Aspects of QED and non-Abelian gauge theories in $S^1\times R^3$ and $S^1\times R^4$ spacetimes

Atsushi Higuchi and Leonard Parker
Department of Physics, University of Wisconsin—Milwaukee, Milwaukee, Wisconsin 53201
(Received 6 August 1987)

We study various aspects of Abelian and non-Abelian gauge field theories in flat spacetime with the topology of $S^1\times R^n$ ($n=3,4$). We first discuss the effective potential for the electromagnetic field in $S^1\times R^3$ spacetime, with the objective of finding the dependence of coupling constants on the size L of the compactified dimension. It is first necessary to determine the stable vacuum. In particular, a charged scalar field satisfies an effective periodic boundary condition, while a charged spinor field satisfies an effective antiperiodic boundary condition at the stable vacuum. Then we study the effective coupling constants which depend on the size L of the compactified dimension at one-loop level for QED in $S^1\times R^3$ spacetime. In the presence of a charged scalar field, the one-loop correction of one of the effective coupling constants for the lowest Fourier component of the electromagnetic field behaves for small L like $1/L$, rather than $\ln L$. This is because the lowest Fourier component of the scalar field is constant in the S^1 direction. The effective coupling constants for higher Fourier components have logarithmic L dependence, but due to collinear singularities (i.e., singularities in self-energy occurring in the limit of zero mass when the virtual particles are on shell and have parallel momenta) the coefficient of the $\ln L$ term differs from component to component. We also point out some ambiguities in defining the effective coupling constants. Finally we discuss the effective potential of non-Abelian gauge theories with a charged spinor field in $S^1\times R^4$ spacetime. We find that if periodic boundary conditions are imposed on the spinor field in the beginning, the local gauge symmetry will be spontaneously broken in some cases, and we give an explicit model in which this occurs. This spontaneous symmetry breaking depends on the gauge group and its representations, and does not occur for an Abelian gauge field.

I. INTRODUCTION

It was shown by studying the renormalization-group scaling behavior of the effective action that effective coupling constants acquire logarithmic curvature dependence in curved spacetime.[1] This fact was shown[2] also by using the partially summed form[3] of the Schwinger-DeWitt proper-time series[4] of the propagator.

These analyses assume implicitly that there is no significant curvature dependence in coupling constants due to the infrared behavior of the theory. This assumption is not always satisfied. In fact, exact one-loop calculations[5] show that models which confirm the above behavior do exist, but that models also exist in which infrared contributions alter the coefficient of the logarithmic curvature-dependent term in the effective gauge coupling constant. Thus, it is interesting to calculate effective coupling constants explicitly in various simple spacetimes. It is also interesting to see how the "size" of the Universe influences effective coupling constants when the spacetime is not necessarily curved.

With the above discussion as the motivation we study quantum electrodynamics (QED) in $S^1\times R^3$ spacetime, i.e., flat spacetime with one compactified spacelike dimension. One can either regard this as a Kaluza-Klein theory with a compactified dimension, or as a theory of a field in spacetime between plates with suitable boundary conditions. In particular, we calculate the renormalized coupling constants to one-loop order to find their dependence on the size of the compactified dimension. The Lagrangian is

$$\mathcal{L}=-\tfrac{1}{4}F_{MN}F^{MN}+\mathcal{L}_{\text{matter}}\quad (M,N=0,1,2,3) \tag{1.1}$$

with

$$\mathcal{L}_{\text{matter}}=(\partial_M+ieA_M)\phi^*(\partial^M-ieA^M)\phi-M^2|\phi|^2 \tag{1.2}$$

or

$$\mathcal{L}_{\text{matter}}=i\bar{\psi}\gamma^M(\partial_M-ieA_M)\psi-M\bar{\psi}\psi\ , \tag{1.3}$$

where

$$F_{MN}=\partial_M A_N-\partial_N A_M\ . \tag{1.4}$$

Here ϕ is a complex scalar field, ψ is a Dirac spinor, and $A_M(M=0,1,2,3)$ is an Abelian gauge field. The metric signature is $(+---)$. The third dimension is compactified; i.e., the spacetime point (x^μ,x^3+L) $(\mu=0,1,2)$ is identified with the point (x^μ,x^3). We study suitably defined on-shell coupling constants because off-shell coupling constants would reflect the momentum scale involved as well as the size of the "third" (compactified) dimension.

