Superrenormalizability in a model with supersymmetry breaking

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A well-known model, due to Zanon, which exhibits dynamical supersymmetry breaking at large N is reexamined. The breaking is known to be pathological because the Goldstone fermion has negative norm. Here the two-loop corrections to the large-N effective potential are computed, and it is seen that the effect of the negative-norm state is to soften ultraviolet divergences beyond what is found in perturbation theory, rendering the model superrenormalizable in the loop expansion.

I. INTRODUCTION

The argument that quantum field theories with indefinite metric exhibit a softening of ultraviolet divergences has a long history.[1] Of course, if there are stable[2] negative-norm states in the physical Hilbert space of a theory the S matrix is not unitary, so the price of reducing divergences in this way is a loss of causality. Supersymmetric theories also exhibit soft ultraviolet behavior, and some time ago Olive and West suggested that the suppression of divergences in these two types of theories might be closely related.[3]

The roots of this relationship lie in the old argument that ultraviolet divergences arise because the simultaneous requirements of causality (Lorentz invariance) and quantum mechanics (the uncertainty principle) result in an infinite probability of finding high-momentum states near the light cone. Olive and West point out that in supersymmetric theories this argument needs modification because the invariant interval has a superspace part

$$(x_{1\mu}-x_{2\mu})^2+(\bar{\theta}_1\sigma_\mu\theta_2)^2 \ .$$

It might therefore be possible to understand the Fermi-Bose cancellation one sees in diagrams in terms of this "smearing" of the light cone in superspace. As these authors note, this discussion touches on some rather timely questions, since it suggests that a four-dimensional field theory which is constrained to have especially soft ultraviolet behavior should either be supersymmetric, or have a noncausal spectrum. This is supported by what is currently known about dynamical supersymmetry breaking. Theories which exhibit supersymmetry at the tree level resist breaking because of the positive semidefiniteness of the vacuum energy. The exceptions are chiral gauge theories, in which it seems possible that infrared divergences destroy supersymmetry.[4]

A simple argument due to Higuchi and Kazama[5] shows the physical reason for the requirement of positive semidefiniteness. The generators of the supersymmetry algebra are Weyl spinors $Q_\alpha, \bar{Q}_{\dot\alpha}$ which obey

$$\{Q_\alpha,\bar{Q}_{\dot\alpha}\}=2\sigma^\mu_{\alpha\dot\alpha}P_\mu \ . \tag{1.1}$$

Writing the Q's in terms of conserved currents in the usual way, $Q_\alpha=\int d^3x\, J^0_\alpha(\mathbf{x},t)$, taking the trace and the vacuum expectation value gives

$$\int d^3x\, d^3y \langle 0 \mid \{J^0_\alpha(\mathbf{x},t),\bar{J}^{0\dot\alpha}(\mathbf{y},t)\} \mid 0\rangle$$
$$=4\langle 0 \mid P_0 \mid 0\rangle\delta^{\dot\alpha}_\alpha=4\int d^3x \langle 0 \mid H(x) \mid 0\rangle\delta^{\dot\alpha}_\alpha \ .$$

From the above follows the famous condition that $\langle 0 \mid P_0 \mid 0\rangle=0$ if and only if $Q_\alpha \mid 0\rangle=Q_{\dot\alpha} \mid 0\rangle=0$. The condition that $P_0\geq 0$ is implied since after the insertion of intermediate states the expression for the vacuum energy density

$$\langle 0 \mid H(x) \mid 0\rangle=\tfrac{1}{2}\sum_n(2\pi)^3\delta^3(\mathbf{P}_n)$$
$$\times\langle 0 \mid J^0_\alpha(0) \mid n\rangle\langle n \mid \bar{J}^{0\dot\alpha}(0) \mid 0\rangle\delta^{\dot\alpha}_\alpha \tag{1.2}$$

is manifestly non-negative. Higuchi and Kazama note that the standard Ward identity for conserved currents,

