PHYSICAL REVIEW D VOLUME 37, NUMBER 11 1 JUNE 1988

Quarkonium annihilation rates

Waikwok Kwong
Enrico Fermi Institute and Department of Physics, University of Chicago, Chicago, Illinois 60637

Paul B. Mackenzie
Fermilab, P.O. Box 500, Batavia, Illinois 60510

Rogerio Rosenfeld and Jonathan L. Rosner
Enrico Fermi Institute and Department of Physics, University of Chicago, Chicago, Illinois 60637
(Received 8 September 1987; revised manuscript received 11 January 1988)

Recent measurements of ratios of quarkonium annihilation rates are used to evaluate the strong fine-structure constant α_s. Expressions are presented for QCD radiative corrections with α_s referred to the quark-mass scale. We find $\alpha_s(m_b)=0.179^{+0.009}_{-0.008}$ from the ratio $\Gamma(\Upsilon\to\gamma gg)/\Gamma(\Upsilon\to ggg)$. The corresponding range of $\Lambda_{\overline{\rm MS}}^{(4)}$ (the QCD scale factor for four light-quark flavors) is 146–210 MeV, where $\overline{\rm MS}$ denotes the modified-minimal-subtraction scheme. The experimentally more precise but theoretically more questionable ratio of the gluonic and muonic widths of J/ψ and Υ yields $\alpha_s(m_c)=0.29\pm0.02$, $\alpha_s(m_b)=0.189\pm0.008$ when v^2/c^2 corrections to these ratios for J/ψ and Υ are parametrized linearly. Further predictions are made for ratios of rates.

The annihilation of a heavy-quark–antiquark pair ("quarkonium") into final states consisting of leptons, photons, and light quarks can provide useful information on the strong fine-structure constant $\alpha_s(\mu)$ (Refs. 1 and 2). Here μ is a renormalization scale, for which various prescriptions have appeared in the literature.[3,4] Annihilation rates typically depend on $\alpha_s(\mu)$ to some power p, times a correction factor:

$$\Gamma(Q\bar{Q}\to(\text{final state}))=A[\alpha_s(\mu)]^p[1+B(\mu)\alpha_s(\mu)+O(\alpha_s^2)]\ . \qquad (1)$$

Unless $p=0$, the coefficient B depends both on the scale μ and on the exact definition of the coupling constant (the "renormalization scheme"). This double ambiguity means that a scale choice which is reasonable in one scheme is unreasonable in another, and has led to some confusion about whether the power-series expansion is well behaved in most processes. In Ref. 4, a technique was introduced for probing physical momentum scales in QCD processes so as to allow an intelligent and informed guess for the renormalization scale in a scheme-independent way. This analysis concluded that with the important exception of the ratio of the gluonic and muonic widths of the Υ, the perturbation series for most QCD processes is quite well behaved and can be used for phenomenology.

The method of Ref. 4 is uniquely specified in QED. In QCD the prescription of Ref. 4 for choosing the scale μ is such that the coefficient of n_f, the number of light-quark flavors, is made to vanish in the constant $B(\mu)$ of Eq. (1). All the dependence on n_f is then absorbed into the term $[\alpha_s(\mu)]^p$. Thus, the choice of scale is form-invariant under a change in the number of flavors. Light fermion loops act as a probe of the scale of the virtual momentum which appears in the argument of α_s. The corresponding scale, defined as $\mu=Q^*$, then appears physical and reasonable for S-wave quarkonium decays. In particular, one can argue that Q^* for such decays corresponds roughly to an expected scale of virtual momentum associated with two- or three-gluon emission. (We have quoted the scales Q^* relative to $\Lambda_{\overline{\rm MS}}$, where $\overline{\rm MS}$ denotes the modified-minimal-subtraction scheme. The physical momentum scales are roughly twice as large, since $\Lambda_{\rm MOM}$ is roughly twice $\Lambda_{\overline{\rm MS}}$.)

This happy situation does not appear to persist for P-wave decays.[5] Radiative corrections to hadronic decay rates of $J^{PC}=0^{++}$ and 2^{++} quarkonium levels contain terms associated with final states consisting of a light-quark pair and a gluon which become infrared singular when the gluon's four-momentum approaches zero. If the n_f dependence associated with such terms is absorbed into the definition of the scale μ, curiously high values of Q^* are obtained. We will return to the interpretation of these large values of Q^* at the end of this paper. For the purpose of comparing expressions of the form (1) for S- and P-wave decays, however, we wished instead to examine the results of making the simpler (but *ad hoc*) choice $\mu=m_Q$. It should be stressed that this choice is not the one dictated by experience with QED (Ref. 4), and that $\mu=m_Q$ in the $\overline{\rm MS}$ scheme is not the same as $\mu=m_Q$ in some other scheme. Moreover, the presence of the infrared logarithms in corrections to P-wave decays points to a role of nonperturbative effects in such corrections, which probably merits further study.

