Brief Reports

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On the magnitude of baryon-to-photon ratio inhomogeneities resulting from a first-order quark-hadron transition

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We show that consideration of the low-lying baryon states in addition to the neutron and proton (specifically, the Λ, Σ, and Δ states) reduces the ratio of baryon-number density in the quark phase to that in the hadronic phase by more than a factor of 2. This implies that inhomogeneities in the local baryon-number-to-photon ratio produced during the quark-hadron transition are likely to be smaller than previous estimates, and therefore unless the transition temperature is less than ~ 150 MeV, the effects upon primordial nucleosynthesis will not be significant.

It has been pointed out that if the quark-hadron transition[1] is strongly first order and if the transition temperature is low enough, then large local fluctuations in the baryon-to-photon ratio can arise,[2] and might significantly modify the predictions of the standard scenario for primordial nucleosynthesis.[3–5] (For a review of the standard scenario of nucleosynthesis see Ref. 6.) Among other things, the potential effect of the quark-hadron transition upon primordial nucleosynthesis depends upon the magnitude of the fluctations in the baryon-to-photon ratio η. The size of the fluctuations is estimated by computing the ratio ($\equiv R$) of the net baryon-number density in the quark phase to that in the hadron phase at the critical temperature T_c, assuming thermal and chemical equilibrium between the quark and hadron phases.[7] In so doing, the only baryonic states in the hadronic phase that have been taken into account are the neutron and proton.[2,3,5] Here we point out that the predicted value of this ratio R decreases significantly when other low-lying baryon states are included:[8] specifically, the $J^P=\frac{3}{2}^+$ Δ^{++}, Δ^+, Δ^0, Δ^- states and the $J^P=\frac{1}{2}^+$, strangeness $=-1$ states Λ, Σ^+, Σ^0, Σ^-.

The net baryon-number density in a species i with internal degrees of freedom g_i and baryon number ± 1 which is very nonrelativistic and in thermal equilibrium with chemical potential μ_i is

$$n^B_{\rm NR}=2g_i(m_iT/2\pi)^{3/2}\exp(-m_i/T)\sinh(\mu_i/T)\ , \tag{1a}$$

where T is the temperature and m_i is the mass of species i.

The net baryon-number density associated with a highly relativistic species with internal degrees of freedom g_i, chemical potential μ_i, and baryon number $\pm\frac{1}{3}$ in thermal equilibrium is

$$n^B_R=(g_iT^3/6\pi^2)[4a(\mu_i/T)+\tfrac{1}{3}(\mu_i/T)^3]\ , \tag{1b}$$

where $a=1-1/2^2+1/3^2-1/4^2+\cdots\simeq 0.822\,46$.

Numerical studies indicate that the transition temperature for the quark-hadron (i.e., deconfinement-confinement) transition is probably in the range[1] $T_c\sim 2\Lambda_{\overline{\rm MS}}\simeq 100$–$400$ MeV. [The quantity $\Lambda_{\overline{\rm MS}}$ is the QCD renormalization scale computed in the modified, minimal-subtraction scheme;[9] recent determinations indicate that $\Lambda_{\overline{\rm MS}}\simeq 100$–$400$ MeV (Ref. 10).] In the quark phase, the u and d quarks (masses $\lesssim 10$ MeV) are most certainly relativistic, while it is uncertain whether or not the s quark (mass ~ 150–300 MeV) is. In the hadronic phase, all baryonic states are more massive than ~ 940 MeV and so are nonrelativistic. Making the assumption of thermal and chemical equilibrium betwen the quark and hadronic phases, and assuming that the quark chemical potential is flavor independent ($\equiv\mu_q$) and that the hadron chemical potential is state independent ($\equiv\mu_h$), it follows that