We also discuss the one-loop effective potential to determine the vacuum expectation value (VEV) of the third component of the gauge potential, i.e., $\langle A_3\rangle$, in the stable vacuum. This is in fact necessary to calculate the coupling constants. The calculation of the effective potential also provides a deeper understanding of QED with charged fields satisfying various boundary conditions,[6] a situation which was discussed by Ford.[7] Spacetimes of the type studied here were also studied by Toms.[8] The dependence of the effective potential on an SU(n) gauge field was studied by Hosotani,[9] who showed that the unstable vacuum can decay to a stable vacuum in which massless spinors coupled to the gauge field acquire a dynamical mass. In the general non-Abelian case, the stability of the vacuum was also investigated by Toms.[10] Here we emphasize the fact that the boundary conditions of the charged matter fields and the value of the VEV, $\langle A_3 \rangle$, are closely related to each other through gauge transformations. We find (for the Abelian gauge field) that if one demands that $\langle A_3 \rangle$ be zero in the stable vacuum, then the boundary condition on the spinor is dynamically determined to be antiperiodic.

We consider the full class of boundary conditions permitted by the form of the Lagrangian of the theory (see Sec. II), without regarding the field as a single-valued cross section on a bundle over $S^1 \times R^n$ ($n=3$ or 4). Restricting ourselves to such single-valued fields on this bundle would allow only the field configurations discussed in Ref. 6, but would not permit us to explore the dynamical connection between such configurations, or the possibility that other configurations may be dynamically favored. In the case of QED, the dynamically favored configurations are of the type considered in Ref. 6. However, in the non-Abelian case, other configurations appear to be, in some cases, dynamically more favorable.

As a natural extension of the Abelian case, we discuss the one-loop effective potential of non-Abelian gauge theories in $S^1 \times R^4$ spacetime. (The dimensionality is increased by one because the calculation here may be relevant to Kaluza-Klein grand unified theories. However, in that case the theory is not renormalizable, although the effective potential is finite at one loop). Unlike the case of Abelian gauge theories, we note that the gauge symmetry can be spontaneously broken by a nonzero VEV of the "fifth" component of the gauge potential. In his well-known work, Hosotani[9] obtained symmetry breaking by introducing a constant phase factor in the fermion boundary condition. We present a model with spontaneous gauge symmetry breaking with charged spinor fields satisfying originally periodic boundary conditions, which was not discussed in Hosotani's work.

The rest of the paper is organized as follows. Sections II–VI deal with QED in $S^1 \times R^3$ spacetime. In Sec. II, the close connection between the boundary conditions of charged fields and the VEV $\langle A_3 \rangle$ is clarified. In Sec. III, the one-loop effective potential is calculated with one charged field (a scalar or a spinor) to determine $\langle A_3 \rangle$. In Sec. IV the general form of the photon inverse propagator is discussed using gauge invariance. Then the on-shell coupling constants are defined. In Sec. V the calculation of these coupling constants is carried out for QED with one charged scalar field. The correction to the mass of the photon arising from the breaking of Lorentz invariance by the compact dimension is also calculated. In Sec. VI, the same quantities are calculated for QED with one charged spinor field. In Sec. VII (which can be read after Sec. III), we turn to SU(2) gauge theories with charged spinor fields satisfying periodic boundary conditions. The one-loop effective potential is calculated to determine whether or not the gauge symmetry is spontaneously broken. In Sec. VIII, the results of the paper are summarized and discussed. In the Appendix some formulas used in the paper are presented.

II. GAUGE INVARIANCE AND BOUNDARY CONDITIONS

Here we discuss the connection between gauge transformations and boundary conditions. First consider scalar QED in this spacetime.

We require that the Lagrangian be single valued:

$$\mathcal{L}(x^0,x^1,x^2,x^3+L)=\mathcal{L}(x^0,x^1,x^2,x^3) . \tag{2.1}$$

Because $\mathcal{L}$ is invariant under the gauge transformation

$$A'_M(x)=A_M(x)+e^{-1}\partial_M \Lambda(x) , \tag{2.2a}$$

$$\phi'(x)=\exp[i\Lambda(x)]\phi(x) , \tag{2.2b}$$

there will be a certain latitude in the form of the boundary conditions relating the values of the fields at $x^3=0$ and L.