$$\frac{\partial}{\partial x^\mu}T(J^\mu_\alpha(x)\bar{J}^{\nu\dot\alpha}(y))=\delta(x^0-y^0)$$
$$\times\{J^0_\alpha(x),\bar{J}^{\nu\dot\alpha}(y)\} \ ,$$

together with the above expression for $\langle 0 \mid H(x) \mid 0\rangle$ impose a constraint on the two-point function

$$S^\mu(P)=i\int d^4x\, e^{-iP\cdot x}\langle 0 \mid T(J^\mu_\alpha(x)\bar{J}^{0\dot\alpha}(0)) \mid 0\rangle \ ,$$

namely,

$$\lim_{P\to 0}P_\mu S^\mu(P)=4\langle 0 \mid H \mid 0\rangle \ . \tag{1.3}$$

Now if $\langle 0 \mid H \mid 0\rangle$ is nonzero there must be a Goldstone fermion among the intermediate states on the right-hand side of (1.2). In this case one may write

$$J^\mu_\alpha(x)=if\sigma^\mu_{\alpha\dot\alpha}\bar{\psi}^{\dot\alpha}(x)+\tilde{J}^\mu_\alpha(x) \ ,$$
$$\bar{J}^{\mu\dot\alpha}(x)=if\bar{\sigma}^{\mu\dot\alpha\alpha}\psi_\alpha(x)+\tilde{J}^{\mu\dot\alpha}(x)$$

with ψ the Goldstone fermion field and $\tilde{J}^\mu$ free of massless poles. The coupling f is real, as can be verified by constructing a Hermitian supercurrent from J^μ_α and $\bar{J}^{\mu\dot\alpha}$. With these definitions the pole in the two-point function

$S^\mu(P)$ is given by

$$\lim_{P\to 0} P_\mu S^\mu(P) = if^2 \lim_{P\to 0} (\bar\sigma\cdot p)^{\dot\alpha\alpha} \times \int d^4x\, e^{-iP\cdot x}\langle 0 \,|\, T(\psi_\alpha(x)\psi_{\dot\alpha}(0)) \,|\, 0\rangle$$
$$= -f^2 \lim_{P\to 0} (\bar\sigma P)^{\dot\alpha\alpha} n \frac{(\sigma\cdot P)_{\alpha\dot\alpha}}{P^2}$$
$$= 2f^2 n \ .$$

The factor n in this expression is the residue of the pole in $\langle 0 \,|\, T(\psi_\alpha\bar\psi_\alpha) \,|\, 0\rangle$, i.e., the norm of the Goldstone fermion state. Now from (1.3) one sees that the loss of positive semidefiniteness after supersymmetry breaking results in a negative form for the Goldstone fermion because $\langle 0 \,|\, H \,|\, 0\rangle < 0$ implies

$$n = \frac{2}{f^2}\langle 0 \,|\, H \,|\, 0\rangle < 0 \ . \tag{1.4}$$

Since (1.4) is inconsistent with unitarity the resulting theory, although not actually mathematically ill defined, is physically unacceptable.

The model I will discuss in this paper exhibits dynamical supersymmetry breaking through just such a loss of positive semidefiniteness. It was first analyzed by Zanon,[5] and subsequently by the other authors of Ref. 5. The Lagrangian is

$$L = \int d^2\theta\, d^2\bar\theta (\bar\phi\phi + \bar\phi_i\phi_i) - \left[\int d^2\theta \left[\frac{m}{2}\phi^2 - \frac{g}{\sqrt{N}}\phi\phi_1^2 \right] + \text{H.c.} \right] . \tag{1.5}$$

The superfields $\{\phi_i,\ i=1,\ldots,N\}$ are regarded as an O(N) vector and ϕ as an O(N) singlet.

