dagger+\partial_\mu\xi^\dagger\xi)]\\ &-\frac{F_\pi^2}{4}(1+k)\mathrm{Tr}(\partial_\mu\xi\partial_\mu\xi^\dagger)\\ &-\frac{F_\pi^2}{4}(1-k)\mathrm{Tr}(\xi^\dagger\partial_\mu\xi^\dagger\xi\partial_\mu\xi) .\end{aligned} \tag{2.10}$$

The identification of m_V^2 and $g_{\rho\pi\pi}$ is straightforward from (2.10). Since we have in mind later on restricting our attention to the nonstrange particles, let us choose parameters so as to fit the $\rho(770)$ meson. Then

$$\begin{aligned}&m_V=(769\pm3)\ \mathrm{MeV},\quad g_{\rho\pi\pi}=8.65\pm0.16 ,\\ &k\approx2.20\pm0.10 .\end{aligned} \tag{2.11}$$

The statement $k=2$ is the Kawarabayashi-Suzuki-

Riazuddin-Fayyazuddin (KSRF) formula. Evidently, it is reasonably well satisfied by experiment, but not required by the present model.

The third piece in $\mathcal{L}$ contains terms proportional to the antisymmetric symbol $\epsilon_{\mu\nu\alpha\beta}$. Only in the last few years have these terms been studied intensively. It is very convenient to use the notation of differential forms to write such terms compactly. A left-handed nonet one-form which we need is

$$\alpha=(\partial_\mu U)U^{-1}dx_\mu=dU\,U^{-1}\ . \tag{2.12}$$

(See Ref. 8 for a more detailed discussion.) Using this language the action Γ_3 is [note that a misprint of a factor of 2 in the last term of (3.6) in Ref. 4 has been corrected here]

$$\Gamma_3=\Gamma_{WZ}(U)+\int \mathrm{Tr}\left[ic_1(A_L\alpha^3)+c_2(dA_L\alpha A_L-A_L\alpha dA_L+A_L\alpha A_L\alpha)+c_3\left[-2iA_L^3\alpha+\frac{1}{g}A_L\alpha A_L\alpha\right]\right]\ , \tag{2.13}$$

where c_1,c_2,c_3 are constants whose values remain to be determined. $\Gamma_{WZ}(U)$ is the Wess-Zumino-Witten term of pseudoscalars:

$$\Gamma_{WZ}(U)=\frac{-iN_c}{240\pi^2}\int_{M^5}\mathrm{Tr}(\alpha^5)\ , \tag{2.14}$$

where the integral is over a five-dimensional manifold whose boundary is ordinary Minkowski space.

It may be of some interest to show how (2.13) is obtained. The first term $\Gamma_{WZ}(U)$ mocks up the non-Abelian axial anomaly with pseudoscalar fields. The c_1, c_2, and c_3 terms are merely chiral symmetric; they can, as we will see in Sec. III, be suitably gauged with external (e.g., electromagnetic) gauge fields so as to make no contribution to the non-Abelian anomaly. Their peculiar form can be seen to arise as follows. Note that the ordinary Skyrme model of pseudoscalars alone can be written completely in terms of α. Let us enumerate the possible ϵ terms (four-forms) which may be constructed out of α as well as A_L. First we see that $\mathrm{Tr}(\alpha^4)=\mathrm{Tr}(\alpha\alpha^3)=-\mathrm{Tr}(\alpha^3\alpha)=0$. Similarly $\mathrm{Tr}(A_L^4)=0$. On the other hand, $\mathrm{Tr}(A_L\alpha^3)$ gives the c_1 term which is both P and C invariant. A term such as $\mathrm{Tr}(A_L^2\alpha^2)$ is not included because it is *odd* under parity. This may be checked by using the following mnemonic for the parity operation:

$$\begin{aligned}&P:\ -A_L\leftrightarrow +A_R=U^{-1}\left[A_L+\frac{i}{g}\alpha\right]U\ ,\\ &d\leftrightarrow -d,\quad \alpha\leftrightarrow\beta=U^{-1}\alpha U,\quad U\leftrightarrow U^{-1}\ ,\\ &(\text{measure})\leftrightarrow-(\text{measure})\ .\end{aligned} \tag{2.15}$$

Neither $\mathrm{Tr}(A_L^3\alpha)$ nor $\mathrm{Tr}(A_L\alpha A_L\alpha)$ goes into itself under parity but the linear combination in the c_3 term does. Next consider terms with the derivative operator. Since $d\alpha=\alpha^2$, nothing new is obtained without dA_L. The c_2 term is the parity-invariant combination involving α, A_L, and dA_L. This exhausts the possibilities although extra derivatives can still be added in a more trivial way (not involving the d operator). C invariance of the terms may be checked with the mnemonic:

$$C:\ A_L\leftrightarrow -A_R^T,\quad \alpha\leftrightarrow\beta^T,\quad U\leftrightarrow U^T\ . \tag{2.16}$$

A little while after the Lagrangian (2.6) + (2.7) + (2.13) was proposed, an interesting Lagrangian of pseudoscalars and vectors based on a "hidden symmetry" was presented by Fujiwara *et al.*[5] That Lagrangian appears to contain more arbitrariness in the ϵ sector than the present one. However, those authors did not take C invariance into account. When that is done, their general model becomes identical to ours.

We have not written the terms which break chiral symmetry nor the SU(3)-breaking terms. Furthermore, terms involving glueball fields to mock up the $U_A(1)$ anomaly [giving the $\eta'(960)$ a mass] and trace anomaly can be added.[9]

The ϵ terms in (2.13) play an important role in decays of vector mesons and as a stabilizing piece (which can replace the Skyrme term) in the nucleon-as-soliton picture. One can try to estimate the values of c_1, c_2, and c_3 by assuming vector-meson dominance in some form but that procedure, as we shall discuss in Sec. III, is not very compelling in the present approach. It seems safer to try to determine c_1, c_2, and c_3 directly from strong-interaction physics. We shall explore this point of view in later sections, assuming that the Lagrangian (2.5) is an adequate approximation at low energies. As a preliminary, we would like to extract the two terms in (2.13)—the vector-vector-pseudoscalar and the vector-(pseudoscalar)^3 ones—which can be most readily related to experimental processes. Using (2.1), (2.2), (2.3a), and (2.12) we eventually find

$$\Gamma_3=\int[-g_{VV\phi}\mathrm{Tr}(d\rho\, d\rho\,\phi)+ih\,\mathrm{Tr}(\rho\, d\phi\, d\phi\, d\phi)]+\cdots\ , \tag{2.17}$$

where

$$g_{VV\phi}=\frac{4ic_2}{F_\pi},\quad h=-\frac{4i}{F_\pi^3}\left[2c_1-\frac{2c_2}{g}-\frac{c_3}{g^2}\right]\ . \tag{2.18}$$

To avoid confusion we rewrite (2.17) explicitly with $\epsilon_{1234}=+1$:

$$\begin{aligned}\Gamma_3=\epsilon_{\mu\nu\alpha\beta}\int d^4x\,[&-ig_{VV\phi}\mathrm{Tr}(\partial_\mu\rho_\nu\partial_\alpha\rho_\beta\phi)\\ &-h\,\mathrm{Tr}(\rho_\mu\partial_\nu\phi\partial_\alpha\phi\partial_\beta\phi)]+\cdots\ .\end{aligned}$$

In an earlier approach,[8] an attempt to uniquely determine the ϵ terms was made based on extending Sakurai's idea.[10] Sakurai postulated that the vector mesons should be introduced by treating them as "external" gauge fields (rather than composite degrees of freedom) and gauging the strong Lagrangian. The generalization suggested for the ϵ terms is to "gauge" the Wess-Zumino term $\Gamma_{WZ}(U)$ with the replacement

$$\Gamma_{WZ}(U, A_L, A_R) - \Gamma_{WZ}(1, A_L, A_R) .$$

This procedure gives a good phenomenological picture (as does Sakurai's original postulate) but suffers from the fact that it breaks chiral symmetry unless additional terms are introduced. For comparison, however, it predicts [see (5.7) and (7.2) of Ref. 8]

$$\tilde{g}_{VV\phi} \equiv F_\pi g_{VV\phi} = \frac{3g_{\rho\pi\pi}^2}{16\pi^2} \approx 1.4, \quad \tilde{h} \equiv F_\pi^3 h \approx 0.1 , \tag{2.19}$$

when the axial-vector mesons are retained. On the other hand, when the axials are eliminated it is important to note that the contact term is four times stronger, i.e.,

$$\tilde{h} = -\frac{m_\rho^2}{\pi^2 g_{\rho\pi\pi} F_\pi^2} \approx -0.4 \tag{2.20}$$

as given by Eq. (3.4) of Ref. 4. Rewriting the coupling of the ω meson to the topological baryon current, i.e., the $\omega \to 3\pi$ contact term, as[11]

$$\mathcal{L}_{\omega\pi} = \beta \omega_\mu B^\mu \tag{2.21}$$

we have

$$\beta = -\frac{3\pi^2 \tilde{h}}{\sqrt{2}} . \tag{2.22}$$

For the linear model (2.19) this leads to $\beta \simeq -1.9$, and $\beta = 8.3$ for the nonlinear model (2.20). Insisting on the KSRF relation in Eq. (2.20), we have $\beta = 8.7$ as used in Refs. 7 and 12.

III. ADDITION OF NONSTRONG GAUGE FIELDS

It is clearly of great practical importance to introduce electromagnetic interactions. When doing so, gauge invariance must be respected. For the terms which result from gauging $\mathcal{L}_2$ an *approximate* vector-meson dominance will automatically arise. Hence it appears to be reasonable to adopt the viewpoint (which we shall do here) that there is no need to impose vector-meson dominance as an explicit assumption. In particular, this viewpoint will be taken for the terms coming from the gauging of Γ_3.

