

$\Delta\sigma_L(pp)$ and jet physics

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We show that there is a positive contribution to $\Delta\sigma_L(pp;s) = \sigma_{\text{tot}}(p(+))p(+);s) - \sigma_{\text{tot}}(p(+))p(-);s)$ (where the \pm refer to proton helicities) associated with the pointlike scattering of fundamental constituents. The magnitude and energy dependence of this hard-scattering component are related to the small- x behavior of spin-weighted parton densities. Simple arguments imply that this positive contribution would, at very large s , be larger in absolute value than the negative contribution to $\Delta\sigma_L$ predicted from the exchange of the A_1 Reggeon. Measurements of $\Delta\sigma_L$ in the energy range $\sqrt{s} = 18-30$ GeV should help clarify theoretical ideas associated with the observation of "minijets" and could aid in the prediction of event structure at future high-energy colliders.

I. INTRODUCTION

To understand hadronic cross sections and particle multiplicities at very high energies, it has proven important to examine the interplay between hard, pointlike scattering mechanisms and soft, coherent dynamics. Attention has been focused on this issue by the observation of a large cross section for "minijets" at the CERN $S\bar{p}pS$ collider.^{1,2} Details of the event structure,³ particle multiplicity distributions,⁴ and the energy dependence of the total inelastic cross section⁵ have been related to the dynamics of hard scattering. Although some aspects of the phenomenological analysis remain controversial, the fact that most events contain a large- p_T jet requires that more study be devoted to the subject.

The phenomenological question concerning the impact the jet physics begins with a breakup of the total inelastic cross section

$$\sigma_{\text{inel}}(pp;s) = \sigma_{\text{soft}}(pp;p_0,s) + \sigma_{\text{jet}}(pp;p_0,s), \quad (1.1)$$

where the jet cross section $\sigma_{\text{jet}}(pp;p_0,s)$ is the cross section for observing at least one large- p_T jet ($p_T > p_0$). The split-up in (1.1) is obviously dependent on the cutoff p_0 used to define the jet cross section. If the cutoff is chosen to be large enough, the jet cross section should be given by the integral of the large- p_T differential cross section which is calculable within the framework of the QCD-aided parton model.⁶

Although the basic theoretical justification for using QCD perturbation theory to calculate a wide variety of hard processes has been given a boost by recent work on perturbative factorization,⁷ the split-up in (1.1) is subject to a considerable amount of theoretical uncertainty. This uncertainty is conveniently discussed in the form of the renormalization prescription dependence and the factorization prescription dependence of the low-order QCD calculation.⁸ As an indication of this uncertainty,

theoretical estimates for the magnitude, cutoff dependence, and energy dependence of $\sigma_{\text{jet}}(pp;p_0,s)$ are highly sensitive to the small- x behavior of the quark and gluon distributions in the proton. Alternate prescriptions can differ significantly concerning these distributions. The experimental jet cross section must be measured as a function of energy and cutoff and compared with the calculated values to test the validity of the overall prescription scheme. Only then can the QCD prediction for the jet cross section be connected to predictions for other hard processes.

A closely related application of the ideas behind the decomposition (1.1) involves the measurement of $\Delta\sigma_L(pp;s) = \sigma(p(+))p(+);s) - \sigma(p(+))p(-);s)$ where the $+$ and $-$ refer to proton helicities. Applying the perturbative factorization hypothesis to individual helicity cross section, the decomposition

$$\Delta\sigma_L(pp;s) = \Delta\sigma_L^{\text{soft}}(pp;p_0,s) + \Delta\sigma_L^{\text{jet}}(pp;p_0,s), \quad (1.2)$$

allows an independent test of the ideas behind the hard-scattering model. The differential cross sections for scattering of quarks and gluons from definite-helicity states are known in the large-momentum-transfer limit from QCD perturbation theory.⁹ Measurement of $\Delta\sigma_L^{\text{jet}}(pp;p_0,s)$ for different values of the jet cutoff can therefore give important information about the nature of the helicity-weighted quark and gluon distributions in a polarized proton. These distributions are poorly known. At present we have only a crude idea about them based on measurements in electroproduction¹⁰ and some simple constraints.¹¹

It is particularly interesting to try to understand the helicity-weighted gluon distribution in a polarized proton. Simple arguments based on the Altarelli-Parisi evolution equations¹² suggest that the amount of angular momentum associated with polarized gluons within a polarized proton should grow as the proton is measured by

a probe with increasing Q^2 . Our numerical studies indicate that $\Delta\sigma_L^{\text{jet}}(pp; p_0, s)$ can be sensitive to the behavior of

$$\Delta G_{g/p}(x, \mu^2) = G_{g(+)/p(+)}(x, \mu^2) - G_{g(-)/p(+)}(x, \mu^2)$$

at small x . Since there have been, to date, no hard experimental results concerning $\Delta G_{g/p}(x, \mu^2)$, it would be very valuable to compare data on $\Delta\sigma_L^{\text{jet}}(pp; p_0, s)$ with models of the constituent distributions based on current theoretical wisdom. It is possible, therefore, that early measurements with the proposed Fermilab polarized proton beam will force us to drastically revise basic theoretical concepts involving hadron structure.¹³ It is certain that these measurements will be the first among a variety of high- p_T spin-asymmetry measurements which are necessary if we are to complete our knowledge of the helicity-weighted quark and gluon distributions.¹⁴

Just as in the case of the unpolarized cross sections, the observation of a sizable $\Delta\sigma_L^{\text{jet}}(pp; p_0, s)$ raises some important issues concerning the interplay of coherent hadronic dynamics and pointlike constituent scattering. Consider, for example, the question of the asymptotic behavior of $\Delta\sigma_L(pp; s)$. Unitarity relates $\Delta\sigma_L(pp; s)$ to the imaginary part of an elastic-scattering amplitude with unnatural parity in the t channel,

$$\Delta\sigma_L(pp; s) = \frac{8\pi}{p} \text{Im} U_0(s, 0), \quad (1.3)$$

where U_0 is defined in terms of the s -channel helicity amplitudes for pp elastic scattering by

$$U_0(s, t) = \frac{1}{2} (\langle ++ | ++ \rangle - \langle +- | +- \rangle). \quad (1.4)$$

Conventional Regge-pole phenomenology¹⁵ suggests that the A_1 pole is the leading singularity with unnatural parity which can couple to $U_0(s, t)$ at $t=0$. If we assume that $U_0(s, t)$ displays coherent, Regge, behavior at high energy we get

$$\Delta\sigma_L(pp; s) \simeq \beta(0)(s/s_0)^{\alpha_{A_1}(0)-1}, \quad (1.5)$$

where the intercept of the A_1 Regge trajectory is approximately

$$\alpha_{A_1}(0) \simeq -0.15. \quad (1.6)$$

This implies an energy dependence

$$\Delta\sigma_L(pp; s) \sim s^{-1.15}. \quad (1.7)$$

This assumption has proven to be adequate for describing the energy dependence of $\Delta\sigma_L$ over the range measured with polarized beam and target at the Argonne Zero Gradient Synchrotron (ZGS).¹⁶ The parametrization (1.5) has been used to provide an estimate for $\Delta\sigma_L$ at higher energies. Normalizing the asymptotic prediction to the measured value at $p_{\text{lab}} = 11.75$ GeV (Ref. 16) of -500 ± 50 μb yields

$$\Delta\sigma_L \simeq -17.5(s/s_0)^{-1.15} \text{ mb}, \quad (1.8)$$

with $s_0 = 1$ GeV². For $p_{\text{lab}} = 200$ GeV/ c , this gives an estimate

$$\Delta\sigma_L(p_{\text{lab}} = 200 \text{ GeV}/c) \simeq -19 \mu\text{b}. \quad (1.9)$$

Equation (1.8) gives the value to which we must compare our expectations for $\Delta\sigma_L^{\text{jet}}$. It is interesting to keep in mind that the value predicted by Regge theory is negative.

