Massless scalar and antisymmetric tensor fields in de Sitter space

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(Received 1 July 1987; revised manuscript received 13 August 1987)

The theory of a massless, minimally coupled scalar field ϕ is closely related to that of a massless, antisymmetric tensor field $A_{\mu\nu}$. It is shown that in de Sitter space the field $A_{\mu\nu}$ has a de Sitter-invariant state with a well-defined two-point function, while it is known that no such states exist for the scalar field ϕ.

I. INTRODUCTION

Quantum field theory in de Sitter space has recently attracted much attention, mainly because of its relevance for the inflationary cosmological models. In particular, there has been much discussion of the peculiar properties of a massless, minimally coupled scalar field in de Sitter space.[1–6] It has been shown that no quantum state of such a field has a de Sitter-invariant two-point function. One can formally define a de Sitter-invariant quantum state, but its two-point function does not exist, because of an infrared divergence. One can also choose a state which is free of infrared divergences, but such states are not de Sitter invariant. If the existence of a two-point function is regarded as necessary for a well-defined quantum state, then we can summarize the situation as follows: a massless, minimally coupled scalar field does not have any de Sitter-invariant states.

It has been known for a long time[7,8] that a massless scalar field is essentially equivalent to a massless antisymmetric tensor field: $A_{\mu\nu}=-A_{\nu\mu}$. It is therefore interesting to develop a quantum field theory for such a field in de Sitter space and to see whether or not it has similar infrared problems. This will be done in this paper. It will be shown that, unlike its scalar counterpart, a massless antisymmetric tensor field has a de Sitter-invariant state with a well-defined two-point function.

The metric and curvature conventions of the paper are $(+---)$, $R^{\mu}{}_{\nu\sigma\tau}=\partial_\tau\Gamma^{\mu}{}_{\nu\sigma}-\cdots$, and $R_{\mu\nu}=R^{\sigma}{}_{\mu\sigma\nu}$. Thus $(\Delta_\mu\Delta_\nu-\Delta_\nu\Delta_\mu)V_\sigma=R^{\tau}{}_{\sigma\mu\nu}V_\tau$.

II. BASIC EQUATIONS

The action for a massless, antisymmetric tensor field $A_{\mu\nu}$ is[7,8]

$$S=\tfrac{1}{12}\int F_{\mu\nu\sigma}F^{\mu\nu\sigma}\sqrt{g}\,d^4x \ , \tag{2.1}$$

where

$$\begin{aligned}F_{\mu\nu\sigma}&=\nabla_\mu A_{\nu\sigma}+\nabla_\nu A_{\sigma\mu}+\nabla_\sigma A_{\mu\nu}\\&=\partial_\mu A_{\nu\sigma}+\partial_\nu A_{\sigma\mu}+\partial_\sigma A_{\mu\nu} \ ,\end{aligned} \tag{2.2}$$

and $g=|\det g_{\mu\nu}|$. ∇_μ and ∂_μ denote covariant and ordinary derivatives, respectively. A variation of (2.1) with respect to $A_{\mu\nu}$ gives the field equations

$$\nabla_\mu F^{\mu\nu\sigma}=0 \tag{2.3}$$

and a variation with respect to $g_{\mu\nu}$ gives the energy-momentum tensor

$$T_\mu^\nu=\tfrac{1}{2}F_{\mu\sigma\tau}F^{\nu\sigma\tau}-\tfrac{1}{12}\delta_\mu^\nu F_{\lambda\sigma\tau}F^{\lambda\sigma\tau} \ . \tag{2.4}$$

The action (2.1) is invariant under gauge transformations

$$A_{\mu\nu}\to A_{\mu\nu}+\nabla_\mu\Lambda_\nu-\nabla_\nu\Lambda_\mu=A_{\mu\nu}+\partial_\mu\Lambda_\nu-\partial_\nu\Lambda_\mu \ , \tag{2.5}$$

where Λ^μ is an arbitrary transverse vector field. A gauge condition analogous to the Lorentz gauge in electromagnetism is

$$\nabla_\nu A^{\mu\nu}=0 \ . \tag{2.6}$$

It does not fix the gauge completely; the remaining gauge freedom is restricted to transformations which satisfy

$$\nabla_\nu\nabla^\nu\Lambda^\mu-\nabla_\nu\nabla^\mu\Lambda^\nu=0 \ . \tag{2.7}$$