In this paper we provide a concise summary of QCD corrections to decay rates based on the choice $\mu=m_Q$ in the $\overline{\rm MS}$ scheme.[6] At the same time we make use of the most recent measurements of Υ annihilation rates to evaluate $\alpha_s(m_b)$ precisely. We find

$$\alpha_s(m_b)=0.179^{+0.009}_{-0.008} \tag{2}$$

corresponding to a QCD scale factor of

$$\Lambda^{(4)}_{\overline{\rm MS}}=175^{+35}_{-29}\ \mathrm{MeV} \tag{3}$$

from the ratio of partial widths into two gluons and a photon and into three gluons. The superscript on $\Lambda_{\overline{\rm MS}}$ denotes the number of light-quark flavors. The result (3) is completely supported by the experimentally more precise but theoretically more questionable ratio of the gluonic and muonic widths, which yields $\alpha_s(m_b)=0.173\pm0.005$. An attempt to describe the ratios of gluonic and muonic widths of J/ψ and Υ simultaneously is made by parametrizing v^2/c^2 corrections to these ratios in a linear fashion. The results are

$$\alpha_s(m_c)=0.29\pm0.02\ ,$$
$$\alpha_s(m_b)=0.189\pm0.008\ , \tag{4}$$

$$\Lambda^{(4)}_{\overline{\rm MS}}=216\pm31\ \mathrm{MeV} \tag{5}$$

in accord with the ranges in Eqs. (2) and (3). Averaging the determinations (3) and (5), we find

$$\Lambda^{(4)}_{\overline{\rm MS}}=196\pm22\ \mathrm{MeV}\ , \tag{6}$$

$$\alpha_s(m_c)=0.276\pm0.014\ , \tag{7}$$

$$\alpha_s(m_b)=0.184\pm0.006\ . \tag{8}$$

We shall compare the result (7) with crude determinations of $\alpha_s(m_c)$ from two-gluon to two-photon partial-width ratios of η_c and χ_2, and predict more precise values for these ratios.

At the end of this paper we shall discuss aspects of the choice $\mu=Q^*$ in more detail. It should be stressed that with this choice, many of the values of $B(Q^*)$ turn out to be quite large. Thus, if such a choice is valid, far fewer quarkonium annihilation processes are amenable to a satisfactory description within the framework of perturbative QCD.

We relate $\alpha_s(m_Q)$ to α_s at some other mass scale μ using the expression

$$\alpha_s^{-1}(m_Q)=\alpha_s^{-1}(\mu)+\frac{\beta_0}{4\pi}\ln(m_Q^2/\mu^2)\ , \tag{9}$$

where

$$\beta_0=11-\tfrac{2}{3}n_f \tag{10}$$

and n_f is the number of light flavors with mass less than m_Q: $n_f=3$ for $m_Q=m_c$, $n_f=4$ for $m_Q=m_b$. Equating

$$[\alpha_s(\mu)]^p[1+B(\mu)\alpha_s(\mu)]=[\alpha_s(m_Q)]^p \times[1+B(m_Q)\alpha_s(m_Q)] \tag{11}$$

to leading order in α_s, we find

$$B(m_Q)=B(\mu)+\frac{p\beta_0}{2\pi}\ln\left[\frac{m_Q}{\mu}\right]. \tag{12}$$

Expressions for QCD radiative corrections are taken from Refs. 4 and 6 for three-gluon and two-gluon plus photon decays of $^3S_1$ states, and from Ref. 5 (see also the last of Ref. 2) for two-gluon decays of $^1S_0$, $^3P_0$, and $^3P_2$ levels. These expressions are summarized in Table I.

In Table I, B is of course independent of μ for purely electromagnetic decays. The number of light flavors is to be taken as $n_f=3$ for charmonium states and 4 for $b\bar{b}$ ones. The logarithmic correction factors $\ln(m_Q R_c)$, where R_c are confinement radii, are taken as approximate expressions for $\ln(4m_Q^2/|M^2-4m_Q^2|)$. The prescription of renormalization at the scale $\mu=m_Q$ leads to some differences in terms $\frac{2}{3}\ln 2$ for two-gluon processes or $\ln 2$ for three-gluon processes from coefficients of α_s/π cited in the literature.

We now calculate the expression quoted in Table I using the quarkonium parameters shown in Table II. We use $m_c=1.5\ \mathrm{GeV}/c^2$ and $m_b=4.9\ \mathrm{GeV}/c^2$ here and in what follows. The results (including familiar expressions for total rates) are summarized in Table III. Ambiguities in the definition of the logarithmic terms in corrections to $\Gamma(^3P_{0,2}\to\text{glue})$ are such that the coefficient of α_s/π should not be regarded as known to better than about ±1.