In contrast, the incoherent pointlike contribution to $\Delta\sigma_L^{\text{jet}}$ associated with quark-quark, quark-gluon, and gluon-gluon scattering in (1.2) should provide a positive contribution to $\Delta\sigma_L$. The reason for the positive sign is straightforward. It depends on the fact that the dominant underlying two-spin asymmetries in perturbative QCD are positive and that, based on our current ideas, there should exist a positive correlation between the spin of the proton and the spin of its constituents. When we look more carefully at the asymptotic behavior of $\Delta\sigma_L^{\text{jet}}$ a surprise emerges. As we shall demonstrate, the asymptotic behavior of $\Delta\sigma_L^{\text{jet}}$ depends on the small- x behavior of the spin-weighted quark and gluon distributions. If the distributions have the behavior near $x=0$,

$$\lim_{x \rightarrow 0} \Delta G_{i/p}(x, \mu^2) = ax^{-J}, \quad (1.10)$$

then the asymptotic behavior of the jet cross section is

$$\Delta\sigma_L^{\text{jet}} = \beta s^{J-1} \ln(s/s_0). \quad (1.11)$$

Simple model estimates of the small- x behavior of the helicity-weighted quark and gluon distributions suggest that the value of J in (1.10) should be near $\alpha_p(0) \simeq \frac{1}{2}$ so $\Delta\sigma_L^{\text{jet}}$ may, in fact, be larger in absolute value than the Regge estimate for $\Delta\sigma_L$, (1.8), at asymptotic values of s . We will investigate this question in more detail below. In principle, the small- x behavior of the constituent distributions can be measured in a variety of processes. Measurement of the energy dependence of $\Delta\sigma_L^{\text{jet}}$ for fixed cutoff can therefore provide important new information concerning the regime where the hard-scattering approximation is valid. Our numerical estimates for the jet contribution to the cross section using the available models¹¹ for the spin-weighted distributions give

$$\Delta\sigma_L^{\text{jet}}(pp; p_0 = \sqrt{5} \text{ GeV}, \sqrt{s} = 20 \text{ GeV}) \simeq 1.4 \mu\text{b}, \quad (1.12)$$

which can be compared to (1.9). While the jet component of $\Delta\sigma_L^{\text{jet}}$ is expected to be smaller than the coherent component in this energy range, it cannot be neglected completely. Based on current models for the constituent distributions, we estimate

$$|\Delta\sigma_L^{\text{jet}}(pp; p_0^2 = 5 \text{ GeV}^2, \sqrt{s})| > |\Delta\sigma_L(\sqrt{s})|_{A_1} \quad (1.13)$$

for $\sqrt{s} > 40$ GeV.

We will organize the remainder of the paper as follows. Section II provides a brief review of the issues in jet physics implied by (1.1). Section III shows the calculation of $\Delta\sigma_L^{\text{jet}}$ within the framework of the simple parton model and discusses the expectations for the spin-weighted distributions. By using some simple models for these distributions, we can calculate the range of possibilities for $\Delta\sigma_L^{\text{jet}}$. Section IV concludes with some discussion about the experimental situation and the interpretation of these estimates.

II. HARD COLLISIONS AND INELASTIC CROSS SECTIONS

Long after the acceptance of QCD perturbation theory for applications in other types of hard processes, there survived specialized *ad hoc* models¹⁷ to account for the production of particles at large transverse momentum in hadron-hadron collisions. In order to make predictions for high- p_T hadron production, it is necessary to understand the factorization of perturbative processes so that distributions obtained from other measurements can be used.⁷ The inclusion of gluons as active constituents of the incident hadron makes cross-section predictions sensitive to the uncertainties in the gluon distribution. It is also important to have a quantitative description of scaling violations in order to predict high- p_T production in hadronic collisions. A considerable body of theoretical work has been done which clarifies the role of QCD perturbation theory in hadronic processes and legitimizes the factorization hypothesis of QCD-aided parton model.¹⁸ In addition, the construction of Monte Carlo event simulation programs^{19,20} which incorporate important aspects of the leading-logarithm perturbative QCD processes has allowed the comparison of chromodynamics with a wide variety of experimental observables accessible in

hadron-hadron collisions. Although there are some remaining uncertainties, QCD calculations are now routinely used to model backgrounds for the production of hypothetical new particles.²¹

The normalization of the hard-scattering component in hadron-hadron collisions has been the focus of renewed theoretical attention. As mentioned in the Introduction, in order to understand the energy dependence of the $\bar{p}p$ cross section,¹ the real part of the elastic-scattering amplitude, multiplicity distributions,⁴ and other features of the event structure in high-energy collisions, it has proved useful to focus on the separation of the dynamics of the hard-scattering of constituents from the coherent effects associated with the whole hadron. The separation indicated in Eq. (1.1) expresses this concern. The important application of (1.1) involves the ability to calculate $\sigma^{\text{jet}}(pp; p_0, \sqrt{s})$ for a range of cutoffs and energies using QCD perturbation theory.

The comparison of any calculation done within the framework of the QCD-augmented parton model with experimental data involves considerable phenomenological judgment. To indicate the type of judgment involved we remind the reader that each prediction involves a choice of factorization and renormalization prescriptions.⁸ Consider a particular hadronic observable, such as

$$d\sigma(pp \rightarrow \text{jet} + X) = \frac{\alpha_s^2(\mu^2)}{\pi} \sum_{ij} \int dx_1 dx_2 G_{i/p}(x_1, \mu^2) G_{j/p}(x_2, \mu^2) \times d\hat{\sigma}(ij \rightarrow \text{jet} + X) \left[1 + K(x_1, x_2, \mu^2) \frac{\alpha_s}{\pi}(\mu^2) + \dots \right], \quad (2.1)$$

which involves subprocesses which can be calculated in QCD perturbation theory. In order to specify the calculation, we must decide the manner in which ultraviolet divergences are regularized and absorbed into the definition of the expansion parameter $\alpha_s(\mu^2)$. This procedure is called a renormalization prescription. In addition, we must pay attention to the manner in which the IR and collinear divergences are absorbed into the definition of the constituent distribution functions $G_{i/p}(x, \mu^2)$. In analogy to the renormalization problem, this procedure is labeled a factorization prescription.⁸ The definitions of α_s and the $G_{i/p}$ enter into the magnitude of the higher-order corrections indicated in (2.1). Only for a limited range of renormalization and factorization prescriptions can a specific calculation have predictive power. In the discussion which follows we will simplify the overall process. The renormalization prescription will be done by choosing $\alpha_s = \alpha_s^{\overline{\text{MS}}}$ ($\overline{\text{MS}}$ denotes the modified minimal-subtraction scheme.) This leaves open only the question of factorization prescription. The constituent distributions are specified to obey the Altarelli-Parisi equations.²³ The factorization scale μ^2 can then be related to the available kinematic variables p_0^2 , and s in a systematic way to complete the prescrip-

tion. There have been two theoretical approaches for controlling this remaining uncertainty. One approach, fastest apparent convergence,²⁴ attempts to push as much as possible of the higher-order corrections into "physical" definitions of the expansion parameter and constituent distributions. The second approach, principle of minimal sensitivity,²⁵ keeps track of the scale parameters which denote the prescriptions and then, for each process, chooses a prescription which produces an extremum. Phenomenological studies on processes where higher-order corrections are available support the idea that either approach can be used to control the impact of higher-order corrections.^{8,26}

For a more complete discussion of the phenomenological impact of prescription specification in a closely related process where the perturbative calculation has been extended beyond leading order, the reader is referred to Aurenche *et al.*²⁷ These considerations are extremely important in the calculation of the hadronic production of high- p_T jets.

