Supernova neutrinos and their oscillations

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Three-neutrino oscillations, both in a vacuum and in matter, are applied to the supernova neutrino flux. It is found that oscillations can change the flux substantially, reducing the number of directional events in the Kamioka nucleon-decay experiment and increasing the number of isotropic events in the Irvine-Michigan-Brookhaven experiment. If any of the observed directional events from SN1987A can be attributed to neutronization, then most of the parameter region which solves the solar-neutrino problem is disfavored. The e-τ nonadiabatic solution would be the most probable.

The recent observations of neutrinos from a supernova[1,2] have implications for astrophysics and particle physics. Besides containing information on the supernova, the signal depends on the properties of neutrinos. In order to interpret the recent observations, the uncertainties in supernova dynamics must be disentangled from the effects of neutrino propagation. In this note we will concentrate on the mixing of neutrino fluxes from neutrino oscillations, both in a vacuum and in matter.

We begin by summarizing some of the general features of stellar core collapse[3–5] which are relevant to neutrino oscillations. There are two mechanisms by which neutrinos are emitted from a supernova: neutronization and thermal emission. Neutronization results from electron capture onto nuclei and free protons and is responsible for a flux of ν_e. Thermal neutrino emission proceeds via the annihilation of real and virtual e^+e^- pairs, forming $\nu\bar{\nu}$ pairs of all flavors. The relative importance of these two processes is not well known[6,7] but the neutronization luminosity is expected to be roughly 10–50 % of the thermal luminosity and it may be emitted on a much shorter time scale.

The relative magnitudes of the different fluxes in thermal emission are less uncertain. For each flavor, the magnitude of the neutrino and antineutrino fluxes must be equal. Also it is known that the thermal electron-neutrino flux will be larger and have a smaller temperature than the other flavors. This is because the production and scattering cross sections are larger for electron neutrinos than for the other flavors so more ν_e's are produced and they are in equilibrium out to a larger radius where the temperature is smaller. Since ν_μ and ν_τ production and scattering only proceed via neutral-current processes, the fluxes of these two flavors are identical.

For the purpose of estimating the size of oscillation effects we will take the fluxes, neutronization and thermal, to be given by static, Fermi-Dirac distributions with a zero chemical potential. The temperature T and relative total luminosities of the initial thermal fluxes, L, are taken to be values typical of theoretical supernova models (Ref. 6):

$$\begin{aligned} &L_\alpha^t = L_{\bar{\alpha}}^t, \quad T_e^t = T_{\bar{e}}^t = 3 \text{ MeV} , \\ &L_\mu^t = L_\tau^t \equiv L_x^t, \quad T_x^t = T_{\bar{x}}^t = 6 \text{ MeV} , \\ &L_e^t = 2L_x^t, \quad T_e^n = 3 \text{ MeV} , \end{aligned} \tag{1}$$

where the superscripts t and n denote thermal and neutronization and the subscripts e, x, and α denote the neutrino flavors electron, muon, or tau, and any flavor, respectively.

In the collapsed core, neutrinos are in equilibrium at densities greater than 3×10^{11} g/cm^3. As the neutrinos leave this region, they travel through a gradually decreasing density and resonant oscillations in matter are important.[8,9] Oscillations in a vacuum, however, are relevant for all species of neutrinos and antineutrinos as they propagate to Earth. Let us now turn to a brief review of the phenomenon of neutrino oscillations.

As is well known, neutrino oscillations will occur if some neutrino mass differences, $\Delta = m_j^2 - m_i^2$, do not vanish.[10] For vacuum oscillations, the amplitude is controlled by the mixing angles and the wavelength, $\lambda = 4\pi E/\Delta$. For distances much larger than the wavelength, the variation over distance can be averaged over to get simple expressions for the probability. Let $U_{\alpha i}$ denote the mixing matrix element from the flavor basis to the mass eigenstate basis:

$$| \nu_\alpha \rangle = U_{\alpha i} | \nu_i \rangle, \quad \alpha = e,\mu,\tau, \quad i = 1,2,3 ; \tag{2}$$

then the averaged oscillation probability in a vacuum is just the classical probability

$$P(\nu_\alpha \to \nu_\beta) = \sum_i | U_{\alpha i} |^2 | U_{\beta i} |^2 . \tag{3}$$

This expression is also valid for the oscillation probability of antineutrinos in a vacuum, by CPT symmetry.