In the large-N limit this model is found to have an effective potential which is unbounded from below. There is also a local minimum below zero for suitable values of m and g (though this feature is regarded as unimportant in what follows; since the theory is unphysical to begin with, its vacuum may as well be unstable). Here I want to show that, apropos of the above discussion, the appearance of the negative-norm state at large N reduces ultraviolet divergences beyond what is found in perturbation theory. The model is actually superrenormalizable order by order in the loop expansion in this limit. This is proved by computing the two-loop correction to the effective potential. It is shown that there are no new divergences at this order; power counting then implies that new divergences cannot arise at any order.

Section II is a review of the one-loop calculation. Section III describes the calculation of the two-loop correction in detail. Section IV contains some concluding remarks.

II. THE EFFECTIVE POTENTIAL AT ONE LOOP

The component field version of (1.5) is

$$L = L_{\text{free}}(A,\psi,F) + L_{\text{free}}(A_i,\psi_i,F_i) + L_{\text{int}}$$

with

$$L_{\text{free}}(A,\psi,F) = \tfrac{1}{2}(-i\psi\sigma\cdot\partial\bar\psi + \bar A\partial^2 A + \bar F F + 2mAF - m\psi\psi + \text{H.c.}) ,$$
$$L_{\text{int}} = \frac{g}{\sqrt{N}}(FA_i^2 + 2AF_iA_i - 2A_i\psi\psi_i - A\psi_i\psi_i + \text{H.c.}) .$$

The tree-level effective potential is easily found to be

$$V_0 = -N\bar F F - mN(AF + \bar A\bar F) - \bar F_i F_i - g(FA_i^2 + 2AF_iA_i + \bar F\bar A_i^2 - 2\bar A\bar F_i\bar A_i) , \tag{2.1}$$

where the singlet fields have been rescaled: $\phi \to \sqrt{N}\phi$. Using the auxiliary field equations of motion this becomes

$$V_0 = \frac{N}{2}|F|^2 + \tfrac{1}{2}|F_i|^2, \quad \langle 0 \,|\, V_0 \,|\, 0\rangle = 0 \ .$$

The development of a large-N expansion for the effective potential of this model is straightforward. The singlet superfield ϕ is shifted by a background value $\phi \to \phi_0 + \phi$, then the effective action $\Gamma[\phi_0]$ is defined in terms of the shifted action

$$\tilde S[\phi_0,\phi,\phi_i] = N\int d^4x \left[\int d^4\theta\, \bar\phi_0\phi_0 - \frac{m}{2}\left[\int d^2\theta\, \phi_0^2 + \text{H.c.} \right] \right] + \int d^4x \left[\int d^4\theta\, \phi_i\phi_i - g\left[\int d^2\theta\, \phi_0\phi_i^2 + \text{H.c.} \right] \right] - g\int d^4x \left[\int d^2\theta\, \phi\phi_i^2 + \text{H.c.} \right]$$
$$= NS[\phi_0] + S[\phi_i] + S_{\text{int}}[\phi,\phi_i] , \tag{2.2}$$

as

$$e^{i\Gamma[\phi_0]} = e^{iNS[\phi_0]} \int d\bar\phi_i d\phi_e e^{iS[\phi_i]} e^{iS_{\text{int}}[\phi,\phi_i]} .$$

To extract the effective potential set $\phi_0 = A_0 + \theta^2 F_0$. Functional integration gives a formula of standard type (with $\Omega = \int d^4x$):

$$V_{\text{eff}}[\phi_0]=-\frac{1}{\Omega}\Gamma[\phi_0]=NV_0(\phi_0)-\frac{i}{2\Omega}\text{Tr}\ln\left[\frac{\delta^2 S[\phi_0]}{\delta\phi_i\delta\phi_j}\right]+\frac{i}{\Omega}\langle T\exp(iS_{\text{int}}[\phi,\phi_i])\rangle\ . \tag{2.3}$$