We want to review briefly the calculation of $\sigma_{\text{jet}}(pp; p_0, \sqrt{s})$ in order to specify our notation. We can write

$$\sigma_{\text{jet}}(pp; p_0, \sqrt{s}) = \frac{\pi \alpha_s^2(\mu^2)}{2s} \sum_{ij} \int_{x_1 x_2 > \xi} dx_1 dx_2 \frac{G_{i/p}(x_1, \mu^2)}{x_1} \frac{G_{j/p}(x_2, \mu^2)}{x_2} H_{ij}(z_0), \quad (2.2)$$

where $\xi = 4p_0^2/s$ and $z_0 = (1 - \xi/x_1x_2)^{1/2}$. The cross-section factor H_{ij} is

$$H_{ij}(z_0) = \int_{-z_0}^{+z_0} dz \left[\frac{2s}{\alpha_s^2} \frac{d\sigma_{ij}}{dx_1 dx_2 dz}(z) \right]. \quad (2.3)$$

The angular factors $H_{ij}(z_0)$ are tabulated for the different subprocesses in Table I. In writing (2.2) we have left open for the time being the choice of the factorization scale μ^2 . We are interested in the behavior of the cross section in the limit $\xi = 4p_0^2/s \rightarrow 0$. The integration in (2.2) is then dominated by the region of small x . The energy dependence and cutoff dependence of the jet cross section defined in (2.2) are therefore seen to be dependent on the small- x behavior of the constituent distribution functions. We can use a simple analytic exercise to demonstrate this behavior. We adopt the single effective subprocess approximation²⁸ for (2.2) with

$$G(x, \mu^2) \simeq G_{g/p}(x, \mu^2) + \frac{4}{9} \sum_{q=u,d,s} [G_{q/p}(x, \mu^2) + G_{\bar{q}/p}(x, \mu^2)] \quad (2.4)$$

and

$$H(z_0) \simeq \frac{144}{1-z_0^2} (1 + \dots). \quad (2.5)$$

We then make a simple power approximation for the small- x behavior of the effective constituent distribution $G(x, \mu^2)$:

$$\lim_{x \rightarrow 0} G(x, \mu^2) = a(\mu^2) x^{-J(\mu^2)}. \quad (2.6)$$

The integral (2.2) then becomes

$$\sigma_{\text{jet}}(pp; p_0, \sqrt{s}) \simeq (18\pi) \frac{\alpha_s^2(\mu^2)}{p_0^2} a^2(\mu^2) \int_{\xi}^1 dx_1 x_1^{-J} \int_{\xi/x_1}^1 dx_2 x_2^{-J} (1 + \dots). \quad (2.7)$$

This leads to

$$\sigma_{\text{jet}}(pp; p_0, \sqrt{s}) \simeq (18\pi) \frac{a^2(\mu^2) \alpha_s^2(\mu^2)}{p_0^2} \left[\frac{1}{J-1} \right] \left[\frac{s}{4p_0^2} \right]^{J-1} \ln \left[\frac{B_S}{4p_0^2} \right], \quad (2.8)$$

TABLE I. Unpolarized angular terms. We write the integrated partonic cross section $H_{ij}(z_0)$ defined in Eq. (2.3) in the form $H_{ij}(z_0) = a_{ij} \ln[(1+z_0)/(1-z_0)] + b_{ij} z_0 + c_{ij} z_0^3 + d_{ij} z_0/(1-z_0^2)$. Cross sections for nonidentical particles have been multiplied by 2 to give the jet cross section. The coefficients of the angular terms are listed below.

Process	$\ln[(1+z_0)/(1-z_0)]$	Coefficients		
		z_0	z_0^3	$z_0/(1-z_0^2)$
$uu \rightarrow uu$	$-\frac{256}{27}$	$\frac{32}{9}$	0	$\frac{256}{9}$
$dd \rightarrow dd$	$-\frac{64}{9}$	$\frac{32}{9}$	0	$\frac{256}{9}$
$ud \rightarrow ud$	$-\frac{112}{9}$	$\frac{88}{9}$	0	64
$du \rightarrow du$	$-\frac{112}{9}$	$\frac{88}{9}$	0	64
$ug \rightarrow ug$	$-\frac{112}{9}$	$\frac{88}{9}$	0	64
$dg \rightarrow dg$	$-\frac{112}{9}$	$\frac{88}{9}$	0	64
$gu \rightarrow gu$	$-\frac{112}{9}$	$\frac{88}{9}$	0	64
$gd \rightarrow gd$	$-\frac{112}{9}$	$\frac{88}{9}$	0	64
$gg \rightarrow gg$	-36	$\frac{99}{2}$	$\frac{3}{2}$	144
$gg \rightarrow uu$	$\frac{8}{3} N_f$	$-\frac{25}{6} N_f$	$-\frac{1}{2} N_f$	0
$uu \rightarrow uu$	$-\frac{128}{27}$	$\frac{16}{9}$	$\frac{16}{27}$	$\frac{256}{9}$
$ud \rightarrow ud$	$-\frac{32}{9}$	$\frac{16}{9}$	0	$\frac{128}{9}$
$uu \rightarrow gg$	$\frac{256}{27}$	$-\frac{400}{27}$	$-\frac{16}{9}$	0
$uu \rightarrow dd$	0	$\frac{16}{9} (N_f - 1)$	$\frac{16}{27} (N_f - 1)$	0

with $B = \exp[-(J-1)^{-1}]$. As has been emphasized by Collins,³ the study of scaling violations of the gluon distribution leads to a singularity as $x \rightarrow 0$ in (2.6) which is above one. Arguments based on QCD perturbation theory²⁹ suggest a leading singularity

$$J(\mu^2) = 1 + (12 \ln 2) \frac{\alpha_s(\mu^2)}{\pi} + O(\alpha_s^2) \approx 1.67 \quad (\alpha_s = 0.25), \quad (2.9)$$

which is significantly above unity. This small- x behavior of the effective distribution is consistent with the rapid growth with energy of the experimentally observed jet cross section measured at the CERN $S\bar{p}p$ S collider.¹⁻⁴

The cross section, σ_{jet} in (2.2), is normalized so that

$$\sigma_{\text{jet}}(pp; p_0, \sqrt{s}) = \langle n_{\text{jet}}(p_0, \sqrt{s}) \rangle \sigma_{\text{inel}}(\sqrt{s}). \quad (2.10)$$

The constraints of unitarity impinge on the calculation of σ_{jet} in two ways. First, they lead to a modification of the power law in (2.8). Specific contributions arising from the shadowing corrections which affect the small- x behavior of the distribution, $G(x, \mu^2)$, have been discussed by Mueller and Qiu.³⁰ Second, the contribution of multiple hard-scattering processes leads to a correction in the identification of the hard-scattering cross section calculated in (2.8) with the jet cross section $\sigma_{\text{jet}}(pp; p_0, s)$. Durand and Pi⁵ have proposed a modification of the separation (1.1) of the hard and coherent processes which is amenable to the inclusion of hadronic unitarity effects and explicitly allows for the possibility of multiple hard scattering.³¹ In the regime under investigation in this paper, however, $\langle n_{\text{jet}} \rangle \ll 1$ and therefore such multiple-scattering effects may be neglected.

