# Strong-interaction effects on the baryon semileptonic decay form factors

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One-gluon QCD corrections to transition amplitudes describing the baryon semileptonic decays $B' \to B + l + \nu$ have been investigated. All six form factors describing the hadronic matrix element of the weak current between states of the spin-parity $\frac{1}{2}^+$ baryon octet have been calculated in the framework of the MIT bag model, including one-gluon vertex corrections to order $\alpha$ . The results provide QCD corrections to the usual phenomenological Cabibbo analyses, including induced second-class form factors. Extensive numerical results are presented and compared with the data, with special attention given to the recent high-statistics results for $\Lambda$ and $\Sigma^-$ $\beta$ decay. The QCD vertex corrections are significant and in general improve the agreement with the data. An exception, however, is $g_1/f_1$ for $\Lambda$ $\beta$ decay, where the QCD vertex corrections do not reconcile the naive predictions with the experimental results. The implications of this and other results for weak-interaction phenomenology are briefly discussed.

## I. INTRODUCTION

The recent high-statistics measurement of polarized $\Sigma^-$ $\beta$ decay[1] and the $\Lambda$ $\beta$ decay[2] results from Fermilab, together with previous CERN (Ref. 3) and BNL (Ref. 4) data on the semileptonic decays of the baryon octet, confront the Cabibbo hypothesis[5] at a level of precision where corrections to the naive predictions become significant. Corrections due to baryon mass differences and other strong-interaction effects, such as induced second-class[6] form factors, have to be taken into account. In the standard $SU(3)\times SU(2)\times U(1)$ gauge theory of strong, weak, and electromagnetic interactions, the weak current $J_\lambda(q)$ certainly satisfies the Cabibbo hypothesis and its matrix elements, taken between confined states of quarks and gluons describing the baryons, give the hadronic part of the baryon semileptonic decay amplitudes. In this framework, some symmetry-breaking effects can be taken into account by allowing for different baryon and quark masses in the hadron wave functions.

The confinement problem has so far proven intractible, being a nonperturbative QCD effect, and in practice some QCD-inspired model must be used for the baryon wave functions. In the context of the MIT bag model,[7] we have previously evaluated the hadronic matrix elements of the quark weak current $J_\lambda(q)$ and have investigated the QCD effects induced by one-gluon corrections to the standard $SU(2)\times U(1)$ electroweak interaction vertex.[8] We considered only the particular process $\Sigma^- \to n + e^- + \bar{\nu}_e$ in view of the recent high-statistics experiment on polarized $\Sigma^-$ $\beta$ decay.[1] The corrections were found to be small, as expected, but sufficiently large to be interesting in view of the precision of the new data. Here we extend our calculations to the semileptonic decays of the entire $\frac{1}{2}^+$ octet of baryons. The QCD effects turn out to be significant, although not large. Several quite interesting features emerge when our results are compared with the data, as well as with some previous calculations. In particular, neither QCD vertex corrections nor mass differences reconcile the SU(6) predictions for $\Lambda$ $\beta$ decay with the data, indicating that color-magnetic effects involving spectator quarks are important. Also, the second-class induced electric dipole form factor $g_2$ is found to be small and not significant in comparison to the precision of the present data. The induced magnetic dipole form factor $f_2$ agrees well with the conserved-vector-current hypothesis, as well as with the data. For the strangeness-changing hyperon decays, flavor-symmetry-breaking effects in the vector form factor $f_1$ are somewhat larger than expected.

In the following we recall some definitions in Sec. II and present the calculations, including numerical results in Sec. III. A summary of our results, a comparison with the experimental data, and comments on related work are given in Sec. IV, along with a discussion of our main conclusions.

## II. PRELIMINARIES

The matrix element of the charged weak current between spin-$\frac{1}{2}^+$ baryon states, following standard conventions,[9] can be expressed in terms of six (real) Lorentz-scalar form factors:

 

$$\langle B_f \mid J_\lambda(q) \mid B_i \rangle = \bar{u}_f \left[ \gamma_\lambda f_1(q^2) - \frac{i\sigma_{\lambda\alpha} q^\alpha f_2(q^2)}{M_f+M_i} + \frac{q_\lambda f_3(q^2)}{M_f+M_i} - \left[ \gamma_\lambda g_1(q^2) - \frac{i\sigma_{\lambda\alpha} q^\alpha g_2(q^2)}{M_f+M_i} + \frac{q_\lambda g_3(q^2)}{M_f+M_i} \right] \gamma_5 \right] u_i \ . \tag{1}$$

Here $|B_i\rangle$, $|B_f\rangle$, $u_i$, $u_f$, $M_i$, and $M_f$ are initial and final baryon states, spinors, and masses, while $q \equiv p_i - p_f$, $p_i$ and $p_f$ being the initial and final four-momenta. The form factors $f_1$ and $g_1$ are the usual vector and axial-vector form factors, $f_2$ and $g_2$ are the weak-magnetic dipole and weak-electric dipole form factors, while $f_3$ and $g_3$ are the induced scalar and pseudoscalar form factors. If SU(3) flavor were exact, $G$ parity, suitably generalized, would require the second-class form factors $f_3$ and $g_2$ to vanish. However, in reality we expect these terms to be present, along with deviations from the exact flavor-symmetry predictions for the other form factors, to the extent that flavor SU(3) is broken.

