Asymmetry between inclusive charmed and anticharmed modes in B^0, $\bar{B}^0$ decay as a measure of CP violation

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The anticharm-charm asymmetry $A(c,\bar{c})$ in the *inclusive* decay of tagged B^0 and $\bar{B}^0$ mesons is calculated within the context of the Kobayashi-Maskawa (KM) model. It is shown that $A(c,\bar{c})$ is independent of the final-state interactions to a very good approximation. By using estimates of branching ratios, of the differences of the masses of the eigenstates of the B_d^0 and the B_s^0, and of matrix elements of the quark currents, we find $|A(c,\bar{c})|<0.01$. If instead of using those estimates of the three sets of parameters we treat them as unknowns, we find that an upper limit on the magnitude of $A(c,\bar{c})$ in the context of the model is of the order of 5%. A measurement of $A(c,\bar{c})$ leading to a value significantly greater than 5% would be a clear indication that the KM model does not provide a *complete* description of CP violation.

I. INTRODUCTION

The only unambiguous experimental evidence for CP violation at the present time is that provided by the decay and interference phenomena of the neutral K mesons.[1] The possibility for gaining insight into the fundamental origins of CP violation is thereby greatly limited. The B mesons appear to be the best candidates for experiments that might provide additional information. Methods using the separation of a long-lived from a short-lived species that have made possible the CP experiments with the $K^0\bar{K}^0$ system are not applicable to the $B^0\bar{B}^0$ system because the lifetimes associated with the two eigenstates of the B mass matrix are expected to be about equal. The near equality is a consequence of the large energy release and the concomitant great variety of modes in B decay, which tend to equalize the total amount of phase space available in the decay of the two eigenstates.

Methods suggested for measuring CP violation for neutral mesons heavier than the K, such as the B mesons, therefore have been based instead on determining an asymmetry in the decay of B^0 and $\bar{B}^0$ into one or another specific mode.[2] Because of the importance attached to obtaining new information concerning CP violation there have been exhaustive studies[3] of particular (exclusive) decay modes of the B^0, $\bar{B}^0$ system on the basis of the Kobayashi-Maskawa (KM) model,[4] to determine which of them are most likely to exhibit B^0-$\bar{B}^0$ asymmetries.

There are difficulties associated with calculating the asymmetries for exclusive modes. They arise primarily from the unknown final-state-interaction effects,[5] that is, the effects of the strong interactions among the hadrons comprising the particular decay mode. The many different decay modes associated with a given set of conserved quantum numbers are mixed by strong final-state interactions. Their unmixing to specify the amplitude of an exclusive mode requires a knowledge of many eigenphases of the strong-interaction S matrix.

There are also experimental problems with exclusive modes associated with picking a particular mode out of the plethora of modes into which the B's may decay. This difficulty might be mitigated in experiments on *inclusive* modes, those including all modes containing one particle of a particular type. The selection criteria are not then so severe and the data rate is much greater.

It is our purpose here to assess the possibility of testing CP invariance by observation of the *inclusive* asymmetry between charmed particles and their antiparticles in B^0, $\bar{B}^0$ decay. We shall show that in the calculation of the inclusive asymmetry on the basis of the KM model the dependence on the final-state interactions is strongly suppressed. Therefore the interpretation of inclusive charm asymmetries is much more straightforward than that of exclusive asymmetries. Because the simplicity of the interpretation is a direct consequence of the KM model, measurement of the inclusive asymmetry may also provide an opportunity to test the model.

There are, of course, uncertainties in the calculation of the inclusive asymmetries. They arise in connection with the evaluation of the matrix element of the effective weak interaction for each mode, the determination of relative branching ratios of the various modes, and estimating the relative production of B_s mesons versus B_d mesons. Again, some advantage is regained by consideration of the inclusive modes because the inclusive asymmetry depends on the sum of products of amplitudes over a complete set of states, which can be estimated with some confidence.

We treat only the case of tagged neutral B mesons since it is the simplest case. However the asymmetries obtained from $B^0\bar{B}^0$ pair production, i.e., by "leptonic tagging"[6] or by determining the like-charmed-particle asymmetry,

$$[N(c,c)-N(\bar{c},\bar{c})]/[N(c,c)+N(\bar{c},\bar{c})]\ ,$$

are expected to be of the same order as the tagged-beam asymmetry.

II. THE INCLUSIVE ASYMMETRY

The quantity to be measured in a determination of the anticharm-charm asymmetry is taken to be

$$A(c,\bar{c})=\frac{N(c)-\bar{N}(\bar{c})+N(\bar{c})-\bar{N}(c)}{N(c)+\bar{N}(\bar{c})+N(\bar{c})+\bar{N}(c)} ,$$

where $N(c)$ $[N(\bar{c})]$ is the total number of decay events in a tagged B^0 beam for which a (single) charmed [anticharmed] particle has been identified and $\bar{N}(c)$ $[\bar{N}(\bar{c})]$ is the corresponding total number for a tagged $\bar{B}^0$ beam. These numbers must be normalized to correspond to B^0 and $\bar{B}^0$ beams of equal intensity. The use of this combination of the more conventionally defined asymmetries based on separate determinations of $N(c)-\bar{N}(\bar{c})$ and $N(\bar{c})-\bar{N}(c)$ simplifies the calculation of the asymmetry in terms of decay amplitudes.