For example, consider gauge transformations with Λ depending only on x^3 [if one were to include in $\mathcal{L}$ a gauge fixing term proportional to $(\partial_\mu A^\mu)^2$, where μ runs from 0 to 2, then only such Λ would be permitted]. Suppose we impose the boundary conditions (suppressing all but x^3 dependence for brevity)

$$\phi(x^3+L)=\exp(i\alpha)\phi(x^3) , \tag{2.3a}$$

$$A_M(x^3+L)=A_M(x^3) . \tag{2.3b}$$

The condition (2.3b) follows from (2.3a) and (2.1) because of terms such as $A_M\phi^*\phi$ in $\mathcal{L}$. Periodic and antiperiodic boundary conditions are special cases of (2.3a), with $\alpha=0$ and π, respectively.

Before proceeding further, we should discuss our viewpoint concerning boundary conditions with arbitrary values of α. We are taking the viewpoint that the physical requirement is Eq. (2.1), and that no values of α in Eq. (2.3) are forbidden. If Eq. (2.1) were not satisfied, then physical amplitudes would depend on the coordinate patch one uses (i.e., whether it contains $x^3=0$ or L). If in addition, one were to require that the field $\phi(x)$ is a single-valued cross section of a fiber bundle with a U(1) structure group and with base space $S^1 \times R^3$, then it would follow[6] that for a scalar field the only value of α is 0 and for a spinor field the only values of α are 0 and π. In order to dynamically study the relation between boundary conditions on ϕ and vacuum expectation values of A_3, we take the viewpoint that α can, in principle, take any value. In terms of fiber bundles, the field $\phi(x)$ is a single-valued cross section of a fiber bundle with a U(1) structure group and with base space being the covering space R^4 of $S^1 \times R^3$. Then α is the angle of twist associated with each change of x^3 by L. Alternatively, one could say we are working on a bundle with base space $S^1 \times R^3$ and a multivalued cross section. To interpret the configuration of ϕ on $S^1 \times R^3$, it is our view that one can use the values of the field on $[0,L) \times R^3$ to calculate physical results [provided Eq. (2.1) holds]. The physical results, which are gauge invariant, are unchanged if the interval $[0,L]$ is translated by a constant value in the x^3 direction, since that would only change the phase of ϕ everywhere by a fixed constant. It is interesting that the minima of the effective potential with a single scalar or spinor field correspond in QED to bundles considered in Ref. 6. (In the non-Abelian case that will no longer be true in general.) We will find that the effective potential V depends on a particular linear combination of α and A_3. This follows from the direct evaluation of V, which we carry out in Sec. III. One can also deduce the dependence of V on α and A_3 from the following argument.

A gauge transformation (2.2) with $\Lambda(x^3)=Kx^3$ gives new physically equivalent fields ϕ' and A'_M satisfying the boundary conditions

$$\phi'(x^3+L)=\exp[i(\alpha+KL)]\phi'(x^3) , \tag{2.4a}$$

$$A'_M(x^3+L)=A'_M(x^3) . \tag{2.4b}$$

(The transformation with $\Lambda=Kx^3$ does not have period L, so strictly speaking it is a gauge transformation on the fiber bundle with base space being the covering space of $S^1\times R^3$, rather than $S^1\times R^3$ itself.) At the same time, the gauge transformation of the vector field is $A'_\mu(x)=A_\mu(x)$ $(\mu=0-2)$ and

$$A'_3(x)=A_3(x)+e^{-1}K . \tag{2.5}$$

Thus, the boundary conditions for ϕ and the field A_3 are not separately invariant under the above gauge transformation, but the following combination is:

$$\alpha L^{-1}-eA_3 . \tag{2.6}$$

This is because the phase α changes to $\alpha'=\alpha+KL$ when A_3 changes as in (2.5).