Since the (timelike) momentum transfer is typically small compared to the masses of the baryons participating in the decay $[0 < q^2 < (M_i - M_f)^2 \ll M_i^2, M_f^2]$, it is convenient to anticipate the nonrelativistic limit and to calculate in the Lorentz frame where $\mathbf{p}_i = -\mathbf{p}_f = \mathbf{q}/2$. It is then straightforward to rewrite Eq. (1) in terms of the rest-frame two-component spinors $\chi_i$ and $\chi_f$ of the initial and final baryons, and rotational covariants formed from $\boldsymbol{\sigma}$ and $\mathbf{q}$, with coefficients that are scalar functions of $\mathbf{q}^2$: namely, $v_0$, $v_V$, $v_M$, $a_0$, $a_S$, and $a_T$. Of these form factors $v_0$, $v_M$, $a_S$, and $a_T$ are first class, while $v_V$ and $a_0$ are second class. We define these functions in terms of $V_\lambda(\mathbf{q})$ and $A_\lambda(\mathbf{q})$ where

$$\langle B_f \mid J_\lambda \mid B_i \rangle \mid_{\mathbf{p}_i = -\mathbf{p}_f = \mathbf{q}/2} = V_\lambda(\mathbf{q}) - A_\lambda(\mathbf{q}) \ . \tag{2}$$

One then has

$$V^0(\mathbf{q}) = \chi_f^\dagger [v_0(\mathbf{q}^2)] \chi_i \ , \tag{3a}$$

$$V^i(\mathbf{q}) = \chi_f^\dagger [q^i v_V(\mathbf{q}^2) + i\epsilon^{ijk} q^j \sigma^k v_M(\mathbf{q}^2)] \chi_i \ , \tag{3b}$$

$$A^0(\mathbf{q}) = \chi_f^\dagger [\boldsymbol{\sigma}\cdot\mathbf{q}\, a_0(\mathbf{q}^2)] \chi_i \ , \tag{3c}$$

and

$$A^i(q) = \chi_f^\dagger [\sigma^i a_S(\mathbf{q}^2) + (q^i q^j - \tfrac{1}{3}\mathbf{q}^2 \delta^{ij}) \sigma_j a_T(\mathbf{q}^2)] \chi_i \ . \tag{3d}$$

In terms of the Lorentz-invariant form factors $f_i$ and $g_i$ one readily finds, in this frame, the following linear relations:

$$v_0 = f_1 + (\Delta M_{fi}/M_{fi}) f_3 \ , \tag{4a}$$

$$v_V = -(\Delta M_{fi}/4M_f M_i)(f_1 + f_2) + (1/M_{fi}) f_3 \ , \tag{4b}$$

$$v_M = (M_{fi}/4M_f M_i)(f_1 + f_2) \ , \tag{4c}$$

$$a_0 = (\Delta M_{fi}/4M_f M_i)(g_3 - g_1) - (1/M_{fi}) g_2 \ , \tag{4d}$$

$$a_S = g_1 - (\Delta M_{fi}/M_{fi}) g_2 \ , \tag{4e}$$

$$a_T = \{ g_3 - \tfrac{1}{2}[g_1 - (\Delta M_{fi}/M_{fi}) g_2] \} / 4M_f M_i \ , \tag{4f}$$

where $\Delta M_{fi} \equiv M_i - M_f$ and $M_{fi} \equiv M_i + M_f$. In Eqs. (4) we have taken the nonrelativistic limit, neglecting all terms explicitly of order $\mathbf{q}^2/M_i^2$ and $\mathbf{q}^2/M_f^2$. In numerical calculations, described below, we shall evaluate $V_\lambda(\mathbf{q})$ and $A_\lambda(\mathbf{q})$, and hence the nonrelativistic form factors, $v_i$ and $a_i$, at $\mathbf{q}^2 = 0$. Inverting Eqs. (4), we thereby determine the relativistic form factors at $\mathbf{q}^2 = 0$, corresponding to $q^2 = q_0^2 = (\Delta M_{fi})^2$. We also note here that, although the induced scalar and pseudoscalar terms $f_3$ and $g_3$ are negligible in the $\beta$-decay amplitude, their contributions being proportional to the small electron mass, we must retain all six form factors in the calculations in order to invert Eqs. (4).

## III. FORM-FACTOR CALCULATIONS

### A. Bare form factors

We shall first outline the calculation of the form factors in the absence of any QCD corrections. The process is schematically shown in Fig. 1. A more detailed discussion, including the calculation of the vertex correction which we describe below, has been presented in Ref. 8.

The bare form factors $f_i^{(0)}$ and $g_i^{(0)}$ are found by evaluating $V_\lambda^{(0)}(\mathbf{q})$ and $A_\lambda^{(0)}(\mathbf{q})$ using

$$2\pi\delta(E_f - E_i - \Delta M_{fi})[V_\lambda^{(0)}(\mathbf{q}) - A_\lambda^{(0)}(\mathbf{q})] = \int d^4x\, e^{iq\cdot x} \langle B_f \mid :\bar{\psi}_f^c(x)\gamma_\lambda(1-\gamma_5)\psi_i^c(x): \mid B_i \rangle \ . \tag{5}$$

Here $\psi_i^c(x)$ and $\psi_f^c(x)$ are the initial and final quark fields and the color index $c$ is summed over. We shall assume the MIT bag model for the baryon wave functions, with the spin and flavor dependence given by SU(6), but shall allow for flavor-symmetry breaking by using the phenomenological bag-model values of the quark masses[7] ($m_u = m_d = 5$ MeV/$c^2$ and $m_s = 280$ MeV/$c^2$). Next, the quark fields are expanded in terms of the MIT bag eigenmodes and the matrix element [Eq. (5)] is calculated assuming all quarks in the initial and final baryon states to be in the ground state. It is then straightforward to expand the exponential in Eq. (5) in powers of $\mathbf{q}\cdot\mathbf{x}$ and identify the bare form factors $v_i^{(0)}$ and $a_i^{(0)}$. These are given by space integrals over quark ground-state wave functions multiplied by the appropriate SU(6) Clebsch-Gordan coefficients. We have ignored slight differences