Under an infinitesimal local $U(3) \times U(3)$ gauge transformation, the "strong" gauge fields of (2.3a) and (2.3b) change by

$$\delta A_{L,R} = -[A_{L,R}, E_{L,R}] - \frac{i}{g} dE_{L,R} , \tag{3.1}$$

where $E_{L,R} = -E_{L,R}^\dagger$. For generality, rather than just the photon, let us introduce a whole $U(3) \times U(3)$ multiplet $B_{L,R}$ of nonstrong gauge fields with a coupling constant h. $B_{L,R}$ transforms just like $A_{L,R}$:

$$\delta B_{L,R} = -[B_{L,R}, E_{L,R}] - \frac{i}{h} dE_{L,R} . \tag{3.2}$$

We specialize to electromagnetism by the replacement

$$hB_{L,R} \to eQ\mathcal{A} , \tag{3.3}$$

where $Q = \mathrm{diag}(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3})$ and $\mathcal{A}$ is the photon field. Of course, kinetic terms

$$-\tfrac{1}{2}\mathrm{Tr}[F_{\mu\nu}(B_L)F_{\mu\nu}(B_L) + F_{\mu\nu}(B_R)F_{\mu\nu}(B_R)]$$

with

$$F_{\mu\nu}(B_L) = \partial_\mu B_\nu^L - \partial_\nu B_\mu^L - ih[B_\mu^L, B_\nu^L]$$

must be included. It is necessary that the local variation of what we get by "gauging" Γ in (2.5) with the fields $B_{L,R}$ reproduce the non-Abelian anomaly. Now this is taken care of by gauging $\Gamma_{WZ}(U)$ as pointed out by Witten:[13]

$$\Gamma_{WZ}(U) \to \Gamma_{WZ}(U, B_L, B_R) , \tag{3.4}$$

where $\Gamma_{WZ}(U, B_L, B_R)$ is given by Eq. (4.18) of Ref. 8. Thus we can mock up the non-Abelian anomaly just by requiring all the other terms in the "gauged" Γ to be exactly invariant under (3.1) and (3.2).

First consider the terms of $\mathcal{L}_2$ as given in (2.7). Since there are no derivatives in (2.7), reference to (3.1) and (3.2) shows that it may be made locally gauge invariant by the simple replacement in (2.7):

$$A_{L,R} \to A_{L,R} - \frac{h}{g} B_{L,R} . \tag{3.5}$$

This prescription yields Eq. (12) of Ref. 6 when restricted to electromagnetism. There it is noted that the model predicts the pion charge radius to be $\sqrt{3k}/m_\rho$ which is actually slightly better [using k as given by (2.9)] than the value $\sqrt{6}/m_\rho$ with exact vector-meson dominance. For $k \neq 2$, the photon couples to the pion partially through the ρ^0 and partially directly. Of course, the difference between the two predictions for the pion charge radius is rather small but it illustrates that there is no great loss in not assuming exact vector-meson dominance.

Next let us turn to making the c_1, c_2, and c_3 terms locally gauge invariant. This may be easily done by building up objects out of field-strength tensors such as $F(A_L) = dA_L - igA_L^2$, etc., and suitable covariant derivatives of the field U. In addition to the two obvious covariant derivatives $dU - igA_L U + igUA_R$ and $dU - ihB_L U + ihUB_R$ there are two "mixed" ones which we choose to employ

$$\begin{aligned} D^{(1)}(U) &= dU - igA_L U + ihUB_R , \\ D^{(2)}(U) &= dU - ihB_L U + igUA_R . \end{aligned} \tag{3.6}$$

From these we may construct two left-handed one-forms:

$$\begin{aligned} \alpha_1 &= (D^{(1)}U)U^\dagger = \alpha - igA_L + ihUB_R U^\dagger , \\ \alpha_2 &= (D^{(2)}U)U^\dagger = igA_L - ihB_L , \end{aligned} \tag{3.7}$$

where (2.4) was used in the last line. We may also construct two related right-handed one-forms:

$$\beta_1 = U^\dagger D^{(1)} U = U^\dagger \alpha_1 U, \quad \beta_2 = U^\dagger D^{(2)} U = U^\dagger \alpha_2 U \; . \tag{3.8}$$

The objects just introduced have the following properties under parity reversal and charge conjugation:

$$P:\ \alpha_1 \to \beta_2,\ \ \alpha_2 \to \beta_1,\ \ F(A_L) \to F(A_R),\ \text{etc.}\ ; \tag{3.9a}$$

$$C:\ \alpha_1 \to \beta_2^T,\ \ \alpha_2 \to \beta_1^T,\ \ F(A_L) \to -[F(A_R)]^T,\ \text{etc.} \tag{3.9b}$$

We now can list P, C, and locally gauge-invariant four-forms:

$$\begin{aligned}
&\mathrm{Tr}(\alpha_1\alpha_2\alpha_1\alpha_2) = \mathrm{Tr}(\beta_1\beta_2\beta_1\beta_2)\ ,\\
&\mathrm{Tr}(\alpha_1^3\alpha_2 - \alpha_2^3\alpha_1) = \mathrm{Tr}(\beta_1^3\beta_2 - \beta_2^3\beta_1)\ ,\\
&\mathrm{Tr}\{F(A_L)[\alpha_1,\alpha_2]\} = \mathrm{Tr}\{F(A_R)[\beta_1,\beta_2]\}\ ,\\
&\mathrm{Tr}\{F(B_L)[\alpha_1,\alpha_2] + F(B_R)[\beta_1,\beta_2]\}\ .
\end{aligned} \tag{3.10}$$

When the nonstrong gauge fields $B_{L,R}$ are set to zero it is easy to see that the first three terms in (3.10) give three linearly independent combinations of the c_1, c_2, c_3 terms in (2.13) and the fourth gives zero. Thus a suitable "gauging" of Γ_3 to introduce electromagnetism in harmony with the anomaly is to replace Γ_3 by

$$\begin{aligned}
\Gamma_{\mathrm{WZ}}(U,B_L,B_R) + \int \mathrm{Tr}\Bigg[&-\frac{c_1}{g}(\alpha_1^3\alpha_2 - \alpha_2^3\alpha_1) + \frac{c_2}{ig}\{F(A_L)[\alpha_1,\alpha_2]\} + \left(-\frac{c_1}{g} + \frac{c_2}{g^2} + \frac{c_3}{g^3}\right)\mathrm{Tr}(\alpha_1\alpha_2\alpha_1\alpha_2)\\
&+ d_1\{F(B_L)[\alpha_1,\alpha_2] + F(B_R)[\beta_1,\beta_2]\}\Bigg]\ ,
\end{aligned} \tag{3.11}$$

where d_1 is a new constant. Equation (3.11) reduces to (2.13) when we keep only the strongly interacting fields.

There are three kinds of electromagnetic interactions of 0^- mesons, 1^- mesons and photons arising from (3.11) which can be related to present experiment. These are the types $\pi^0 \to \gamma\gamma$, $\gamma\pi \to 2\pi$, and $\rho \to \pi\gamma$. The first two are predicted by famous low-energy theorems which follow from the non-Abelian anomaly and hold by construction in (3.11). Specifically, $\pi^0 \to \gamma\gamma$ and $\gamma\pi \to 2\pi$ receive their contribution from $\Gamma_{\mathrm{WZ}}(U,B_L,B_R)$. The remainder of the terms in (3.11), which are all gauge invariant under the full chiral group, make no contribution. This is to be expected since historically[14] one predicted a vanishing $\pi^0 \to \gamma\gamma$ amplitude (and related ones) when no anomaly (i.e., exact local chiral-gauge invariance) was present. On the other hand, there is no low-energy theorem for processes of the type $\rho \to \pi\gamma$. Suppose one fixes c_1, c_2, and c_3 from the strong interactions. Then, since the d_1 term contributes to $\rho \to \pi\gamma$, we can always choose d_1 (which clearly cannot be determined from the strong interactions) to fit the experimental value of $\rho \to \pi\gamma$. Thus, strictly speaking, the electromagnetic processes will not enable us to practically obtain information about c_1, c_2, c_3.

It may be interesting to write the relevant pieces of the d_1 term and to verify that it makes no contribution to $\pi^0 \to \gamma\gamma$ and to $\gamma\pi \to \pi\pi$ in the zero-energy limit. The d_1 term expands out to

$$d_1 \int \mathrm{Tr}\left[-\frac{e}{F_\pi}(Qd\mathcal{A}\, Q\mathcal{A}\, d\phi) + \frac{4g}{F_\pi}(Qd\mathcal{A}[\rho, d\phi]) - \frac{8i}{F_\pi^3}[Q\mathcal{A}(d\phi)^3] + \cdots \right]. \tag{3.12}$$

For the amplitude for $\pi^0(p) \to \gamma(\epsilon,k) + \gamma(\epsilon',k')$ the first term of (3.12) contributes

$$\frac{16ed_1}{3\sqrt{2}F_\pi}\epsilon_{\mu\nu\alpha\beta}k_\mu\epsilon_\nu\epsilon'_\alpha p_\beta\ .$$

This is exactly canceled by the contribution from the second term which gives $\pi^0 \to \gamma + \rho^0$ or $\pi^0 \to \gamma + \omega$ to be followed by $\rho^0 \to \gamma$ or $\omega^0 \to \gamma$ as extracted from (2.7) "gauged" according to (3.5). The amplitude for $\gamma(k,\epsilon) + \pi^+(p) \to \pi^+(p') + \pi^0(q)$ receives a direct contribution from the third term of (3.12) and s, t, and u ρ-meson pole contributions utilizing the second term of (3.12):

$$\frac{8d_1}{\sqrt{2}F_\pi^3}\epsilon_{\mu\nu\alpha\beta}\epsilon_\mu q_\nu p_\alpha p'_\beta\left[1 - \frac{m_\rho^2}{3}\left(\frac{1}{m_\rho^2 + (k+p)^2} + \frac{1}{m_\rho^2 + (k-q)^2} + \frac{1}{m_\rho^2 + (k-p')^2}\right)\right].$$

This clearly vanishes as the four-momenta go to zero. For finite momenta there are some corrections which might in the future be used, together with the corresponding pieces from the other parts of (3.11), for a more accurate comparison with experiment.