TABLE I. Numerical or analytic expressions for first-order corrections to decay rates, of the form $\Gamma/\Gamma^{(0)}=1+B(\mu)\alpha_s/\pi$.

Process	$B(\mu)$
$^1S_0\to\gamma\gamma$	$\pi^2/3-\frac{20}{3}=-3.38$
$^1S_0\to$glue	$\beta_0\ln(\mu/m_Q)+\frac{159}{6}-31\pi^2/24-11\ln2+n_f(-\frac{8}{9}+\frac{2}{3}\ln2)$
$^3S_1\to e^+e^-$	$-\frac{16}{3}=-5.33$
$^3S_1\to\gamma\gamma\gamma$	-12.61 ± 0.03
$^3S_1\to$glue	$(3\beta_0/2)\ln(\mu/m_Q)-0.26-1.16n_f$
$^3S_1\to\gamma+$glue	$\beta_0\ln(\mu/m_Q)-4.37-0.77n_f$
$^3P_0\to\gamma\gamma$	$\pi^2/3-\frac{28}{9}=0.18$
$^3P_0\to$glue	$\beta_0\ln(\mu/m_Q)+\frac{370}{27}+5\pi^2/16-11\ln2+n_f[-\frac{16}{27}+\frac{2}{3}\ln2+\frac{4}{27}\ln(m_QR_c)]$
$^3P_2\to\gamma\gamma$	$-\frac{16}{3}=-5.33$
$^3P_2\to$glue	$\beta_0\ln(\mu/m_Q)+\frac{1855}{72}-337\pi^2/128-6\ln2+n_f[-\frac{11}{18}+\frac{2}{3}\ln2+\frac{5}{9}\ln(m_QR_c)]$

TABLE II. Quarkonium parameters affecting first-order corrections to P-wave annihilation rates.

State	R_c (GeV^{-1})
$c\bar{c}(1P)$	3.17
$b\bar{b}(1P)$	1.86
$b\bar{b}(2P)$	3.05

We next summarize relevant partial decay widths and branching ratios of quarkonium, and give the values of α_s extracted from various ratios. The results are shown in Table IV. We quote a few details of the determinations. Unless otherwise noted, experimental values are taken from Ref. 7.

(1) η_c *decays*. The total width of η_c can be assumed to be dominated by two-gluon decay:

$$\Gamma(\eta_c \to gg) \approx \Gamma_{\rm tot}(\eta_c) = 11.5 \pm 4.3 \text{ MeV} . \quad (13)$$

An average of several experiments involving e^+e^- (Ref. 8) and $\bar{p}p$ collisions[9] gives[8]

$$\Gamma(\eta_c \to \gamma\gamma) = 9 \pm 4 \text{ keV} . \quad (14)$$

The predicted ratio of these two quantities is

$$\frac{\Gamma(\eta_c \to gg)}{\Gamma(\eta_c \to \gamma\gamma)} = \frac{9[\alpha_s(m_c)]^2}{8\alpha^2}\left[1 + 8.2\frac{\alpha_s}{\pi}\right] . \quad (15)$$

Here and subsequently we omit the argument (μ) of α_s in the correction term, since a change in μ only affects higher-order corrections.

The $\eta_c \to \gamma\gamma$ width can be expressed in terms of that for $J/\psi \to \mu^+\mu^-$ if $|\Psi(0)|^2$ is the same for the two states. The magnetic transition $J/\Psi \to \gamma\eta_c$ is substantially weaker than one estimates nonrelativistically, however. This suggests that the J/ψ and η_c wave functions may not be identical, with their overlap reduced by hyperfine and coupled-channel effects. Ignoring such effects, we would predict

$$\frac{\Gamma(\eta_c \to \gamma\gamma)}{\Gamma(J/\psi \to \mu^+\mu^-)} = \frac{4}{3}\left[1 + 1.96\frac{\alpha_s}{\pi}\right] \quad (16)$$

or $\Gamma(\eta_c \to \gamma\gamma) \approx 7$ keV for $\Gamma(J/\psi \to \mu^+\mu^-) = 4.7 \pm 0.3$ keV (Ref. 8). This value is compatible with the present experimental range (14).

(2) J/ψ *decays*. The total decay width $\Gamma_{\rm tot}(J/\psi)$ is composed of e^+e^-, $\mu^+\mu^-$, $\gamma^* \to q\bar{q}$, $\gamma\eta_c$, ggg, and γgg contributions. From Ref. 7 we find

TABLE III. Lowest-order expressions and first-order QCD corrections with α_s computed at the mass scale of the constituent quark ($m_c = 1.5$ GeV, $m_b = 4.9$ GeV) for decay processes of $c\bar{c}$ and $b\bar{b}$ quarkonium states. Here we assume three colors of quarks. Note that corrections to ratios of ^3P_J decay widths are known more precisely than individual values.