Because the issues remain of theoretical interest, it is important to approach the overall subject using another experimental observable. As we shall see, the closely related observable $\Delta\sigma_L^{\text{jet}}$ provides some important parallels which may help resolve some of these dynamical questions. Our numerical estimate of $\sigma_{\text{jet}}(pp; p_0, \sqrt{s})$ is shown in Fig. 1 for $p_0^2 = 5 \text{ GeV}^2$ and $\mu^2 = 9 \text{ GeV}^2$ as a

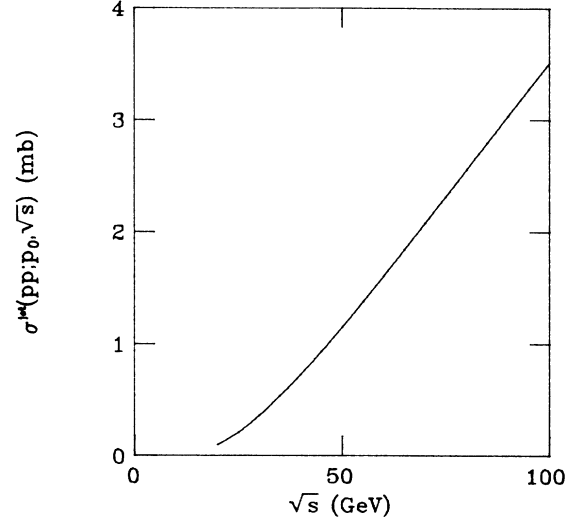


FIG. 1. The unpolarized jet cross section $\sigma^{\text{jet}}(pp; p_0, \sqrt{s})$ is shown as a function of \sqrt{s} with cutoff $p_0^2 = 5 \text{ GeV}^2$ and renormalization scale $\mu^2 = 9 \text{ GeV}^2$. The distributions used are those of Gluck, Hoffmann, and Reya (Ref. 32).

function of energy.³² Because of the caveats mentioned above we have faith in the split-up proposed in (1.1) only when $\sigma_{\text{jet}} \ll \sigma_{\text{inel}}$ and, for these assumptions, this condition begins to fail at energies above $\sqrt{s} = 200 \text{ GeV}$. While a more robust theoretical formalism may be necessary at TeV energies, we will pattern our initial approach to spin-weighted cross sections on (2.2) above.

III. AN ESTIMATE FOR $\Delta\sigma_L^{\text{jet}}(pp; p_0, \sqrt{s})$

As discussed in the Introduction, the theoretical framework of the QCD-based parton model can be used to estimate a contribution to $\Delta\sigma_L(pp; s)$ associated with the hard scattering of fundamental constituents. We hypothesize the split-up of $\Delta\sigma_L$ given in (1.2). Following the procedures for the unpolarized jet cross section we can then obtain the formula

$$\Delta\sigma_L^{\text{jet}}(pp; p_0, \sqrt{s}) = \frac{\pi\alpha_s^2(\mu^2)}{2s} \sum_{i,j} \int_{x_1 x_2 > \xi} dx_1 dx_2 \frac{\Delta G_{i/p}(x_1, \mu^2)}{x_1} \frac{\Delta G_{j/p}(x_2, \mu^2)}{x_2} \Delta H_{ij}(z_0). \quad (3.1)$$

The basic kinematic variables $\xi = 4p_0^2/s$ and $z_0 = (1 - \xi/x_1 x_2)^{1/2}$ are the same as in (2.2). The spin-weighted factors ΔH_{ij} are obtained from the integral over the range of allowed angles of the basic $2 \rightarrow 2$ processes in the same way that they are found for the unpolarized case. Table II gives these factors for the different quark and gluon processes. The spin-weighted distribution functions $\Delta G_{i/p}(x, \mu^2)$ give the probability for finding a given constituent in the proton with its helicity aligned with that of the proton minus the probability with helicity opposite that of the proton

$$\Delta G_{i/p}(x, \mu^2) = G_{i(+)/p(+)}(x, \mu^2) - G_{i(-)/p(+)}(x, \mu^2). \quad (3.2)$$

We will be dealing with massless quarks and gluons so that the (+) and (-) are the only allowed helicities. As discussed above, we will leave the choice of factorization scale, μ^2 , open.

A major aspect in the comparison of the formalism for jets from unpolarized beams with that of $\Delta\sigma_L^{\text{jet}}(pp; p_0, \sqrt{s})$ hinges on our considerable lack of knowledge concerning the $\Delta G_{i/p}(x, \mu^2)$. There has been recent interesting in understanding of spin structure of the proton as measured by the $\Delta G_{i/p}(x, \mu^2)$ but the results are not yet conclusive.

In unpolarized electroproduction, measurements have been performed over a range of Q^2 and x so that the data provide a nontrivial test of ideas concerning the constituent distribution functions. Although electroproduction

TABLE II. Spin-weighted angular terms. The spin-weighted, integrated, partonic cross sections, $\Delta H_{ij}(z_0)$, are given in terms of their angular coefficients below.

Process	$\ln[(1+z_0)/(1-z_0)]$	Coefficients	
		z_0	z_0^3
$uu \rightarrow uu$	$\frac{128}{27}$	$-\frac{32}{9}$	0
$dd \rightarrow dd$	$\frac{64}{9}$	$-\frac{32}{9}$	0
$ud \rightarrow ud$	$\frac{176}{9}$	$-\frac{88}{9}$	0
$dg \rightarrow dg$	72	$-\frac{99}{2}$	$-\frac{3}{2}$
$gu \rightarrow gu$	$-\frac{8}{3}N_f$	$\frac{25}{6}N_f$	$\frac{1}{2}N_f$
$gd \rightarrow gd$	$\frac{128}{27}$	$-\frac{16}{9}$	$-\frac{16}{27}$
$gg \rightarrow gg$	$\frac{64}{9}$	$-\frac{32}{9}$	0
$gg \rightarrow u\bar{u}$	$-\frac{256}{27}$	$\frac{400}{27}$	$\frac{16}{9}$
$u\bar{u} \rightarrow u\bar{u}$	0	$-\frac{16}{9}(N_f-1)$	$-\frac{16}{27}(N_f-1)$
$u\bar{u} \rightarrow g\bar{g}$			
$u\bar{u} \rightarrow d\bar{d}$			

experiments do not provide a direct measurement of the gluon distribution, the shape of the gluon distribution is constrained by data on the scaling violations of the quark distributions. There are in existence a number of fits to the electroproduction data.^{32,33} The different fits (using different underlying assumptions) then give a good indication of the experimental and theoretical uncertainties.

Data on electroproduction with a polarized beam and target¹⁰ can, in principle, be used to extract the same type of information about the $\Delta G_{i/p}(x, \mu^2)$. However, to date, experimental information is available only in the limited range of Q^2 and x . The shape of the gluon distribution is not constrained by the data. This means that the results of calculations involving $\Delta G_{i/p}(x, \mu^2)$ are considerably more “model dependent” than are those which involve the $G_{i/p}(x, \mu^2)$. There have been a number of efforts to estimate the spin-weighted distributions using the polarized electroproduction data plus some simple theoretical ideas. The range of allowable theoretical models is quite large and there are few useful constraints.^{9,11}

Before proceeding with numerical results, we want to show that there exists an interesting puzzle concerning the asymptotic behavior of $\Delta\sigma_L$.