$$3\mu_q=\mu_h\ . \tag{2}$$

The net baryon-number density in the quark and hadronic phases is, respectively,

$$n_{Bq}=(2\text{ or }3)(4a/3\pi^2)\mu_hT^2\ , \tag{3a}$$

$$n_{Bh}=2\mu_hT^2(2\pi)^{-3/2}(4x_N^{3/2}e^{-x_N}+2x_\Lambda^{3/2}e^{-x_\Lambda}+6x_\Sigma^{3/2}e^{-x_\Sigma}+16x_\Delta^{3/2}e^{-x_\Delta})\ , \tag{3b}$$

where $x_N = m_N/T$ ($g_N = 4$, $m_N \simeq 940$ MeV), $x_\Lambda = m_\Lambda/T$ ($g_\Lambda = 2$, $m_\Lambda \simeq 1120$ MeV), $x_\Sigma = m_\Sigma/T$ ($g_\Sigma = 6$, $m_\Sigma \simeq 1190$ MeV), $x_\Delta = m_\Delta/T$ ($g_\Delta = 16$, $m_\Delta \simeq 1230$ MeV), and $g_{\text{quark}} = 6$ (for each flavor). Only terms of $O(\mu_h/T)$ have been retained in Eqs. (3) since $\mu_h/T \sim 10^{-10} \ll 1$. In the expression for n_{Bq} the 2 (3) pertains if u and d (u, d, and s) quarks are considered. The baryon-number density contrast R between the two phases at the critical temperature is

$$R = (n_{Bq}/n_{Bh})\,|_{T_c}$$
$$\simeq (2 \text{ or } 3)(2/\pi)^{1/2}(2a/3)$$
$$\times (2x_N^{3/2}e^{-x_N} + x_\Lambda^{3/2}e^{-x_\Lambda} + 3x_\Sigma^{3/2}e^{-x_\Sigma}$$
$$+ 8x_\Delta^{3/2}e^{-x_\Delta})^{-1},$$

where $x_i = m_i/T_c$. The baryon-number contrast R as a function of T_c is shown in Fig. 1.

From Fig. 1 the importance of including the $I = \frac{3}{2}$, $J = \frac{3}{2}$ Δ resonance is very clear: it leads to a reduction in R by a factor of more than 2 for $T_c \gtrsim 150$ MeV. The inclusion of the strangeness $= -1$ states is less important. In addition, there is some question whether or not the strangeness $= -1$ states would be in thermal equilibrium in the hadronic phase. [We have also considered the effect on R of including the next-lowest-mass baryon state, the Ξ^-, Ξ^0 (strangeness $= -2$) states. For $T_c \gtrsim 100$ MeV, their effect upon R is less than $\sim 10\%$.] Note that for a sufficiently high value of T_c, ($\gtrsim 250$ MeV) it becomes thermodynamically favorable for the baryon number to reside predominantly in the hadronic phase and $R < 1$.

In sum, for $T_c \gtrsim 150$ MeV the inclusion of the low-mass states beyond the neutron and proton ensures that R is less than 10, and for $T_c \geq 200$ MeV that R is less than 2. Thus it is very unlikely that sufficiently large inhomogeneities arise to affect primordial nucleosynthesis, unless T_c is very low.

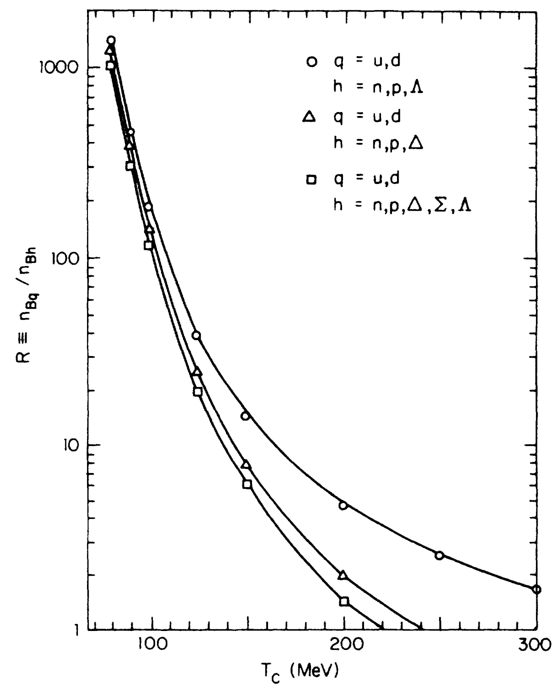

FIG. 1. Equilibrium ratio of net baryon-number density in the quark phase to that in the hadron phase as a function of the critical temperature T_c. In the quark phase only two quark flavors (u and d) have been used; if the s quark is also in thermal equilibrium and relativistic ($m_s \ll T_c$), then these results must be scaled upward by a factor of $\frac{3}{2}$.