Process	Rate	Correction factor
$^1S_0 \to \gamma\gamma$	$12\pi e_Q^4 \alpha^2 \lvert\Psi(0)\rvert^2/m_Q^2$	$1 - 3.4\alpha_s/\pi$
$^1S_0 \to$ glue	$8\pi\alpha_s^2 \lvert\Psi(0)\rvert^2/3m_Q^2$	$1 + 4.8\alpha_s/\pi$ (η_c)
		$1 + 4.4\alpha_s/\pi$ (η_b)
$^3S_1 \to e^+e^-$	$16\pi\alpha^2 e_Q^2 \lvert\Psi(0)\rvert^2/M^2$	$1 - 16\alpha_s/3\pi$
$^3S_1 \to \gamma\gamma\gamma$	$16(\pi^2 - 9)\alpha^3 e_Q^6 \lvert\Psi(0)\rvert^2/3m_Q^2$	$1 - 12.6\alpha_s/\pi$
$^3S_1 \to$ glue	$40(\pi^2 - 9)\alpha_s^3 \lvert\Psi(0)\rvert^2/81m_Q^2$	$1 - 3.7\alpha_s/\pi$ (J/ψ)
		$1 - 4.9\alpha_s/\pi$ (Υ)
$^3S_1 \to \gamma +$ glue	$32(\pi^2 - 9)e_Q^2 \alpha\alpha_s^2 \lvert\Psi(0)\rvert^2/9m_Q^2$	$1 - 6.7\alpha_s/\pi$ (J/ψ)
		$1 - 7.4\alpha_s/\pi$ (Υ)
$^1P_1 \to$ glue[a]	$(20/9\pi)\alpha_s^3 \lvert R'_{nP}(0)\rvert^2 \ln(m_Q\langle R_c\rangle)/m_Q^4$	Not known
$^3P_0 \to \gamma\gamma$	$27 e_Q^4 \alpha^2 \lvert R'_{nP}(0)\rvert^2/m_Q^4$	$1 + 0.2\alpha_s/\pi$
$^3P_0 \to$ glue	$6\alpha_s^2 \lvert R'_{nP}(0)\rvert^2/m_Q^4$	$1 + 9.5\alpha_s/\pi$ (χ)
		$1 + 10.0\alpha_s/\pi$ (χ_b)
		$1 + 10.2\alpha_s/\pi$ (χ'_b)
$^3P_1 \to q\bar{q} +$ glue[a]	$(8/9\pi)n_f\alpha_s^3 \lvert R'_{nP}(0)\rvert^2/m_Q^4 \ln(m_Q\langle R_c\rangle)$	Not known
$^3P_2 \to \gamma\gamma$	$36 e_Q^4 \alpha^2 \lvert R'_{nP}(0)\rvert^2/5m_Q^4$	$1 - 16\alpha_s/3\pi$
$^3P_2 \to$ glue	$8\alpha_s^2 \lvert R'_{nP}(0)\rvert^2/5m_Q^4$	$1 - 2.2\alpha_s/\pi$ (χ_c)
		$1 - 0.1\alpha_s/\pi$ (χ_b)
		$1 + 1.0\alpha_s/\pi$ (χ'_b)

[a] $\langle R_c\rangle$ is the average radius of the $^1P_1$ or $^3P_1$ state.

TABLE IV. Information on α_s obtained from various ratios of quarkonium annihilation rates.

Ratio	Expression (Eq. No.)	Value[a]	Parameter	Value
$\frac{\Gamma(\eta_c\to gg)}{\Gamma(\eta_c\to\gamma\gamma)}$	(15)	$(1.28\pm0.74)\times10^3$ $[(2.8\pm0.4)\times10^3]$	$\alpha_s(m_c)$	$0.20^{+0.04}_{-0.06}$
$\frac{\Gamma(J/\psi\to ggg)}{\Gamma(J/\psi\to\mu\mu)}$	(22)	9.0 ± 1.3	$\alpha_s(m_c)$	0.175 ± 0.008
$\frac{\Gamma(J/\psi\to\gamma gg)}{\Gamma(J/\psi\to ggg)}$	(23)	0.10 ± 0.04 $[0.063\pm0.005]$	$\alpha_s(m_c)$	$0.19^{+0.10}_{-0.05}$
$\frac{\Gamma(\chi_{c2}\to gg)}{\Gamma(\chi_{c2}\to\gamma\gamma)}$	(26)	$(0.91\pm0.62)\times10^3$ $[(2.1\pm0.3)\times10^3]$	$\alpha_s(m_c)$	$0.19^{+0.05}_{-0.08}$
$\frac{\Gamma(\Upsilon\to ggg)}{\Gamma(\Upsilon\to\mu\mu)}$	(33)	$28.4^{+2.7}_{-2.4}$	$\alpha_s(m_b)$	0.173 ± 0.005
$\frac{\Gamma(\Upsilon\to\gamma gg)}{\Gamma(\Upsilon\to ggg)}$	(37)	$(2.79\pm0.15)\%$ $[(2.70\pm0.09)\%]$	$\alpha_s(m_b)$	$0.179^{+0.009}_{-0.008}$