Using (2.3), (2.6), and the relation

$$(\nabla_\mu\nabla_\nu-\nabla_\nu\nabla_\mu)A_{\sigma\tau}=R^{\lambda}{}_{\sigma\mu\nu}A_{\lambda\tau}+R^{\lambda}{}_{\tau\mu\nu}A_{\sigma\lambda} \ , \tag{2.8}$$

we obtain the following equation for $A_{\mu\nu}$:

$$\nabla_\sigma\nabla^\sigma A_{\mu\nu}+2R^{\sigma}{}_{\mu\nu}{}^{\tau}A_{\sigma\tau}+R_\nu{}^\sigma A_{\mu\sigma}-R_\mu{}^\sigma A_{\nu\sigma}=0 \ . \tag{2.9}$$

To establish the relation between $A_{\mu\nu}$ and a scalar field, we define

$$F_\mu=\tfrac{1}{6}\epsilon_{\mu\nu\sigma\tau}F^{\nu\sigma\tau}, \quad F^{\mu\nu\sigma}=\epsilon^{\mu\nu\sigma\tau}F_\tau \ . \tag{2.10}$$

Then it follows from (2.3) that

$$\partial_\mu F_\nu-\partial_\nu F_\mu=0 \ , \tag{2.11}$$

and so F_μ has to be gradient of a scalar:

$$F_\mu=\partial_\mu\phi \ . \tag{2.12}$$

To show that ϕ is a minimally coupled massless field, we write

$$\nabla_\mu\nabla^\mu\phi=\nabla_\mu F^\mu=\tfrac{1}{2}\epsilon^{\mu\nu\sigma\tau}\nabla_\mu\nabla_\nu A_{\sigma\tau}=0 \ , \tag{2.13}$$

where in the last step we have used Eq. (2.8) and the relation

$$\epsilon^{\mu\nu\sigma\tau}R^{\lambda}{}_{\nu\sigma\tau}=0 \ . \tag{2.14}$$

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III. MODE FUNCTIONS

The metric of de Sitter space can be written as

$$g_{\mu\nu}=a^2(\eta)\eta_{\mu\nu} , \tag{3.1}$$

where

$$\eta_{\mu\nu}=\text{diag}(+1,-1,-1,-1) , \tag{3.2}$$

$a(\eta)=-(H\eta)^{-1}$, and η is the conformal time, which is related to the comoving time t by $\eta=-H^{-1}\exp(-Ht)$.

To simplify certain equations, it will be convenient to introduce "barred" contravariant tensors $\bar{A}^{\mu\nu},\bar{F}^{\mu\nu\sigma}$, and an operator $\bar{\partial}^{\mu}$ which are obtained from their covariant counterparts using the flat-space metric $\eta^{\mu\nu}$:

$$\bar{A}^{\mu\nu}=\eta^{\mu\sigma}\eta^{\nu\tau}A_{\sigma\tau}=a^4A^{\mu\nu}, \quad \bar{\partial}^{\mu}=\eta^{\mu\nu}\partial_\nu=a^2\partial^{\mu} , \tag{3.3}$$

etc. In terms of the barred quantities, the field equations (2.3) and the gauge conditions (2.6) take the form

$$\partial_\mu(a^{-2}\bar{F}^{\mu\nu\sigma})=0 , \tag{3.4}$$

$$\partial_\nu\bar{A}^{\mu\nu}=0 . \tag{3.5a}$$

As we already mentioned, Eq. (3.5a) does not fix the gauge completely. It is shown in the Appendix that in a conformally flat metric of the form (3.1) one can always impose an additional condition:

$$A^{0\mu}=0 . \tag{3.5b}$$

Note that Eqs. (3.5) give only five independent constraints on the six components of $A_{\mu\nu}$. [Equation (3.5b) reduces the number of components to three, and the three conditions of (3.5a) are not all independent due to $\partial_\mu\partial_\nu\bar{A}^{\mu\nu}=0$.] The remaining degree of freedom corresponds to the scalar field ϕ. The gauge specified by the conditions (3.5) can be called the Coulomb gauge.