For neutrinos propagating in matter, resonant oscillations can occur[11–15] [the Mikheyev, Smirnov,[11] and Wolfenstein[12] (MSW) effect]. In this case, maximal oscillation amplitudes can be reached even for small mixing angles. As we will see later, the relevant quantity for supernova neutrino oscillations is $P(\nu_e \to \nu_e)$, the probability of a produced ν_e to be a ν_e at the detector. In the limit of small mixing angles, this MSW probability is given by[14,15]

$$P(\nu_e \to \nu_e) = \theta(E_A - E) + \theta(E - E_A) P_{\rm LZ} \tag{4}$$

in the case of two flavors. Here, θ is the step function and E_A defines the (adiabatic) energy threshold for flavor conversion and is given by

$$E_A = \frac{\Delta C_{2\theta}}{2\sqrt{2}\, G_F N_{e\,\max}} \ . \tag{5}$$

$N_{e\,\max}$ is the electron density at production and $C_{2\theta}$ is the cosine of the vacuum mixing angle. $P_{\rm LZ}$ is the Landau-Zener transition probability and is given by

$$P_{\rm LZ} = \exp\left[-\frac{\pi}{4} \frac{\Delta}{E\,|d\rho/\rho\, dr\,|_0} \frac{S_{2\theta}^2}{C_{2\theta}} \right] , \tag{6}$$

where $|d\rho/\rho\, dr\,|_0$ is evaluated at the resonance.

The crucial quantity in $P_{\rm LZ}$ is this rate of density change factor. Immediately after the core of a supernova collapses, model calculations generally yield a density function in the core and mantle of the form

$$\rho(r) = C/r^3 \ , \qquad 10^{12}\ {\rm g/cm^3} < \rho < 10^{-5}\ {\rm g/cm^3} \ , \tag{7}$$

with C varying weakly with r over the range (for later numerical calculations, we will take C from Refs. 7 and 5)

$$1 < C/10^{31}\ {\rm g} < 15 \ . \tag{8}$$

With this $\rho(r)$, the Landau-Zener transition probability takes on the simple form

$$P_{\rm LZ} = \exp[-(E_{\rm NA}/E)^{2/3}] \ , \qquad E_{\rm NA} \approx (\Delta/1\ {\rm eV}^2)\,|\,U_{ei}\,|^3 1.8\times 10^{10}\ {\rm MeV} \times (Y_e C/1.5\times 10^{31}\ {\rm g})^{1/2} \ . \tag{9}$$

$E_{\rm NA}$ is the energy threshold where resonant conversion ceases to be adiabatic; above $E_{\rm NA}$ the probability of a ν_e remaining a ν_e approaches 1. The range of neutrino energies for which conversion occurs is approximately given by

$$E_A < E < E_{\rm NA} \ . \tag{10}$$

For three species of neutrinos,[14,15] the main modification is that there are now two resonances, an upper (e-τ) and a lower (e-μ) resonance. In the limit of small mixing angles, the result is a product

$$P(\nu_e \to \nu_e) = P^u(\nu_e \to \nu_e) P^l(\nu_e \to \nu_e) \ , \tag{11}$$

where P^l (P^u) is the two-flavor probability function given above for the lower (upper) resonance. Each resonance will have its own energy range over which depletion occurs and the two energy ranges may overlap. For a supernova, it is likely that the two energy ranges will overlap because of the large range of slowly varying density described in Eqs. (7) and (8) (see Fig. 1).