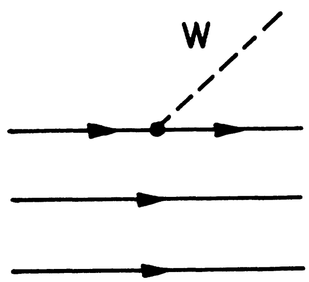


FIG. 1. $B' \rightarrow B + l + \nu$ in the absence of QCD corrections. The solid lines represent the constituent quarks and the dashed line is the $W$ boson.

in bag radii encountered in fits[7] of the baryon mass spectrum, adopting an average value, $R = 5.0$ GeV$^{-1}$. We have also circumvented the problematic issue of "boosted bags" by evaluating $v_i^{(0)}$ and $a_i^{(0)}$ at $\mathbf{q}^2 = 0$, taking both initial and final baryons to be at rest in this frame. Finally, using Eqs. (4) one obtains the bare form factors $f_i^{(0)}$ and $g_i^{(0)}$ in the absence of any perturbative QCD corrections. The numerical values of these bare form factors for the various semileptonic decay processes are given in Table I. These values agree with a similar calculation of Kohyama, Oikawa, Tsushima, and Kubodera.[10] We defer further discussion of these results until the QCD corrections have also been computed.

### B. QCD corrections

As in the calculation of higher-order corrections to any composite system one must take care not to double-count effects which are already included in the bound-state wave function. There is no systematic procedure for treating the gluon exchanges between confined quarks, shown in Fig. 2, which rigorously eliminates double counting and we shall, therefore, necessarily be guided by the conventional wisdom on the subject: One presumes the problem of double-counting soft-gluon effects is ameliorated by the fact that the energy of confined gluons is bounded from below by a minimum zero-point energy, providing for a *de facto* separation between the (hard) gluons in the bag and the soft gluons responsible for the formation of the bag. Soft-gluon effects are then conventionally assumed to be implicitly included in the bag wave functions.

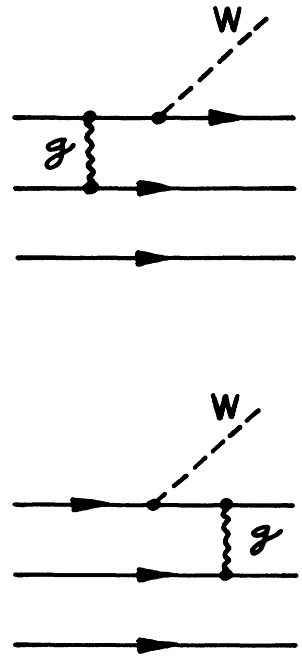


FIG. 2. Gluon-exchange diagrams contributing to $B' \rightarrow B + l + \nu$. Quarks, gluons, and $W$ bosons are represented by solid, wavy, and dashed lines, respectively.

Similarly, the phenomenologically determined bag-model values of the quark masses already include the contributions of quark self-energy diagrams, shown in Fig. 3, inside the bag. We will follow the common practice, or so-called minimal prescription,[7] and drop all quark self-energy diagrams, except those which, together with the gluon-exchange diagrams (Fig. 2) involving all quarks in the bag, will produce zero radial color flux at the bag surface. This eliminates well-known infrared divergences which, in the Coulomb gauge, are associated with the $s$-wave component of the bag Coulomb propagator $D_{00}$ (Ref. 11). However, since we are using baryon wave functions in which the quarks have well-defined energies, the intermediate quark states may be off mass shell, since the $W$ boson carries away energy in the $\beta$ decay. Consequently, only the on-mass-shell intermediate quark states should be absorbed into wave-function and mass renormalization.

TABLE I. Bare form factors evaluated at $q^2 = \Delta M_{fi}^2$, with quark masses $m_u = m_d = 5$ MeV/$c^2$ and $m_s = 280$ MeV/$c^2$. For baryon masses the isospin-averaged values, $M_N = 938.9$ MeV/$c^2$, $M_\Lambda = 1115.6$ MeV/$c^2$, $M_\Sigma = 1193.1$ MeV/$c^2$, and $M_\Xi = 1318.1$ MeV/$c^2$ were used.

| Process | $f_1^{(0)}$ | $f_2^{(0)}$ | $f_3^{(0)}$ | $g_1^{(0)}$ | $g_2^{(0)}$ | $g_3^{(0)}$ |
|---|---|---|---|---|---|---|
| $n \rightarrow p$ | 1.000 | 2.152 | 0.0 | 1.093 | 0.0 | −2.844 |
| $\Sigma^+ \rightarrow \Lambda$ | −0.002 | 1.898 | 0.064 | 0.532 | −0.093 | −2.240 |
| $\Sigma^- \rightarrow \Lambda$ | 0.002 | −1.898 | −0.064 | −0.532 | 0.093 | 2.240 |
| $\Sigma^- \rightarrow n$ | −1.015 | 1.636 | 0.244 | 0.232 | −0.039 | −0.555 |
| $\Lambda \rightarrow p$ | 1.208 | 1.005 | −0.008 | 0.868 | −0.038 | −1.878 |
| $\Xi^0 \rightarrow \Sigma^+$ | 0.986 | 2.713 | −0.015 | 1.189 | 0.067 | −4.134 |
| $\Xi^- \rightarrow \Sigma^0$ | 0.697 | 1.918 | −0.010 | 0.840 | 0.047 | −2.923 |
| $\Xi^- \rightarrow \Lambda$ | −1.221 | 0.346 | 0.164 | −0.288 | 0.024 | 0.937 |

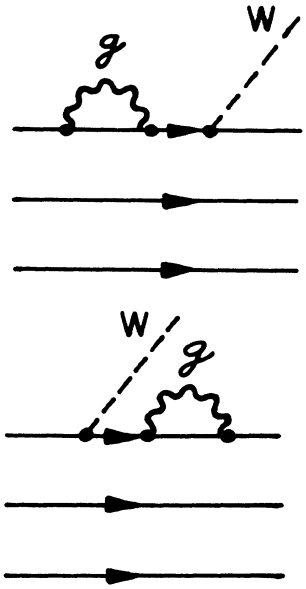


FIG. 3. Confined quark self-energy diagrams. Quarks, gluons, and $W$ bosons are represented by solid, wavy, and dashed lines, respectively.