With gauge conditions (3.5), the field equations (3.4) take the form

$$\Box A_{ij}-2(\dot{a}/a)\dot{A}_{ij}=0 , \tag{3.6}$$

where latin indices take values from 1 to 3, dots stand for differentiation with respect to η, and $\Box=\eta^{\mu\nu}\partial_\mu\partial_\nu$. The solution of Eqs. (3.5) and (3.6) for the mode with a wave vector $\mathbf{k}$ is

$$(\psi_{\mathbf{k}})_{ij}=N\epsilon_{ij}(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}}\frac{e^{-ik\eta}}{\sqrt{k}\,\eta}=N\epsilon_{ij}(\mathbf{k})\frac{e^{i(\mathbf{k}\cdot\mathbf{x}-k\eta)}}{\sqrt{k}\,\eta} , \tag{3.7}$$

where $k=|\mathbf{k}|$, N is a normalization factor, and $\epsilon_{ij}(\mathbf{k})$ is a three-dimensional antisymmetric tensor satisfying

$$\epsilon_{ij}k^j=0 . \tag{3.8}$$

In general, one could write a linear combination of solutions with $\exp(\pm ik\eta)$, but we shall see that the simplest choice of the mode functions (3.7) already gives a de Sitter-invariant state. A possible choice of ϵ_{ij} is

$$\epsilon_{ij}(\mathbf{k})=\frac{1}{\sqrt{2}k}\epsilon_{ijl}k^l , \tag{3.9}$$

where ϵ_{ijl} is the three-dimensional Levi-Civita tensor.

The form of the functions (3.7) creates an impression that the field $A_{\mu\nu}$ is conformally invariant. This impression, however, is deceptive: the mode functions are proportional to the flat-space functions only in de Sitter space and only for a particular choice of gauge.

IV. QUANTIZATION

To quantize the field A_{ij}, we first expand it in annihilation and creation operators $a_{\mathbf{k}}$ and $a_{\mathbf{k}}^\dagger$:

$$A_{ij}=(2\pi)^{-3/2}\int d^3k[a_{\mathbf{k}}(\psi_{\mathbf{k}})_{ij}+\text{H.c.}] , \tag{4.1}$$

where the mode functions $\psi_{\mathbf{k}}$ are given by Eq. (3.7). $a_{\mathbf{k}}$ and $a_{\mathbf{k}}^\dagger$ satisfy the usual commutation relations

$$\begin{aligned}&[a_{\mathbf{k}},a_{\mathbf{k}'}^\dagger]=\delta^{(3)}(\mathbf{k}-\mathbf{k}') ,\\ &[a_{\mathbf{k}},a_{\mathbf{k}'}]=[a_{\mathbf{k}}^\dagger,a_{\mathbf{k}'}^\dagger]=0 .\end{aligned} \tag{4.2}$$

To fix the normalization constant N, we demand that A_{ij} should satisfy the equation

$$[A_{mn}(\mathbf{x},\eta),H(\eta)]=i\dot{A}_{mn}(\mathbf{x},\eta) , \tag{4.3}$$

where $H(\eta)$ is the canonical Hamiltonian:

$$H(\eta)=\int(\tfrac{1}{2}\dot{A}_{ij}\pi^{ij}-\mathcal{L})d^3x . \tag{4.4}$$

Here $\mathcal{L}=\frac{1}{12}F_{\mu\nu\sigma}F^{\mu\nu\sigma}\sqrt{g}$ is the Lagrangian density and π^{ij} is the canonical momentum conjugate to A_{ij}:

$$\pi^{ij}=\frac{\partial\mathcal{L}}{\partial\dot{A}_{ij}}=a^{-2}\dot{\bar{A}}^{ij} . \tag{4.5}$$

Hence

$$H(\eta)=\frac{a^{-2}}{4}\int(\dot{\bar{A}}^{ij}\dot{A}_{ij}-\bar{\partial}^i\bar{A}^{jk}\partial_iA_{jk})d^3x . \tag{4.6}$$

From (4.1), (4.2), and (4.6) we find

$$[A_{mn}(\mathbf{x},\eta),H(\eta)]=iN^2H^2\dot{A}_{mn}(\mathbf{x},\eta) , \tag{4.7}$$

and thus $N=H^{-1}$.