In order to obtain a crude estimate for the asymptotic behavior of $\Delta\sigma_L^{\text{jet}}(pp; p_0, \sqrt{s})$ we will initially assume that the process is demonstrated by the uu scattering contribution. We will also assume a simple approximation for the small- x region of the spin-weighted distribution

$$\Delta u(x, \mu^2) \simeq a_u(\mu^2)x^{-J(\mu^2)}. \quad (3.3)$$

Based on other ideas, we would expect $J \simeq \alpha_p(0) + O(\alpha_s/\pi)$ in analogy to the association of the small- x behavior of the nonsinglet distributions with the intercept of the Pomeron. Using the form of the angular factor

$$\Delta H_{uu} = \frac{8}{9} \left[c_0 + a_0 \ln \left[\frac{x_1 x_2}{\xi} \right] + \sum_{n=1}^{\infty} c_n \xi_n x_1^{-n} x_2^{-n} \right], \quad (3.4)$$

with $c_0 = (\frac{16}{3}\ln 4 - 4)$, $a_0 = \frac{16}{3}$, ... from Table II we use (3.3) and (3.4) to get

$$\Delta\sigma_L^{\text{jet}}(pp; \xi, s) \simeq \frac{\pi\alpha_s^2}{2s} a_u^2(\frac{8}{9}) \xi^{-J} [A(J) + B(J)\ln\xi], \quad (3.5)$$

where

$$A(J) = -\frac{32}{3} \frac{1}{J^3} - (\frac{16}{3}\ln 4 - 4) \frac{1}{J^2} - \sum_{n=1}^{\infty} \frac{c_n}{(J+n)^2}, \quad (3.6)$$

$$B(J) = -\frac{16}{3} \frac{1}{J^2} - (\frac{16}{3}\ln 4 - 4) \frac{1}{J} - \sum_{n=1}^{\infty} \frac{c_n}{J+n}.$$

If our current ideas concerning the small- x behavior of the structure functions are found to be correct and (3.3) is valid with $J = \alpha_p(0) \simeq \frac{1}{2}$, then we have

$$\Delta\sigma_L^{\text{jet}} \simeq s^{-1/2} (A + B \ln s). \quad (3.7)$$

Note that this dominates asymptotically any contribution to $\Delta\sigma_L$ associated with the exchange of the A_1 trajectory. One possibility which must be considered to allow the smooth merger of the hard-scattering contribution and the coherent contribution is that the small- x behavior of the spin-weighted quark distribution is, instead, related to the intercept of the A_1 trajectory

$$\lim_{x \rightarrow 0} \Delta u(x, \mu^2) \simeq a_u(\mu^2) x^{-\alpha_{A_1}^{(0)}}, \quad (3.8)$$

$$\lim_{x \rightarrow 0} \Delta d(x, \mu^2) \simeq a_d(\mu^2) x^{-\alpha_{A_1}^{(0)}}.$$

Although this behavior is not postulated in the models for these functions, it is not ruled out by the data. It is a symptom of our lack of knowledge concerning these distributions that this possibility is still viable.

The assumption that one subprocess dominates the asymptotic behavior of $\Delta\sigma_L^{\text{jet}}$ is considerably less believable than is the case for the single effective process approximation for the unpolarized scattering. However, the puzzle concerning the asymptotic behavior of $\Delta\sigma_L$ only becomes more complicated when these many processes are considered. The underlying contradiction is not resolved.

As can be seen from Table II, different subprocesses provide both positive and negative contributions to $\Delta\sigma_L^{\text{jet}}(pp; p_0, s)$. At very large \sqrt{s} , the gg process is significant and the behavior of the helicity-weighted gluon distribution near $x=0$ is particularly important. From a study of the evolution equation it has been conjectured¹²

$$\lim_{x \rightarrow 0} \frac{\Delta G(x, \mu^2)}{G(x, \mu^2)} \sim x \exp\{c [\ln \ln(\mu^2/\mu_0^2) \ln(1/x)]^{1/2}\}. \quad (3.9)$$

The estimate above can be combined with the hypothetical small- x behavior of the gluon distribution given in (2.6) to give the behavior of the gluon-gluon and gluon-quark processes. We then arrive at the conclusion that the contributions to $\Delta\sigma_L^{\text{jet}}$ from these processes should also fall less rapidly than the Regge prediction for $\Delta\sigma_L$ from A_1 exchange.

These arguments indicate that there exists an interesting relationship between the hard-scattering mechanisms of QCD and the coherent processes which lead to $\Delta\sigma_L$. The situation is similar to that of the unpolarized cross section except that the ‘‘contradiction’’ concerning the asymptotic behavior of the hard component here involves the spin-weighted valence-quark distributions. Unlike the gluon distribution, these can be measured in polarized electroproduction experiments and the small- x behavior, (3.3), can be checked. The shadowing corrections, multiparton processes, and unitarity constraints involved in the interpretation of the unpolarized jet data^{5,31} should have their place in understanding $\Delta\sigma_L^{\text{jet}}$. The final resolution may also involve new concepts.

To understand the question of the importance of the hard component to $\Delta\sigma_L$ at a given energy, we must do more than the simple analytic estimates above. In the following numerical estimates, our approach will be to calculate using (3.1) with our best guess concerning the constituent distributions.

We will employ the parametrizations of the structure functions obtained by Chiappetta and Soffer,³⁴ which we will review briefly here. The polarized valence-quark distributions

$$\Delta q(x, \mu^2) = \Delta G_{q/p}(x, \mu^2) \quad (3.10)$$

are given by

$$\Delta u_v(x, \mu^2) = [u_v(x, \mu^2) - \frac{2}{3}d_v(x, \mu^2)] \times \left[1 + H_0(\mu^2) \frac{(1-x)^2}{\sqrt{x}} \right]^{-1}, \quad (3.11a)$$

$$\Delta d_v(x, \mu^2) = -\frac{1}{3}d_v(x, \mu^2) \left[1 + H_0(\mu^2) \frac{(1-x)^2}{\sqrt{x}} \right]^{-1}, \quad (3.11b)$$

where $u_v(x, \mu^2)$ and $d_v(x, \mu^2)$ are the unpolarized up and down valence-quark distributions extracted by Gluck, Hoffmann, and Reya.³² The initial polarized gluon and sea-quark distributions are assumed to be generated from the polarized valence distributions by a process of bremsstrahlung and quark-antiquark pair creation, with the normalization determined by requiring that the sum of the third components of the spins of the constituents be $\frac{1}{2}$ (Ref. 9). Explicitly the gluon and sea distributions are given by

$$\begin{aligned} \Delta g_s(x, \mu_0^2) &= 0.0327x(2-x)(1-x)^{6.5}, \\ \Delta g(x, \mu_0^2) &= 0.141x(5-2x)(1-x)^5, \end{aligned} \quad (3.12)$$

with

$$\mu_0^2 = 5 \text{ GeV}^2.$$

The distributions are evolved according to the Altarelli-Parisi equations, and the energy dependence of the spin dilution parameter H_0 of Eq. (3.11) is extracted by requiring the the Bjorken sum rule be satisfied. The value used is $H_0 = 0.114$ at $\mu^2 = 5 \text{ GeV}^2$.