These amplitudes are given as matrix elements between the pure state $|B^0\rangle$ and $|\bar{B}^0\rangle$ and final states $|f\rangle$ that describe specific modes of decay. Therefore the calculated asymmetry is given by

$$A(c,\bar{c})=\frac{\Sigma_f[N(f)-\bar{N}(\bar{f})+N(\bar{f})-\bar{N}(f)]n_c(f)}{\Sigma_f[N(f)+\bar{N}(\bar{f})+N(\bar{f})+\bar{N}(f)]n_c(f)} , \quad (1)$$

where $N(f)$ $(\bar{N}(f))$ is the calculated total number of decays into state $|f\rangle$ per tagged B^0 $(\bar{B}^0)$ particle, $n_c(f)$ is the number of charmed and anticharmed particles in mode f, and the sums are carried out over all modes f $(\bar{f})$ that include a charmed (anticharmed) particle such as a D $(\bar{D})$, D^* $(\bar{D}^*)$, or D_s $(\bar{D}_s)$. A single mode f may of course include both a c quark and its antiquark $\bar{c}$, but in that case to be included in the sum f must have "open" charm; that is, $n_c(f)=2$ $[n_c(f)>2$ is excluded by energy conservation] because an inclusive event is defined as one for which either a single charmed or a single anticharmed particle is identified. Note, however, that when such a mode is self-conjugate, that is, $f\equiv\bar{f}$, it is counted doubly in Eq. (1) so that we must substitute $n_c(\text{effective})\equiv 1$ for open self-conjugate channels.

The calculation of $N(f)$ and $\bar{N}(f)$ is straightforward and yields

$$N(f)=\left[(2+x^2)|\langle f|B^0\rangle|^2-2x\,\mathrm{Im}\left[\frac{q}{p}\langle f|B^0\rangle^*\langle f|\bar{B}^0\rangle\right]+x^2\left|\frac{q}{p}\langle f|\bar{B}^0\rangle\right|^2\right]\Big/2(1+x^2) , \quad (2a)$$

$$\bar{N}(f)=\left[(2+x^2)|\langle f|\bar{B}^0\rangle|^2-2x\,\mathrm{Im}\left[\frac{p}{q}\langle f|\bar{B}^0\rangle^*\langle f|B^0\rangle\right]+x^2\left|\frac{p}{q}\langle f|B^0\rangle\right|^2\right]\Big/2(1+x^2) , \quad (2b)$$

where

$$x=\Delta m/\Gamma . \quad (3)$$

Δm is the mass difference and Γ the decay rate of the eigenstates of the $B^0,\bar{B}^0$ mass matrix, $\langle f|B^0\rangle$ and $\langle f|\bar{B}^0\rangle$ are the decay amplitudes of the states $|B^0\rangle$ and $|\bar{B}^0\rangle$, and p,q are the mixing coefficients describing the eigenstates of the mass matrix:

$$|B^0_\pm\rangle=p|B^0\rangle\pm q|\bar{B}^0\rangle .$$

The distinct values of these parameters for the B_d^0 and B_s^0 will be denoted by x_d,x_s, p_d,p_s, and q_d,q_s. In obtaining Eqs. (2a) and (2b) use has been made of the assumption that the relative difference between the decay rates of the two eigenstates is very small.

The inclusive decay properties are obtained by summing $N(f)$ and $\bar{N}(f)$ over states $|f\rangle$. These sums take on a particularly simple form for the KM model. The simplification arises for two reasons. One is the direct result of summing over all states $|f\rangle$ having the appropriate total quantum numbers (energy, momentum, angular momentum, charm, etc.). From the form of Eqs. (2a) and of (2b) it is clear that the sums are independent of the choice of representation of the states $|f\rangle$ within the set corresponding to the given quantum numbers as long as *all* states of the set are included. Since the total quantum numbers are conserved by the strong interactions, the states $|f\rangle$ may then be chosen to be the eigenstates of the strong-interaction S matrix. With that choice the effect of the final-state interactions, which appears as a common phase factor in $\langle f|B^0\rangle$ and $\langle f|\bar{B}^0\rangle$, drops out of the expressions for $N(f)$ and $\bar{N}(f)$. In the following, $|f\rangle$ and $|\bar{f}\rangle=CP|f\rangle$ denote such eigenstates of the S matrix unless we specify otherwise.