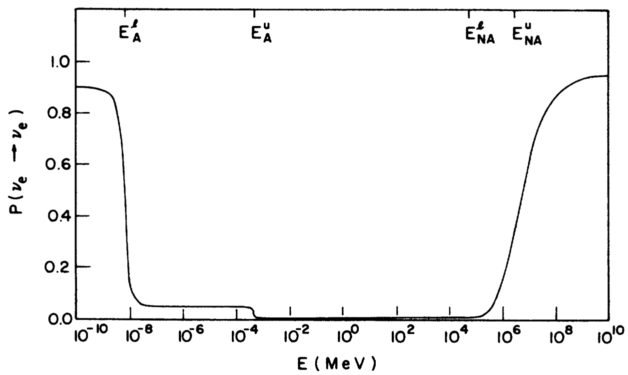

FIG. 1. The probability of a ν_e, produced in a supernova, reaching Earth as a function of energy. Here we take three neutrino flavors and "typical" vacuum parameters, $m_2^2 - m_1^2 = 10^{-4}$ eV^2, $m_3^2 - m_1^2 = 6.3$ eV^2, $|\,U_{e2}\,|^2 = 5\times 10^{-2}$, and $|\,U_{e3}\,|^2 = 5\times 10^{-4}$. The adiabatic ($A$) and nonadiabatic (NA) energy thresholds for the e-τ upper (u) and e-μ lower (l) resonances are shown.

When the mixing angles are not small, one can still work out approximate, analytic formulas for the two-flavor and three-flavor oscillation probabilities in matter. We will not quote these explicit expressions here [see Eqs. (2.18) and (3.22) of Ref. 14].[16]

For antineutrinos, resonant oscillations do not occur for $\bar{\nu}_e$ (for typical mixing parameters). Radiative corrections can induce a resonant flavor conversion between $\bar{\nu}_\mu$ and $\bar{\nu}_\tau$ (Ref. 17). However, the fluxes of $\bar{\nu}_\mu$ and $\bar{\nu}_\tau$ are expected to be equal and thus no net effect will ensue.

The $\bar{\nu}_\mu$ and $\bar{\nu}_\tau$ (ν_μ and ν_τ) fluxes are equal because in the supernova these neutrinos are produced only via the neutral current. The neutral current is also the only process by which the μ and τ neutrino species can be detected on Earth (to leading order in G_F). Using these two observations we can simplify the expressions for the effect of neutrino oscillations on the supernova neutrino fluxes. The neutrino flux of species α at Earth, F_α, can be expressed in terms of the initially produced neutrino flux of species β, F_β^0, as

$$F_\alpha = \sum_\beta P(\nu_\beta \to \nu_\alpha) F_\beta^0 \ . \tag{12}$$

$P(\nu_\beta \to \nu_\alpha)$ is the probability that if species β is produced in the supernova, species α is observed on Earth. Both the P's and F^0's are, in principle, functions of time and energy and the P's include matter and vacuum oscillation effects. However all the P's for neutrino oscillations are constrained by unitarity to satisfy

$$\sum_\beta P(\nu_\alpha \to \nu_\beta) = 1, \quad \sum_\beta P(\nu_\beta \to \nu_\alpha) = 1 \ , \tag{13}$$

for any neutrino species α. When we combine these constraints with the two observations that followed from the neutral-current interaction of neutrinos, that $F_\mu^0 = F_\tau^0 \equiv F_x^0$ and that the only detectible flux is $F_\mu + F_\tau$, we get

$$F_e = F_e^0 - [1 - P(\nu_e \to \nu_e)](F_e^0 - F_x^0) \ , \qquad F_\mu + F_\tau = 2F_x^0 + [1 - P(\nu_e \to \nu_e)](F_e^0 - F_x^0) \ . \tag{14}$$

We see that the only relevant oscillation probability for supernova neutrinos is $P(\nu_e \to \nu_e)$.

The above argument is equally valid for antineutrinos, and the relations between the produced and detected fluxes are identical to the above expressions with the neutrinos replaced everywhere by antineutrinos. For antineutrinos the only relevant probability is $P(\bar{\nu}_e \to \bar{\nu}_e)$. These two probabilities, $P(\nu_e \to \nu_e)$ and $P(\bar{\nu}_e \to \bar{\nu}_e)$, are thus the only quantities that must be specified to determine how neutrino oscillations mix the fluxes.

The previous formulas relate the produced and detected neutrino fluxes from a supernova. In order to make quantitative statements about oscillation effects and the relevant neutrino parameters, we shall now work through some explicit examples. We calculate the total number of events expected in a water Cherenkov detector with and without oscillations and define this ratio to be R:

$$R \equiv \frac{\text{No. of events in detector with oscillations}}{\text{No. of events in detector without oscillations}} . \tag{15}$$

This will be a measure of oscillation effects while many of the details of the supernova model will cancel out in this ratio.