[a]Quantities in brackets denote predictions based on Eqs. (7) and (8).

$$\Gamma(ee)=\Gamma(\mu\mu)=(6.9\pm0.9)\%\ \Gamma_{\rm tot}\ , \tag{17a}$$

$$\Gamma(\gamma^*\to q\bar{q})=(2.4\pm0.2)\Gamma(\mu\mu) \tag{17b}$$

(the latter value is estimated from e^+e^- cross section measurements around the J/ψ mass); and

$$\Gamma(\gamma\eta_c)=(1.27\pm0.36)\%\ \Gamma_{\rm tot}\ . \tag{18}$$

Then

$$\Gamma(\gamma gg)+\Gamma(ggg)=(68.4\pm4.2)\%\ \Gamma_{\rm tot}\ . \tag{19}$$

Now we use the measured value[10]

$$\Gamma(\gamma gg)=(10\pm4)\%\ \Gamma(ggg) \tag{20}$$

to conclude

$$\Gamma(ggg)=(62.2\pm4.4)\%\ \Gamma_{\rm tot}\ . \tag{21}$$

Combining this with Eq. (17a), we find the ratio shown in Table IV. The theoretical expectation is

$$\frac{\Gamma(J/\psi\to ggg)}{\Gamma(J/\psi\to\mu\mu)}=\frac{5}{18}\left[\frac{M}{2m_c}\right]^2\frac{(\pi^2-9)[\alpha_s(m_c)]^3}{\pi\alpha^2}\times\left[1+1.6\frac{\alpha_s}{\pi}\right]\ . \tag{22}$$

The rather tightly constrained value of $\alpha_s(m_c)$ noted in Table IV is probably in fact a crude estimate, since v^2/c^2 corrections have been neglected in Eq. (22).

The $\gamma gg/ggg$ ratio in Eq. (20) is expected to be

$$\frac{\Gamma(J/\psi\to\gamma gg)}{\Gamma(J/\psi\to ggg)}=\frac{16}{5}\frac{\alpha}{\alpha_s(m_c)}\left[1-2.9\frac{\alpha_s}{\pi}\right]\ . \tag{23}$$

(3) χ *decays*. The most precise measurements of the total and $\gamma\gamma$ widths of χ_2 (the $J^{PC}=2^{++}$ $c\bar{c}$ state at 3556 MeV) come from a CERN ISR experiment. They yield[9]

$$\Gamma(\chi_2\to gg)=2.6^{+1.4}_{-1.0}\ \text{MeV}\ , \tag{24}$$

$$\Gamma(\chi_2\to\gamma\gamma)=2.9^{+1.3}_{-1.0}\pm1.7\ \text{keV}\ . \tag{25}$$

The Crystal Ball Collaboration[11] obtains 2.8 ± 2.0 keV for this last value. Averaging the two, we obtain $\Gamma(\chi_2\to\gamma\gamma)=2.85\pm1.43$ keV. The predicted ratio of the gg and $\gamma\gamma$ rates is

$$\frac{\Gamma(\chi_2\to gg)}{\Gamma(\chi_2\to\gamma\gamma)}=\frac{9[\alpha_s(m_c)]^2}{8\alpha^2}\left[1+3.2\frac{\alpha_s}{\pi}\right]\ . \tag{26}$$

We also present in Table IV predictions for experimental ratios based on the value of $\alpha_s(m_c)$ quoted in Eq. (7). Future measurements[12] of η_c and χ widths to gg and $\gamma\gamma$ will be able to check these predictions much more closely than in the past.

(4) Υ *decays*. We use the branching ratios

$$\Gamma(ee)=\Gamma(\mu\mu)=(2.8\pm0.2)\%\ \Gamma_{\rm tot} \tag{27}$$

and calculate[13]

$$\Gamma(\tau\tau)=(2.76\pm0.2)\%\ \Gamma_{\rm tot}\ , \tag{28}$$

$$\Gamma(\gamma^*\to q\bar{q})=(10.1\pm0.9)\%\ \Gamma_{\rm tot}\ . \tag{29}$$

This implies

$$\Gamma(\gamma gg)+\Gamma(ggg)=(81.6\pm1.4)\%\ \Gamma_{\rm tot}\ . \tag{30}$$

There are three determinations of $\Gamma(\gamma gg)/\Gamma(ggg)$:

$$\frac{\Gamma(\Upsilon\to\gamma gg)}{\Gamma(\Upsilon\to ggg)}=\begin{cases}(3.00\pm0.13\pm0.18)\% & (\text{ARGUS}^{14})\\ (2.54\pm0.18\pm0.14)\% & (\text{CLEO}^{15})\\ (2.99\pm0.59)\% & (\text{CUSB}^{16})\ .\end{cases} \tag{31}$$