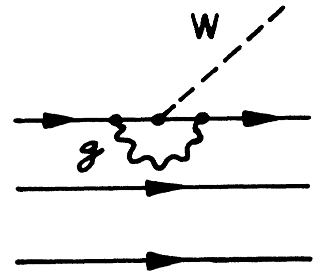


FIG. 4. One-gluon vertex corrections. Quarks, gluons, and $W$ bosons are represented by solid, wavy, and dashed lines, respectively.

Moreover, it has been argued[12] that inside the bag the transverse part of the gluon propagator dominates the Coulomb part and explicit calculations support this argument by showing the Coulomb gluon contributions to the gluon-exchange diagrams, the self-energy diagrams, and the vertex corrections tend to cancel.

In the present calculation we shall, therefore, consider only the one-gluon vertex corrections to the form factors due to transverse gluon exchange, shown in Fig. 4. Implicitly we have assumed that all gluon-exchange diagrams (Fig. 2) and quark self-energy diagrams (Fig. 3) have been absorbed into the baryon wave functions and quark masses.

The one-gluon vertex correction (Fig. 4) to the bare vertex (Fig. 1) is given to order $\alpha_s = g_s^2/4\pi$ by

$$2\pi\delta(E_f - E_i - \Delta M_{fi})[V_\lambda^{(1)}(\mathbf{q}) - A_\lambda^{(1)}(\mathbf{q})] = ig_s^2 \int d^4x \int d^4y \int d^4z \, e^{iq\cdot y} D_{ab}^{jk}(z,x) \langle B_f \,|\, :\bar{\psi}_f^c(z)\gamma_j \lambda_{cc'}^a S_F^f(z,y)\gamma_\lambda(1-\gamma_5) \times S_F^i(y,x)\gamma_k \gamma_{c'c''}^b \psi_i^{c''}(x): \,|\, B_i \rangle \,. \quad (6)$$

As above in Eq. (5) $\psi_i^c$ is the quark field operator for flavor $i$ and color $c$ while $S_F^i$ and $D_{ab}^{jk}$ are, respectively, the quark and transverse gluon propagators inside the bag. Explicit expressions for these propagators are given in our previous work.[8] The factors $\lambda_{cc'}^a$ are color-SU(3) matrices. Summations over the color indices $(c,c',c''=1,2,3$ and $a,b=1,2,\ldots,8)$ and transverse indices $(j,k=1,2,3)$ are understood in Eq. (6).

The calculation of the matrix element in Eq. (6) proceeds as described above for the bare vertex. Using rest-frame MIT bag ground-state wave functions for the baryons, the exponential is expanded in powers of $\mathbf{q}\cdot\mathbf{y}$ and the rotationally invariant form factors $v_i^{(1)}$ and $a_i^{(1)}$, evaluated at $\mathbf{q}^2=0$, are identified. These calculations, which involve energy denominators coming from the quark and gluon propagators, quark-quark-gluon wave-function overlaps, and an overall color factor of $\mathrm{Tr}(\lambda^a\lambda^a)=\frac{16}{3}$, were necessarily carried out numerically. The number of eigenmodes required in the expansion of the quark and gluon propagators was typically 7 or 8 to ensure numerical accuracy, and thus entailed the evaluation of a very large number of integrals over the various intermediate states. Finally, the order-$\alpha_s$ QCD corrections to the form factors $f_i^{(1)}$ and $g_i^{(1)}$ are found using Eqs. (4). The numerical results are presented in Table II. Of course, there are ultraviolet divergences expected in these calculations, and we deal with this issue in the next subsection. In fact, the summation over various intermediate quark and gluon states, while large, was truncated in our numerical calculation, providing a large but finite effective cutoff energy, $\Lambda \simeq 1$ GeV.

### C. Renormalization

As in the case of a free quark,[13] the one-gluon vertex correction calculated for a quark confined inside a bag is expected to be ultraviolet divergent, although this point has never been rigorously demonstrated as it has been for the one-gluon corrections to the quark self-energy inside the MIT bag.[11,14] To carry out the renormalization, thereby removing the implicit cutoff dependence of the calculations, we must first recognize that the bare form factors $f_i^{(0)}$ and $g_i^{(0)}$ already include some strong-interaction contributions which are also QCD effects; i.e., effects due to the influence of the bag boundary. Therefore, in the renormalization procedure, these should be

TABLE II. One-gluon (unrenormalized) vertex corrections to the form factors at $q^2=\Delta M_{fi}^2$, using the same mass parameters as for Table I.