There will be more than one type of R to calculate. In a water Cherenkov detector there are two kinds of neutrino events which can be distinguished from each other. One type of event is inverse beta decay where the positron is emitted almost isotropically:

$$\sigma(\bar{\nu}_e + p) \approx 89 \times 10^{-43} \text{ cm}^2 (E_\nu / 10 \text{ MeV})^2 . \tag{16}$$

The other type of event is neutrino-electron scattering which is extremely directional:

$$\frac{d\sigma}{dE_e} = \frac{\sigma_0}{m_e}\left[g_L^2 + g_R^2 \left[1 - \frac{E_e}{E_\nu}\right]^2 - g_L g_R \left[\frac{m_e E_e}{E_\nu^2}\right]\right] ,$$

$$\sigma_0 = 88 \times 10^{-46} \text{ cm}^2 , \tag{17}$$

$$g_L = (\pm\tfrac{1}{2} + \sin^2\theta_W), \quad g_R = \sin^2\theta_W .$$

The upper sign applies to ν_e-e scattering, the lower sign to ν_x-e scattering (here ν_x represents either the μ or τ neutrino), and for $\bar{\nu}$-e scattering, g_L and g_R are interchanged. These two classes of events, directional and isotropic, combined with the two supernova emission processes, give us three relevant R's to calculate. There are two R's for thermal emission: one for directional and one for isotropic events, and one R for the directional events from neutronization.

Using the formulas for the fluxes, Eq. (14), the expression for the directional events from neutronization is

$$R_d^n = 1 + \frac{\int dE_\nu F_e^{0n}[1 - P(\nu_e \to \nu_e)] \int \varepsilon [d\sigma(\nu_x + e) - d\sigma(\nu_e + e)]}{\int dE_\nu F_e^{0n} \int \varepsilon\, d\sigma(\nu_e + e)} . \tag{18}$$

Here $\int \varepsilon\, d\sigma$ denotes an integral over electron energies (from 0 to E_ν) of the differential cross section times ε, the detector efficiency, a known function of E_e (Refs. 1 and 2). Independent of any supernova details, the range of R is approximately $1 > R_d^n > 0.14$, where 0.14 is the ratio of ν_x to ν_e cross sections. To calculate where R_d^n lies in this range we must first specify some relevant quantities. We use the detector efficiency of the Kamioka nucleon-decay experiment (Kamiokande). We must also fix the neutrino flux so we use a static Fermi-Dirac distribution as discussed previously. For $P(\nu_e \to \nu_e)$ we use the three-flavor expression (including large-angle effects) discussed previously. This probability depends on four independent neutrino parameters so, in order to make a two-dimensional plot, we constrain two of the four parameters consistent with the theoretical expectations of grand-unified theories.[18] Using these we get contour plots for R_d^n as given in Figs. 2 and 3 (these graphs are qualitatively similar to those graphs showing the effects of three-flavor neutrino oscillations on the solar-neutrino flux[14]).

The range of parameters for which oscillations can affect the supernova flux is amazingly large because of the large range of densities in the supernova. For any reasonably expected neutrino parameters the two energy ranges where flavor conversion occurs overlap. To a good approximation, the top, horizontal contours of Fig. 2 are given by $E_A^l \approx$ a constant, where E_A^l is the adiabatic threshold of the lower e-μ resonance. Similarly, the lower, diagonal contours of Fig. 2 and all the contours shown in Fig. 3 are given by $(E_{\rm NA}^u)^{2/3} + (E_{\rm NA}^l)^{2/3} \approx$ a constant, where $E_{\rm NA}^{u(l)}$ is the nonadiabatic threshold energy of the upper e-τ (lower e-μ) resonance. A general expression for the nonadiabatic, 50% contour is

$$m_2^{4/3} \, |U_{e2}|^2 + m_3^{4/3} \, |U_{e3}|^2 \approx 8 \times 10^{-7} \text{ eV}^{4/3} .$$

This expression comes from the overlap of the Landau-Zener probability factors of the upper e-τ times lower e-μ resonances and is applicable when $E > E_A^u|_{\max}$. Using typical core densities and neutrino energies to evaluate this condition gives $m_3 < 200$ eV.