Finally, let us compare with the hidden symmetry model of Fujiwara *et al.*[5] In that approach one writes $U = \xi_L \xi_R^\dagger$ and eventually makes the gauge choice $\xi_L = \xi_R^\dagger = \xi$. (Note that our definitions differ slightly from those of Ref. 5.) Covariant derivatives are defined as

$$D\xi_L = d\xi_L + ig\xi_L\rho - ihB_L\xi_L\ ,$$

$$D\xi_R = d\xi_R + ig\xi_R\rho - ihB_R\xi_R\ ,$$

from which we may construct the one-forms

$$\alpha_L = \xi_L^\dagger D_L \xi_L, \quad \alpha_R = \xi_R^\dagger D_R \xi_R \ . \tag{3.13}$$

(These correspond to $-\hat{\alpha}_L$ and $-\hat{\alpha}_R$ in the notation of Ref. 5.) Notice that α_L and α_R in (3.13) are chiral invariants which transform nontrivially under the hidden-symmetry group. This may be contrasted with our α_1 and α_2 in (3.7) which transform nontrivially in the left-handed chiral space. Nevertheless, it is easy to verify that once the gauge choice $\xi_L = \xi_R^\dagger = \xi$ is made to eliminate the extraneous fields in the hidden-symmetry approach, (3.13) is simply related to (3.7) by

$$\alpha_L = \xi^\dagger \alpha_2 \xi, \quad \alpha_R = -\xi^\dagger \alpha_1 \xi \ . \tag{3.14}$$

Furthermore, one has $F(\rho) = \xi^\dagger F(A_L)\xi$. Since the trace of a string of matrices is invariant if each matrix is subjected to a similarity transformation, each of our invariants in (3.10) should be in one-to-one correspondence with one of the invariants in (4.5) of Ref. 5. For example, $\mathrm{Tr}(\alpha_1\alpha_2\alpha_1\alpha_2) = \mathrm{Tr}(\hat{\alpha}_R\hat{\alpha}_L\hat{\alpha}_R\hat{\alpha}_L)$. However, we have no analogs of their terms $\mathcal{L}_3$ and $\mathcal{L}_5$. Using (3.9b) (or their analogs) one may see that these terms are both odd under C and hence must be discarded. (Actually, when the nonstrong gauge fields are set to zero $\mathcal{L}_5$ vanishes and $\mathcal{L}_3$ becomes a total divergence.) Their term $\mathcal{L}_6$ should be modified to look like the last of (3.10) to make it C invariant.

IV. DETERMINATION OF PARAMETERS FROM STRONG-INTERACTION EXPERIMENTS

In the previous section we have pointed out that the relevant electromagnetic processes can be fit regardless of the values of c_1, c_2, and c_3—the parameters of the ϵ piece of the chiral-invariant Lagrangian. Hence the most reliable way, from a theoretical point of view, to obtain information about c_1, c_2, c_3 is to restrict our attention to strong-interaction processes. There are two relevant decays which we will study: $\omega \to 3\pi$ and $\phi \to 3\pi$. The related processes of the form $K^* \to K\pi\pi$ have not been fully measured and would also involve us in details of SU(3)-symmetry breaking which we wish to avoid. The present experimental data are[14,15]

$$\Gamma(\omega \to 3\pi) = 8.78 \pm 0.32 \text{ MeV} \ , \tag{4.1a}$$

$$\Gamma_{\rm tot}(\phi \to 3\pi) = 0.63 \pm 0.08 \text{ MeV} \ , \tag{4.1b}$$

$$\frac{\Gamma(\phi \to \rho\pi)}{\Gamma_{\rm tot}(\phi \to 3\pi)} \geq 0.8 \ . \tag{4.1c}$$

Note that $\Gamma_{\rm tot}(\phi \to 3\pi)$ includes both the $\phi \to \pi\rho$ and $\phi \to 3\pi$ pieces, added incoherently. The separate decay $\omega \to \rho\pi$ is, of course, not energetically allowed.

It is important to note that the decays $\phi \to 3\pi$ and $\phi \to \rho\pi$ violate the Okubo-Zweig-Iizuka (OZI) rule. This rule holds by construction in our Lagrangian since every term is a single trace in U(3)-flavor space (rather than a product of traces). As usual, we will assume that these decays go because the ω field $\equiv (\rho_{11} + \rho_{22})/\sqrt{2}$ contains a small admixture of the physical ϕ:

$$\omega \approx \omega_p + \epsilon\phi_p, \quad \phi \approx \phi_p - \epsilon\omega_p \ , \tag{4.2}$$

where ϵ is the ω-ϕ mixing angle and the subscript p denotes physical. The conventional determination of ϵ is based on the analysis of the SU(3) mass splitting in the vector-meson nonet. As reviewed in the Appendix one gets

$$|\epsilon| \approx 0.055 \pm 0.030 \ . \tag{4.3}$$

It is clear that there is a rather large uncertainty in this determination. One may look for confirmation of this value by determining ϵ "experimentally" from the formula

$$\frac{\Gamma(\phi \to \pi^0\gamma)}{\Gamma(\omega \to \pi^0\gamma)} = \epsilon^2(|\mathbf{p}_\phi| / |\mathbf{p}_\omega|)^3 \ , \tag{4.4}$$

where $|\mathbf{p}_\phi|$ and $|\mathbf{p}_\omega|$ are the final particle center-of-mass momenta in the ϕ and ω decays. [$\Gamma(\omega \to \pi^0\gamma) = (0.853 \pm 0.077)$ MeV, $\Gamma(\phi \to \pi^0\gamma) = (5.53 \pm 0.72 \times 10^{-3})$ MeV.] This yields

$$|\epsilon| = 0.053 \pm 0.005 \ , \tag{4.5}$$

in agreement with (4.3) but having a considerably smaller uncertainty.

The analysis of this section uses the $\pi\rho\omega$ and $\omega\pi^3$ vertices given in (2.17) and (2.18). Hence we can determine only c_2 and the linear combination of c_1, c_2, and c_3 called h. The $\omega(p) \to \pi^+(q^+) + \pi^-(q^-) + \pi^0(q^0)$ rate is calculated from a contact diagram as well as three ρ-exchange diagrams to be[4]

$$\Gamma(\omega \to \pi^+\pi^0\pi^-) = \frac{m_\omega}{192\pi^3} \times \int dE^+ dE^- [(\mathbf{q}^-)^2(\mathbf{q}^+)^2 - (\mathbf{q}^+ \cdot \mathbf{q}^-)^2] \, |F|^2 \ ,$$

$$F = -3h + 2g_{VV\phi}g_{\rho\pi\pi} \times \sum_{a=+,0,-} \frac{1}{(p - q^a)^2 + (m_\rho - i\Gamma_\rho/2)^2} \ , \tag{4.6}$$

where Γ_ρ is the ρ-meson width. Carrying out the numerical integration in (4.6) gives us following ellipse in the $\tilde{g}_{VV\phi} = g_{VV\phi}F_\pi$, $\tilde{h} = hF_\pi^3$ plane:

$$\Gamma(\omega \to \pi^+\pi^-\pi^0) = 8.78 \text{ MeV} = (6.05\tilde{h}^2 + 4.01\tilde{g}_{VV\phi}^2 - 9.29\tilde{g}_{VV\phi}\tilde{h}) \text{ MeV} \ . \tag{4.7}$$

Evidently, any choice of $\tilde{g}_{VV\phi}$ and $\tilde{h}$ satisfying (4.7) leads to a fit for the ω decay width. A *different* ellipse can be constructed for the $\phi(1020)$ decay into $\pi^+\pi^-\pi^0$. The ϕ case differs in a useful way from the ω case in that for the ϕ, the $\pi\rho$ modes may be observed separately. The cleanest model for $\Gamma_{\rm tot}(\phi \to 3\pi)$, and the one which is used in the analysis[16] of the experimental data, is to add incoherently the widths into $\pi\rho$ and 3π. This yields

$$\Gamma_{\text{tot}}(\phi \to \pi^+\pi^-\pi^0) = \Gamma(\phi \to \pi\rho) + \Gamma(\phi \to 3\pi) , \tag{4.8a}$$

$$\Gamma(\phi \to \pi\rho) = \frac{|\epsilon|^2 \tilde{g}_{VV\phi}^2 |\mathbf{K}|^3}{2\pi} , \tag{4.8b}$$

$$\Gamma(\phi \to 3\pi) = \frac{9m_\phi |\epsilon|^2 h^2}{192\pi^3} \times \int dE^+ dE^- \int [(\mathbf{q}^-)^2(\mathbf{q}^+)^2 - (\mathbf{q}^+\cdot\mathbf{q}^-)^2] , \tag{4.8c}$$

where

$$|\mathbf{K}| = \frac{m_\phi}{2}\{[1-(m_\rho - m_\pi)^2/m_\phi^2] \times [1-(m_\rho + m_\pi)^2/m_\phi^2]\}^{1/2}$$

and the $\mathbf{q}^\pm$ are defined as for (4.6). Equations (4.8a)–(4.8c) lead to the following ellipses in the $\tilde{g}_{VV\phi}$-$\tilde{h}$ plane:

$$\Gamma(\phi \to \pi^+\pi^-\pi^0) = 0.63 \text{ MeV} = |\epsilon|^2(74.5\tilde{h}^2 + 56.2\tilde{g}_{VV\phi}^2) \text{ MeV} . \tag{4.9}$$

The ellipses (4.7) and (4.9) are illustrated in Fig. 1 for the "central" choice $|\epsilon| = 0.053$. It is seen that there are four possible intersections. Two of them, however, may be ruled out from the experimental result (4.1c) which translates to the condition

$$|\tilde{h}/\tilde{g}_{VV\phi}| < 0.43 . \tag{4.10}$$

Thus we have a unique "central" solution up to a *common* overall sign:

$$\tilde{g}_{VV\phi} = \pm 1.9, \quad \tilde{h} = \pm 0.4 . \tag{4.11}$$

We see that solution (4.11) is, as expected, qualitatively similar to those obtained—(2.19) and (2.20)—from "gauging" the Wess-Zumino term in reasonable ways.

The accuracy of the above determination is clearly a crucial issue. It is possible that other contributions (e.g., radially excited vector mesons[17]) to these processes exist. However, we are attempting here to fit the process completely within the framework of the Lagrangian (2.5). Considering other sources of error, the uncertainties in the ω,ρ,ϕ masses and widths have been found not to play an important role. For example, the effect of putting the ρ meson width to zero in (4.6) is not important. The main source of error appears to be the uncertainty in the ω-ϕ mixing angle ϵ. In Table I we show the dependence of the predicted $\tilde{g}_{VV\phi}$ and $\tilde{h}$ as $|\epsilon|$ is varied slightly around its central value. From Table I we see that $\tilde{g}_{VV\phi}$ suffers a small percentage change as $|\epsilon|$ varies while $\tilde{h}$, though clearly small, changes more. However, the relative $\tilde{g}_{VV\phi}$-$\tilde{h}$ sign is clearly positive in the present determination. This sign corresponds to the ρ pole and contact term contribution to $\omega \to 3\pi$ interfering destructively with each other.