| Process | $f_1^{(1)}$ | $f_2^{(1)}$ | $f_3^{(1)}$ | $g_1^{(1)}$ | $g_2^{(1)}$ | $g_3^{(1)}$ |
|---|---|---|---|---|---|---|
| $n\to p$ | 0.606 | −2.175 | 0.0 | −0.002 | 0.0 | −2.777 |
| $\Sigma^+\to\Lambda$ | 0.001 | −0.945 | −0.032 | −0.004 | −0.069 | −2.054 |
| $\Sigma^-\to\Lambda$ | −0.001 | 0.945 | 0.032 | 0.004 | 0.069 | 2.054 |
| $\Sigma^-\to n$ | −0.515 | 0.270 | 0.024 | −0.015 | −0.046 | −0.489 |
| $\Lambda\to p$ | 0.639 | −1.513 | −0.140 | −0.044 | −0.101 | −1.682 |
| $\Xi^0\to\Sigma^+$ | 0.519 | −1.980 | −0.136 | −0.053 | −0.101 | −3.423 |
| $\Xi^-\to\Sigma^0$ | 0.367 | −1.400 | −0.096 | −0.038 | −0.071 | −2.421 |
| $\Xi^-\to\Lambda$ | −0.636 | 0.981 | 0.104 | 0.016 | 0.048 | 0.784 |

treated in the same sense as the QCD vertex corrections $f_i^{(1)}$ and $g_i^{(1)}$, which are explicitly of order $\alpha_s$. Adopting this approach, we shall assume that soft-gluon contributions to $f_i^{(0)}$ and $g_i^{(0)}$ can be formally handled as corrections of the same order as the QCD vertex corrections. That is, we identify the values of the form factors in the free quark limit, given by their standard SU(6) values,[15] and treat the deviations $\Delta f_i^{(0)}$ and $\Delta g_i^{(0)}$ of the bare form factors from these values as formally being of the same order as the explicit order-$\alpha_s$ vertex corrections $f_i^{(1)}$ and $g_i^{(1)}$.

The renormalization then proceeds in the usual manner. We choose the renormalization point to be the SU(3)-symmetric limit of equal quark masses $\overline{m}$ and equal baryon masses $\overline{M}$. Moreover, we shall assume that this limit is realized at $\overline{m}=5$ MeV/$c^2$ and $\overline{M}=939$ MeV/$c^2$, where the renormalization constants can be determined from neutron-$\beta$-decay data. We define the values of the unrenormalized form factors, $f_i^{\text{unren}}=f_i^{(0)}+f_i^{(1)}$ and $g_i^{\text{unren}}=g_i^{(0)}+g_i^{(1)}$, calculated at this SU(3) symmetry point to be $\overline{f}_i$ and $\overline{g}_i$ and give their numerical values in Table III. To lowest order in the QCD corrections, the multiplicative renormalization constants can be expanded leading to renormalized form factors $f_i$ and $g_i$ given by

$$f_i(q^2)=f_i^{\text{unren}}(q^2)-\overline{f}_i+F_i \tag{7a}$$

and

$$g_i(q^2)=g_i^{\text{unren}}(q^2)-\overline{g}_i+G_i\ . \tag{7b}$$

Here, the renormalization constants $F_i$ and $G_i$ are determined for the entire octet from neutron-$\beta$-decay data using SU(6) Clebsch-Gordan coefficients. We use the following data[16] as input:

$$F_1(n\to p)=1\ , \tag{8a}$$

$$F_2(n\to p)=(\kappa_p-\kappa_n)=3.706\ , \tag{8b}$$

$$F_3(n\to p)=0\ , \tag{8c}$$

$$G_1(n\to p)=1.254\ , \tag{8d}$$

$$G_2(n\to p)=0\ , \tag{8e}$$

$$G_3(n\to p)=0\ . \tag{8f}$$

It is then straightforward to calculate all the other constants $F_i(B_i\to B_f)$ and $G_i(B_i\to B_f)$ in terms of $F_i(n\to p)$ and $G_i(n\to p)$ and SU(6) Clebsch-Gordan coefficients.

Here it should be emphasized that in using full SU(6) spin-flavor symmetry, as opposed to the lesser restrictive SU(3)-flavor symmetry, we have implicitly fixed the $F/D$ ratio for both $f_2$ and $g_1$ to be $\frac{2}{3}$. Also, in Eq. (8f), we have ignored any strong-interaction contributions to $g_3$ in the SU(6)-symmetry limit; specifically, Nambu-Goldstone meson pole contributions. We shall discuss both of these points further after giving our numerical results.

In Table IV the numerical values of the renormalized form factors, Eqs. (7), for the various semileptonic decay processes are presented. Since the recoil of the baryons, $\mathbf{q}=\mathbf{p}_i-\mathbf{p}_f$, was neglected in all calculations, these form factors refer to a four-momentum transfer of $q_\lambda^2=(M_i-M_f)^2$ in each case.

TABLE III. Unrenormalized form factors, including (unrenormalized) vertex corrections at the flavor-SU(3)-symmetric point $q^2=0$ with equal quark masses $\overline{m}=5$ MeV/$c^2$ and equal baryon masses $\overline{M}=939$ MeV/$c^2$.

| Process | $\overline{f}_1$ | $\overline{f}_2$ | $\overline{f}_3$ | $\overline{g}_1$ | $\overline{g}_2$ | $\overline{g}_3$ |
|---|---|---|---|---|---|---|
| $n\to p$ | 1.606 | −0.023 | 0.0 | 1.091 | 0.0 | −5.621 |
| $\Sigma^+\to\Lambda$ | 0.0 | 0.775 | 0.0 | 0.534 | 0.0 | −2.753 |
| $\Sigma^-\to\Lambda$ | 0.0 | −0.775 | 0.0 | −0.534 | 0.0 | 2.753 |
| $\Sigma^-\to n$ | −1.606 | 1.922 | 0.0 | 0.217 | 0.0 | −1.124 |
| $\Lambda\to p$ | 1.967 | −0.803 | 0.0 | 0.801 | 0.0 | −4.130 |
| $\Xi^0\to\Sigma^+$ | 1.606 | −0.023 | 0.0 | 1.091 | 0.0 | −5.621 |
| $\Xi^-\to\Sigma^0$ | 1.135 | −0.016 | 0.0 | 0.771 | 0.0 | −3.975 |
| $\Xi^-\to\Lambda$ | −1.967 | 1.579 | 0.0 | −0.267 | 0.0 | 1.376 |