The equal-time commutation relations between A_{ij} and π_{ij} are

$$\begin{aligned}&[\pi^{ij}(\mathbf{y},\eta),A_{ml}(\mathbf{x},\eta)]=\frac{-2i}{(2\pi)^3}\int d^3k\,e^{i\mathbf{k}\cdot(\mathbf{y}-\mathbf{x})}\delta^{*ij}{}_{ml}(\mathbf{k}) ,\\ &[\pi^{ij}(\mathbf{y},\eta),\pi_{ml}(\mathbf{x},\eta)]=[A^{ij}(\mathbf{y},\eta),A_{ml}(\mathbf{x},\eta)]=0 ,\end{aligned} \tag{4.8}$$

where

$$\delta^{*ij}_{ml}(\mathbf{k})=\frac{1}{2}\left[(\delta^i_m\delta^j_l-\delta^i_l\delta^j_m)+\frac{1}{k^2}(\delta^i_mk_lk^j+\delta^j_lk_mk^i-\delta^j_mk_lk^i-\delta^i_lk_mk^j)\right] . \tag{4.9}$$

Here, we regard the components of $\mathbf{k}$ as contravariant and obtain the corresponding covariant components using η_{ij}, so that $k_j=-k^j$. Note that Eq. (4.8) does not have a standard canonical form. In fact, it is easily seen that the canonical commutation relations are inconsistent with Eq. (3.5a). Our quantization procedure here is similar to the quantization of the electromagnetic field in the Coulomb gauge. In Minkowski space, the commutation relations (4.8) have been obtained by Kalb and Ramond.[7,9]

V. TWO-POINT FUNCTIONS

The two-point function for A_{ij} is

$$\langle A_{ij}(x)A_{m'l'}(x')\rangle=\frac{(H^2\eta\eta')^{-1}}{(2\pi)^3}\int\frac{d^3k}{k}e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')ik(\eta-\eta')}\left[\tfrac{1}{2}(\eta_{im'}\eta_{jl'}-\eta_{il'}\eta_{jm'})+\frac{1}{2k^2}(\eta_{l'j}k_{m'}k_i+\eta_{im'}k_jk_{l'}-\eta_{l'j}k_jk_{m'}-\eta_{jm'}k_ik_{l'})\right]. \quad (5.1)$$

It is a bitensor with unprimed indices belonging to point x and primed indices to point x'. After performing the $\mathbf{k}$ integration, we obtain

$$\langle A_{ij}(x)A_{m'l'}(x')\rangle=\frac{(H^2\eta\eta')^{-1}}{2(2\pi)^3}[(\eta_{m'i}\eta_{jl'}-\eta_{m'j}\eta_{l'i})(2A+B)+(3A+B)(\eta_{l'j}e_{m'}e_i+\eta_{m'i}e_je_{l'}-\eta_{l'i}e_je_{m'}-\eta_{jm'}e_ie_{l'})]\,, \quad (5.2)$$

where

$$A=\frac{2\pi}{(\Delta x)^2}\left[\frac{\Delta\eta}{\Delta x}\ln\left[\frac{\Delta\eta+\Delta x}{\Delta\eta-\Delta x}\right]-2\right],\quad B=\frac{4\pi}{(\Delta x)^2-(\Delta\eta)^2},\quad \Delta\mathbf{x}=\mathbf{x}-\mathbf{x}',\quad \Delta\eta=\eta-\eta',\quad \Delta x=|\Delta\mathbf{x}|\,,$$

$$\mathbf{e}=\Delta\mathbf{x}/\Delta x,\quad e_i=-e^i\,.$$

An obvious improvement compared to the scalar field case with a similar choice of state is that the two-point function (5.2) is finite. However, it does not have a de Sitter-invariant form. This is not surprising, since our choice of gauge (3.5b) breaks the de Sitter invariance. The two-point function for the field $A_{\mu\nu}$ is gauge dependent, and in Sec. VI we show that in the Feynman gauge it takes a completely de Sitter-invariant form. The gauge-invariant object $\langle F_\mu F_{\alpha'}\rangle$ *is* de Sitter invariant. To show this we write

$$\langle F_\mu(x)F_{\alpha'}(x')\rangle=\tfrac{1}{4}\epsilon_\mu{}^{\nu\lambda\tau}\epsilon_{\alpha'}{}^{\beta'\gamma'\sigma'}\partial_\tau\partial_{\sigma'}\langle A_{\nu\lambda}(x)A_{\beta'\gamma'}(x')\rangle\,. \quad (5.3)$$