The other reason for simplification is that in the KM model the amplitudes $\langle f|B^0\rangle,\langle f|\bar{B}^0\rangle$ consist of a factor depending on the KM matrix elements multiplying the matrix element of a CP-invariant and T-invariant operator. All effects of CP (and T) violation on an ampli-

TABLE I. Flavor decay channels and associated KM matrix elements of the B meson. (Entries for the $\bar{B}$ are obtained by replacing all quarks by their antiquarks and taking the conjugate complex of the vertex matrix elements.)

ϕ	$\bar{b}$ to	Vertex	W^+ to	Vertex
1	$\bar{c}$	V_{cb}^*	$u\bar{d}$	V_{ud}
2			$u\bar{s}$	V_{us}
3			$c\bar{d}$	V_{cd}
4			$c\bar{s}$	V_{cs}
5			$l^+\nu$	
6	$\bar{u}$	V_{ub}^*	$u\bar{d}$	V_{ud}
7			$u\bar{s}$	V_{us}
8			$c\bar{d}$	V_{cd}
9			$c\bar{s}$	V_{cs}
10			$l^+\nu$	
11	$\bar{t}$	V_{tb}^*	$t\bar{d}$	V_{td}
12			$t\bar{s}$	V_{ts}

tude are determined by the phases of the KM matrix elements in the factor, which is the same for all final states arising from a weak vertex having a given flavor composition, denoted as the "flavor channel" ϕ. Thus the sum over final states may be divided into sums over all modes f_ϕ associated with a given flavor channel ϕ, each such sum having a common CP-violating phase. For example, f_1 represents any hadronic mode $\bar{D}\pi$, $\bar{D}\pi\pi$, etc., of B_d^0 that can be formed from $\bar{c}du\bar{d}$ ($\phi=1$ in Table I) by hadronization. Except for one important class of states, the self-conjugate states to be treated later, the $|f_\phi\rangle$ may be taken to be eigenstates of the S matrix because total flavor is conserved by the strong interactions.

The quark flavor channels ϕ for B decay are illustrated in Fig. 1 (where a "spectator" d or s quark for the B_d or B_s, respectively, must be added to each of the indicated three-flavor channels and included in the definition of ϕ) and Table I. Since the V_{jk} for a given $|f_\phi\rangle$ depend only[7] on ϕ, the factorization of decay amplitudes means that

$$\langle f_\phi | B^0\rangle = U_\phi a(f_\phi) e^{i\delta(f_\phi)} ,$$
$$\langle \bar{f}_\phi | \bar{B}^0\rangle = U_\phi^* \bar{a}(\bar{f}_\phi) e^{i\delta(f_\phi)} , \tag{4}$$

where U_ϕ is the product of the V_{jk} associated with channel ϕ in Table I and $2\delta(f_\phi)$ is the eigenphase of the S matrix associated with the eigenstate $|f_\phi\rangle$. The CP invariance of the amplitudes $a(f_\phi)$ implies that

$$\bar{a}(\bar{f}_\phi) = a(f_\phi) \tag{5}$$

if we adopt the phase convention $CP|B^0\rangle = |\bar{B}^0\rangle$.

Furthermore, since $ae^{i\delta}$ and $\bar{a}e^{i\delta}$ are matrix elements of a T-invariant operator, the reduced amplitudes a and $\bar{a}$ are real quantities (under the assumption of spin 0 for the B meson). Finally we note that application of Eq. (5) to Eq. (4) yields

$$|\langle f_\phi | B^0\rangle| = |\langle \bar{f}_\phi | \bar{B}^0\rangle| . \tag{6}$$

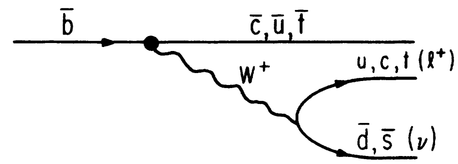

FIG. 1. The generic flavor channels into which the B mesons decay. The accompanying (spectator) d or s quarks (for the B_d or B_s, respectively) are not shown. The contribution of the $t\bar{t}$ pair is limited to their annihilation into gluons leading to lower-mass hadrons (penguin diagrams).

It is apparent that the interference terms in Eqs. (2a) and (2b) occur only if there is a common final state for the decay of $|B^0\rangle$ and $|\bar{B}^0\rangle$. Since the flavor channels available to $|\bar{B}^0\rangle$ are the charge conjugates $\bar{\phi}$ to those shown in Fig. 1 and Table I, interference arises only between those channels ϕ for which there is another channel ϕ' such that $\bar{\phi}'\equiv\phi$. For such a ϕ we have

$$(q/p)\langle f_\phi | \bar{B}^0\rangle / \langle f_\phi | B^0\rangle = \lambda(\phi)\bar{a}(f_\phi)/a(f_\phi) , \tag{7a}$$

where

$$\lambda(\phi) = \frac{q}{p}\frac{U_{\phi'}^*}{U_\phi} \tag{7b}$$

and

$$(p/q)\langle \bar{f}_\phi | B^0\rangle / \langle \bar{f}_\phi | \bar{B}^0\rangle = \lambda^*(\phi)\bar{a}(f_\phi)/a(f_\phi) \tag{7c}$$

under the generally accepted assumption[8] that $|p/q|^2-1$ is negligible for the B_d^0 and B_s^0.