As far as we know, this is the first time an attempt has been made to determine the parameters of the minimal pseudoscalar-vector chiral Lagrangian from strong-interaction processes alone.

V. U(2) REDUCTION

The formula (2.13) for the ϵ terms in the action [wherein A_L should be replaced by the expression in (2.3a)] is actually a fairly complicated one. A simplification can be made, however, since we wish to study the ordinary nonstrange baryons as soliton excitations. We simply delete all reference to the strange-quark index, decreasing the symmetry of Γ_3 to chiral U(2)×U(2). Then the vector-meson nonet is replaced by the 2×2 matrix

$$\rho = \frac{\omega}{\sqrt{2}}1 + \frac{1}{\sqrt{2}}\tau\cdot\rho , \tag{5.1}$$

with the usual isospin notation. Similarly the meson-nonet field ϕ is to be replaced by

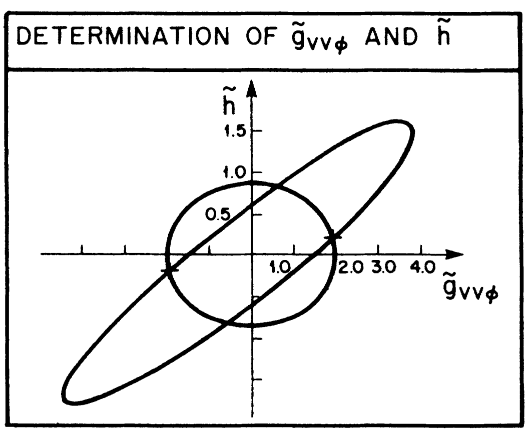

FIG. 1. Determination of the parameters $\tilde{g}_{VV\phi}$ and $\tilde{h}$ from the decays $\omega \to \pi^+\pi^0\pi^-$ and $\phi \to \pi^+\pi^0\pi^-$ for the ω-ϕ mixing angle $|\epsilon| = 0.053$. Both decays define an ellipse in the $\tilde{g}_{VV\phi}$-$\tilde{h}$ plane with four intersections. Two intersections are ruled out by the relative branching ratio $\Gamma(\phi \to \rho\pi)/\Gamma_{\text{tot}}(\phi \to 3\pi)$. The two allowed intersections give $\tilde{g}_{VV\phi}$ and $\tilde{h}$ up to an overall minus sign ($\tilde{g}_{VV\phi} = \pm 1.9$, $\tilde{h} = \pm 0.4$ for this value of ϵ).

TABLE I. Predicted values of $\tilde{g}_{VV\phi}$ and $\tilde{h}$ for various values of the ω-ϕ mixing angle $|\epsilon|$. (Note that $\tilde{g}_{VV\phi}$ and $\tilde{h}$ may both be multiplied by a minus sign.)

$\lvert\epsilon\rvert$	$\tilde{g}_{VV\phi}$	$\tilde{h}$
0.045	2.2	0.7
0.050	2.0	0.5
0.053	1.9	0.4
0.060	1.7	0.2
0.065	1.6	0.1
0.070	1.5	0.0
0.080	1.3	−0.15

$$\phi=\frac{\text{“}\eta\text{”}}{\sqrt{2}}1+\frac{1}{\sqrt{2}}\boldsymbol{\tau}\cdot\boldsymbol{\pi}\ . \tag{5.2}$$

In this formula, “η” is a mathematical isosinglet pseudoscalar which does not have any $s\bar{s}$ component. It is not expected to play as important a role as the π, ρ, and ω fields in the study of the nucleon. This is based on one's intuition from nuclear physics wherein the π, ρ, and ω provide (together with a possible scalar, which may be accommodated in the present formalism as discussed elsewhere)[18] the main contribution to the nuclear force.[19]

For our present purpose it is convenient to explicitly separate the η by setting

$$\xi=\hat{\xi}\exp\left[\frac{i\eta}{\sqrt{2}F_\pi}\right],\quad \hat{\xi}=\exp\left[\frac{i\boldsymbol{\pi}\cdot\boldsymbol{\tau}}{\sqrt{2}F_\pi}\right], \tag{5.3}$$

and to define the one-forms

$$\begin{aligned} p&=\xi^\dagger d\xi+d\xi\,\xi^\dagger=\hat{p}+\frac{\sqrt{2}i}{F_\pi}d\eta\ ,\\ v&=\xi^\dagger d\xi-d\xi\,\xi^\dagger=\hat{\xi}^\dagger d\hat{\xi}-d\hat{\xi}\,\hat{\xi}^\dagger\ , \end{aligned} \tag{5.4}$$

wherein $\hat{p}=\hat{\xi}^\dagger d\hat{\xi}+d\hat{\xi}\,\hat{\xi}^\dagger$. These objects enter our formula for Γ_3 naturally since $\xi^\dagger\alpha\xi=p$ and $\xi^\dagger A_L\xi=\rho-i(p+v)/2g$. Under parity $p\rightarrow+p$ and $v\rightarrow-v$. This means that the expansion of $\hat{p}$ contains an odd number of pion fields and has negative G parity while v contains an even number of pion fields and has positive G parity. These properties make it very easy to accomplish the U(2) reduction of Γ_3. An identity which we use is

$$vp+pv=-2dp\ . \tag{5.5}$$

Making the indicated substitutions and using (5.5), Γ_3 becomes, in the two-flavor limit, the integral of the four-form:

$$\begin{aligned} &\frac{i}{\sqrt{2}}\left[c_1-\frac{c_2}{g}-\frac{c_3}{2g^2}\right]\omega\,\mathrm{Tr}(\alpha^3)+2c_2 d\omega\,\mathrm{Tr}(p\boldsymbol{\tau}\cdot\boldsymbol{\rho})+\frac{c_3}{g}\omega\,\mathrm{Tr}(dp\boldsymbol{\tau}\cdot\boldsymbol{\rho})-\frac{i}{\sqrt{2}}\left[\frac{c_2}{g}+\frac{c_3}{2g^2}\right]\omega\,\mathrm{Tr}(dpv)\\ &\quad\frac{-ic_3}{\sqrt{2}}\omega\,\mathrm{Tr}[(\boldsymbol{\tau}\cdot\boldsymbol{\rho})^2p]+\frac{1}{\sqrt{2}F_\pi}d\eta\,\mathrm{Tr}\left[-i\left[c_1-\frac{c_2}{g}-\frac{c_3}{2g^2}\right]\left[\frac{1}{g}v\hat{p}^{\,2}+\sqrt{2}i\boldsymbol{\tau}\cdot\boldsymbol{\rho}\hat{p}^{\,2}\right]\right.\\ &\qquad\qquad+ic_2\left[2d(\boldsymbol{\tau}\cdot\boldsymbol{\rho})(\boldsymbol{\tau}\cdot\boldsymbol{\rho})+\frac{1}{g^2}vdv\right]+\left[2c_2+\frac{3}{g}c_3\right]\left[iv(\boldsymbol{\tau}\cdot\boldsymbol{\rho})^2+\frac{1}{\sqrt{2}g}\boldsymbol{\tau}\cdot\boldsymbol{\rho}v^2\right]\\ &\qquad\qquad\left.-2ic_3\left[-\frac{i}{\sqrt{2}}(\boldsymbol{\tau}\cdot\boldsymbol{\rho})^3+\frac{1}{4g^3}v^3\right]\right]\ . \end{aligned} \tag{5.6}$$

It is interesting to compare the expansion of this formula in terms of the pseudoscalars with (2.17) and (2.18). The first term of (5.6) starts as $\omega\pi^3$ and indeed its coefficient is proportional to h. Similarly the second term of (5.6) starts out as $\pi\rho\omega$ and its coefficient is proportional to $g_{VV\phi}$. The third, fourth, and fifth terms of (5.6) start out as $\pi^3\rho\omega$, $\omega\pi^5$, and $\omega\rho^2\pi$, respectively. Hence these three terms are not easily “measured” by observed meson reactions. The remaining terms in (5.6) are all linearly proportional to $d\eta$ and also are difficult to “measure” in observed meson reactions. These η terms will not contribute when we make the usual $\mathbf{K}=\mathbf{T}+\mathbf{J}=0$ static parity-invariant hedgehog *Ansatz* for the nucleon because they do not contain the field ω_μ which is the only one with a needed nonzero fourth component in the static limit. The η terms may, however, make some contribution to the “collective quantization” of the soliton, but that is beyond the scope of this paper.

Note that the “minimal” model discussed in Ref. 12 corresponds to the present Lagrangian with $c_2=c_3=0$. The complete model discussed in Ref. 7 is not a special case of the present one since it not chiral symmetric (without additional terms). However, it has ϵ terms with coefficients $g_{VV\phi}$ and h which are numerically similar to the ones we find. To the extent that the leading terms in the pseudoscalars determine the structure and behavior of the soliton solutions, the present model will be similar to the complete model. In the present case, however, we also have the freedom to arbitrarily adjust c_3. The natural question is whether this freedom can improve the predictions of the nucleon's properties. This will be investigated in the following sections.