Solar densities occur in the supernova models at solar distance scales. Thus there is some overlap of the nonadiabatic contours of the supernova with the nonadiabatic solutions to the solar-neutrino problem.[19,11–15] The overlap occurs when the same resonance dominates for both situations. This is most naturally the case when the resonance that occurs at solar densities is the heaviest possible resonance. Any heavier resonance would almost certainly occur in the higher densities of the supernova and would tend to dominate the situation because it would occur first and because it would be expected to more readily satisfy the adiabatic criterion. This is the situation in Fig. 2 where the e-τ nonadiabatic solar and supernova contours overlap, but the solar, e-μ, nonadiabatic contours lie inside those of the supernova contours. Only in the case of extremely small $|U_{e3}|$, when the e-τ resonance decouples, $m_2^2|U_{e2}|^3 \gg m_3^2|U_{e3}|^3$, can there be some overlap of the solar and supernova e-μ nonadiabatic contours.

Away from the adiabatic and nonadiabatic energy thresholds, the probability can be taken as a constant, independent of energy (see Fig. 1). Then Eq. (18) yields, independent of the supernova and detector details,

$$
\begin{aligned}
R_d^n &= 1+[1-P(\nu_e\to\nu_e)]B_d^n\,,\\
B_d^n &\approx [\sigma(\nu_x+e)-\sigma(\nu_e+e)]/\sigma(\nu_e+e)\\
&\approx -0.86\,.
\end{aligned}
\tag{19}
$$

This equation describes the large-angle contours for R_d^n. For the form of $P(\nu_e\to\nu_e)$ for large mixing parameters, see Refs. 14 and 15.

The calculation of R for the directional events from thermal emission R_d^t is similar to the previous case

$$
R_d^t = 1+\frac{\int dE_\nu \left[[1-P(\nu_e\to\nu_e)](F_e^{0t}-F_x^{0t})\int \varepsilon[d\sigma(\nu_x+e)-d\sigma(\nu_e+e)]+(\nu\to\bar\nu)\right]}{\int dE_\nu \left[F_e^{0t}\int \varepsilon\, d\sigma(\nu_e+e)+2F_x^{0t}\int \varepsilon\, d\sigma(\nu_x+e)+(\nu\to\bar\nu)\right]}\,. \tag{20}
$$

Now there is more than one flavor of initial flux present and oscillations can either increase or decrease R_d^t. R_d^t is more sensitive to the details of the supernova model than R_d^n because we must specify the temperatures and the relative magnitudes of each produced flux [Eq. (1)].

As in the neutronization case, the important flux energies do not overlap thresholds of $P(\nu_e\to\nu_e)$ for most neutrino parameters. Then taking the probability to be

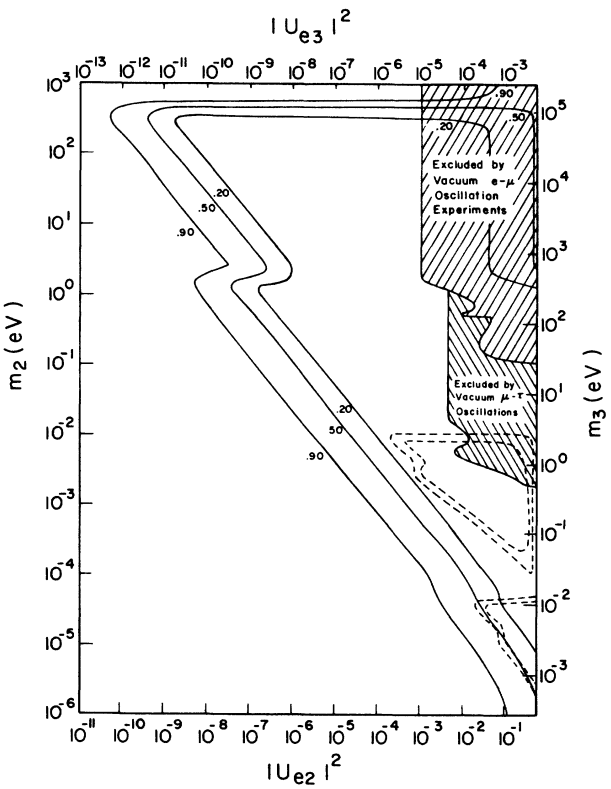

FIG. 2. A contour plot of R_d^n, the ratio of the number of events with oscillations to those without for directional events from neutronization, for the Kamiokande detector. Here we use three flavors and "typical" constraints on the relevant neutrino vacuum parameters: $m_1=0$, $(m_3/m_2)=250$, and $(|U_{e2}|^2/|U_{e3}|^2)=100$. The dashed lines show the (3σ) solutions to the solar-neutrino problem. The shaded regions are excluded by reactor oscillation experiments where we have taken $|U_{\mu3}|^2\approx|U_{e2}|^2$ (Ref. 22).