If we denote by N_ϕ and $\bar{N}_{\bar{\phi}}$ the sums of $N(f_\phi)$ and $\bar{N}(\bar{f}_\phi)$ over all final states associated with a given ϕ and $\bar{\phi}$, N_ϕ is then found from Eq. (2a) to be

$$N_\phi = |\langle \phi | B^0\rangle|^2 [2+x^2+x^2|\lambda(\phi)|^2 \langle |\bar{a}/a|^2\rangle_\phi - 2x\,\mathrm{Im}\lambda(\phi)\langle \bar{a}/a\rangle_\phi]/2(1+x^2) , \tag{8a}$$

while Eqs. (2b), (6), and (7) give

$$\bar{N}_{\bar{\phi}} = |\langle \phi | B^0\rangle|^2 [2+x^2+x^2|\lambda(\phi)|^2 \langle |\bar{a}/a|^2\rangle_\phi + 2x\,\mathrm{Im}\lambda(\phi)\langle \bar{a}/a\rangle_\phi]/2(1+x^2) \tag{8b}$$

with

$$\langle \bar{a}/a\rangle_\phi = \sum_{f_\phi} B(f_\phi)\bar{a}(f_\phi)/a(f_\phi) , \tag{9a}$$

$$\langle |\bar{a}/a|^2\rangle_\phi = \sum_{f_\phi} B(f_\phi)|\bar{a}(f_\phi)/a(f_\phi)|^2 , \tag{9b}$$

and

$$|\langle \phi | B^0\rangle|^2 = \sum_{f_\phi} |\langle f_\phi | B^0\rangle|^2 . \tag{9c}$$

$B(f_\phi)$ is the branching ratio for the mode f_ϕ relative to all modes of the same ϕ. These averages, Eqs. (9a) and (9b), are independent of final-state interactions.

By replacing ϕ by ϕ' and again making use of Eq. (6b) we also find

$$N_{\bar{\phi}} = |\langle \phi | B^0\rangle|^2 [(2+x^2)|\lambda(\phi)|^2 \langle |\bar{a}/a|^2\rangle_\phi - 2x\,\mathrm{Im}\lambda(\phi)\langle \bar{a}/a\rangle_\phi + x^2]/2(1+x^2) \tag{10a}$$

and

$$\bar{N}_\phi = |\langle \phi | B^0\rangle|^2 [(2+x^2)|\lambda(\phi)|^2 \langle |\bar{a}/a|^2\rangle_\phi + 2x\,\mathrm{Im}\lambda(\phi)\langle \bar{a}/a\rangle_\phi + x^2]/2(1+x^2) . \tag{10b}$$

Therefore if the inclusive asymmetry between channels ϕ and $\bar{\phi}$ is defined as

$$A_{\phi\bar{\phi}}=\frac{N_\phi-\bar{N}_{\bar{\phi}}+N_{\bar{\phi}}-\bar{N}_\phi}{N_\phi+\bar{N}_{\bar{\phi}}+N_{\bar{\phi}}+\bar{N}_\phi}$$

we find that

$$A_{\phi\bar{\phi}}=-\frac{2x\,\mathrm{Im}\lambda(\phi)\langle\bar{a}/a\rangle_\phi}{(1+x^2)[1+|\lambda(\phi)|^2\langle|\bar{a}/a|^2\rangle_\phi]} . \tag{11}$$

We have noted that interference between the hadronic decay modes will occur only for those flavor channels satisfying the condition $\bar{\phi}\equiv\phi'$. For the B_d^0, it can be seen from Table I that this condition is satisfied for $\phi=1$, $\phi'=8$, while for the B_s^0 it is satisfied for $\phi=2$, $\phi'=9$. Therefore their contributions to the inclusive asymmetry are given by inserting $\lambda_d(1)$ and $\lambda_s(2)$ into Eq. (11).

The other flavor channels for which $\phi'\equiv\bar{\phi}$ is satisfied are the self-conjugate channels having $\bar{\phi}\equiv\phi$. These are $\phi=3,6$ for the B_d^0 and $\phi=4,7$ for the B_s^0. A modification of Eq. (11) is needed for these cases because the modes f_3 and f_6 of the B_d^0 and the modes f_4 and f_7 of the B_s^0 are flavorless and may have the same total quantum numbers. Therefore they will in general be mixed by the strong interactions so that the contributions to the asymmetry of the pairs of flavor channels cannot be separated as in Eq. (11).