(Here, the covariant derivatives have been replaced by ordinary derivatives because of the antisymmetry of $\epsilon_{\mu\nu\lambda\tau}$.) Substitution of (5.1) into (5.3) gives

$$\langle F_\mu(x)F_{\alpha'}(x')\rangle=(32\pi^3H^2)^{-1}\epsilon_\mu{}^{ij\nu}\epsilon_{\alpha'}{}^{m'l'\beta'}\partial_\nu\partial_{\beta'}\int\frac{d^3k}{k}(\eta\eta')^{-1}\left[e^{-ik\Delta\eta+i\mathbf{k}\cdot\Delta\mathbf{x}}\eta_{im'}\left[\eta_{jl'}+\frac{2}{k^2}k_jk_{l'}\right]\right], \quad (5.4)$$

and after performing the $\mathbf{k}$ integration we obtain

$$\langle F_\mu(x)F_{\alpha'}(x')\rangle=\frac{H^2}{4(2\pi)^3}\left[-B\left[2+\frac{B\eta\eta'}{\pi}\right]\eta_{\mu\alpha'}-\frac{B^2\Delta x^2}{\pi}\left[1+\frac{B\eta'\eta}{\pi}\right]e_{\alpha'}e_\mu+\frac{B^2\Delta x}{\pi}(\eta e_\mu\eta_{\alpha'0}-\eta' e_{\alpha'}\eta_{\mu0})+2\eta_{\mu0}\eta_{\alpha'0}B\right], \quad (5.5)$$

where

$$e^\mu=\left[\frac{\Delta\eta}{\Delta x},\mathbf{e}\right],\quad e_\mu=\left[\frac{\Delta\eta}{\Delta x},-\mathbf{e}\right].$$

To express (5.5) in a manifestly de Sitter-invariant form, we introduce the geodesic separation $\mu(x,x')$ between the points x and x' (Ref. 5)

$$\mu(x,x')=H^{-1}\text{arccosh}(2z-1)\,, \quad (5.6)$$

where

$$z(x,x')=\frac{(\eta+\eta')^2-\Delta x^2}{4\eta\eta'}\,. \quad (5.7)$$

Then

$$\langle F_\mu(x)F_{\alpha'}(x')\rangle=\frac{H^2}{8\pi^2(z-1)^2}\left[\left[\frac{z-2}{z-1}\right]z_{,\alpha'}z_{,\mu}-(z-\tfrac{3}{2})z_{,\alpha'\mu}\right], \quad (5.8)$$

indicating that the state we have chosen is indeed de Sitter invariant. We note that Eq. (5.8) coincides with the scalar field function $\langle\phi_{,\mu}(x)\phi_{,\alpha'}(x')\rangle$ calculated for the formally de Sitter-invariant state with a divergent two-point function $\langle\phi(x)\phi(x')\rangle$. This is not surprising, in view of relation (2.12).

The two-point function (5.8) can also be expressed in the form

$$\langle F_\mu(x)F_{\alpha'}(x')\rangle=\frac{-H^4}{32\pi^2}\{[2(1-z)^{-1}+(1-z)^{-2}]g_{\mu\alpha'}+[2(1-z)^{-1}+4(1-z)^{-2}]n_\mu n_{\alpha'}\}\ , \tag{5.9}$$

where $g_{\mu\alpha'}(x,x')$ is the bivector of parallel transport and the vectors n_μ and $n_{\alpha'}$ are defined by $n_\mu=\nabla_\mu\mu(x,x')$ and $n_{\alpha'}=\nabla_{\alpha'}\mu(x,x')$.

VI. TWO-POINT FUNCTION IN THE FEYNMAN GAUGE

The two-point function $\langle A_{ij}A_{m'l'}\rangle$ (5.2) is not de Sitter invariant because the gauge condition $A_{0\mu}=0$ breaks de Sitter invariance by picking out a preferred time direction. In this section, we show that in the Feynman gauge, which does *not* break de Sitter invariance, the two-point function *is* de Sitter invariant, and gives rise to the same expectation value for $\langle F_a F_{b'}\rangle$. To show this, we make use of the formalism of Allen and Jacobson[10] with the metric signature changed to $(+---)$.

We assume that $\langle A_{ab}(x)A_{c'd'}(x')\rangle$ is (1) de Sitter invariant, (2) has the same $x\to x'$ behavior as in flat space, and (3) has only one singularity, when x approaches the light cone of x' (plus possibly singularities at infinity). In the case of a massless minimally coupled scalar field in de Sitter space, these assumptions lead to a contradiction, because there is no vacuum state which is de Sitter invariant. In the present case, we will see that they are entirely consistent and lead to a unique propagator.