VI. SOLITON SOLUTIONS

The action (2.5) describes both mesons and solitons, the latter being finite-energy configurations with nonvanishing winding number $B=\int d^3r\,B_0(r)$. Here, we are interested in investigating the baryon number $B=1$ sector. For that, let us specialize to the following hedgehog *Ansätze:*

$$U(\mathbf{r})=\exp[i\boldsymbol{\tau}\cdot\hat{\mathbf{r}}F(r)]\ , \tag{6.1a}$$

$$\xi(\mathbf{r})=\exp[i\boldsymbol{\tau}\cdot\hat{\mathbf{r}}F(r)/2]\ , \tag{6.1b}$$

$$\omega_\mu(\mathbf{r})=i\omega(r)\delta_{\mu4}\ , \tag{6.1c}$$

$$\rho_i^a(\mathbf{r})=\epsilon_{ika}\hat{r}_k\frac{G(r)}{\sqrt{2}gr}\ , \tag{6.1d}$$

where $\mu=1,2,3,4$ is a Lorentz index and $a=1,2,3$ refers to isospin. With the static *Ansatz* (6.1a) the baryon-number density reduces to

$$B_0(r) = -\frac{1}{2\pi^2}\frac{F'}{r^2}\sin^2 F \ . \tag{6.2}$$

To ensure unit baryon number, we have to choose

$$F(0)=\pi, \quad F(\infty)=0 \ . \tag{6.3}$$

In what follows, we will refer to the radial functions $F(r)$, $G(r)$, and $\omega(r)$ as the pion profile, the ρ-meson profile, and the ω-meson profile, respectively. The static-soliton energy, to be identified with the Skyrmion or hedgehog mass M_H, is obtained with the help of (5.6) and reads

$$M_H = E[F,G,\omega] = -\int d^3r\, \mathcal{L}(F,G,\omega) \ ,$$

$$\begin{aligned} E[F,G,\omega] = 4\pi\int_0^\infty dr \Bigg[& \frac{F_\pi^2}{4}(r^2F'^2+2\sin^2F)+\tfrac{1}{2}m_\pi^2F_\pi^2r^2(1-\cos F)+F_\pi^2(G-1+\cos F)^2 \\ & +\frac{1}{4g^2}\left[2G'^2+\frac{G^2(G-2)^2}{r^2}\right]-\tfrac{1}{2}r^2(m^2\omega^2+\omega'^2)-\gamma_1\omega F'\sin^2F \\ & -\{(\gamma_2+\gamma_3)\omega F'[1+2(G-1)\cos F+\cos^2F]+2\gamma_2G'\omega\sin F+\gamma_3\omega F'G(G-2)\}\Bigg] \\ = {} & E_\pi+E_\pi^m+E_{\pi\rho}+E_\rho+E_\omega+E_{\mathrm{WZ}}^{\min}+E_{\mathrm{WZ}}^{\pi\rho\omega} \ , \end{aligned} \tag{6.4}$$

where we have assumed, for simplicity, the KSFR relation $m_\rho^2=m_\omega^2=m^2=2F_\pi^2g^2$. The constants γ_i, $i=1,2,3$, are defined by

$$\begin{aligned} \gamma_1 &= -\frac{3\tilde{h}}{2\sqrt{2}}=\frac{g_\omega}{2\pi^2} \ , \\ \gamma_2 &= \frac{4ic_2}{\sqrt{2}g}=\frac{\tilde{g}_{VV\phi}}{\sqrt{2}g} \ , \\ \gamma_3 &= \frac{i\sqrt{2}c_3}{g^2} \ . \end{aligned} \tag{6.5}$$

We have added in (6.4) the canonical pion mass term

$$\mathcal{L}_\pi = \tfrac{1}{8}m_\pi^2F_\pi^2\mathrm{Tr}(U+U^\dagger-2) \ .$$

The $\pi\rho$ interaction energy $E_{\pi\rho}$ embodies the ρ-meson mass term, and the ϵ terms (loosely designated in the literature as Wess-Zumino terms) have been split into two parts: the first coming from the $\omega_\mu B^\mu$ interaction[11] and the second giving the $\omega\rho\pi$ correlations. To arrive at (6.4), we have performed two partial integrations, since possible surface terms do not contribute due to the asymptotic behavior of the meson profiles given in the end of this section. Functional minimization of (6.4) leads to the coupled equations of motion for the meson profiles. They read

$$\begin{aligned} F'' = & -\frac{2F'}{r}-\frac{1}{r^2}[4(G-1)\sin F+\sin 2F] \\ & +m_\pi^2\sin F+\frac{2\gamma_1}{F_\pi^2}\frac{\omega'}{r^2}\sin^2F \\ & +\frac{2(\gamma_2+\gamma_3)}{F_\pi^2r^2}\omega'[1+2(G-1)\cos F+\cos^2F] \\ & +\frac{2\gamma_3}{F_\pi^2r^2}[\omega'G(G-2)+2G'\omega(G-1+\cos F)] \ , \end{aligned} \tag{6.6a}$$

$$\begin{aligned} G'' = & \frac{G(G-1)(G-2)}{r^2}+m^2(G-1+\cos F) \\ & +2g^2\gamma_2\omega'\sin F-2\gamma_3g^2F'\omega(G-1+\cos F) \ , \end{aligned} \tag{6.6b}$$

$$\begin{aligned} \omega'' = & -\frac{2\omega'}{r}+m^2\omega+\frac{\gamma_1}{r^2}F'\sin^2F \\ & +2\gamma_2\frac{G'}{r^2}\sin F+\gamma_3\frac{F'}{r^2}G(G-2) \\ & +\frac{\gamma_2+\gamma_3}{r^2}F'[1+2(G-1)\cos F+\cos^2F] \ . \end{aligned} \tag{6.6c}$$

The pertinent boundary conditions to ensure finite-energy solutions are

$$G(0)=2, \quad G(\infty)=0 , \tag{6.7a}$$

$$\omega'(0)=0, \quad \omega(\infty)=0 . \tag{6.7b}$$

Of particular interest is the large-distance behavior of the pion profile $F(r)$; it falls off exponentially:

$$F(r) \sim \frac{b}{r^2} e^{-m_\pi r}(1+m_\pi r) \quad \text{as } r \to \infty . \tag{6.8}$$

The vector-meson profiles $G(r)$ and $\omega(r)$ fall off exponentially with $\exp(-2m_\pi r)$ and $\exp(-3m_\pi r)$, respectively. Although the numerical evaluation of (6.6) is straightforward, we perform the following checks. First, for $\gamma_2=\gamma_3=0$, and $\gamma_1=(1.5\times 5.8545/2\pi^2)=0.4449$ we have to recover the results of the minimal model presented in Ref. 12. Second, since the Wess-Zumino term is linear in $\omega(r)$ and $\omega'(r)$, the virial theorem

$$E_{\rm WZ}=-2E_\omega \tag{6.9}$$

has to be satisfied, with $E_{\rm WZ}=E_{\rm WZ}^{\rm min}+E_{\rm WZ}^{\pi\rho\omega}$ and E_ω as given by (6.4). As a measure of the numerical accuracy, let us introduce

$$\Delta V = -\frac{E_{\rm WZ}+2E_\omega}{2E_\omega} . \tag{6.10}$$

For a solution to be accepted, we require $\Delta V \le 10^{-5}$. Furthermore, the solutions have to satisfy the generalized version of Rafelski's virial theorem in the presence of vector mesons,[20] $E_\rho - E_\pi - E_\omega^{\rm kin} = E_{\pi\rho} + 3E_\omega^m + 3E_\pi^m$ [where $E_\omega = E_\omega^{\rm kin} + E_\omega^m$ can be read off from (6.4)]. In the following section, we will present numerical solutions to the equations of motion (6.6). Before doing that, let us define two observables which will be of interest in what follows. The hedgehog radius r_H measures the extension of baryonic charge of the soliton

$$r_H^2 = 4\pi \int_0^\infty r^4 B_0(r) dr = -\frac{2}{\pi}\int_0^\infty r^2 F' \sin^2 F \, dr \tag{6.11}$$

and the axial-vector coupling constant g_A can be read off from the pion tail (6.8) via

$$g_A = \frac{4\pi}{3} b F_\pi^2 . \tag{6.12}$$

VII. NUMERICAL RESULTS AND DISCUSSION

We will now present numerical solutions to the set of coupled equations (6.6) subject to the boundary conditions (6.3) and (6.7). From Sec. IV we expect the parameter $\tilde{g}_{VV\phi}$ to be around ± 1.9 and the parameter $\tilde{h}$ to be around ± 0.4. Reference to Table I shows that $\tilde{g}_{VV\phi}$ and $\tilde{h}$ obey the empirical correlation $|\tilde{g}_{VV\phi}-\tilde{h}| \approx 1.5$. From Eq. (4.5) we see that the range $|0.2| < |\tilde{h}| < |0.6|$ is reasonable. Remember that c_3 is an undetermined parameter. Note from (5.6) that all the contributing terms are proportional to ω, so changing the sign of $\omega(r)$ corresponds to reversing the signs of all of c_1, c_2, and c_3.

For orientation, let us discuss in some detail cases with $c_3=0$ ($\gamma_3=0$). In Fig. 2, we show the meson profiles for the central choice of parameters, $\tilde{h}=-0.4$ and $\tilde{g}_{VV\phi}=-1.9$, in comparison with the "minimal model" ($\tilde{h}=-0.4, \tilde{g}_{VV\phi}=0$). In what follows, we adopt the notation of Ref. 7, referring to models with $\gamma_1 \neq 0$, $\gamma_2=\gamma_3=0$ as "minimal models." These profiles are rather similar to the ones of the minimal model of Ref. 12 and the complete model of Ref. 7. In Table II, we compare the static hedgehog mass M_H, the baryon charge radius r_H, and the axial-vector coupling constant g_A for these three cases. It appears that this set of parameters gives a soliton with reasonable properties very similar to the complete model. The differences in M_H, r_H, and g_A stem, on one hand, from the different terms in the $\pi\rho\omega$ correlations and from the fact that the ω-coupling constant used here,

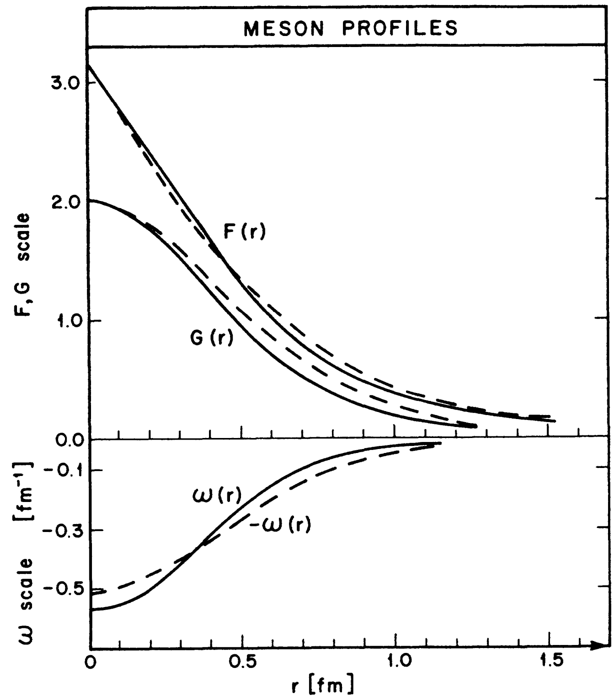

FIG. 2. Meson profiles for the central choice of $\tilde{h}=-0.4$, $\tilde{g}_{VV\phi}=-1.9$, and $\gamma_3=0$ (dashed lines). The solid lines give the result for the "minimal" model with $\tilde{g}_{VV\phi}=0$, $\tilde{h}=-0.4$. Notice the different scale for the ω meson. The standard parameters $g=4.1248$, $F_\pi=132$ MeV, and $m_\pi=139$ MeV are used.

$$g_\omega = -\frac{3\pi^2 \tilde{h}}{\sqrt{2}} = 8.37 \tag{7.1}$$

is somewhat smaller than the one used in Ref. 7 (there, $g_\omega=8.78$).