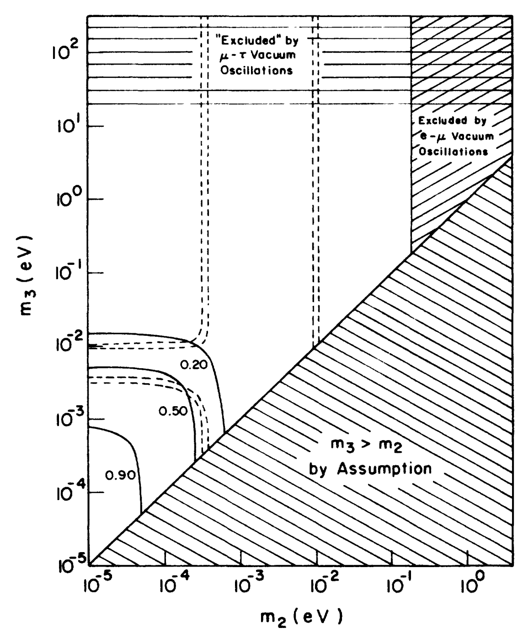

FIG. 3. A contour plot of R_d^n, the ratio of the number of events with oscillations to those without for directional events from neutronization, for the Kamiokande detector. Here we use three flavors and "typical" values for the relevant neutrino vacuum parameters: $m_1=0$, $|U_{e2}|^2=5\times10^{-2}$, and $|U_{e3}|^2=5\times10^{-4}$. The dashed lines show the (3σ) solutions to the solar-neutrino problem. The shaded regions are excluded by reactor oscillation experiments where we have taken $|U_{\mu3}|^2\approx|U_{e2}|^2$ (Ref. 22).

a constant, independent of energy, in Eq. (20),

$$R_d^t = 1 + [1 - P(\nu_e \to \nu_e)] B_d^t \; ,$$
$$B_d^t = 0.02 \text{ Kamiokande} \; , \tag{21}$$
$$B_d^t = 1.1 \text{ Irvine-Michigan-Brookhaven (IMB) experiment} \; .$$

B_d^t depends on the detector because the different temperature fluxes have different energy dependencies, they fall off as $E^2 \exp(-E/T)$. Oscillations produce larger changes at higher energies where the relative difference between the fluxes is largest. The different temperatures of the fluxes also means that judiciously placed energy thresholds can sometimes enhance or reduce R_d^t outside of the range given by Eq. (21). For the Kamiokande detector the range is $1.26 > R_d^t > 0.95$ with the former (latter) limit occurring when the adiabatic (nonadiabatic) threshold overlaps the important flux energies. For IMB, the range does not exceed that of Eq. (21).

The calculation of R for the isotropic events follows from

$$R_i^t = 1 + \frac{\int dE \; \varepsilon\sigma(\bar{\nu}_e + p)[1 - P(\bar{\nu}_e \to \bar{\nu}_e)](F_{\bar{x}}^{0t} - F_{\bar{e}}^{0t})}{\int dE \; \varepsilon\sigma(\bar{\nu}_e + p) F_{\bar{e}}^{0t}} \; . \tag{22}$$

As in the previous case, more than one species is present so the calculation of R_i^t will again depend on the temperatures and relative magnitude of the thermal fluxes. Here the probability $P(\bar{\nu}_e \to \bar{\nu}_e)$ is given by the vacuum oscillation expression [Eq. (3)] and is independent of energy. This allows us to easily evaluate R_i^t. Taking the supernova parameters given, we find

$$R_i^t = 1 + [1 - P(\bar{\nu}_e \to \bar{\nu}_e)] B_i^t \; ,$$
$$B_i^t = 0.20 \text{ Kamiokande} \; , \tag{23}$$
$$B_i^t = 4.5 \text{ IMB} \; .$$