To fix the gauge, we add to the action (2.1) a gauge-fixing term

$$S_{\rm gf}=\frac{\lambda}{2}\int(A^{bc}{}_{;c})^2\sqrt{g}\,d^4x\ . \tag{6.1}$$

The action can then be written in de Sitter space as

$$S+S_{\rm gf}=\tfrac{1}{4}\int A_{cd}[-\Box A^{cd}+2(\lambda-1)A^{b[c}{}_{;b}{}^{;d]}-4H^2A^{cd}]\sqrt{g}\,d^4x\ . \tag{6.2}$$

In the Feynman gauge $(\lambda=1)$ the equation of motion satisfied by A^{bc} is then

$$(\Box+4H^2)A^{bc}=0\ . \tag{6.3}$$

This equation of motion is also obeyed by the two-point function.

The assumption of de Sitter invariance implies that the propagator can be expressed as

$$\langle A^{ab}A^{c'd'}\rangle=\alpha g^{a[c'}g^{d']b}+\beta n^{[a}g^{b][c'}n^{d']}\ , \tag{6.4}$$

where α and β are functions of the geodesic distance $\mu(x,x')$ from x to x'. The unit vectors $n^a=\nabla^a\mu$ and $n^{a'}=\nabla^{a'}\mu$ are tangent to the geodesic joining x to x', and point away from the geodesic. The bivector of parallel transport is denoted $g_{ab'}(x,x')$.

The equation of motion obeyed by the propagator is

$$(\Box+4H^2)\langle A^{ab}A^{c'd'}\rangle=0\ . \tag{6.5}$$

Using the method of Allen and Jacobson[10] one can show from (6.5) that the coefficient functions α and β of (6.4) obey the ordinary differential equations

$$\frac{d^2\alpha}{d\mu^2}+3A\frac{d\alpha}{d\mu}-[2(A+C)^2-4H^2]\alpha-2AC\beta=0\ , \tag{6.6a}$$

$$\frac{d^2\beta}{d\mu^2}+3A\frac{d\beta}{d\mu}-[2(A-C)^2-4H^2]\beta=0\ . \tag{6.6b}$$

Here $A=H\coth(H\mu)$ and $C=-H\,\mathrm{csch}(H\mu)$ differ from Ref. 10 because the metric signature is different. The general solution to these equations may be written as

$$\alpha=C_1[-\tfrac{1}{2}(z-1)^{-1}+\tfrac{1}{2}z^{-2}+\tfrac{1}{4}z^{-1}+\tfrac{1}{2}(z-1)^{-2}\ln z]+C_2[\tfrac{1}{4}z^{-2}-\tfrac{1}{4}(z-1)^{-2}]+C_3[-(z-1)^{-1}+2z^{-1}+z^{-2}+2z^{-2}\ln(1-z)]+C_4(z^{-2})\ , \tag{6.7a}$$

$$\beta=C_1[2(z-1)^{-1}-z^{-1}-2(z-1)^{-2}\ln z]+C_2(z-1)^{-2}\ , \tag{6.7b}$$

where C_1, C_2, C_3, and C_4 are arbitrary constants, and $z(\mu)$ is defined by Eq. (5.6). For comparison, the minimally coupled massless scalar field propagator, obeying the equation of motion $\Box G=0$, has a de Sitter-invariant solution:

$$G=d_1[z^{-1}+(z-1)^{-1}+2\ln(z-1)-2\ln z]+d_2\ , \tag{6.8}$$

where d_1 and d_2 are arbitrary constants.

One can now see that conditions (2) and (3) above determine a unique solution (i.e., unique values of $C_1\to C_4$) in the antisymmetric tensor case. Near $z=1$ (corresponding to x on the light cone of x') and near $z=0$ (corresponding to x on the light cone of the *antipodal* point of x') one finds

$$\alpha=-\tfrac{1}{4}C_2(z-1)^{-2}-C_3(z-1)^{-1}+\cdots\ , \tag{6.9a}$$

$$\beta=C_2(z-1)^{-2}+\cdots \tag{6.9b}$$

near $z=1$ and

$$\alpha=(\tfrac{1}{2}C_1+\tfrac{1}{4}C_2+C_3+C_4)z^{-2}+\tfrac{1}{4}C_1z^{-1}+\cdots\ , \tag{6.9c}$$

$$\beta=-C_1z^{-1}-2C_1\ln z+\cdots \tag{6.9d}$$

near $z=0$. Thus, the condition that α and β have their only singularities at $z=1$ implies that $C_1=0$ and $C_4=-\tfrac{1}{2}C_1-\tfrac{1}{4}C_2-C_3$. The condition that α and β have flat-space behavior near the light cone implies that $C_2=0$ and $C_3=(8\pi^2)^{-1}H^2$, because for small μ one finds that $\mu^2\simeq 4H^2(1-z)$.