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[1]There are actually two transitions associated with the SU(3)-color gauge theory (or QCD): the chiral-symmetry-breaking transition and the deconfinement-confinement (or quark-hadron) transition. In the pure SU(3) theory (no dynamical quarks), both are strongly first-order phase transitions and occur at the same temperature. When the dynamical effects (e.g., screening due to quark loops) of colored fermions (i.e., quarks) are included, the situation is far from being clear. In the very massive quark limit ($m_q \gg \Lambda_{\overline{\text{MS}}}$), the quark-hadron transition is strongly first order (equivalent to the pure gauge theory case), but there is no chiral-symmetry-breaking transition. In the opposite limit, very light quarks ($m_q \ll \Lambda_{\overline{\text{MS}}}$), with three or more flavors of quarks, the chiral transition is first order. The effect of "light" quarks is to soften the quark-hadron transition, apparently making it second order (or perhaps not even a phase transition at all). The effect of "sufficiently heavy" quarks is to eliminate the chiral phase transition. In fact, it is possible that there exists a range of quark masses for which neither transition is first order (or a phase transition at all). The nature of these transitions for realistic quark masses (i.e., two very light quarks and at least one intermediate mass quark) is still uncertain because of the difficulties of including the effects of dynamical quarks, especially quarks of differing masses. Recent numerical work suggests that the deconfinement-confinement transition is either weakly first order or second order, with a transition temperature of $\sim 2\Lambda_{\overline{\text{MS}}} \sim 100$–$400$ MeV. In other studies the transition temperature is determined relative to a hadron mass: e.g., $T_c \simeq (0.18$–$0.33)m_\rho \simeq 140$–250 MeV, or $T_c \simeq (0.11$–$0.29)m_N \simeq 100$–220 MeV [S. A. Gottlieb *et al.*, Phys. Rev. Lett. **59**, 1513 (1987); M. Fukugita *et al.*, *ibid.* **58**, 2515 (1987)]. The following are recent numerical studies of the SU(3) gauge theory and some relevant theoretical papers (note, no attempt was made at completeness): J. Kogut *et al.*, *ibid.* **50**, 393 (1983); J. Engles, F. Karsch, and H. Satz, Phys. Lett. **113B**, 398 (1982); J. Engles, F. Karsch, H. Satz, and I. Montvay, Nucl. Phys. **B205** [FS5], 545 (1982); B. Svetitsky and F. Fucito, Phys. Lett. **131B**, 165 (1983); T. Celik, J. Engles, and H. Satz, *ibid.* **129B**, 323 (1983); S. A. Gottlieb *et al.*, Phys. Rev. Lett. **55**, 1958 (1985); N. H. Christ and A. E. Ter-

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[7]Applegate and Hogan have emphasized that the value of R may not be determined so much by equilibrium thermodynamics, but rather more by nonequilibrium processes, such as shock dissipation, neutrino heat conduction, and expansion of an inhomogeneous, two-phase mixture. See, J. Applegate and C. J. Hogan, Phys. Rev. D **31**, 3037 (1985); **34**, 1938(E) (1986).

[8]The importance of the low-lying baryonic resonances has also been pointed out independently by Alcock, Fuller, and Mathews (Ref. 4).

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[10]E.g., see the recent review of experimental determinations of $\Lambda_{\overline{MS}}$ by D. W. Duke and R. G. Roberts, Phys. Rep. **120**, 275 (1985).