The three terms on the right-hand side of (2.3) are, respectively, the tree-level, quantum corrections at infinite N, and quantum corrections to all finite orders in $1/N$. The large-N term is most easily evaluated in terms of component fields.[5] Defining

$$\Phi_i^T=(A_i\bar{A}_iF_i\bar{F}_i),\quad \Psi_i^T=(\psi_{i\alpha}\bar{\psi}_{i\dot{\alpha}})$$

one finds

$$S[\Phi_i]=\tfrac{1}{2}\int d^4x\,\Phi^T(iM^{-1})\Phi+\tfrac{1}{2}\int d^4x\,\Psi^T(iN^{-1})\Psi\ ,$$

where the inverse propagators M^{-1}, N^{-1} of the component fields are

$$iM^{-1}=\begin{bmatrix}-2gF_0 & \partial^2 & -2gA_0 & 0\\ \partial^2 & -2g\bar{F}_0 & 0 & -2g\bar{A}_0\\ -2gA_0 & 0 & 0 & 1\\ 0 & -2g\bar{A}_0 & 1 & 0\end{bmatrix},\quad iN^{-1}=\begin{bmatrix}2gA_0\epsilon^{\alpha\beta} & -i\bar{\sigma}^{\mu\dot{\beta}\alpha}\partial_\mu\\ -i\bar{\sigma}^{\mu\dot{\alpha}\beta}\partial_\mu & 2g\bar{A}_0\epsilon^{\dot{\alpha}\dot{\beta}}\end{bmatrix}.$$

The resulting integrals are trivial and give

$$V_{\text{eff}}[\phi_0]=-\frac{N}{g^2}\left[|a_0|^4|x|^2+m(a_0^3x+\bar{a}_0^3\bar{x})\right.$$
$$\left.-\frac{g^2|a_0|^4}{64\pi^2}\left[(1+|x|)^2\ln(1+|x|)+(1-|x|)^2\ln(1-|x|)+2|x|^2\ln\frac{|a_0|^2}{4\pi\mu^2}\right]\right].$$

In this equation the change of notation $a_0=2gA_0$, $x=F_0/2gA_0^2\equiv f/a_0^2$ has been made for later convenience. V_{eff} is already renormalized, as can be seen from the appearance of the scale factor μ. Bare and renormalized quantities are related through

$$A_{0B}=Z^{1/2}A_{0R},\quad F_{0B}=Z^{1/2}F_{0R},\quad m_B=Z^{-1}m_R,\quad g_B^2=Z^{-1}g_R^2$$

with

$$Z=1-\frac{g^2}{32\pi^2}\left[\frac{1}{\epsilon}+\psi(1)\right].$$

Now, as many of the authors of Ref. 5 point out, V_{eff} is unbounded from below for large $|a_0|$. One is free to choose a_0 and f_0 to be real and the auxiliary field equation $(\partial V_{\text{eff}}/\partial f_0)=0$ gives

$$V_{\text{eff}}\simeq\frac{Ng^2m^2a_0^2}{1-\frac{g^2}{32\pi^2}\ln\frac{a_0^2}{4\pi\mu^2}} \tag{2.4}$$

for $f_0\ll a_0^2$. Obviously, the potential goes negative above the pole in this expression and is unbounded from below as $a_0\to\infty$.

III. TWO-LOOP CORRECTIONS

There is no reason to expect that the pathological behavior of V_{eff} will be cured by computing the next term in the expansion (2.2). Cancellations of large-N effects by nonleading corrections is not known in the $1/N$ expansion, but the circumstances under which they occur are very special. In the present case it is apparent that, to any finite order, corrections will only serve to shift the location of the pole in V_{eff}. If N is not large, however, one cannot be sure that the pole even exists. Higher-order corrections may move it toward successively larger value of a_0 so that it disappears when all orders in N are summed. This is a lot to infer from lowest-order corrections. But one can expect the next term to at least give an indication of whether such a scenario is possible.