A first step in the required modification is to identify the eigenmodes associated with such a pair of channels as CP eigenstates, since CP is conserved by the strong interactions. To simplify the notation, we omit the ϕ label and denote these states by $|f_\pm\rangle$ with

$$CP\,|f_\pm\rangle=\pm|f_\pm\rangle . \tag{12}$$

Then, in place of Eq. (4), we have

$$\langle f_\pm|B_d^0\rangle=[U_3a_3(f_\pm)+U_6a_6(f_\pm)]e^{i\delta(f_\pm)} , \tag{13a}$$

where, as before, $a_3(f_\pm)$ and $a_6(f_\pm)$ are real matrix elements of a CP-invariant (and T-invariant) operator. From this CP invariance and Eq. (12) it follows that

$$\langle f_\pm|\bar{B}_d^0\rangle=\pm[U_3^*a_3(f_\pm)+U_6^*a_6(f_\pm)]e^{i\delta(f_\pm)} . \tag{13b}$$

Similarly we find

$$\langle f_\pm|B_s^0\rangle=[U_4a_4(f_\pm)+U_7a_7(f_\pm)]e^{i\delta(f_\pm)} , \tag{13c}$$

$$\langle f_\pm|\bar{B}_s^0\rangle=\pm[U_4^*a_4(f_\pm)+U_7^*a_7(f_\pm)]e^{i\delta(f_\pm)} . \tag{13d}$$

In order to obtain the contribution to the anticharm-charm asymmetry from these amplitudes we must determine the linear combinations of the states $|f_\pm\rangle$ corresponding to states $|f_\pm^0\rangle$ having only open charm, that is, $n_c(f_\pm^0)=2$. Closed charm modes $[n_c(f)=0]$ such as $\psi\pi$, $K\bar{K}$, and purely pionic modes must be excluded. But the states $|f_\pm^0\rangle$ are not, in general, eigenstates of the S matrix because the open and closed modes can be mixed by the final-state interactions.

We note (see Table I) that for the B_d (B_s) channel 6 (7) does not include charmed quarks. Therefore these interactions can produce open charm modes only through the intervention of gluon interactions, which may be treated as a final-state interaction effect to the extent that gluon production at the weak-interaction vertex can be neglected.[7] Therefore it is a good approximation to assume that for these two cases the contributions to the matrix elements directly into the open channels are negligible:

$$\langle f_\pm^0|B_d^0\rangle_{\phi=6}\approx0,\quad \langle f_\pm^0|B_s^0\rangle_{\phi=7}\approx0 . \tag{14}$$

The states $|f_\pm^0\rangle$ may be written as

$$|f_\pm^0\rangle=\cos\beta_\pm|f_\pm^{(1)}\rangle+\sin\beta_\pm|f_\pm^{(2)}\rangle , \tag{15}$$

where $|f_\pm^{(j)}\rangle$ with $j=1,2$ are the two independent eigenstates of the S matrix required to form the open state $|f^0\rangle$. The eigenphases associated with these eigenstates are $2\delta_j$, and $\beta_\pm$ defines the unitary transformation to the corresponding pairs of open and closed states. Therefore, Eqs. (13a), (13c), and (14) lead to

$$\langle f_\pm^0|B_d^0\rangle\approx U_3[a_3(f_\pm^{(1)})e^{i\delta_1}\cos\beta_\pm+a_3(f_\pm^{(2)})e^{i\delta_2}\sin\beta_\pm] , \tag{16a}$$

$$\langle f_\pm^0|B_s^0\rangle\approx U_4[a_4(f_\pm^{(1)})e^{i\delta_1}\cos\beta_\pm+a_4(f_\pm^{(2)})e^{i\delta_2}\sin\beta_\pm] , \tag{16b}$$

and $\langle f_\pm^0|\bar{B}_d^0\rangle$ and $\langle f_\pm^0|\bar{B}_s^0\rangle$ are obtained from Eqs. (13b) and (13d) by replacing U_3 and U_4 in Eqs. (16a) and (16b) with $\pm U_3^*$ and $\pm U_4^*$, respectively. Therefore

$$\langle f_\pm^0|\bar{B}_d^0\rangle/\langle f_\pm^0|B_d^0\rangle=\pm U_3^*/U_3 \tag{17a}$$

and

$$\langle f_\pm^0|\bar{B}_s^0\rangle/\langle f_\pm^0|B_s^0\rangle=\pm U_4^*/U_4 \tag{17b}$$

to a very good approximation.

From these results it follows that the sums of $N(f^0)$ and $\bar{N}(f^0)$ over all self-conjugate channels, which we denote again by N_ϕ and $\bar{N}_\phi$ with $\phi=3$ (4) for the B_d^0 (B_s^0), take the form of Eqs. (8a) and (8b) with $\bar{a}/a$ replaced by ±1 for the even or odd states, respectively. Thus $\langle|\bar{a}/a|^2\rangle_\phi=1$ for $\phi=3,4$. On the other hand, $\langle\bar{a}/a\rangle_\phi$ is small in this case because for almost every even mode there corresponds an odd mode and, on the average, it is to be expected that the branching ratios for these modes will be almost equal as a consequence of the many possible combinations of states. The only exceptions are the pure $D\bar{D}$ and $D_s\bar{D}_s$ modes and the pure $D\bar{D}\pi^0$ and $D_s\bar{D}_s\pi^0$ modes. The former modes must be in CP-even states and the latter in CP-odd states because the D and presumably the D_s have spin 0 and are assumed to satisfy Bose statistics. Therefore their net contribution to $\langle\bar{a}/a\rangle_\phi$ is also expected to be small.