B_i^t is always positive for the given supernova parameters. The isotropic cross section increases faster with energy than the directional cross section so B_i^t is more sensitive to the small amount of a higher-temperature flux that oscillation adds. IMB's threshold is much larger than the temperature so the expected number of events is very sensitive to the temperature(s) of the flux. If we evaluate the probability for mixing parameters equal to the corresponding Kobayashi-Maskawa (KM) angles, we get $P(\bar{\nu}_e \to \bar{\nu}_e) \approx 0.9$. Thus, even for small angles such as those in the hadronic sector, vacuum mixing can still yield a sizable effect in IMB. However this sensitivity to the temperature also means that uncertainties in the supernova theoretical models can have large effects. The expected rates without mixing are given in Table I and show this sensitivity of the IMB signal to the temperature. Resolving this ambiguity will require very good data or a reliable theoretical model for the neutrino fluxes.

We turn now to a discussion of SN1987A and what implications there are for neutrino oscillations. The supernova is 170 000 light years from Earth so neutrinos will undergo vacuum oscillations if $\Delta > 10^{-20}$ eV^2. The timing of the fluxes will be altered if the relevant neutrino mass is larger than a few eV (Ref. 20). Then heavier-mass eigenstates would travel more slowly and reach Earth at a different time. The observed value for the R's might then be smaller than estimated above because part of the signal would be delayed and probably missed (because of the small number of events).

The observed neutrino signal consists of 11 events in the Kamiokande and 8 events in the IMB experiment. The first two events in the Kamiokande data appear to be directional events and the subsequent events are consistent with isotropic events. The IMB data cannot reliably be resolved into directional and isotropic events but from Table I and our discussion we can conclude that they are mostly isotropic events.

It is tempting to attribute one or both of the first two directional events in Kamiokande to be from ν_e emitted during neutronization. From Table I we see that this interpretation is barely compatible with theoretical estimates with no oscillations, and if the neutronization

TABLE I. The number of neutrino events expected from stellar collapse, without neutrino oscillations. The thermal flux is normalized to ten isotropic events in Kamiokande and the neutronization flux is normalized to a relative luminosity of $L^n/L^t = 10\%$–50%, as described in the text.

	Neutronization Directional	Thermal emission Directional	Thermal emission Isotropic	$T_e^t = T_e^n$ (MeV)	T_x^t (MeV)
Kamiokande	0.06–0.28	0.24	10	3	6
IMB	0.003–0.02	0.04	2.6		
Kamiokande	0.05–0.27	0.22	10	4	8
IMB	0.01–0.04	0.06	5.6		
Kamiokande	0.05–0.25	0.21	10	5	10
IMB	0.01–0.07	0.08	8.7		

luminosity is at or above the largest theoretical estimates. With neutrino oscillations the expected event rate can be reduced substantially, up to a factor of $\frac{1}{7}$. If one or two neutronization events are contained in the Kamiokande sample then it appears likely that neutrino parameters that yield maximal suppression are ruled out. In particular, the reduction is near maximal for most of the parameter region that solves the solar-neutrino problem. For Figs. 2 and 3 this includes all of the e-μ resonance solutions (adiabatic, nonadiabatic, large-angle, and Earth effect), and also the e-τ adiabatic solution. This is also the case for choices of neutrino parameters other than those in Figs. 2 and 3, as long as there is a hierarchy of masses and mixing angles [as expected from grand-unified-theory (GUT) seesaw mechanisms] and $|U_{e3}|$ does not vanish. Only the e-τ nonadiabatic solutions to the solar-neutrino problem are still allowed. These solutions are partially suppressed but we note that the supernova contours in this region depend on the density change at the edge of the mantle and are especially model dependent and so cannot be excluded.

We now consider events that may have originated from the thermal phase. They probably comprise the majority of the data sample. As far as the directional events in Kamiokande are concerned, from Eq. (21), oscillation does not have any major impact on them. For most of the possible neutrino parameters, there is an enhancement of about 2%. There are small regions where the signal can be enhanced by about 20%. Overall, because of the very limited data available, one can safely ignore the effects of oscillation on directional events which originate from the thermal phase.