Here it appears important[14] to use the photon energy spectrum calculated by Field;[17] others[18] are not in accord with the data. Averaging these values, we find the $\gamma gg/ggg$ ratio quoted in Table IV, and so

$$\Gamma(ggg)=(79.4\pm1.4)\%\ \Gamma_{\rm tot} \tag{32}$$

leading to the $ggg/\mu\mu$ ratio cited in Table IV. We expect

$$\frac{\Gamma(\Upsilon\to ggg)}{\Gamma(\Upsilon\to\mu^+\mu^-)}=\frac{10}{9}\left[\frac{M}{2m_b}\right]^2\frac{(\pi^2-9)[\alpha_s(m_b)]^3}{\pi\alpha^2}\times\left[1+0.43\frac{\alpha_s}{\pi}\right]. \tag{33}$$

The implied experimental value of $\alpha_s(m_b)$ in Table IV is in accord with the estimates of Ref. 19:

$$\Upsilon:\ \alpha_s(0.48M_\Upsilon)=0.172^{+0.008}_{-0.007}\ , \tag{34}$$

$$\Upsilon':\ \alpha_s(0.48M_{\Upsilon'})=0.177^{+0.015}_{-0.012}\ , \tag{35}$$

$$\Upsilon'':\ \alpha_s(0.48M_{\Upsilon''})=0.170^{+0.015}_{-0.012}\ . \tag{36}$$

The theoretical expression for the $\gamma gg/ggg$ ratio is

$$\frac{\Gamma(\Upsilon\to\gamma gg)}{\Gamma(\Upsilon\to ggg)}=\frac{4}{5}\frac{\alpha}{\alpha_s(m_b)}\left[1-2.6\frac{\alpha_s}{\pi}\right]. \tag{37}$$

The experimental average for this ratio leads to a value of $\alpha_s(m_b)$ almost precisely equal to that obtained from Eq. (33).

The agreement of the two determinations of α_s is especially important since concerns have been raised about the convergence of the perturbation series for the ratio of Eq. (33), but not for the ratio of Eq. (37). The prediction for the ratio of Eq. (37) based on Eq. (8), shown in the last line of Table IV in square brackets, is also in accord with the present experimental value.

We now present details of the calculation that led to Eqs. (4) and (5). The relation between $\alpha_s(\mu)$ and $\Lambda_{\overline{\rm MS}}^{(n_f)}$, to two-loop accuracy, is

$$\alpha_s(\mu)=\frac{4\pi}{\beta_0\ln\frac{\mu^2}{\Lambda_{\overline{\rm MS}}^2}}\left[1-\frac{\beta_1\ln\ln\frac{\mu^2}{\Lambda_{\overline{\rm MS}}^2}}{\beta_0^2\ln\frac{\mu^2}{\Lambda_{\overline{\rm MS}}^2}}\right], \tag{38}$$

where

$$\beta_0=11-\tfrac{2}{3}n_f,\quad \beta_1=102-\tfrac{38}{3}n_f \tag{39}$$

and we use $n_f=4$ for $m_c\le\mu\le m_b$.

We parametrize v^2/c^2 corrections to Eqs. (22) and (33) by a factor $(1+Cv^2/c^2)$, with $v^2/c^2=0.24$ for charmonium, 0.073 for Υ. (See the first of Ref. 3.) The experimental $ggg/\mu\mu$ ratios are both reproduced to within one standard deviation over the range of parameters in Eqs. (4) and (5), with C ranging between ≈-2.9 and ≈-3.5. The large magnitude of the v^2/c^2 correction for charmonium means this exercise is at best a qualitative one, but it does lead to a value of $\Lambda_{\overline{\rm MS}}^{(4)}$ [Eq. (5)] consistent with that implied by the $\gamma gg/ggg$ ratio [Eq. (3)]. Other determinations[20] of $\Lambda_{\overline{\rm MS}}^{(4)}$ are consistent with values around 200 MeV, but with wide error limits.

To summarize, the choice $\mu=m_Q$ for defining the scale of strong interactions in quarkonium decays leads to a value of $\Lambda_{\overline{\rm MS}}^{(4)}=196\pm22$ MeV, when one averages determinations from $\gamma gg/ggg$ and $ggg/\mu\mu$ ratios of decay rates. This determination does not properly reflect the large systematic error associated with a choice of scale, however. We conclude this paper with a brief summary of the correction terms in Eq. (1) that arise when one chooses the scale $\mu=Q^*$ discussed in Ref. 4.