TABLE IV. Renormalized form factors, including one-gluon vertex corrections, evaluated at $q^2=\Delta M_{fi}^2$.

| Process | $f_1$ | $f_2$ | $f_3$ | $g_1$ | $g_2$ | $g_3$ |
|---|---|---|---|---|---|---|
| $n\to p$ | 1.0 | 3.706 | 0.0 | 1.254 | 0.0 | 0.0 |
| $\Sigma^+\to\Lambda$ | −0.001 | 2.483 | 0.032 | 0.608 | −0.162 | −1.541 |
| $\Sigma^-\to\Lambda$ | 0.001 | −2.483 | −0.032 | −0.608 | 0.162 | 1.541 |
| $\Sigma^-\to n$ | −0.924 | 1.925 | 0.268 | 0.249 | −0.085 | 0.080 |
| $\Lambda\to p$ | 1.105 | 2.528 | −0.148 | 0.944 | −0.139 | 0.570 |
| $\Xi^0\to\Sigma^+$ | 0.899 | 4.462 | −0.151 | 1.299 | −0.034 | −1.936 |
| $\Xi^-\to\Sigma^0$ | 0.636 | 3.154 | −0.106 | 0.918 | −0.024 | −1.369 |
| $\Xi^-\to\Lambda$ | −1.115 | −0.162 | 0.268 | −0.312 | 0.072 | 0.345 |

## IV. DISCUSSION

All values of the renormalized form factors in Table IV have been calculated for each process at the relevant value of $q^2=\Delta M_{fi}^2$. To compare with the experimental data, as well as with other calculations, we shall assume the usual, empirical dipole form for the $q^2$ dependence of the vector and axial-vector form factors:

$$f_1(q^2)=f_1(0)(1-q^2/M_V^2)^{-2}\ , \tag{9a}$$

$$g_1(q^2)=g_1(0)(1-q^2/M_A^2)^{-2}\ . \tag{9b}$$

Fitting the data one finds[3] $M_V=0.84$ GeV/$c^2$ and $M_A=1.08$ GeV/$c^2$ for $\Delta S=0$ transitions. The convention of rescaling by the vector-meson mass ratio, $m_{K^*}/m_\rho$, then gives $M_V'=0.98$ GeV/$c^2$ and $M_A'=1.25$ GeV/$c^2$ for $\Delta S=1$ processes. We shall neglect the (unknown) $q^2$ dependence of the other form factors since it is unimportant at the current level of experimental precision.

The most precisely measured quantity in hyperon $\beta$ decay is the ratio $g_1(0)/f_1(0)$. In Table V we have listed the experimental values of this ratio and our results, both neglecting any QCD effects (bare) and including the one-gluon vertex corrections (renormalized). We also present the corresponding results of Ushio and Konashi[12] who included one-gluon-exchange effects (Fig. 2) but neglected the transverse gluon propagator in calculating the vertex correction. For comparison, we also list the results for the MIT bag model of Lie-Svendsen and Høgaasen,[7] who have considered recoil effects, but not QCD corrections.

There are a number of other calculations[18–21] which use various other quark-confinement models and/or input parameters but do not include the order-$\alpha_s$ QCD corrections; for the most part, these are all in agreement with the bare calculations listed in Table V. We have also chosen not to make comparisons with those quark-confinement models which augment the quark degrees of freedom in the MIT bag model with pseudoscalar meson fields, usually to implement PCAC (partial conservation

TABLE V. $g_1(0)/f_1(0)$, except for $\Sigma\to\Lambda$ where $g_1(0)$ is given since $f_1(0)$ vanishes. The experimental values assume $f_2/f_1$ is given by CVC and $g_2=0$.

| Process | Ushio and Konashi Bare | Ushio and Konashi QCD | Lie-Svendsen and Høgaasen Static | Lie-Svendsen and Høgaasen Recoil | Present calculation Bare | Present calculation QCD | Experiment |
|---|---|---|---|---|---|---|---|
| $n\to p$ | 1.09 | 1.12 | 1.09 | 1.09 | 1.09 | 1.25 | 1.254±0.001[a] |
| $\Sigma^-\to\Lambda$ | −0.53 | −0.56 | −0.52 | −0.52 | −0.53 | −0.60 | 0.589±0.016[b] |
| $\Sigma^-\to n$ | −0.24 | −0.31 | −0.24 | −0.24 | −0.24 | −0.29 | −0.34 ±0.05[c] |
| | | | | | | | −0.29 ±0.07[d] |
| | | | | | | | 0.327±0.020[e] |
| $\Lambda\to p$ | 0.72 | 0.73 | 0.74 | 0.73 | 0.74 | 0.88 | 0.71 ±0.03[b] |
| | | | | | | | 0.70 ±0.03[c] |
| $\Xi^-\to\Sigma^0$ | 1.20 | 1.23 | 1.21 | 1.21 | 1.22 | 1.46 | 0.73 ±0.03[f] |
| | | | | | | | 1.25 ±0.15[b] |
| $\Xi^-\to\Lambda$ | 0.24 | 0.22 | 0.25 | 0.25 | 0.24 | 0.29 | 0.30 ±0.04[b] |
| | | | | | | | 0.25 ±0.05[c] |
| $\dfrac{(g_1/f_1)_{\Lambda\to p}}{(g_1/f_1)_{\Sigma^-\to n}}$ | −3.00 | −2.35 | −3.08 | −3.04 | −3.06 | −3.08 | −2.21 ±0.21[g] |