We note further in this case that $|\lambda_\phi|^2=1$ in the approximation $|p/q|^2=1$ so that the contribution to the asymmetry, Eq. (11), for $\phi=3$ or 4 is

$$A_{\phi\bar{\phi}}\equiv A_{\phi\phi}=-[x/(1+x^2)]\mathrm{Im}\lambda(\phi)F , \tag{18}$$

where F is the relative difference between the average branching ratios for the CP-even and the CP-odd modes.

The inclusive asymmetry is obtained from the weighted average of the contributions of Eq. (11) with $\phi=1$ for the B_d^0 and $\phi=2$ for the B_s^0 and the contributions of Eq. (18) with $\phi=3$ and $\phi=4$ for each of these cases. The weights are given by the approximate branching ratios for each term and the B_s^0 to B_d^0 production ratio.

III. THE ASYMMETRY IN TERMS OF KM PARAMETERS

We make use of the parametrization of the KM matrix suggested by Wolfenstein[9] with Wolfenstein's λ replaced by $s_C=\sin\theta_C$ where θ_C is the Cabibbo angle. This is used to find expressions for the V_{ij} shown in Table I. The phase of q/p appearing in $\lambda(\phi)$, given by Eq. (6b), may also be expressed[8] directly in terms of these parameters in the approximation $|p/q|=1$. The resulting values of $\lambda(\phi)$ are as follows:

$$\lambda^d(1)=-s_C^2(\rho-i\eta)[1-(\rho+i\eta)]/[1-(\rho-i\eta)] , \tag{19a}$$

$$\lambda^d(3)=[1-(\rho+i\eta)]/[1-(\rho-i\eta)] , \tag{19b}$$

$$\lambda^s(2)=\rho-i\eta , \tag{19c}$$

$$\lambda^s(4)=e^{2i\eta s_C^2} . \tag{19d}$$

By inserting these expressions into Eq. (11) and Eq. (18) we find

$$A_{18}^d=\frac{-2x_d}{1+x_d^2}\frac{s_C^2\eta(1-\rho^2-\eta^2)}{(1-\rho)^2+\eta^2}\langle\bar{a}/a\rangle_1 , \tag{20a}$$

$$A_{29}^s=\frac{2x_s\eta}{1+x_s^2}\frac{\langle\bar{a}/a\rangle_2}{1+(\rho^2+\eta^2)\langle|\bar{a}/a|^2\rangle_2} , \tag{20b}$$

$$A_{33}^d=\frac{2x_d}{1+x_d^2}\frac{\eta(1-\rho)}{(1-\rho)^2+\eta^2}F^d , \tag{20c}$$

$$A_{44}^s=\frac{-2x_s}{1+x_s^2}\eta s_C^2F^s , \tag{20d}$$

where terms of higher order in $s_C^2\approx 0.05$ have been neglected.

The total inclusive anticharm-charm asymmetry for B^0, $\bar{B}^0$ decay is then given by

$$A(c,\bar{c})=P(B_d^0)A^d(c,\bar{c})+P(B_s^0)A^s(c,\bar{c}) , \tag{21a}$$

where $P(B_d^0)$ and $P(B_s^0)$ are the relative probabilities of producing B_d^0 and B_s^0 and

$$A^d(c,\bar{c})=[(N_1^d+\bar{N}_1^d)A_{18}^d+(N_3^d+\bar{N}_3^d)A_{33}^d]\Big/\sum_{\phi=1}^{12}n_c(f_\phi)(N_\phi^d+\bar{N}_\phi^d) , \tag{21b}$$

with $n_c(f_\phi)=1$ for $\phi=1,2,3,5,9$, $n_c(f_4)=2$, and $n_c(f_\phi)=0$ otherwise. Also

$$A^s(c,\bar{c})=[(N_2^s+\bar{N}_2^s)A_{29}^s+(N_4^s+\bar{N}_4^s)A_{44}^s]\Big/\sum_{\phi=1}^{12}n_c(f_\phi)(N_\phi^s+\bar{N}_\phi^s) \tag{21c}$$

with $n_c(f_\phi)=1$ for $\phi=1,2,4,5,8$, $n_c(f_3)=2$, and $n_c(f_\phi)=0$ otherwise. Use has been made of the relationships $N_\phi+\bar{N}_\phi=N_{\bar{\phi}}+\bar{N}_{\bar{\phi}}$ and $\phi'=\bar{\phi}$ for $(\phi,\phi')_d=(1,8)$ and $(\phi,\phi')_s=(2,9)$. The N_ϕ may be determined from the branching ratios for the channels ϕ except in the cases of the self-conjugate channels $\phi=3,4$. In those special cases N_ϕ is to be determined from the branching ratio for *open* charm modes.