We now address the choice of renormalization and factorization prescription. Any comprehensive treatment of this question must await the calculation of higher-order corrections to the processes tabulated in Table II. However it is clear that the renormalization scale should be chosen to be of order p_0^2 , the typical momentum transfer occurring in the interaction. Furthermore, if we set $\mu^2 = \lambda p_0^2$ then we wish $\Delta\sigma_L^{\text{jet}}$ to be reasonably insensitive to changes in λ . For our numerical calculations our philosophy is just that of the principle of minimal sensitivity²⁵ alluded to earlier. This issue is well illustrated in Fig. 2 where we show $\Delta\sigma_L^{\text{jet}}$ as a function of \sqrt{s} at fixed cutoff $p_0^2 = 5 \text{ GeV}^2$ for $\mu^2 = 5, 9, \text{ and } 20 \text{ GeV}^2$, the first value being just that of the starting distributions of Eq. (3.11). Whereas the calculations employing the higher two energy scales exhibit qualitatively similar behavior, the calculation at $\mu^2 = p_0^2$ peaks at a very much smaller value of \sqrt{s} . This qualitative difference in the behavior is attributable purely to the large increase in the gluon distribution at small x as the energy scale μ^2 is increased. Thus to the extent that the dichotomy into a hard and a soft cross section exhibited in Eq. (2.1) is indeed valid at p_0^2 as low as 5 GeV^2 , the choice of factorization scale $\mu^2 = 2p_0^2$ would seem to yield more stable results than $\mu^2 = p_0^2$, and

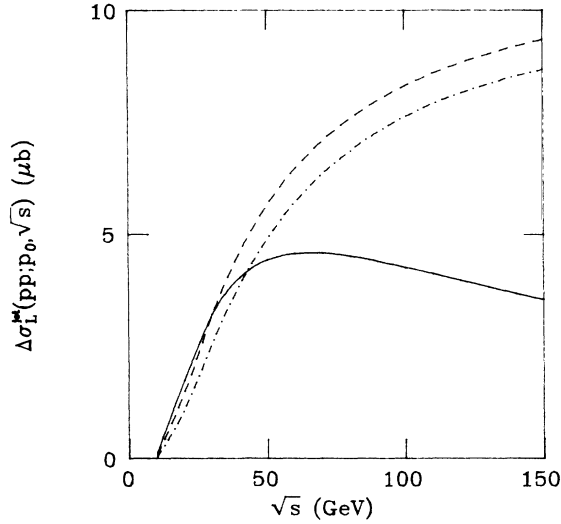


FIG. 2. The polarized jet cross section $\Delta\sigma_L^{\text{jet}}(pp; p_0, \sqrt{s})$ is shown as a function of \sqrt{s} with cutoff $p_0^2 = 5 \text{ GeV}^2$. The three curves correspond to $\mu^2 = 5 \text{ GeV}^2$ (solid line), $\mu^2 = 9 \text{ GeV}^2$ (dashed line), and $\mu^2 = 20 \text{ GeV}^2$ (dot-dash line).

we shall use this factorization prescription in the remainder of this section. A rather more informative, but equivalent, interpretation of this choice is that our numerical results should not depend strongly on the energy scale at which the starting distributions of Eq. (3.11) are defined.

Given this prescription choice, we plot the energy dependence of $\Delta\sigma_L^{\text{jet}}(pp; p_0, \sqrt{s})$ for $p_0^2 = 5 \text{ GeV}^2$ in Fig. 3 and compare it to the energy dependence for $\Delta\sigma_L(pp; \sqrt{s})$ predicted from the Regge plot fit in Eq. (1.8). As discussed above, the hard-scattering contribution dominates for $\sqrt{s} > 40 \text{ GeV}$. At $\sqrt{s} = 20 \text{ GeV}$, where some data should be available within the next

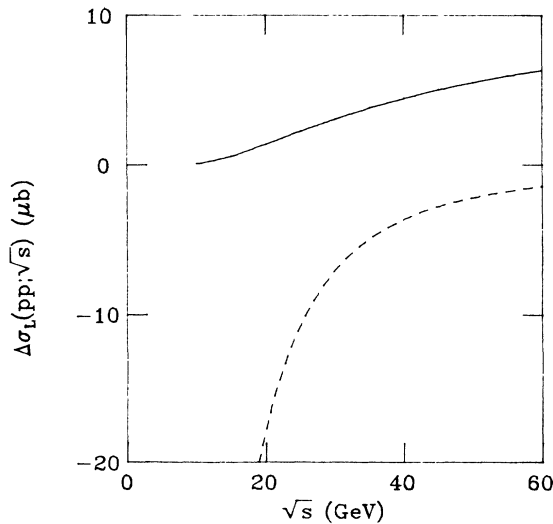


FIG. 3. The dashed line represents the expectation for $\Delta\sigma_L(pp; \sqrt{s})$ as a function of \sqrt{s} using Eq. (1.8). The solid line is $\Delta\sigma_L^{\text{jet}}(pp; p_0, \sqrt{s})$ at fixed cutoff $p_0^2 = 5 \text{ GeV}^2$ and $\mu^2 = 10 \text{ GeV}^2$ as a function of \sqrt{s} .

Rel. Contributions to $\Delta\sigma_L^{\text{jet}}$ at $\sqrt{s} = 20 \text{ GeV}$

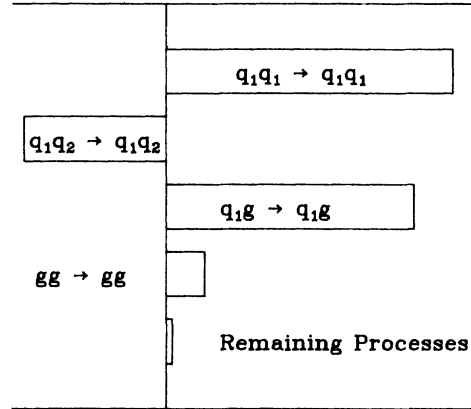


FIG. 4. The relative contributions of different processes to $\Delta\sigma_L^{\text{jet}}(pp; p_0, \sqrt{s})$ are shown at $p_0^2 = 5 \text{ GeV}^2$, $\mu^2 = 10 \text{ GeV}^2$, and $\sqrt{s} = 20 \text{ GeV}$.

year,¹³ the contributions to $\Delta\sigma_L^{\text{jet}}(pp; p_0, \sqrt{s})$ are dominated by the valence-quark subprocess. This is shown in Fig. 4 where the contributions from the different subprocesses are separated. In fact, as can be seen in Fig. 5, the valence-quark contribution to the unpolarized jet cross section $\sigma_{\text{jet}}(pp; p_0, \sqrt{s})$ is also large at this energy. Measurements of $\Delta\sigma_L^{\text{jet}}(pp; p_0, \sqrt{s})$ in this energy range should give a good indication of the basic viability of the hard-scattering formalism involving polarized constituents. In addition, the behavior of the cross sections for a limited range of \sqrt{s} and p_0 should give some idea whether the constituent distributions follow the expected pattern.

Rel. Contributions to σ^{jet} at $\sqrt{s} = 20 \text{ GeV}$

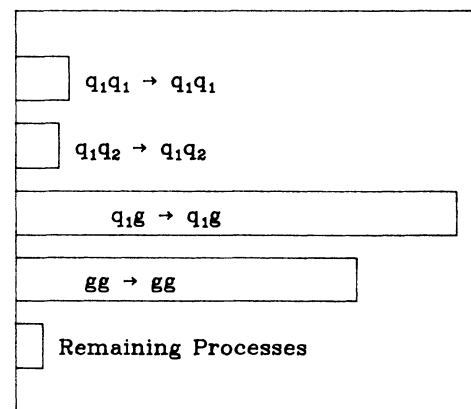


FIG. 5. The relative contributions of different processes to $\sigma^{\text{jet}}(pp; p_0, \sqrt{s})$ are shown with the same parameters as in Fig. 4.