In Table III, we compare in detail the different contri-

TABLE II. Static hedgehog properties. The Skyrmion mass M_H, the baryon charge radius r_H, and the axial-vector coupling constant g_A are given for the minimal[12] and the complete model,[7] as well as for the Lagrangian (6.4) with $\tilde{h}=-0.4$, $\tilde{g}_{VV\phi}=-1.9$, and $\gamma_3=0$.

	Minimal model	Complete model	$\tilde{h}=-0.4$ $\tilde{g}_{VV\phi}=-1.9$
M_H (MeV)	1474	1465	1422
r_H (fm)	0.50	0.48	0.42
g_A	0.88	0.99	0.79

TABLE III. Various contributions to the static soliton energy as defined in (6.4). The first column gives the results (in MeV) of the minimal and the complete model of Ref. 7, the last column exhibits the results of the Lagrangian used here with $\tilde{h}=-0.4$, $\tilde{g}_{VV\phi}=-1.9$, and $\gamma_3=0$. In all three cases, the input parameters are $F_\pi=132$ MeV, $g=4.1248$, $m_\rho=m_\omega=770$ MeV, and $m_\pi=138$ MeV.

	Minimal model	Complete model	$\tilde{h}=-0.4$ $\tilde{g}_{VV\phi}=-1.9$
E_π	751	733	645
E_π^m	39	43	32
$E_{\pi\rho}$	49	33	160
E_ρ	369	370	350
E_ω	-257	-256	-234
E_{WZ}	513	512	468

butions to the static energy as defined in (6.4) of our model with $\tilde{h}=-0.4$, $\tilde{g}_{VV\phi}=-1.9$ with the ones of Refs. 7 and 12. The present model is again seen to be similar to the complete model. Of course, the model considered here exhibits some pertinent differences from the complete model. First, in our approach the Goldberger-Treiman relation holds exactly, whereas in the complete model it is violated by 15% due to the lack of chiral symmetry in the Wess-Zumino term. Second, in the complete model the Wess-Zumino energy is completely dominated by the $\omega_\mu B^\mu$ term, whereas in our model there is an intricate cancellation between positive and negative contributions of the same order of magnitude from the γ_1 and γ_2 terms. Nonetheless, the net contribution of the Wess-Zumino term to the static energy is approximately 500 MeV for the model considered here as well as the complete model. We will come back to this point later on.

In Table IV, static properties are given for a range of the ω-ϕ mixing angle $|\epsilon|$. We can read off the following trend from this table. For larger values of $|\tilde{g}_{VV\phi}|$ (and corresponding $|\tilde{g}_{VV\phi}-\tilde{h}|\approx 1.5$) both the mass and the radius of the soliton tend to decrease. Allowing for a variation of $\tilde{g}_{VV\phi}$ of the order of 10%, we see that the mass and the radius change by 10% or less in the whole range of the mixing angle $|\epsilon|$. Notice that since g_A measures the extension of the pion source, a decrease in r_H induces a decrease in g_A. Using $\tilde{g}_{VV\phi}$ on the smaller

TABLE IV. Bulk properties of the soliton for $\gamma_3=0$. The Skyrmion mass M_H, the baryon charge radius r_H, and the axial-vector coupling constant g_A are given as a function of the mixing angle $|\epsilon|$, i.e., the parameters $\tilde{h}$ and $\tilde{g}_{VV\phi}$. The standard input $F_\pi=132$ MeV, $m_\pi=138$ MeV, $g=4.1248$, and $m_\omega=m_\rho=770$ MeV is used throughout.

$(\tilde{h},\tilde{g}_{VV\phi})$	M_H (GeV)	r_H (fm)	g_A
$(-0.7,-2.2)$	1.225	0.35	0.54
$(-0.5,-2.0)$	1.348	0.39	0.69
$(-0.4,-1.9)$	1.422	0.42	0.79
$(-0.2,-1.7)$	1.581	0.50	1.03
$(-0.1,-1.6)$	1.660	0.53	1.15
$(0.0,-1.5)$	1.788	0.57	1.29
$(+0.15,-1.3)$	1.824	0.61	1.44

side gives a too heavy soliton, with an increased radius and reasonable g_A.

In Fig. 3 we show the dependence of our results for the uncorrelated situation with fixed $\tilde{h}$ equal to its central value -0.4 and various values of $\tilde{g}_{VV\phi}$ (not necessarily the determined one). Only a drastic reduction in $\tilde{g}_{VV\phi}$ gives an appreciable change of the soliton properties. In any case, we also observed that lowering the mass to less than 1200 MeV by lowering $\tilde{g}_{VV\phi}$ inevitably leads to a small radius ($r_H\sim 0.3$ fm) and a small g_A (≤ 0.50).

Before discussing our results for $\gamma_3\neq 0$ it may be of interest to remark on the case when the $\omega 3\pi$ contact term $\tilde{h}$ is set to zero and $\tilde{g}_{VV\phi}$ is taken from $\pi^0\rightarrow 2\gamma$ computed with vector-meson dominance (VMD). Assuming that $\pi^0\rightarrow 2\gamma$ is also described by the anomaly, this yields

$$\tilde{g}_{VV\phi}=\frac{3m_\rho^4}{4\pi^2 g_{\rho\pi\pi}^2 F_\pi^4}\ . \tag{7.2}$$

Essentially this is the ancient Gell-Mann–Sharp-Wagner model.[21] Using (2.11), (7.2) gives $\tilde{g}_{VV\phi}=1.2$. We find $M_H=1574$ MeV, $r_H=0.5$ fm and $g_A=1.02$, i.e., results similar to the model with the choice of $\tilde{g}_{VV\phi}=-1.7$ and $\tilde{h}=-0.2$. (For other models incorporating exact VMD, see, e.g., Refs. 22–24.)

Now let us see if the nucleon properties can be improved by choosing a nonzero value for γ_3 (or c_3). It is convenient to introduce

$$\kappa=\frac{\gamma_3}{\gamma_2}=\frac{c_3}{c_2}\frac{1}{2g}=0.1212\frac{c_3}{c_2} \tag{7.3}$$

to measure the relative strength of the γ_3 terms with respect to the γ_2 terms. Before giving specific results, we can already read off some trends from the relative sign of κ. For κ position, the repulsive terms in (6.4) proportional to $(\gamma_2+\gamma_3)\omega F'[1+2(G-1)\cos F+\cos^2 F]$ will become even stronger, leading to an increase of the soliton size and its mass. Inversely, for $\kappa<0$, the repulsion due to the Wess-Zumino term will be weakened and lead to a decrease in the soliton mass and its radius.

Let us concentrate first on the central choice of param-

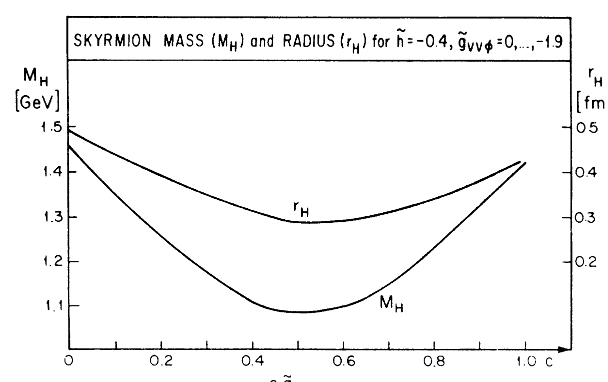

FIG. 3. Skyrmion mass (M_H) and the baryon charge radius (r_H) for $\tilde{h}=-0.4$ and $0\geq\tilde{g}_{VV\phi}\geq-1.9$. Notice that only a drastic change of $\tilde{g}_{VV\phi}$ as compared to its value determined in Sec. IV gives considerable changes in the bulk properties of the soliton.

eters, $\tilde{h}=-0.4$ and $\tilde{g}_{VV\phi}=-1.9$. In Fig. 4 we show the static properties as a function of κ, for $-1\le\kappa\le 1$. For $\kappa=1$, we have $M_H=1475$ MeV, $r_H=0.48$, and $g_A=0.94$, almost identical to the complete model. On the other hand, for $\kappa\lesssim -1$, the mass tends to fall, whereas the radius r_H levels off at around $r_H\sim 0.25$ fm, with g_A being unacceptably small. So a lowering of the soliton mass can only be obtained at the expense of the other static properties (see also discussion below). In Fig. 5 we give the soliton mass as a function of $\tilde{g}_{VV\phi}$ (with correlated $\tilde{h}$) and κ for $1.3\le\tilde{g}_{VV\phi}\le 2.2$ and $-1\le\kappa\le 0$. The plot shows a smooth decrease of M_H with increasing $\tilde{g}_{VV\phi}$ and decreasing κ. We have monitored an even wider range in parameter space, with the following results: For the largest values of the mixing angle $|\epsilon|$ in Table I, the soliton is very heavy (~ 1.8 GeV), extended ($r_H\sim 0.6$ fm), and g_A is close to its empirical value. It appears that vector-meson propagator effects are frozen out and the physics is very similar to the "VMD-inspired modified Skyrme model" discussed in Refs. 25–27. Around the central choice of parameters, the soliton mass is generally of the order of 1.4 GeV, $r_H\sim 0.5$ fm, and $g_A\lesssim 1.0$, similar to the complete model of Ref. 7 (for $0\lesssim\kappa\lesssim 1$). For $|\epsilon|$ on the small end of its allowed values, the soliton mass is approximately 1.2 GeV, with a too small radius and g_A. Only for $\kappa\gtrsim 4$, can we come back to a reasonable radius ($r_H\le 0.4$ fm and $g_A\le 0.8$), with the mass increasing beyond 1350 MeV. It therefore appears that there is no magic set of parameters leading to a realistically low mass, a reasonable r_H and g_A at the same time. This is a common problem in all soliton models.

Additional static properties of the nucleon, such as its mass, the Δ-N splitting, and the electromagnetic charge radii, may be calculated in a standard way using the collective-coordinate quantization. For our present purpose, considering the similarity of this model's predictions to those of the "complete" model, it seems sufficient to give estimates of these quantities.