The $\bar{\nu}_e$ (isotropic, thermal) events can, however, be enhanced substantially by vacuum oscillations. To resolve experimentally vacuum oscillations of $\bar{\nu}_e$ from uncertainties in $T^t_{\bar{e}}$, one must fit the $\bar{\nu}_e$ spectrum[21] with the flux described below Eq. (14). There are at least four parameters to be extracted—the temperature and the magnitude of the $\bar{\nu}_e$ and $\bar{\nu}_x$ fluxes. The small number of events and the sensitivity to the experimental efficiency make it difficult to do this reliably. However one can hope that forthcoming improvements in the thecretical supernova models will increase the reliability of any such analysis. We do not attempt such an analysis here.

In this paper we have studied the effects of oscillations on supernova neutrinos. For supernovas, it is important to use the formalism of three neutrino oscillations since all three flavors are produced and since the density spans a very wide range. However, because the ν_μ and ν_τ ($\bar{\nu}_\mu$ and $\bar{\nu}_\tau$) flavors interact only via neutral currents, the only relevant probability is $P(\nu_e \to \nu_e)[P(\bar{\nu}_e \to \bar{\nu}_e)]$. We found that oscillation has little impact on directional, thermal events. It can enhance the isotropic, thermal events substantially for detectors with high threshold energies. Finally, for a large range of neutrino parameters, directional events from neutronization are greatly suppressed ($\approx \frac{1}{7}$). If one or two neutronization events are contained in the Kamiokande sample, then the $e-\tau$ nonadiabatic solution to the solar-neutrino problem is most probable.

Note added

(1) We would like to clarify the implications of the first two Kamiokande events by estimating the probability of these events using the "standard" supernova model assumed here. One expects about 0.1 (or more) events from neutronization (with a large uncertainty,[23] Table I) so the probability that two events occur is about $(0.1)^2 \approx \frac{1}{100}$ [if full flavor conversion occurs, Fig. 2, there is a further suppression of $(\frac{1}{7})^2$]. The two events are observed to be separated by 0.107 sec which is consistent with estimates for the duration of significant neutronization emission[23] (a 0.01-sec collapse burst and 0.1–1-sec diffusive emission). Thermal emission follows neutronization and occurs on a much larger time scale, of order 10 sec. If the two events are from proton capture of thermal $\bar{\nu}_e$, as the other ten events, the probability is approximately the product of two factors: the probability for two of the ten electrons being emitted in the forward direction ($\pm 20°$) times the probability that these events are the first two events, $[10 \times (20 \times \pi/180)^2/4]^2 \times (\frac{1}{100}) \approx \frac{1}{1000}$. The probability of these two events being from thermal neutrino-electron scattering, for which one expects 0.23 events (Table I), is about $(0.23)^2 \times (\frac{1}{100}) \approx \frac{1}{2000}$. Thus all standard-model explanations have small probabilities but the most likely scenario is that the two events are due to neutrinos from neutronization, without flavor conversion. Any other scenario has a probability roughly an order of magnitude smaller at the level of $\frac{1}{1000}$.

(2) After completion of this work we learned of other papers on this topic [J. Arafune, M. Fukugita, T. Yanagida, and M. Yosimura, Phys. Rev. Lett. **59**, 1864 (1987), Dirk Notzold, Phys. Lett. B **196**, 315 (1987), and S. P. Rosen, Los Alamos Report No. 87-1296 (unpublished)]. While we agree with many points made in these papers, they treat MSW flavor conversion of neutrinos in the supernova in a two-flavor framework and do not properly generalize their conclusions to the physical case of three neutrino flavors. The three-flavor MSW effect is characterized by the occurrence of two resonances. The two-flavor approximation is adequate only if one resonance decouples. Compared to the solar neutrinos, decoupling of the supernova neutrinos is harder to realize. For instance, the e-μ solution of the solar problem corresponds to having a large ν_τ mass so that the solar core density is not high enough for the e-τ resonance to occur. Because the central supernova density is so high, an e-τ resonance almost always precedes an e-μ resonance. A proper treatment can only be done with the three-flavor formalism. We emphasize that with three flavors and a hierarchy of masses and mixing angles, (a) the e-μ solution to the solar-neutrino problem is disfavored over the e-τ nonadiabatic solution and (b) flavor conversion of supernova neutrinos in Earth does not occur.[16]

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