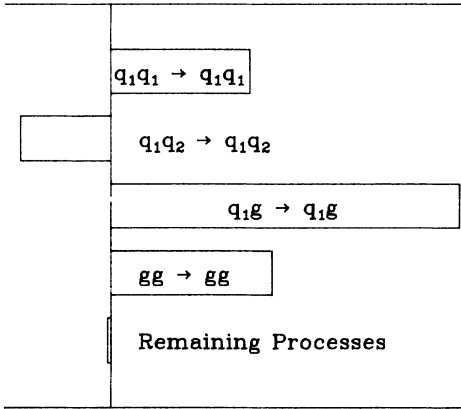
Rel. Contributions to $\Delta\sigma_L^{\text{jet}}$ at $\sqrt{s}=100$ GeV

FIG. 6. The relative contributions of different processes to $\Delta\sigma_L^{\text{jet}}(pp;p_0,\sqrt{s})$ are shown at $p_0^2=5$ GeV², $\mu^2=10$ GeV², and $\sqrt{s}=100$ GeV.

It would be very desirable if experiments with polarized hadron beams can be performed at significantly higher energies, comparable to the energies where the data^{1,2} on unpolarized jet cross sections show the strong-energy thresholds. At $\sqrt{s}=100$ GeV, our estimate for $\Delta\sigma_L^{\text{jet}}$ is

$$\Delta\sigma_L^{\text{jet}}(pp;p_0^2=5 \text{ GeV}^2, \sqrt{s}=100 \text{ GeV})=8.2 \mu\text{b} \quad (\mu^2=10 \text{ GeV}^2). \quad (3.13)$$

This prediction, however, involves the extrapolation of the $\Delta G_{i/p}(x,\mu^2)$ into x regions where they are not well known. The break-up of the cross section at this energy into the contribution of the various subprocesses is shown in Fig. 6. As can be seen there, most of the growth of the cross section in our calculation can be attributed to the

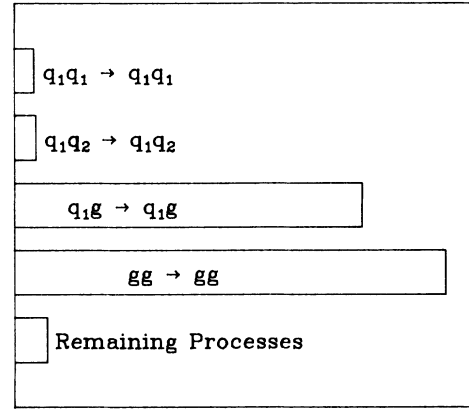
Rel. Contributions to σ^{jet} at $\sqrt{s}=100$ GeV

FIG. 7. The relative contributions of different processes to $\sigma^{\text{jet}}(pp;p_0,\sqrt{s})$ are shown with the same parameters as in Fig. 6.

gluon processes. Since our parametrization builds in the expected growth of $\langle s \rangle_z$ associated with small- x gluons, this energy dependence is an important feature of the theory. By comparison, the data for the unpolarized jet cross section is also dominated by the gluon processes at this energy. This is shown in Fig. 7.

IV. EXPERIMENTAL CONSEQUENCES AND DISCUSSION

Our numerical calculations above indicate that it should be possible to observe interesting structure associated with $\Delta\sigma_L^{\text{jet}}(pp;p_0^2,s)$ in experiments using high-energy polarized proton beams. The interpretation of our calculation involves some further discussion. Consider the hadronic multiplicity sum rule

$$\int \frac{d^3p}{E} A_{LL} \frac{E d\sigma}{d^3p} (pp \rightarrow hX) = \int \frac{d^3p}{E} \left[\frac{E d\sigma}{d^3p} (p(+)p(+) \rightarrow hX) - \frac{E d\sigma}{d^3p} (p(+)p(-) \rightarrow hX) \right] \\ = \langle n_h^{++} \rangle \sigma(p(+)p(+);s) - \langle n_h^{+-} \rangle \sigma(p(+)p(-);s). \quad (4.1)$$

We now write the average multiplicities in the form

$$\langle n_h^{++} \rangle = \bar{n}_h + \langle \delta n_h \rangle, \\ \langle n_h^{+-} \rangle = \bar{n}_h - \langle \delta n_h \rangle, \quad (4.2)$$

so that

$$\int \frac{d^3p}{E} A_{LL} \frac{E d\sigma}{d^3p} (pp \rightarrow hX) = \bar{n}_h \Delta\sigma_L(pp;s) \\ + \langle \delta n_h \rangle \sigma_{\text{inel}}(pp;s). \quad (4.3)$$

Because of the conservation of transverse momentum, an event with a high- p_T jet will always have at least one other jet balancing it. Absorbing this constraint into the definition of jet multiplicities and using the QCD parton-model expression (3.1) gives, in analogy to (4.3),

$$\Delta\sigma_L^{\text{jet}}(pp;p_0,\sqrt{s}) = \bar{n}_{\text{jet}}(p_0,\sqrt{s}) \Delta\sigma_L(pp;\sqrt{s}) \\ + \langle \delta n_{\text{jet}}(p_0,\sqrt{s}) \rangle \sigma_{\text{inel}}(pp;\sqrt{s}) \quad (4.4)$$

and

$$\begin{aligned} \Delta\sigma_L^{\text{soft}}(pp;p_0,\sqrt{s}) &= [1 - \bar{n}_{\text{jet}}(p_0,\sqrt{s})] \Delta\sigma_L(pp;\sqrt{s}) \\ &\quad - \langle \delta n_{\text{jet}}(p_0,\sqrt{s}) \rangle \sigma_{\text{inel}}(pp;\sqrt{s}). \end{aligned} \quad (4.5)$$

If we assume that our theoretical assumptions concerning *both* the small- x behavior of the distribution functions *and* the Regge behavior of $\Delta\sigma_L$ are correct, we have a energy regime where

$$\begin{aligned} \lim_{s \rightarrow \infty} |\Delta\sigma_L^{\text{jet}}(pp;p_0,\sqrt{s})| \\ \gg |\bar{n}_{\text{jet}}(p_0,\sqrt{s}) \Delta\sigma_L(pp,\sqrt{s})|, \end{aligned}$$

so that

$$\begin{aligned} \lim_{s \rightarrow \infty} \Delta\sigma_L^{\text{jet}}(pp;p_0,\sqrt{s}) &\simeq \langle \delta n_{\text{jet}}(p_0,\sqrt{s}) \rangle \sigma_{\text{inel}}(pp,\sqrt{s}), \\ \lim_{s \rightarrow \infty} \Delta\sigma_L^{\text{soft}}(pp;p_0,\sqrt{s}) &\simeq - \langle \delta n_{\text{jet}}(p_0,\sqrt{s}) \rangle \\ &\quad \times \sigma_{\text{inel}}(pp,\sqrt{s}). \end{aligned} \quad (4.6)$$

The perturbative expression, (3.5), has the form

$$\Delta\sigma_L^{\text{jet}}(pp;p_0,\sqrt{s}) = C_1 \frac{s^{+J_{\text{eff}}-1}}{(p_0^2)^{J_{\text{eff}}}} \ln \left[C_2 \frac{s}{p_0^2} \right]. \quad (4.7)$$