To calculate this term I make a simplifying assumption. One can see by inspection of the one-loop result, that supersymmetry will be unbroken in V_{eff} if $m=0$. The existence of the pole, however, does not require any particular value for m; thus one may take m small and retain only those terms which are at most linear in m when computing V_{eff}. This just amounts to setting $m^2=0$ in the singlet propagators $\langle\phi\phi\rangle$ and $\langle\phi\bar{\phi}\rangle$, and considerably simplifies the evaluation of integrals.

The $O(\hbar^2)$ term in (2.1) is

$$V_{\text{eff}}^{(2)} = -\frac{ig^2}{2\Omega N}\langle T(S_{\text{int}}S_{\text{int}})\rangle$$
$$= -\frac{ig^2}{2\Omega}\int d^4x_1 d^4x_2 \left[\int d^2\theta_1 d^2\theta_2 \langle\phi(1)\phi(2)\rangle\langle\phi_i(1)\phi_i(2)\rangle^2 + \int d^2\theta_1 d^2\bar\theta_2 \langle\phi(1)\bar\phi(\bar 2)\rangle\langle\phi_i(1)\bar\phi_i(\bar 2)\rangle^2 + \text{H.c.}\right] . \tag{3.1}$$

The appearance of the background fields a_0 and f_0 in $S[\phi_i]$ results in rather complicated expressions for the vector propagators in (3.9). Consider, for example, the propagator $\langle\phi_i(z_1)\bar\phi_i(\bar z_2)\rangle$. Using the superfield expansion and the inverse propagators of Sec. II, one finds

$$\langle\phi_i(1)\bar\phi_i(\bar 2)\rangle = -\frac{i}{\Delta p}\left[\frac{\bar D_1^2 D_1^2}{16}\delta^4(\theta_{12})\right]\Phi(p,\theta_1,\bar\theta_2) , \tag{3.2}$$

where

$$\Phi(p,\theta_1,\bar\theta_2) = p^2 + |a_0|^2 + \frac{2|f_0|^2}{p^2+|a_0|^2}\theta_1\sigma^\mu\bar\theta_2 p_\mu - \theta_1^2 f_0\bar a_0 - \bar\theta_2^2 \bar f_0 a_0 + \theta_1^2\bar\theta_2^2\frac{|f_0|^2|a_0|^2}{p^2+|a_0|^2} ,$$
$$\Delta p = (p^2+|a_0|^2)^2 - |f_0|^2 .$$

The corresponding two-point function for the singlet field is of standard form:

$$\langle\phi(1)\bar\phi(\bar 2)\rangle = -\frac{i}{p^2+m^2}\frac{\bar D_1^2 D_1^2}{16}\delta^4(\theta_{12}) . \tag{3.3}$$

I will refer to the contribution involving (3.2) and (3.3) is the chiral part of V_{eff}. The propagators $\langle\phi_i\phi_i\rangle$ and $\langle\phi\phi\rangle$ in the nonchiral part have a somewhat different form. The singlet field is again standard:

$$\langle\phi(1)\phi(2)\rangle = \frac{im}{p^2+m^2}\left[\frac{\bar D_1^2}{4}\delta(\bar\theta_{12})\right]\delta(\theta_{12}) . \tag{3.4}$$

The factor $\delta(\theta_{12})$ in (3.4) ensures that loop graphs contributing to the nonchiral part of V_{eff} will vanish. This nonrenormalization theorem will be spoiled in the presence of background fields, so that the vector propagator must have the form

$$\langle\phi_i(1)\phi_i(2)\rangle = \langle\phi_i(1)\phi_i(2)\rangle^{(0)} + \langle\phi_i(1)\phi_i(2)\rangle^{(b)} ,$$

where $\langle\phi_i(1)\phi_i(2)\rangle^{(0)}$ looks like (3.4) with $m\to\bar a_0$, $m^2\to|a_0|^2$ and

$$\langle\phi_i(1)\phi_i(2)\rangle^{(b)} = -\frac{i}{\Delta p}\left[\frac{\bar D_1^2}{4}\delta(\bar\theta_{12})\right]\chi(p,\theta_1,\theta_2) ,$$
$$\chi(p,\theta_1,\theta_2) = \bar f_0 - (\theta_1^2+\theta_2^2)\left[\frac{\bar a_0|f_0|^2}{p^2+|a_0|^2}\right] + \theta_1^2\theta_2^2\bar a_0^2 f_0 .$$