If K is chosen to be $2\pi nL^{-1}$ with n an integer, the boundary conditions on the fields are unchanged, while A_3 is shifted by $2\pi n(eL)^{-1}$. As this is a gauge transformation in the sense that it is an invariance of the Lagrangian, it implies, for example, that the gauge-invariant effective potential (or more generally, the effective action) must be periodic in A_3 with period $2\pi(eL)^{-1}$. Because of the interplay described above between boundary conditions and gauge transformations, we can regard the phase α in (2.3a) as arbitrary. Then the vacuum expectation value, $\langle A_3\rangle$, will be dynamically determined [modulo $2\pi(eL)^{-1}$] by the minima of the effective potential, as we show explicitly in the next section. (Alternatively, one could regard $\langle A_3\rangle$ as freely chosen and the boundary condition, α, on ϕ as dynamically determined so as to minimize the effective potential. The physical consequences are the same. We adopt the viewpoint in which α is freely chosen and $\langle A_3\rangle$ fixed by the dynamics.) The possible objection that configurations with different globally constant values of $\langle A_3\rangle$ cannot be dynamically connected because they differ by infinite energy, can be met in the same way as in the usual Higgs mechanism. Namely, in an actual physical system the field configuration reached dynamically will be such that $\langle A_3\rangle$ is constant (at a value which minimizes V) over a large but finite region of space, and $\langle A_3\rangle$ may correspond to different minima of V in widely separated domains.

When there is more than one charged scalar field present, the relative phases in the boundary conditions will also be gauge invariant and physically meaningful. The case of more than one charged scalar field will not be discussed further in this paper.

Fermions may also be present, in which case the Lagrangian has additional terms of the form

$$\mathcal{L}=i\bar\psi(\partial\!\!\!/-ieA\!\!\!/)\psi-M\bar\psi\psi . \tag{2.7}$$

The previous discussion is also valid for the fermion field ψ, if one simply replaces ϕ by ψ in Eqs. (2.3a) and (2.4a). Before turning to the calculation of the A_3 dependence of the effective potential we remark that because A_3 is taken to be constant in the effective potential, by definition, we cannot expect the result obtained for V [i.e., Eq. (3.4)] to be manifestly invariant under gauge transformations $\Lambda(x)$ which make A_3 nonconstant. However, we can expect manifest invariance of V under gauge transformations with $\Lambda=Kx^3$ which change A_3 to a new constant value. In the case of QED, the dynamics will not give rise to symmetry breaking. However, as we show in Sec. VII (which may be read after Sec. III), the dynamics can give rise to spontaneous symmetry breaking in the non-Abelian case.

III. EFFECTIVE POTENTIAL FOR A_3

The minima of the effective potential V for A_3 determine the vacuum expectation value $\langle A_3\rangle$ (Ref. 11). The lowest Fourier component of the field A_3 is a scalar field in the three-dimensional spacetime having coordinates (x^0,x^1,x^2). A nonzero value of $\langle A_3\rangle$ would shift the value of the dimensionally reduced mass of the charged fields present in the Lagrangian. This will also give rise in the dimensionally reduced theory to massive vector bosons when the gauge field is non-Abelian.

In calculating V, we take A_3 constant and $A_\mu=0$ $(\mu=0-2)$. Then $\mathcal{L}$ of Eq. (1.2) is quadratic in ϕ and to within a four-divergence has the form

$$\mathcal{L}=-\phi^*D\phi , \tag{3.1}$$

where

$$D=\eta^{\mu\nu}\partial_\mu\partial_\nu-(\partial_3-ieA_3)^2+M^2 . \tag{3.2}$$

Upon Euclideanization $(x^0\to ix^0)$, the eigenvalues of the operator D are

$$p^2+\left[\frac{1}{L}(2\pi m+\alpha)-eA_3\right]^2+M^2 \quad (m=0,\pm1,\ldots) , \tag{3.3}$$

where $p^2=p_\mu p^\mu=\delta^{\mu\nu}p_\mu p_\nu$. Here α appears in the boundary condition of Eq. (2.3a) on the scalar field. The Gaussian functional integral for the effective potential gives $V=\mathrm{Tr}\ln D$ divided by the spacetime volume, or

$$V=(2\pi)^{-3}L^{-1}\sum_{m=-\infty}^{\infty}\int d^3p\,\ln\left[p^2+\left[\frac{2\pi m}{L}-eA_3+\frac{\alpha}{L}\right]^2+M^2\right]$$
$$=(2\pi)^{-3}L^{-1}\sum_{m=-\infty}^{\infty}\int d^3p\,\ln\left[\left[\frac{L}{2\pi}\right]^2\omega_p^2+\left[m-\frac{L}{2\pi}eA_3+\frac{\alpha}{2\pi}\right]^2\right], \quad (3.4)$$

where $d^3p=dp_0dp_1dp_2$. Here $\omega_p\equiv(p^2+M^2)^{1/2}$, and an infinite constant term has been dropped, since it will not affect the value of A_3 which minimizes V. The periodicity of V in A_3 with period $2\pi(eL)^{-1}$ is manifest in Eq. (3.4).