Thus one obtains a unique solution for the two-point function

$$\langle A^{ab}A^{c'd'}\rangle_{\lambda=1}=\frac{-H^2}{8\pi^2}[(1-z)^{-1}+2z^{-1}+2z^{-2}\ln(1-z)]g^{a[c'}g^{d']b}\ . \tag{6.10}$$

For comparison, in the case of a massless minimally coupled scalar field, these three conditions lead to a contradiction: absence of a singularity as $z \to 0$ implies that $d_1=0$, but flat-space behavior as $z \to 1$ implies that $d_1=-(16\pi^2)^{-1}H^2$. This contradiction is how the lack of a de Sitter-invariant state manifests itself; in the present case there are no such difficulties.

In the complex z plane, the two-point function (6.10) has a branch cut along the real-z axis for $1 \le z < \infty$. One may obtain different correlation functions by evaluating (6.10) above or below this branch cut. The Feynman function is obtained in this way, as the limiting value above the branch cut

$$\lim_{\epsilon \to 0^+} G(z+i\epsilon) \ . \tag{6.11}$$

The symmetric function is the average of the values above and below the cut

$$\lim_{\epsilon \to 0^+} \tfrac{1}{2}[G(z+i\epsilon)+G(z-i\epsilon)] \ . \tag{6.12}$$

This has the effect of replacing $\ln(1-z)$ in (6.10) by $\ln|1-z|$. The commutator function is given by $\epsilon(t,t')\Delta G(z)$ where $\Delta G(z)$ is the discontinuity across the branch cut

$$\Delta G(z)=\lim_{\epsilon \to 0} \tfrac{1}{2}[G(z+i\epsilon)-G(z-i\epsilon)] \ . \tag{6.13}$$

Note that although (6.10) does possess a singularity at $z=\infty$, this point does not lie on the spacetime since $|z(x,x')|$ is finite (although unbounded) for any choice of points x and x' in de Sitter space.

The two-point function for the field vector $F_\mu(x)$ calculated from (6.10) using Eq. (5.3) coincides with Eqs. (5.8) and (5.9) as expected. Thus we have shown explicitly that in the Feynman gauge the two-point functions of the potential and the field are well behaved and de Sitter invariant.

We note that in addition to the gauge-fixing term (6.1) a compensating ghost field must also be added to the action. This has been shown to be equivalent to canonical quantization.[11] The ghost field is a Maxwell field which obeys fermionic statistics, and subtracts four degrees of freedom from the action. One of the remaining two degrees of freedom then decouples automatically. Because the ghost field (like the electromagnetic field) possesses as de Sitter-invariant ground state, it does not adversely affect de Sitter invariance.

VII. CONCLUDING REMARKS

Despite a very close connection between the massless minimally coupled scalar and antisymmetric tensor fields, there are also some important differences. We have seen that in de Sitter space the field $A_{\mu\nu}$ has a de Sitter-invariant quantum state, while it is known that no such states exist for the scalar field ϕ.

We note that the massive theories of the two fields are entirely different. In the presence of interactions, no symmetry prevents the scalar field ϕ from acquiring a mass, while the field $A_{\mu\nu}$ cannot become massive without breaking the gauge invariance. This makes the antisymmetric tensor representation particularly useful for describing Goldstone bosons in theories with spontaneously broken global symmetries. The simplest theory of this kind with a spontaneously broken U(1) symmetry has classical solutions describing "cosmic strings."[12] In this case the Goldstone field is a phase variable which changes by 2π around a string. In the scalar representation, the Goldstone field is multiple valued in the presence of strings. The antisymmetric tensor representation does not have this problem; it has been recently used to describe the interaction of cosmic strings with Goldstone bosons in Ref. 13.