[a]Particle Data Group (Ref. 16).
[b]CERN WA2 result for $|g_1/f_1|$ from branching-ratio measurement (Ref. 3).
[c]CERN WA2 result from form factor analysis (Ref. 3).
[d]Fermilab E715 result from electron asymmetry (Hsueh *et al.*, Ref. 1).
[e]Fermilab E715 result for $|g_1/f_1|$ from neutron spectrum (Winston, Ref. 1).
[f]BNL result (Ref. 4).
[g]Average value for data quoted above.

of axial-vector current) in some form. Through the inclusion of pion ($\Delta S=0$) or kaon ($\Delta S=1$) poles (long-distance effects which are neglected in this calculation), more realistic values for $g_3$ are presumably obtained, at least in the $\Delta S=0$ case. However, spurious meson poles are simultaneously introduced in the axial-vector ($g_1$) and pseudotensor ($g_2$) form factors as well.[17] Although such artifacts of these extended bag models can be subtracted away by hand, they pose a serious inconsistency in principle when QCD corrections are included and we shall not consider them here.

Comparing the results in Table V, all of which have been extrapolated to $q^2=0$ as described above, it is clear that there is good agreement among all the calculations when QCD effects are neglected, thus providing a check on the numerical computations. And Lie-Svendsen and Høgaasen's results show that we can safely neglect recoil effects.

Turning next to one-gluon QCD effects, we give in Table V the QCD-corrected results for $g_1(0)/f_1(0)$ of the present calculation and those of the Ushio and Konashi. Recall that in the present calculation the one-gluon-exchange effects (Fig. 2) were assumed to be implicitly contained in the quark wave function and the one-gluon vertex correction was calculated using the transverse gluon propagator, which gives the dominant part. Ushio and Konashi, on the other hand, included one-gluon-exchange effects but neglected the transverse part of the gluon propagator in calculating their QCD corrections. Observe from Table V that, with the exception of $\Lambda$ $\beta$ decay, the QCD corrections in both calculations improve the agreement with the data. In the case of $\Lambda$ $\beta$ decay, however, there is a substantial disagreement with the data when the QCD effects we have calculated are included. This is particularly puzzling since the QCD corrections significantly improve the agreement with the data for the process most similar to $\Lambda$ $\beta$ decay, namely, $\Sigma^-$ $\beta$ decay. The mass differences are relatively small and in both $\Lambda$ and $\Sigma^-$ $\beta$ decays a strange quark decays to an up quark. But the spectator quarks, while both light, are in quite different spin-isospin states in the $\Lambda$ and $\Sigma^-$ hyperons. Therefore, comparing our calculations with Ushio and Konashi's and with the data we conclude that there must be important effects associated with the spectator quarks.

TABLE VI. $g_2/g_1(0)$.

| Process | Lie-Svendsen and Høgaasen Static | Recoil | Present calculation Bare | QCD |
|---|---|---|---|---|
| $n\to p$ | 0 | 0 | 0 | 0 |
| $\Sigma^-\to\Lambda$ | −0.15 | −0.15 | −0.18 | −0.27 |
| $\Sigma^-\to n$ | −0.05 | −0.16 | −0.18 | −0.37 |
| $\Lambda\to p$ | 0.04 | −0.06 | −0.04 | −0.15 |
| $\Xi^-\to\Sigma^0$ | 0.10 | 0.0 | 0.06 | −0.03 |
| $\Xi^-\to\Lambda$ | 0.0 | −0.10 | −0.09 | −0.24 |

To emphasize this point we give in the bottom row of Table V the ratio

$$\frac{[g_1(0)/f_1(0)]_{\Lambda\to p}}{[g_1(0)/f_1(0)]_{\Sigma^-\to n}}\,, \tag{10}$$

a quantity which one would expect to be insensitive to systematic errors. Note that except for the QCD-corrected results of Ushio and Konashi, all the calculated values of this ratio are quite close to the naive SU(6) prediction of $-3$, which is in substantial conflict with the data. Thus we have unequivocal evidence for SU(6)-symmetry breaking beyond that which can be accommodated by quark/hadron mass splittings, recoil corrections, or QCD vertex corrections. Indeed, the agreement between the results of Ushio and Konashi and the experimental value for this ratio suggests that SU(6)-breaking perturbative corrections to the initial and final baryon wave functions, of the sort induced by gluon exchange (Fig. 2), are quite essential.

It is worth emphasizing the sensitivity of the ratio (10) to SU(6)-symmetry-breaking effects. Taking the expression for this ratio as a function of the $F/D$ ratio and expanding in powers of the deviation of $F/D$ from its SU(6) value, $\epsilon\equiv F/D-\frac{2}{3}$, we have

$$\frac{(g_1/f_1)_{\Lambda\to p}}{(g_1/f_1)_{\Sigma^-\to n}}=\frac{F+D/3}{F-D}=-3\left[\frac{1+\epsilon}{1-3\epsilon}\right]$$
$$=-3-12\epsilon+\cdots\,. \tag{11}$$

TABLE VII. $f_2/f_1(0)$, except for $\Sigma\to\Lambda$ where $f_2$ is given since $f_1(0)$ vanishes.