The part of the vector propagator containing the δ function drops out of (3.1). Also, because there are no $\bar\theta$ integrations in this term, the covariant derivatives only contribute an overall factor of -1. The nonchiral term can be written

$$V_{\text{eff}}^{(2)}(\text{nonchiral}) = -\frac{g^2}{2}m(a_0 f_0 + \bar a_0\bar f_0)|f_0|^2 \times \int \frac{d^4p}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}\frac{1}{p^2+m^2}\left[\frac{1}{(q^2+|a_0|^2)\Delta_{p+q}\Delta_q} + \frac{1}{[(p+q)^2+|a_0|^2]\Delta_q\Delta_{p+q}}\right] . \tag{3.5}$$

The evaluation of the chiral part of (3.1) is no more difficult. This term is (schematically)

$$V_{\text{eff}}^{(2)}(\text{chiral}) = -\frac{g^2}{2}\int\frac{d^4p}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}d^2\theta_1 d^2\bar\theta_2\left[\frac{\bar D_1^2 D_1^2}{16}\delta^4(\theta_{12})\right]^3\frac{\Phi(q)\Phi(-p-q)}{(p^2+m^2)\Delta_q\Delta_{p+q}} .$$

This may be simplified by expressing the covariant derivative factors as exponentials. These cancel because of momentum conservation, so the result is

$$V_{\text{eff}}^{(2)}(\text{chiral}) = \frac{g^2}{2}|f_0|^2\int\frac{d^4p}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}\frac{|a_0|^2[q^2+|a_0|^2+(p+q)^2+|a_0|^2]^2+2|f_0|^2(q^2+p\cdot q)}{(p^2+m^2)(q^2+|a_0|^2)[(p+q)^2+|a_0|^2]\Delta_q\Delta_{p+q}} . \tag{3.6}$$

Setting $m^2=0$ in (3.5) and (3.6) and expanding the integrands, one finds that all integrals can be expressed in terms of

$$J(\alpha+\beta) = \int\frac{d^4p}{(2\pi)^4}\frac{1}{p^2+\alpha+\beta} = \frac{\alpha}{(4\pi)^2}\left[\frac{4\pi\mu^2}{\alpha}\right]^\epsilon\left[1+\frac{\beta}{\alpha}\right]^{1-\epsilon}\Gamma(-\epsilon+1)$$

and

$$K(\alpha,\beta)=\int \frac{d^4p}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}\frac{1}{p^2[(p+q)^2+\alpha](q^2+\beta)}$$
$$=\frac{\alpha}{(4\pi)^4}\left[\frac{4\pi\mu^2}{\alpha}\right]^{2\epsilon}\frac{\Gamma(-1+2\epsilon)}{1-\epsilon}B(\epsilon,\epsilon)\,{}_2F_1(2\epsilon-1,\epsilon;2\epsilon;1-\beta/\alpha)\ .$$