To calculate the N and Δ rotational energies and hence the Δ-N mass splitting, it is sufficient to find the moment

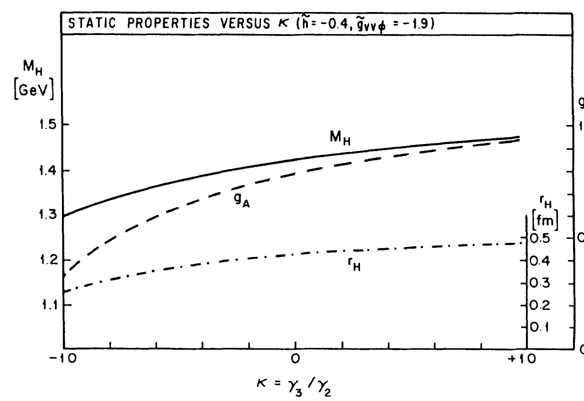

FIG. 4. Static baryon properties as a function of $\kappa(\gamma_3)$ for $\tilde{g}_{VV\phi}=-1.9$ and $\tilde{h}=-0.4$. For κ positive, the static properties are only mildly varying with κ, whereas for κ approaching -1, the mass, the baryon charge radius, and g_A decrease considerably.

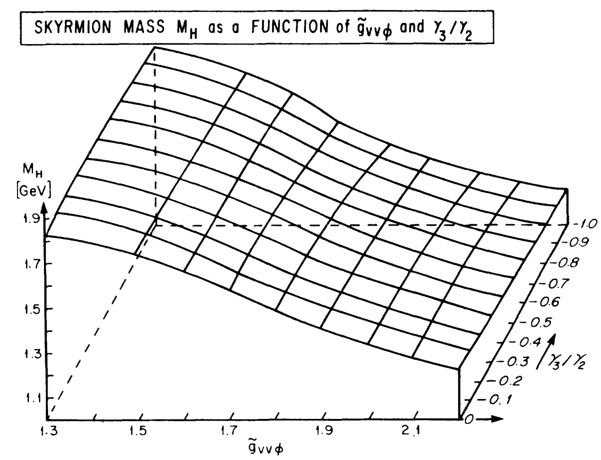

FIG. 5. Three-dimensional plot of the static soliton energy as a function of $\tilde{g}_{VV\phi}$ (and therefore $\tilde{h}$) and $\kappa=\gamma_3/\gamma_2$, with $-1\le\kappa\le 0$. The soliton mass varies smoothly in both directions, indicating that there is no set of "magic" values for c_1, c_2, and c_3 leading to a reasonably low mass, a reasonable baryon charge radius, and the axial-vector coupling constant g_A.

of inertia. This is proportional to the Skyrmion radius. As has been shown in Refs. 3, 22, and 23, the moment of inertia is dominated by its pionic part, and can thus be simply estimated as

$$\lambda_\pi=\frac{4\pi}{3}F_\pi^2\int_0^\infty r^2\left[\sin^2F+8\sin^4\left[\frac{F}{2}\right]\right]dr\ . \tag{7.4}$$

For $\tilde{g}_{VV\phi}=-1.9$, $\tilde{h}=-0.4$, and $c_3=0$, we find $\lambda_\pi=0.55$ fm. As κ increases, λ_π also increases. For $\kappa=+1$, we have $\lambda_\pi=0.73$ fm. For comparison, in the complete model, $\lambda_\pi=0.69$ fm and $\lambda_{\text{tot}}=0.68$ fm. The dependence of λ_π on $\tilde{g}_{VV\phi}$, $\tilde{h}$, and κ agrees with our observations concerning the classical mass, the baryon charge radius and g_A. For the "low mass parameters" $\tilde{g}_{VV\phi}=-2.2$, $\tilde{h}=-0.7$, λ_π is less than 0.5 fm, i.e., considerably too small for all values of κ. On the other hand, for $\tilde{g}_{VV\phi}=-1.3$ and $\tilde{h}=+0.15$, λ_π is somewhat too large, it lies between 1.1 and 1.5 fm as κ varies from -1 to $+1$. In general, for κ positive and increasing, λ_π increases weakly. For κ negative, we observe a sharp drop in the approximate moment of inertia as κ approaches -1 (or smaller) for all values of $\tilde{g}_{VV\phi}$ and $\tilde{h}$. The physical N-Δ splitting is given by $\lambda=0.99$ fm. Assuming $\lambda\simeq\lambda_\pi$, we find this value for $\tilde{g}_{VV\phi}=-1.7$, $\tilde{h}=-0.2$, and $\kappa=1$, together with $M_H=1671$ MeV, $r_H=0.54$ fm, and $g_A=1.16$.

Now, the electromagnetic charge radii can be approximated to within 10% by the VMD formula

$$r_c^2=r_H^2+\frac{6}{m_\omega^2}\ , \tag{7.5}$$

which for $r_H=0.5$ fm implies $r_c\simeq 0.8$ fm (Ref. 12). Of course, there are modifications to this estimate for each particular channel (isoscalar, isovector, charge, magnetic), but it is safe to conclude that $r_H\sim 0.4$–0.6 fm leads to reasonable electromagnetic charge radii, i.e., the overall picture of the soliton emerging from our Lagrangian will lead to a satisfactory description of the nucleon electromagnetic properties, and a too high mass for the nucleon and the $\Delta(1233)$. These findings give us confidence to state that the results of the complete model in Ref. 7 should hold for any realistic $\pi\rho\omega$ Lagrangian. This is certainly a very important point, since it means that the seeming uncertainties in constructing the anomalous action lead to a rather unique description in the soliton sector. Of course, this does not mean that the ϵ terms in the action play no role in the baryon sector, indeed the "WZ action" is vital for the soliton stability (not necessarily through the $\omega_\mu B^\mu$ coupling).

Our findings concerning the dependence of the soliton properties on κ, i.e., c_3, allow us to give a suitable range for c_3. For the central range of $\tilde{h}$ and $\tilde{g}_{VV\phi}$, we conclude that $\kappa \approx 1$ (i.e., $c_3 \approx 8.3$) gives the best description of the nucleon as a solitonic excitation. More conservatively, if we vary $\tilde{h}$ and $\tilde{g}_{VV\phi}$ around their central values, we find a fair description of the nucleon properties for $0 \lesssim \kappa \lesssim 1.5$, i.e., $0 \lesssim c_3 \lesssim 12$.

VIII. SUMMARY AND OUTLOOK

It is generally felt that a suitable chiral Lagrangian of vectors and pseudoscalars should provide a realistic testing ground for the notion that the nucleon is a soliton excitation. In the past, the ϵ terms [Eq. (2.13)] of such a Lagrangian were determined[28] by a kind of heuristic "gauging" principle or by a similar extrapolation of vector-meson dominance and consideration of meson electromagnetic amplitudes. Detailed discussions and references are given in Ref. 3.

Here, after a careful discussion of the addition of electromagnetism to the chiral Lagrangian (2.6) + (2.7) + (2.13), we have concluded that the most reliable procedure for determining the coefficients c_1, c_2, c_3 in (2.13) is to use strong-interaction processes exclusively. In this way we found $\tilde{g}_{VV\phi}$ [related to c_2 by (2.18) and (2.19)] to have the central value

$$|\tilde{g}_{VV\phi}| \approx 1.9 \ . \tag{8.1}$$

Here the sign is undetermined and the accuracy is about 15%. The quantity $\tilde{h}$ [related to a linear combination of c_1, c_2, and c_3 by (2.18) and (2.19)] which measures the strength of the $\omega\pi^3$ "contact" term has the central value:

$$|\tilde{h}| \approx 0.4 \ . \tag{8.2}$$

The sign of the central determination of $\tilde{h}$ is the same as that of $\tilde{g}_{VV\phi}$. $\tilde{h}$ has a fairly large *percentage* uncertainty but our treatment gives the empirical correlation

$$|\tilde{g}_{VV\phi} - \tilde{h}| \approx 1.5 \ . \tag{8.3}$$

The remaining coefficient κ [related to c_3/c_2 by (7.3)] is difficult to determine from mesonic processes. An initial hope was that variation of this parameter could solve the main characteristic difficulty of the soliton models—the too large nucleon mass (other predictions of the nucleon properties are quite reasonable). However, we found here that variation of the κ did not allow us to significantly lower the nucleon mass without disturbing the good predictions of the soliton approach. The overall "best fit" is similar to that of the "complete" model.[7] From a consideration of the nucleon properties we found a central value:

$$\kappa \approx 1.0 \ . \tag{8.4}$$

However, since varying κ has a relatively small effect, (8.4) should be interpreted as an indication of the signs and order of magnitude of κ. Of course, one would like to have further experimental constraints to eventually pin down the parameter c_3.

Two additional problems can be straightforwardly treated in the present framework. We have argued that the η does not contribute to the classical soliton, but it might give a contribution when one quantizes the spinning modes. The effect should be investigated, although from one's knowledge of nuclear physics we do not expect it to be sizable. Second, one should also investigate the properties of the strange baryons in the present model. A detailed analysis of this feature might pave the way to an eventual understanding of the foundation of the subject.

Finally, we remark that a number of proposals are present in the literature for modification of the chiral model and its treatment with the implied intent of lowering the nucleon mass. Including the axial-vector mesons[29] is a natural suggestion, but then why not include all the other p-wave $\bar{q}q$ bound states in a similar energy range? A consistently truncated chiral Lagrangian with all the p-wave states appears extremely complicated. Soft-pion corrections[18] are another possibility, but it is not clear how they will affect all static properties. Of course, more sophisticated quantization schemes[30] may always be entertained. The explicit introduction of quarks[31] with a "Cheshire Cat"[32] or other philosophy is an interesting possibility but there are some very serious problems to be straightened out.[33]

ACKNOWLEDGMENTS

One of us (U.-G.M.) would like to thank Norbert Kaiser for checking parts of the algebra and stimulating discussions. This work was supported in part by funds provided by the U.S. Department of Energy (DOE) under Contract Nos. DE-AC02-76ER03069 and DE-FG-85ER40231.

APPENDIX

Here we review the SU(3) analysis[34] leading to the determination of the $\omega\phi$ mixing angle ϵ in (4.3).