In Table V we show the values of Q^* (in the $\overline{\rm MS}$ scheme and in units of $2m_Q$) and $B(\mu)$ [in terms of $\ln(\mu/Q^*)$] appropriate to the S- and P-wave quarkonium decays discussed above. The values of Q^* are approximately $(2m_Q)/4$ and $(2m_Q)/6$ for $^1S_0$ and $^3S_1$ decays. As noted, since $\Lambda_{\rm MOM}$ is roughly twice as large as $\Lambda_{\overline{\rm MS}}$, we expect the physical momentum scales to be roughly twice Q^*, or $(2m_Q)/2$ and $(2m_Q)/3$ for $^1S_0$ and $^3S_1$ decays to two and three gluons, respectively. Thus, for these states the appropriate scale indeed seems to be the correct fraction of the mass of the decaying state. The large values of $B(Q^*)$ for $^3S_1$ decays could be interpreted pessimistically as indicating that perturbative QCD is

TABLE V. Quarkonium decay processes and corresponding values of Q^* (scale defined in Ref. 4) for setting α_s. Also shown are correction terms $B(\mu)$ in Eq. (1).

Process	$Q^*/2m_Q$	$B(\mu)$
$^1S_0\to$glue	0.264	$\beta_0\ln\frac{\mu}{Q^*}-0.92$
$^3S_1\to$glue	0.157	$\frac{3}{2}\beta_0\ln\frac{\mu}{Q^*}-19.35$
$^3S_1\to\gamma+$glue	0.157	$\beta_0\ln\frac{\mu}{Q^*}-17.07$
$^3P_0\to$glue		
$c\bar{c}(1P)$	0.58	$\beta_0\ln\frac{\mu}{Q^*}+10.8$
$b\bar{b}(1P)$	0.67	$\beta_0\ln\frac{\mu}{Q^*}+12.4$
$b\bar{b}(2P)$	0.75	$\beta_0\ln\frac{\mu}{Q^*}+13.6$
$^3P_2\to$glue		
$c\bar{c}(1P)$	1.47	$\beta_0\ln\frac{\mu}{Q^*}+7.5$
$b\bar{b}(1P)$	2.52	$\beta_0\ln\frac{\mu}{Q^*}+13.4$
$b\bar{b}(2P)$	3.81	$\beta_0\ln\frac{\mu}{Q^*}+18.0$

very risky for describing these decays. At the very least, the quantities for which perturbative QCD could be valid appear to be ratios such as $\gamma gg/ggg$, since the coefficients $B(Q^*)$ for $\Upsilon \to \gamma gg$ and $\Upsilon \to ggg$ are nearly equal.[4]

A measure of the systematic uncertainty attending the choice of scale is that on the basis of $\Gamma(\Upsilon \to \gamma gg)/\Gamma(\Upsilon \to ggg) = (3.00 \pm 0.13 \pm 0.18)\%$, the ARGUS Collaboration[14] quotes $\alpha_s(0.157 M_\Upsilon) = 0.225 \pm 0.011 \pm 0.019$, $\Lambda^{(4)}_{\overline{\rm MS}} = 115 \pm 17 \pm 28$ MeV, whereas using the ARGUS result we would find $\alpha_s(m_b) = 0.168^{+0.011}_{-0.010}$, $\Lambda^{(4)}_{\overline{\rm MS}} = 137^{+41}_{-31}$ MeV.

The values of Q^* in $^3P_0$ and particularly in $^3P_2$ decays do not appear to be related to any obvious physical scale. The terms proportional to $n_f \ln(m_q R_c)$ in Table I arise from infrared singularities mentioned at the outset of this article. Their magnitudes dictate the choice of Q^* in what appears to be an arbitrary manner. It is notable that no $\ln(m_Q R_c)$ terms appear in n_f-independent contributions to $B(\mu)$ in Table I. One would expect such terms to appear if n_f-dependent terms are an accurate probe of gluon virtuality. In fact, however, since the $\ln(m_Q R_c)$ terms are of infrared origin, perturbative QCD is on shaky grounds for describing both the corrections to $^3P_0$ and $^3P_2$ decays, and the *leading*-order contributions to $^3P_1$ decays. The same conclusion may be drawn from the large size of the coefficients $B(Q^*)$ for $^3P_0$ and $^3P_2$ decays to gluons. A resummation of higher-order contributions to such decays may be called for.

We wish to thank C. Quigg for helpful comments. This work was supported in part by the U.S. Department of Energy under Grants Nos. DE-AC02-82ER 40073 and DE-AC02-80ER 10587. One of us (R.R.) wishes to gratefully acknowledge financial support from Coordenacão de Aperfeicoamento de Pessoal de Ensino Superior (Brazil).