After some algebra, one finds that the dimensionally regularized form of $V_{\rm eff}^{(2)}$ is

$$V_{\rm eff}^{(2)}=-\frac{g^2}{2}J^2(|a_0|^2)\Bigg[G(\epsilon)\Bigg\{(1+x)^{1-2\epsilon}\left[(1+x)+2(1+x)F\left[\frac{x}{1+x}\right]-2F\left[\frac{2x}{1+x}\right]\right]$$
$$+(1+x)(1-x)^{1-2\epsilon}+2(1-x)F(x)-4\Bigg\}+\tfrac{1}{2}[(1+x)^{1-\epsilon}+(1-x)^{1-\epsilon}-2]^2$$
$$-m\frac{a_0f_0+\bar{a}_0\bar{f}_0}{|f_0||a_0|^2}G(\epsilon)\left\{(1+x)^{1-2\epsilon}\left[1-2F\left[\frac{x}{1+x}\right]\right]-(1-x)^{1-2\epsilon}+2F(x)\right\}\Bigg]\ . \qquad (3.7)$$

Here I have used an abbreviated notation for the hypergeometric function and omitted absolute value signs for x. The function

$$G(\epsilon)=\frac{\Gamma(-1+2\epsilon)B(\epsilon,\epsilon)}{(1-\epsilon)\Gamma^2(-1+\epsilon)}$$

appears in (3.7) because of the relation

$$K(\alpha,\beta)=\frac{J^2(\alpha)}{\alpha}G(\epsilon)F\left[1-\frac{\beta}{\alpha}\right]\ .$$

Finally, the expansion of the hypergeometric function to $O(\epsilon^2)$ is

$$F(x)=F_0-\epsilon F_1-\epsilon^2F_2=1-\frac{x}{2}-\epsilon\left[\frac{x}{2}+(1-x)\ln(1-x)\right]-\epsilon^2[2x+(1-x)\ln(1-x)-\tfrac{1}{2}(1-x)^2\ln^2(1-x)-L_{i_2}(x)]\ .$$

Note, for small x,

$$F(x)\simeq 1-\frac{x}{2}-\epsilon\frac{x(1-x)}{2}+\epsilon^2\frac{x^2}{4}\ .$$

After expanding (3.7) one finds that the first nonvanishing contribution to the term in bold parentheses is $O(\epsilon)$; it is

$$-2\epsilon G(\epsilon)[(1+x)\ln(1+x)+(1-x)\ln(1-x)]$$

so there is a nonlocal divergence. The contribution to $V_{\rm eff}^{(2)}$ is

$$-\frac{g^2|a_0|^4}{(4\pi)^4}\left[\frac{1}{\epsilon}+2\psi(2)+1-2\ln\frac{|a_0|^4}{4\pi\mu^2}\right][(1+x)\ln(1+x)+(1-x)\ln(1-x)]\ .$$

The divergent part is canceled by one-loop counterterm diagrams as in Fig. 1. Explicitly these are

$$\frac{ig^2}{32\pi^2}\left[\frac{1}{\epsilon}+\psi(1)\right]\int\frac{d^4p}{(2\pi)^4}\left[\int d^4\theta_1\langle\phi_i(1)\bar{\phi}_i(\bar{1})\rangle+\int d^4\theta_2\langle\phi_i(2)\bar{\phi}_i(\bar{2})\rangle\right]$$
$$=\frac{g^2|a_0|^4}{(4\pi)^4}\Bigg[\left[\frac{1}{\epsilon}+2\psi(2)-1-\ln\frac{|a_0|^4}{4\pi\mu^2}\right][(1+x)\ln(1+x)+(1-x)\ln(1-x)]$$
$$+\tfrac{1}{2}[(1+x)\ln^2(1+x)+(1-x)\ln^2(1-x)]\Bigg]\ .$$

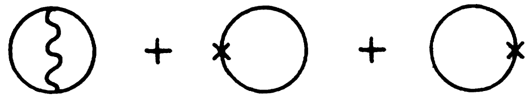

FIG. 1. Two-loop effective potential plus counterterms.

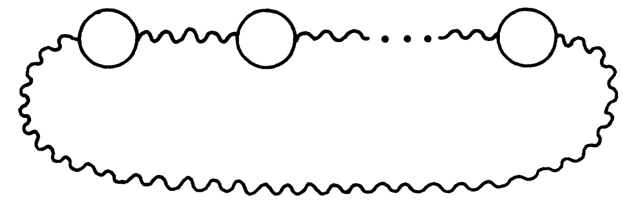

FIG. 2. Remaining $O(N^0)$ diagrams.