Also, note that V depends only on $\alpha/L-eA_3$, not separately on α and A_3, as discussed in Sec. II. Therefore, we will set $\alpha=0$ below without loss of generality. A nonzero α would only shift the value of A_3 at the minimum of V by α/eL.

By using the identity [see Eq. (A5) in the Appendix]

$$\sum_{\substack{m=-\infty\\(m\neq 0)}}^{\infty}\{\ln[x^2+(m-a)^2]-\ln m^2\}+\ln(x^2+a^2)+2\ln\pi$$
$$=\ln\sinh[\pi(x-ia)]+\ln\sinh[\pi(x+ia)], \quad (3.5)$$

we obtain

$$V=(2\pi)^{-3}L^{-1}\int d^3p\left[\ln\sinh\left[\frac{L}{2}(\omega_p-ieA_3)\right]+\ln\sinh\left[\frac{L}{2}(\omega_p+ieA_3)\right]\right], \quad (3.6)$$

up to an infinite constant. From

$$\frac{\partial V}{\partial A_3}=(2\pi)^{-3}e\,\mathrm{Im}\int d^3p\,\coth\left[\frac{L}{2}(\omega_p-ieA_3)\right], \quad (3.7)$$

we find that V has extrema at $A_3=0$ and $A_3=\pi(eL)^{-1}$. As we mentioned above, the lowest Fourier component of A_3 after dimensional reduction is a scalar in the three-dimensional spacetime. Its (mass)^2 is

$$\frac{\partial^2 V}{\partial A_3^2}=(16\pi^3)^{-1}e^2L\int d^2p[\sinh^2(L\omega_p/2)]^{-1} \quad \text{for } A_3=0, \quad (3.8)$$

or

$$\frac{\partial^2 V}{\partial A_3^2}=-(16\pi^3)^{-1}e^2L\int d^3p[\cosh^2(L\omega_p/2)]^{-1} \quad \text{for } A_3=\pi(eL)^{-1}. \quad (3.9)$$

Thus, the minimum of V occurs at $A_3=0$ and there is no shift of the dimensionally reduced mass for the scalar field at the tree level. The general shape of V will be as in Fig. 1. Notice its periodic dependence on A_3.

For a charged Dirac spinor ψ with boundary condition $\psi(x^3+L)=\exp(i\alpha)\psi(x^3)$ one has the same result as for

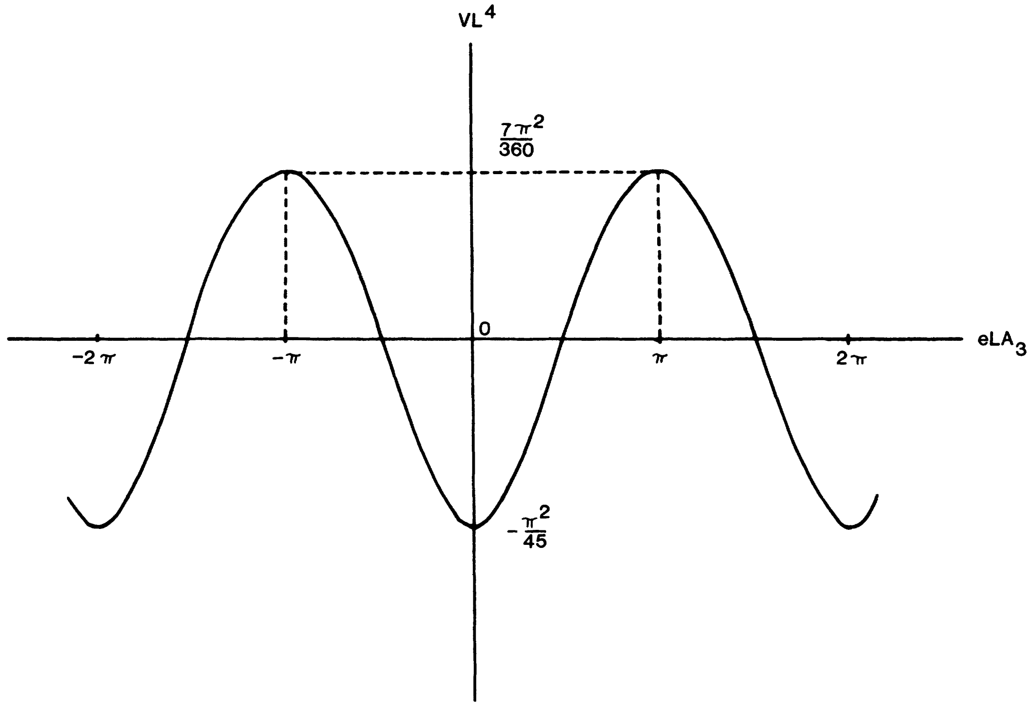

FIG. 1. The effective potential V with massless charged scalar field.