IV. ESTIMATE OF THE ASYMMETRY

On the basis of phase-space considerations and estimates of the magnitudes of the KM matrix elements the expected orders of magnitude of the branching ratios required to evaluate Eq. (21) can be estimated. In order to obtain the order of magnitude of $A(c,\bar{c})$ we make use of the estimated open channel branching ratios[10] for the *pure* $B^0,\bar{B}^0$ states:

$$B(\phi=1)=\bar{B}(\phi=\bar{1})\approx 0.45 ,$$

$$B(\phi=2)=\bar{B}(\phi=\bar{2})\approx 0.40s_C^2 ,$$

$$B(\phi=3)=\bar{B}(\phi=\bar{3})\approx 0.20s_C^2 ,$$

$$B(\phi=4)=\bar{B}(\phi=\bar{4})\approx 0.20 ,$$

where

$$B(\phi_0)=|\langle\phi_0(\text{open})|B^0\rangle|^2\Big/\sum_\phi|\langle\phi(\text{open})|B^0\rangle|^2$$

and $\bar{B}(\phi_0)$ is the corresponding expression with B^0 replaced by $\bar{B}^0$.

The factors $x/(1+x^2)$ are a measure of the mixing and, since strong mixing in the B_d^0, $\bar{B}_d^0$ system has been observed,[11] we take $x_d\approx 1$ to maximize the mixing effect. Conventional wisdom based on Hagelin's[12] estimate of box-diagram contributions to Δm_s would suggest that[13] $x_s\approx x_d s_C^{-2}\approx 20$, leading to very small mixing. Therefore the contributions of B_s to the asymmetry are of the order of 10^{-1} of those associated with the B_d and may be neglected. Thus we find

$$A(c,\bar{c}) \approx -P(B_d^0) s_C^2 \eta [0.4(1-\rho^2-\eta^2)\langle \bar{a}/a \rangle_1 - 0.3(1-\rho)F^d]/[(1-\rho)^2+\eta^2] , \tag{22}$$

where we have made use of

$$(N_{\phi_0}+\bar{N}_{\phi_0}) \Big/ \sum_\phi n_c(f_\phi)(N_\phi+\bar{N}_\phi) = [B(\phi_0)+\bar{B}(\phi_0)] \Big/ \sum_\phi n_c(f_\phi)[B(\phi)+\bar{B}(\phi)] ,$$

which follows from Eqs. (2a) and (2b) in our approximation $|p/q|^2=1$.

The fraction F representing the relative difference between the CP-even and CP-odd modes is expected to be very small. Therefore, in order to obtain an estimate we take $F=0$. We also choose $\rho=\pm\frac{1}{2}$, $\eta=\pm\frac{1}{2}$, values that are consistent with available data on CP violation and meson decay rates.[14]

Finally, it is necessary to determine $\langle \bar{a}/a \rangle_1$. From their definitions (4), $a(f_1)$ and $\bar{a}(f_1)$ are given by

$$e^{i\delta(f_1)} a(f_1) = \langle f_1 | \bar{b}\gamma_\mu(1+\gamma_5)c\bar{u}\gamma_\mu(1+\gamma_5)d | B_d^0 \rangle \tag{23a}$$

and

$$e^{i\delta(f_1)} \bar{a}(f_1) = \langle f_1 | \bar{u}\gamma_\mu(1+\gamma_5)b\bar{d}\gamma_\mu(1+\gamma_5)c | \bar{B}_d^0 \rangle , \tag{23b}$$

where b,c,d,u are spinor field operators for quarks of the indicated flavors and $|f_1\rangle$ is an eigenstate of the S matrix for modes belonging to the flavor channel $\phi=1$. Since

$$\langle \bar{a}/a \rangle = \Sigma_f A_f^* \bar{A}_f / \Sigma_f |A_f|^2 ,$$

where

$$A(f) = a(f)e^{i\delta(f)}$$

and

$$\bar{A}(f) = \bar{a}(f)e^{i\delta(f)} ,$$

we may replace the set of eigenstates in the sum by any complete set of states having the same flavor character. We choose the free-quark spinor states confined to a bag of dimensions no smaller than m_b^{-1} as the complete set. The matrix elements (23a) and (23b) are easily calculated in this case by standard methods. After summing over the quark spins and the directions of the three independent momenta we find

$$\langle \bar{a}/a \rangle_1 = \frac{-\langle (E_b+m_b+E_d-m_d)^2 - [(E_u-m_u)-(E_c-m_c)]^2 \rangle}{(E_b+m_b+E_d-m_d+E_u-m_u-E_c+m_c)^2} , \tag{24}$$

the minus sign resulting from the anticommutation of the c- and u-quark operators that are interchanged in going from $\phi=1$ to $\phi'=8$.