| Process | CVC | Lie-Svendsen and Høgaasen Static | Recoil | Present calculation Bare | QCD | Experiment |
|---|---|---|---|---|---|---|
| $n\to p$ | 3.71 | 2.17 | 1.88 | 2.15 | 3.71 | 3.71 (input) |
| $\Sigma^-\to\Lambda$ | −2.34 | −1.94 | −1.79 | −1.90 | −2.48 | $-3.51\pm3.51$[a] |
| $\Sigma^-\to n$ | −2.03 | −1.85 | −1.80 | −1.85 | −2.39 | $-1.82\pm0.61$[a] |
| | | | | | | $-1.71\pm0.27$[b] |
| $\Lambda\to p$ | 1.79 | 0.90 | 0.74 | 0.89 | 2.44 | $2.43\pm1.49$[a] |
| $\Xi^-\to\Sigma^0$ | 3.71 | 2.86 | 2.59 | 2.84 | 5.12 | |
| $\Xi^-\to\Lambda$ | −0.12 | −0.30 | −0.36 | −0.31 | 0.16 | $-0.44\pm0.46$[a] |

[a]CERN WA2 (Ref. 3).
[b]Fermilab E715 (Ref. 1).

Thus, first-order deviations from SU(6) symmetry are amplified by an order of magnitude making this ratio, which is quite well measured, extremely sensitive to the value of $F/D$.

One can understand the physical significance of the one-gluon-exchange diagrams (Fig. 2) by recalling that the QCD tensor force, a color magnetic effect, is well known to be quite important for the hadron mass spectrum;[22] specifically, the $P$ states of charmonium and the $\Sigma^0$-$\Lambda$ mass splitting. Thus, it is not so surprising that in the strangeness-changing processes $\Lambda \to p + e^- + \bar{\nu}_e$ and $\Sigma^- \to n + e^- + \bar{\nu}_e$, although quite similar, the different spin states of the light spectator quarks in the two processes is very significant. We can conclude that any quark confinement model based on only a central potential and taking the spin-flavor part of the wave function from SU(6) is most certainly going to predict the ratio (10) to be close to $-3$, the SU(6) value. It is imperative to take into account the configuration mixing in the spin-flavor part of the wave function induced by the one-gluon-exchange diagrams of Fig. 2.

In Table VI we present results for $g_2/g_1(0)$, where $g_2$ is the induced second-class electric dipole form factor. Our calculation indicates the QCD effects are significant while Lie-Svendsen and Høgaasen find recoil effects are less important. However, the main point is that neither calculation finds very large induced second-class effects for any of the hyperon $\beta$ decays. The recent high-precision $\Sigma^-$ $\beta$-decay experiment[1] suggests the presence of such a second-class term, finding

$$g_2/g_1(0) = -6.7 \pm 4.8 \ . \tag{12}$$

While the experimental uncertainty is quite large, at least the sign agrees with the calculations. We also point out that the magnitudes of $g_2/g_1(0)$, particularly for the processes $\Lambda \to p$ and $\Sigma^- \to n$, are not large enough to significantly influence the comparison of $g_1/f_1$ with the experimentally measured ratio $g_A/g_V$, which differs from $g_1/f_1$ by terms of order $(\Delta M_{fi}/M_{fi})(g_2/f_1)$. The second-class form factor $g_2$ is too small to significantly affect our above discussion of the values of $g_1(0)/f_1(0)$ and our conclusion that the effects of the spectator quarks are important.

Table VII gives our results for $f_2/f_1(0)$ and compares them with the data as well as the calculations of Lie-Svendsen and Høgaasen and the conserved-vector-current (CVC) predictions. It is clear that both recoil and QCD corrections are important. Unfortunately, the experimental data are not very precise, but the general agreement is good.

Table VIII gives the ratio of $f_1(0)$ to the naive predictions $f_1^{\text{CVC}}$ of the conserved-vector-current hypothesis.

TABLE VIII. $f_1(0)/f_1^{\text{CVC}}$ except for $\Sigma \to \Lambda$ where $f_1(0)$ is given since $f_1^{\text{CVC}}$ vanishes.

| | Lie-Svendsen and Høgaasen | | Present calculation | |
|---|---|---|---|---|
| Process | Static | Recoil | Bare | QCD |
| $n \to p$ | 1.000 | 1.000 | 1.000 | 1.000 |
| $\Sigma^- \to \Lambda$ | | | 0.002 | 0.001 |
| $\Sigma^- \to n$ | 0.896 | 0.887 | 0.883 | 0.804 |
| $\Lambda \to p$ | 0.929 | 0.924 | 0.923 | 0.844 |
| $\Xi^- \to \Sigma^0$ | 0.956 | 0.953 | 0.954 | 0.870 |
| $\Xi^- \to \Lambda$ | 0.920 | 0.913 | 0.914 | 0.834 |

Again, as a check on the numerical computations, we note that our calculations agree with Lie-Svendsen and Høgaasen when QCD effects are ignored. The principal observation here, however, is that the deviations of $f_1(0)$ from the CVC values are substantial for the $\Delta S = 1$ processes in both calculations. In fact, QCD corrections exacerbate the disagreement, while the recoil corrections make little difference. This is contrary to expectations based on the Ademollo-Gatto theorem,[23] which states that the flavor-symmetry-breaking effects in $f_1$ are second order, although it is not so clear precisely how large this means. This could have significant implications for the determination of the Cabibbo angle, as well as other Kobayashi-Maskawa matrix elements.[24] Certainly, the issue should be explored further, systematically including QCD corrections in all the experimental information used; specifically, meson semileptonic decay data. Incidentally, the ratio (10) is not significantly affected since for the $\Lambda$ and $\Sigma$ $\beta$ decays the corrections to $f_1$ are nearly in the same ratio.

We conclude that the hadronic $\beta$-decay data, taken together, are now sufficiently precise to challenge the standard model at the level of the order-$\alpha_s$ corrections and more ambitious calculations are warranted.

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