The matrix of squared masses for the neutral nonstrange vector mesons is conventionally written (in a basis $\rho_{11}, \rho_{22}, \rho_{33}$) as

$$(M^2)_{ij} = A_i \delta_{ij} + b_i b_j \ . \tag{A1}$$

Here the A_i correspond to the OZI-rule-conserving piece and the $b_i b_j$ to a factorizable OZI-rule-violating piece. In the isospin limit

$$A_1 = A_2, \quad b_1 = b_2 \ . \tag{A2}$$

It is convenient to go to the ρ^0, $\omega \equiv (\rho_{11}+\rho_{22})/\sqrt{2}$, $\phi \equiv \rho_{33}$ basis by the transformation $M^{2'} = Q^T M^2 Q$ with

$$Q = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & \sqrt{2} \end{bmatrix} , \tag{A3}$$

which yields the symmetric matrix

$$M'^2 \approx \frac{1}{2} \begin{bmatrix} A_1 + A_2 & A_1 - A_2 + 2\bar{b}(b_1 - b_2) & \sqrt{2} b_3 (b_1 - b_2) \\ & A_1 + A_2 + 4\bar{b}^2 & 2\sqrt{2}\,\bar{b} b_3 \\ \text{etc.} & & 2A_3 + 2b_3^2 \end{bmatrix} , \tag{A4}$$

where $\bar{b} = (b_1 + b_2)/2$. We shall consider the isospin limit (A2). From (A4) we read off that

$$|\bar{b}| = \tfrac{1}{2}(m_\omega^2 - m_\rho^2)^{1/2} = (71 \pm 16) \text{ MeV} , \tag{A5}$$

where the error primarily results from the uncertainty in the ρ^0 mass. The unknown quantity A_3 may be related to the K^* mass as $A_3 = m_{K^*}^2 - m_\rho^2/2$. [This follows from an OZI-rule-conserving mass Lagrangian of the form $-\mathcal{L} = A_1 \mathrm{Tr}(\rho^2) + (A_3 - A_1)\mathrm{Tr}(\rho\rho S)$, with $S = \mathrm{diag}(0,0,1)$.] Knowing A_3 we read off from (A4) that

$$|b_3| = \frac{1}{\sqrt{2}}(m_\phi^2 + m_\rho^2 - 2m_{K^*}^2)^{1/2} = (116 \pm 37) \text{ MeV} . \tag{A6}$$

Note that we have assigned a rather sizable uncertainty to $|b_3|$ owing to the uncertainty in the ρ mass as well as to the choice of either the K^{0*} or K^{+*} masses. Finally, from (A4) we get the ω-ϕ mixing angle as defined in (4.2):

$$\epsilon = \frac{2\sqrt{2}\,\bar{b} b_3}{m_\phi^2 - m_\omega^2} . \tag{A7}$$

This leads, using (A5) and (A6) to the result (4.3). An independent theoretical check of the quantity $2\sqrt{2}\,\bar{b} b_3$ (the coefficient of $-\omega\phi$ in the Lagrangian) may be made by comparing it to an effective determinantal term which gives OZI-rule violation for the pseudoscalars. That procedure [see Eq. (5.8) of Ref. 35] yields $2\sqrt{2}\bar{b} b_3 \approx (122 \text{ MeV})^2$ in rough agreement with $(153 \pm 40 \text{ MeV})^2$ obtained by use of (A5) and (A6).

*Present address: Department of Physics, Iowa State University, Ames, Iowa 50011.

[1]For reviews, see I. Zahed and G. E. Brown, Phys. Rep. **142**, 1 (1986); U.-G. Meissner and I. Zahed, Adv. Nucl. Phys. **17**, 143 (1986); G. Holzwarth and B. Schwesinger, Rep. Prog. Phys. **49**, 825 (1986); G. S. Adkins, in *Chiral Solitons*, edited by K. F. Liu (World Scientific, Singapore, 1987).

[2]N. I. Karchev and A. A. Slavnov, Teor. Mat. Fiz. (USSR) **65**, 192 (1985); R. Ball, in *Skyrmions and Anomalies*, edited by M. Jezabek and M. Praszalowicz (World Scientific, Singapore, 1987); A. A. Andrianov *et al.*, Phys. Lett. B **186**, 401 (1987).

[3]U.-G. Meissner, Phys. Rep. (to be published).

[4]Ö. Kaymakcalan and J. Schechter, Phys. Rev. D **31**, 1109 (1985).

[5]T. Fujiwara, T. Kugo, H. Terao, S. Uehara, and K. Yamawaki, Prog. Theor. Phys. **73**, 926 (1985).

[6]J. Schechter, Phys. Rev. D **34**, 868 (1986).

[7]U.-G. Meissner, N. Kaiser, and W. Weise, Nucl. Phys. **A466**, 685 (1987).

[8]Ö. Kaymakcalan, S. Rajeev, and J. Schechter, Phys. Rev. D **30**, 594 (1984).

[9]J. Schechter, Phys. Rev. D **21**, 3393 (1980); H. Gomm, P. Jain, R. Johnson, and J. Schechter, *ibid.* **33**, 3476 (1986); P. Jain, R. Johnson, and J. Schechter, *ibid.* **34**, 2230 (1987); U.-G. Meissner and N. Kaiser, *ibid.* **35**, 2859 (1987); U.-G. Meissner, R. Johnson, N. W. Park, and J. Schechter, *ibid.* **37**, 1285 (1988).

[10]J. J. Sakurai, *Currents and Mesons* (University of Chicago Press, Chicago, IL, 1969); J. J. Sakurai, Ann. Phys. (N.Y.) **11**, 1 (1960).

[11]G. S. Adkins and C. R. Nappi, Phys. Lett. **137B**, 251 (1984).

[12]U.-G. Meissner, N. Kaiser, A. Wirzba, and W. Weise, Phys. Rev. Lett. **57**, 1676 (1986).

[13]E. Witten, Nucl. Phys. **B223**, 442 (1983); **B223**, 433 (1983).

[14]D. G. Sutherland, Nucl. Phys. **B2**, 433 (1967); M. Veltman, Proc. R. Soc. London **A301**, 107 (1967); S. L. Adler, Phys. Rev. **177**, 2476 (1969); J. Bell and R. Jackiw, Nuovo Cimento **60A**, 47 (1969).

[15]For Eqs. (4.1a) and (4.1b), see Particle Data Group, M. Aguilar-Benitez *et al.*, Phys. Lett. **170B**, 1 (1986).

[16]For Eq. (4.1c), see, S. Jullian, in *Proceedings of the XVIII International Conference on High Energy Physics*, Tbilisi, 1976, edited by N. N. Bogolubov *et al.* (Joint Institute for Nuclear Research, Dubna, USSR, 1977), Vol. II, p. 319.

[17]A recent discussion of this possibility is given by C. A. Dominguez, Mod. Phys. Lett. **A2**, 983 (1987).

[18]For some ways of introducing scalars, see, e.g., M. Lacombe, B. Loiseau, R. Vinh Mau, and W. Cottingham, Phys. Rev. Lett. **57**, 170 (1986); I. Zahed, A. Wirzba, and U.-G. Meissner, Phys. Rev. D **33**, 830 (1986); M. Marshall, T. N. Pham, and T. Truong, Phys. Rev. Lett. **56**, 436 (1986).

[19]For a review, see R. Machleidt, K. Holinde, and Ch. Elster, Phys. Rep. **149**, 1 (1987).

[20]J. Rafelski, Phys. Rev. D **16**, 1980 (1977); W. Broniowski and M. K. Banerjee, Phys. Lett. **158B**, 335 (1985) (we thank Nor-

bert Kaiser for reminding us of this).

[21]M. Gell-Mann, D. Sharp, and W. Wagner, Phys. Rev. Lett. **8**, 261 (1962).

[22]M. Chemtob, Nucl. Phys. **A466**, 509 (1987).

[23]U.-G. Meissner and N. Kaiser, Z. Phys. A **325**, 267 (1986); Phys. Lett. B **180**, 129 (1986); Phys. Rev. D **36**, 203 (1987).

[24]J. Kunz, D. Masak, and T. Reitz, Phys. Lett. B **195**, 459 (1987).

[25]N. Kaiser, University of Regensburg Report No. TPR-87-1, 1987 (unpublished).

[26]A. Wirzba and W. Weise, Phys. Lett. B **188**, 6 (1987).

[27]U.-G. Meissner and W. Weise, in *Proceedings of the Workshop on Low-Energy Effective Theory of QCD,* edited by S. Saito and K. Yamawaki (Nagoya University Press, Japan, 1987).

[28]A recent attempt to obtain the c_1 and c_2 coefficients from such an approach is discussed by M. Chemtob, Saclay Report No. SPhT/87/117, 1987 (unpublished).

[29]Some attempts have been made by, e.g., U.-G. Meissner and I. Zahed, Z. Phys. A **327**, 5 (1987); M. Chemtob, in Ref. 22; M. Lacombe, B. Loiseau, R. Vinh Mau, and W. Cottingham, Orsay Report No. IPNO/TH 87-28, 1987 (unpublished); J. Baacke, D. Pottinger, and M. Golterman, Phys. Lett. B **185**, 421 (1987). The calculation of Baake, Pottinger, and Golterman is based on a model of Golterman and Hari Dass [M. Golterman and N. Hari Dass, Nucl. Phys. **B277**, 739 (1986)] which also discusses some aspects of eliminating axial-vector mesons and anomalous terms.

[30]For a recent attempt, see S. H. Lee and I. Zahed, Phys. Rev. D **37**, 1963 (1988).

[31]Some references are G. Ripka, S. Kahana, and V. Soni, Nucl. Phys. **A415**, 351 (1984); W. Broniowski and M. K. Banerjee, Phys. Rev. D **34**, 3472 (1986); D. Klabucar and G. E. Brown, Nucl. Phys. **A454**, 589 (1986); S. Yoro and T. Tatsumi, Kyoto report, 1987 (unpublished).

[32]A recent review is given by H. B. Nielsen and A. Wirzba, La Structure de la Matèrie, proceedings of the workshop, Les Houches, France, 1987 (Springer, Berlin, to be published).

[33]These are addressed in, e.g., I. Zahed, A. Wirzba, and U.-G. Meissner, Ann. Phys. (N.Y.) **165**, 408 (1985), and references therein.

[34]A recent reference is M. Scadron, Phys. Rev. D **29**, 2076 (1984).

[35]J. Kandaswamy, J. Schechter, and M. Singer, Phys. Rev. D **17**, 1430 (1978).