[1]T. Appelquist and H. D. Politzer, Phys. Rev. Lett. **34**, 43 (1975).

[2]For reviews see, e.g., V. A. Novikov *et al.*, Phys. Rep. **41C**, 1 (1978); T. Appelquist, R. M. Barnett, and K. Lane, Annu. Rev. Nucl. Sci. **28**, 387 (1978); D. W. Duke and R. G. Roberts, Phys. Rep. **120**, 277 (1985); S. Cooper, in *Proceedings of the 23rd International Conference on High Energy Physics*, Berkeley, 1986, edited by S. Loken (World Scientific, Singapore, 1987), p. 67; W. Kwong, J. L. Rosner, and C. Quigg, Annu. Rev. Nucl. Part. Sci. **37**, 325 (1987); W. Buchmüller and S. Cooper, Report No. MIT-LNS-159, 1987 (unpublished).

[3]P. B. Mackenzie and G. P. Lepage, Phys. Rev. Lett **47**, 1244 (1981); in *Perturbative Quantum Chromodynamics*, proceedings of the Conference, Tallahassee, 1981, edited by D. W. Duke and J. F. Owens (AIP Conf. Proc. No. 74) (AIP, New York, 1981), p. 176; G. P. Lepage, in *Proceedings of the 1983 International Symposium on Lepton and Photon Interactions at High Energies,* Ithaca, New York, 1983, edited by D. G. Cassel and D. L. Kreinick (Newman Laboratory of Nuclear Studies, Cornell University, Ithaca, NY, 1983), p. 565.

[4]S. J. Brodsky, G. P. Lepage, and P. Mackenzie, Phys. Rev. D **28**, 228 (1983).

[5]$^1S_0$ decays: R. Barbieri, G. Curci, E. d'Emilio, and E. Remiddi, Nucl. Phys. **B154**, 535 (1979); ^3P decays: R. Barbieri, M. Caffo, R. Gatto, and E. Remiddi, Phys. Lett. **95B**, 93 (1980); Nucl. Phys. **B192**, 61 (1981). For a review, see E. Remiddi, in *Theory of Fundamental Interactions,* proceedings of the International School of Physics "Enrico Fermi," Course LXXX, Varenna, Italy, 1980, edited by G. Costa and R. R. Gatto (North-Holland, Amsterdam, 1982), p. 18.

[6]W. A. Bardeen, A. J. Buras, D. W. Duke, and T. Muta, Phys. Rev. D **18**, 3998 (1978).

[7]Particle Data Group, M. Aguilar-Benitez *et al.*, Phys. Lett. **170B**, 1 (1986).

[8]S. Cooper, in *Proceedings of the 23rd International Conference on High Energy Physics* (Ref. 2).

[9]C. Baglin *et al.*, Phys. Lett. B **172**, 455 (1986); **187**, 191 (1987).

[10]D. L. Scharre *et al.*, Phys. Rev. D **23**, 43 (1981), as quoted in last of Ref. 3.

[11]Crystal Ball Collaboration, as quoted in the second of Ref. 9.

[12]E. Menichetti, in *Proceedings of the First Workshop on Antimatter Physics at Low Energy,* Batavia, 1986, edited by B. E. Bonner and L. S. Pinsky (Fermilab, Batavia, Illinois, 1986), p. 95.

[13]Kwong, Rosner, and Quigg (Ref. 2). In that work a crude estimate was made of $\Gamma(\gamma^* \to q\bar{q})$. A more accurate value comes from the measured value of $R = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$, which we estimate from the figure quoted in Ref. 7 to be 3.6 ± 0.2 for $E_{\rm c.m.} \approx M_\Upsilon$. We then calculate $\Gamma(\gamma^* \to q\bar{q}) = R\,\Gamma(ee)$.

[14]ARGUS Collaboration, H. Albrecht *et al.*, Phys. Lett. B **199**, 291 (1987).

[15]S. E. Csorna *et al.*, Phys. Rev. Lett. **56**, 1222 (1986).

[16]R. D. Schamberger *et al.*, Phys. Lett. **138B**, 225 (1984).

[17]R. D. Field, Phys. Lett. **133B**, 248 (1983).

[18]D. M. Photiadis, Phys. Lett. **164B**, 160 (1985); K. Koller and T. F. Walsh, Nucl. Phys. **B140**, 449 (1978).

[19]C. Yanagisawa *et al.*, in *Proceedings of the Second International Conference on Hadron Spectroscopy,* KEK, Tsukuba, Japan, 1987, edited by Y. Oyanagi, K. Takamatsu, and T. Tsuru (KEK, Tsukuba, 1987).

[20]See, e.g., Duke and Roberts (Ref. 2).