It was shown by Duff and van Nieuwenhuizen[14] that the theories of a minimally coupled scalar field ϕ and an antisymmetric tensor field $A_{\mu\nu}$ are inequivalent if the topology of the spacetime is nontrivial. More precisely if the integral of the Gauss-Bonet invariant $R^{abcd}R_{abcd}-4R^{ab}R_{ab}+R^2$ does not vanish, then the equivalence between ϕ and $A_{\mu\nu}$ fails to be one to one, and the integrated trace anomalies of ϕ and $A_{\mu\nu}$ differ by an integer, corresponding to the missing mode.

The inequivalence can arise in two ways. If the first Betti number b_1 is not zero, then a vector with vanishing curl (2.11) is not necessarily the gradient of a scalar (2.12). In this case F^μ contains additional "harmonic" modes beyond those of ϕ. However, in de Sitter space where $b_1=0$ this complication does not arise.

The second way in which the scalar and antisymmetric tensor theories can fail to be equivalent occurs in spacetimes with compact spatial sections, as in de Sitter space. In this case the scalar mode $\phi=\text{const}$ has no antisymmetric tensor equivalent. Thus F^μ lacks one discrete degree of freedom in comparison to the scalar case. Of course both theories contain (roughly speaking) one degree of freedom at each point of spacetime and they are therefore equivalent in the sense that $\infty=\infty-1$.

The antisymmetric tensor field forms a representation of the de Sitter group with one mode left out, relative to the scalar case. One might think that the representation is not complete in the absence of this mode, but this is not the case. While it is no longer complete over the set of scalar square-integrable functions, it is complete over the set of square-integrable antisymmetric tensors. Thus the demonstration that the de Sitter group has no complete massless scalar representation[15] does not apply to the antisymmetric tensor case.

The infrared properties of a minimally coupled massless scalar field in de Sitter spacetime are similar to those in a flat two-dimensional spacetime,[4,6] and one can wonder whether or not such a similarity exists for the field $A_{\mu\nu}$. The answer is very simple. All totally antisymmetric tensors of rank greater than 2 vanish identically in a two-dimensional spacetime. Hence, $F_{\mu\nu\sigma}\equiv 0$ and the theory of the form (2.1) does not exist in two dimensions.

ACKNOWLEDGMENTS

We are grateful to Larry Ford for very helpful discussions. This research was supported in part by the National Science Foundation under Grant No. PHY 87-

05107 and by contributions from the General Electric Company and Dennison Company.

APPENDIX

Here it will be shown that in a conformally flat metric of the form (3.1) it is always possible to choose the gauge condition (3.5b) in addition to the "Lorentz condition" (3.5a). Suppose we have already imposed the gauge (3.5a),

$$\partial_\nu \bar{A}^{\mu\nu}=0 . \tag{A1}$$

Then the field equations (3.4) can be written as

$$\bar{\Box}\bar{A}^{\mu\nu}-2\frac{\dot{a}}{a}\bar{F}^{0\mu\nu}=0 . \tag{A2}$$

Hence, the components $\bar{A}^{0i}$ satisfy

$$\bar{\Box}\bar{A}^{0i}=0,\quad \partial_i \bar{A}^{0i}=0 . \tag{A3}$$

The general solution of (A3) is a linear combination of terms of the form

$$A_{0i}(x)=c_i(\mathbf{k})e^{-ik\cdot x}+d_i(\mathbf{k})e^{ik\cdot x} , \tag{A4}$$

where c_i and d_i satisfy

$$k^i c_i=k^i d_i=0 . \tag{A5}$$

To impose the additional gauge conditions (3.5b), let us consider the gauge transformation

$$A'_{\mu\nu}=A_{\mu\nu}+\partial_\mu\Lambda_\nu-\partial_\nu\Lambda_\mu \tag{A6}$$

with Λ_μ satisfying Eq. (2.7), so that $A'_{\mu\nu}$ is still in the gauge (A1). Equation (2.7) can be rewritten as

$$\partial_\nu(\bar{\partial}^\mu\bar{\Lambda}^\nu-\bar{\partial}^\nu\bar{\Lambda}^\mu)=0 . \tag{A7}$$

One of its solutions is $\Lambda_0=0$ and

$$\Lambda_i(x)=f_i(\mathbf{k})e^{-ik\cdot x}+h_i(\mathbf{k})e^{ik\cdot x} \tag{A8}$$

with

$$k^i f_i=k^i h_i=0 . \tag{A9}$$

If we choose

$$ikf_i=c_i,\quad ikh_i=d_i , \tag{A10}$$

where c_i and d_i are from Eq. (A4), then

$$A'_{0i}(x)=0 . \tag{A11}$$

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