The resulting contribution to $V_{\text{eff}}^{(2)}$ is (for small x)

$$-\frac{g^2|a_0|^4x^2}{(4\pi)^4}\left[1-\ln\frac{|a_0|^4}{4\pi\mu^2}\right] . \tag{3.8}$$

Next consider the finite parts of V_{eff}^2 coming from the $O(\epsilon^2)$ piece of the term in bold parentheses in (3.7). The mass-dependent terms give a small-x contribution

$$-\frac{2g^2}{(4\pi)^4}ma^3x^2 .$$

Because this term is proportional to x^2 it will contribute to the denominator of V_{eff} after x is eliminated through $\partial V_{\text{eff}}/\partial x=0$, it must therefore be dropped in order to be consistent with the small-m^2 approximation used to evaluate (3.5) and (3.6). The small-x contribution of the remaining terms is

$$\frac{g^2|a_0|^4x^2}{(4\pi)^4} ;$$

putting this together with (3.8) and the lower-order result, one finds that the corrected value for V_{eff} is

$$V_{\text{eff}}\simeq\frac{g^2Nm^2|a_0|^4}{1-\frac{g^2}{32\pi^2}\left[1+\frac{4}{N}\frac{g^2}{32\pi^2}\right]\ln\frac{|a_0|^4}{4\pi\mu^2}} . \tag{3.9}$$

The sign of the correction implies that the pole is merely shifted by $O(1/N)$ to a *smaller* value of $|a_0|/4\pi\mu^2$. If this trend continues the pole will not be removed by higher-order corrections. A more spectacular feature of (3.9) is that it has only linear $\ln\mu$ dependence which originates from the subtraction of one-loop counterterms; thus the β function receives no $O(\hbar^2)$ correction. This phenomenon must continue in higher orders, so that the model is superrenormalizable. This is peculiar because there are interaction monomials of dimension four in the Lagrangian and the superficial degree of divergence of diagrams does not decrease in perturbation theory. However, one can see that it is true by turning the latter fact around: the momentum integrals (3.5) and (3.6) already have negative superficial degrees of divergence; since the theory is at least renormalizable, all other diagrams must be superficially convergent as well. Any divergences which arise at higher order can be gotten rid of by renormalizing subdiagrams. Then, by induction, there are no new divergences. Thus there is at most linear logarithmic dependence in higher orders, coming from the effect of the one-loop divergence. Another way to see this is to note that the diagrams which contribute leading logarithmics are the $O(N^0)$ diagrams (Fig. 2). Since the coefficient of the leading power of the logarithm depends on the second-order correction to the β function, it must vanish.

IV. CONCLUSIONS

The unexpected suppression of divergences found in this model fits rather neatly with the folklore discussed in the Introduction. In this connection it might be desirable to understand the breaking of supersymmetry from a more modern point of view. The Lagrangian (1.5) has a $U(1)_R$ symmetry, and at the tree level there are Goldstone bosons on the coset space

$$U(1)_R\times O(N)/U(1)'\times O(N-2) .$$

This manifold is almost the compact Kahler manifold

$$O(N)/U(1)\times O(N) .$$

Supersymmetric theories defined on such manifolds are known to be anomalous, so in a sense, R symmetry must be broken in the tree effective potential to evade the anomaly. On the other hand, the large-N limit breaks R symmetry [but not O(N) symmetry] explicitly because the singlet field must be given an expectation value. This is reminiscent of the situation in gauge theories described in Ref. 4, where there are no physical negative-norm states, but where also some global-symmetry breaking is supposed to accompany the breakdown of supersymmetry. It would be intersting to know if this parallel runs deeper.

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