This quantity is positive and blows up as $p_0 \rightarrow 0$. However, for small p_0 , we are not allowed to use the hard-scattering expression (3.1). For the hard-scattering expression, (3.1), to be valid we require $s \gg p_0^2 \gg m^2$ where $m^2 \simeq 1 \text{ GeV}^2$ is a hadronic mass scale. For finite values of s , we therefore expect that

$$\begin{aligned} A_h(k_T) &= \int \frac{d^3p}{E} A_{LL} \frac{E d\sigma}{d^3p}(pp \rightarrow hX) \delta(p_T - k_T) \\ &\quad (h = \pi, \dots) \end{aligned} \quad (4.8)$$

$$\begin{aligned} \sigma(p(+)p(+) \rightarrow \text{jet}; p_0, \sqrt{s}) &= \sigma_0(p_0, \sqrt{s}) \left[1 + A_{++} \xi^\epsilon [1 + o(1)] + B_{++} \frac{m^2}{4p_0^2} + \dots \right], \\ \sigma(p(+)p(-) \rightarrow \text{jet}; p_0, \sqrt{s}) &= \sigma_0(p_0, \sqrt{s}) \left[1 + A_{+-} \xi^\epsilon [1 + o(1)] + B_{+-} \frac{m^2}{4p_0^2} + \dots \right], \end{aligned} \quad (4.9)$$

where $\sigma_0(p,\sqrt{s})$ is the ‘‘leading’’ contribution to the jet cross section. For fixed p_0 at high energy we expect that it will become approximately independent of s . The terms involving $\xi = 4p_0^2/s$ are the spin-dependent effects we are attempting to calculate. The possibility of spin-dependent higher-twist contributions is indicated by the terms proportional to $m^2/4p_0^2$. The requirement $s \gg 4p_0^2 \gg m^2$ mentioned above guarantees that both types of subleading terms are negligible in the unpolarized cross section. However, if we now consider the polarized case we have

$$\Delta\sigma_L^{\text{jet}}(pp;p_0,\sqrt{s}) = \sigma_0 \left[\Delta A \xi^\epsilon + \Delta B \frac{m^2}{4p_0^2} + \dots \right]. \quad (4.10)$$

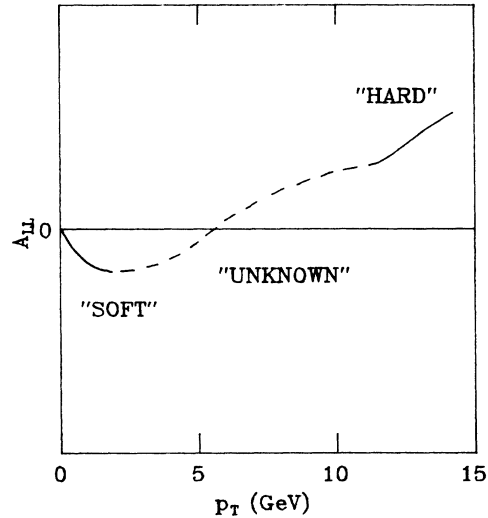


FIG. 8. The expected shape for the integrated spin asymmetry $A_{LL}(p_T)$ as a function of transverse momentum p_T .

should have the form sketched in Fig. 8. The negative region is associated with the negative value of $\Delta\sigma_L$ from A_1 exchange while the positive region corresponds to the onset of the hard-scattering regime. The question of where the transition between soft and hard scattering takes place is potentially very interesting but cannot be answered at the present time.

Our discussion of $\Delta\sigma_L^{\text{jet}}$ has, so far, ignored the contribution of subasymptotic ‘‘higher-twist’’ contributions to the hard-scattering cross section. Since our estimates involve the extrapolation of structure functions to small x , we have to estimate the possible contribution of such terms. We can begin by writing the definite-helicity cross sections

Thus at fixed p_0 , with $\epsilon > 0$, it is clear that the spin-dependent leading-twist calculation outlined above can be swamped by unknown higher-twist effects as s increases.

It is possible to avoid this problem by increasing the cutoff. For example, our expectation discussed in Sec. III would be that (4.10) is valid with $\epsilon \simeq \frac{1}{2}$. The condition for choosing the cutoff, is then

$$p_0 > \frac{1}{2} \left[\frac{\Delta B}{\Delta A} \right]^{1/3} \sqrt{s^{1/3}} m^{2/3}. \quad (4.11)$$

The general condition

$$p_0 > \frac{1}{2} s^{\epsilon/(2+2\epsilon)} m^{2/(2+2\epsilon)} \left[\frac{\Delta B}{\Delta A} \right]^{1/(2+2\epsilon)}, \quad (4.12)$$

assures that we can take $\xi \rightarrow 0$ and stay away from possible higher-twist effects. If we adopt the stringent condition

$$(4p_0^2)^2 = m^2 s, \quad (4.13)$$

which corresponds to $\epsilon=1$ in (4.12) then we expect the higher-twist contributions to be relatively more strongly suppressed as \sqrt{s} increases. We can estimate the numerical impact. Figure 9 shows $\Delta\sigma_L^{\text{jet}}(pp; p_0(\sqrt{s}), \sqrt{s})$ with $p_0 = \frac{1}{2}m^{1/2}\sqrt{s}^{1/2}$. It should be emphasized that this conservative approach may be necessary to evade problems with unknown dynamics but there is no indication that higher-twist effects of the type indicated in (4.9) are necessary.

One final intriguing experimental possibility is that $\Delta\sigma_L^{\text{soft}}(pp; p_0, \sqrt{s}) \ll \Delta\sigma_L^{\text{jet}}(pp; p_0, \sqrt{s})$ for some choice of available p_0 and \sqrt{s} . This would be associated with the contribution of a "new" Regge contribution to $U_0(s, t)$. The interpretation of this singularity is uncertain. An indication that something like this might happen would be the measurement of $A_{LL}(pp \rightarrow \pi X) > 0$ for small p_T .

It should be kept in mind that all the models for reconciling the data for unpolarized jet physics with the asymptotic behavior of total cross sections, will have application to $\Delta\sigma_L$. Measurements of $\Delta\sigma_L^{\text{jet}}$ can provide important checks of the various theoretical ideas which have been proposed. We will not explore this possibility in detail here but it adds an important extra reason for pursuing an experimental program with polarized beams and targets. It is possible that an enlightened experimen-

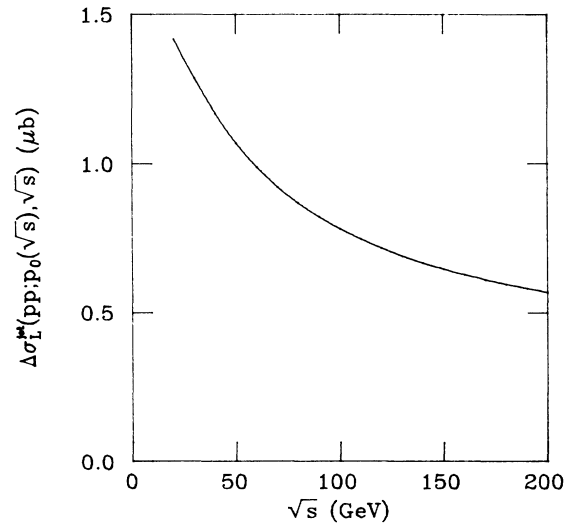


FIG. 9. We show $\Delta\sigma_L^{\text{jet}}(pp; p_0(\sqrt{s}), \sqrt{s})$ as a function of \sqrt{s} with p_0 given by Eq. (4.13) and $\mu^2 = 2p_0^2$.

tal program could show that our current ideas are woefully inadequate.

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