Since the expectation values of the magnitude of the momenta for the plane waves confined to the bag are expected to be of order m_b or smaller we arrive at the estimates

$$E_b \lesssim 2^{1/2} m_b ,$$

$$E_c - m_c \lesssim E_d - m_d \approx E_u - m_u \lesssim m_b ,$$

so that

$$0.40 \lesssim |\langle \bar{a}/a \rangle_1| < 1 . \tag{25}$$

Among these values of the parameters the choice in Eq. (22) of the positive sign for ρ leads to the largest magnitude of the asymmetry (assuming pure B_d production):

$$0.004 \lesssim |A(c,\bar{c})| < 0.01 . \tag{26a}$$

The suggested[14] choice of the minus sign for ρ would lead to

$$0.0008 \lesssim |A(c,\bar{c})| < 0.002 . \tag{26b}$$

V. CONCLUSIONS

We have found that the calculation of the inclusive anticharm-charm asymmetry in the decay of tagged B^0 and $\bar{B}^0$ mesons based on the KM model leads to a result that is independent of final-state interactions to a very good approximation. However, on the basis of current estimates of branching ratios for the various modes and other parameters, such as $x_s=\Delta m_s/\Gamma_s$, the maximum magnitude of the asymmetry to be expected is found to be at most of the order of 1%.[15]

It should be kept in mind that some of these estimates may be in error by significant factors because our choices of the values of ρ,η,x_s,F and the branching ratios are not firmly established. Furthermore, our assumption that $|p/q|^2-1$ is negligible is also based on an unconfirmed theoretical estimate and even then the neglected contributions would be of the same order as those given by Eq. (26b). Therefore a value of $|A(c,\bar{c})|$ smaller than the lower limit given above would be perfectly consistent with the KM model, as would a value somewhat larger than the upper limit given by Eq. (26a). However, we can obtain an absolute upper limit on the magnitude permitted by the KM model by returning to Eq. (21) and inserting only the most general restrictions on the parameters. Since from the Wolfenstein representation of the KM matrix it can be seen that both N_2/N_1 and N_3/N_4 are of the order s_C^2 and since the other factors appearing in Eq. (21) are of order of magnitude 1 or less, we conclude that, on the basis of the KM model,

$$|A(c,\bar{c})| \lesssim s_C^2 \approx 0.05 \ . \tag{27}$$

Thus observation of an inclusive anticharm-charm asymmetry in tagged B^0, $\bar{B}^0$ decay much larger than 5% would raise a serious question about the KM model as a *complete* description of *CP* violation.

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[5]An exception is the cascade mode $B^0 \to \psi K^0 \to \psi\pi\pi$ for which the $\bar{B}^0$ has the same ultimate state, $\bar{B}^0 \to \psi\bar{K}^0 \to \psi\pi\pi$, so that there can be B^0, $\bar{B}^0$ interference resulting in a difference in the cascade decay rates between the physical (tagged) B^0 and $\bar{B}^0$ states. Because the total decay rate of the K is slow relative to the B, the K and ψ are well separated at the time of the $K \to 2\pi$ transitions so that the strong interactions that would mix the $\psi\pi\pi$ state with other self-conjugate states can be ignored. We discuss this mixing of self-conjugate states produced in direct (rather than cascade) transitions in Sec. II.

[6]This refers to the "*CP* taster" method of Bigi and Sanda (Ref. 3).

[7]Contributions of vector-boson loops (penguin) diagrams are neglected here, as are other higher-order radiative gluon corrections to the weak vertices. In particular, the production of $c\bar{c}$ pairs in addition to those identified in Table I is neglected.

[8]R. G. Sachs, *The Physics of Time Reversal* (University of Chicago Press, Chicago, 1987), p. 257. See, also, Wolfenstein (Ref. 3).

[9]L. Wolfenstein, Phys. Rev. Lett. **51**, 1945 (1983).

[10]The estimates of branching ratios for B decays that we use are taken from R. Rückl, Habilitationschrift, University of Munich, 1983 (unpublished); and J. D. Bjorken (private communication).

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[14]The sign of ρ is not determined by these observations but J. Rosner (private communication) suggests that consistency between Hagelin's (Ref. 12) value of Δm_B and the choice $x_d \approx 1$ would require that ρ be negative if an excessively large top-quark mass is to be avoided. However, since Hagelin's calculation is approximate and completely neglects penguin diagrams, this is not a firm conclusion.

[15]Note that, because we have used the approximation $|p/q|^2 = 1$ throughout, a small contribution to the asymmetry of the order of $|p/q|^2 - 1 \lesssim 0.005$ has been neglected. See Sachs, *The Physics of Time Reversal* (Ref. 8), p. 254.