the scalar case except for a factor of 2 coming from the spin degrees of freedom and a minus sign coming from the functional integration over anticommuting Grassman fields. Thus, the effective potential for a spinor field with periodic boundary conditions is

$$V_{1/2}=-2V\ , \tag{3.10}$$

where V is given by Eq. (3.6). As before, with $\alpha=0$, one has extrema of $V_{1/2}$ at $A_3=0$ and $A_3=\pi(eL)^{-1}$, but $V_{1/2}$ has a maximum at $A_3=0$ and a minimum at $A_3=\pi(eL)^{-1}$, with $(\text{mass})^2$ given by

$$\frac{\partial^2 V_{1/2}}{\partial A_3^2}=(2\pi)^{-3}e^2L\int d^3p[\cosh^2(L\omega_p/2)]^{-1} \quad \text{for } A_3=\pi(eL)^{-1}\ . \tag{3.11}$$

By a gauge transformation (Sec. II), this minimum of $V_{1/2}$ with the same mass would occur at $A_3=0$ if we were to impose antiperiodic or twisted boundary conditions on the spinor field. This agrees with Ford[7] who found a positive $(\text{mass})^2$ for A_3, with $\langle A_3\rangle=0$, in the presence of a twisted spinor field.

However, from the general shape of V in Fig. 1 and Eq. (3.10) it is clear that if we impose periodic boundary conditions on the spinor field then $A_3=0$ is an unstable extremum, so that the vacuum will spontaneously develop a nonzero expectation value, $\langle A_3\rangle=\pi(eL)^{-1}$, or one physically equivalent to that value. Therefore, A_3 does not develop a negative $(\text{mass})^2$ and become tachyonic, even if periodic boundary conditions are imposed on the spinor field. Instead, it develops a nonzero vacuum expectation value, rendering the vacuum stable and the $(\text{mass})^2$ of A_3 positive. Later, we will show how this mechanism leads to spontaneous breakdown of gauge symmetry in Yang-Mills theories.

Finally, we mention in passing the correspondence of this problem to one in thermal plasma physics. By well-known methods,[12] the free energy F of a charged scalar field in thermal equilibrium in four-dimensional Minkowski space in the presence of a static, but spatially slowly varying electric potential A_0 can be obtained from the previous effective potential V by making the changes $A_3\to -iA_0$ and $L\to T^{-1}$, where T is the temperature (in units with Boltzmann's constant $=1$). The periodicity of the scalar field in the Euclideanized x^0 direction imposes the necessary thermal averaging. Thus, from Eq. (3.6), we have

$$F=(2\pi)^{-3}T\int d^3p\{\ln\sinh[(2T)^{-1}(\omega-eA_0)] +\ln\sinh[(2T)^{-1}(\omega+eA_0)]\}\ , \tag{3.12}$$

where $\omega=(\mathbf{p}^2+M^2)^{1/2}$. The charge density is

$$\begin{aligned}\rho=\frac{\partial F}{\partial A_0}&=16\pi^{-3}e\int d^3p\{\coth[(2T)^{-1}(\omega+eA_0)]-\coth[(2T)^{-1}(\omega-eA_0)]\}\\ &=8\pi^{-3}e\int d^3p(\{\exp[T^{-1}(\omega+eA_0)]-1\}^{-1}-\{\exp[T^{-1}(\omega-eA_0)]-1\}^{-1})\\ &\approx -16\pi^3T^{-1}e^2A_0\int d^3p\{\sinh[(2T)^{-1}\omega]\}^{-2}\ .\end{aligned} \tag{3.13}$$

The Maxwell's equations give

$$\nabla^2A_0=-\rho=\lambda_D^{-2}A_0\ , \tag{3.14a}$$

where

$$\lambda_D^{-2}=16\pi^3T^{-1}e^2\int d^3p\{\sinh[(2T)^{-1}\omega]\}^{-2}\ . \tag{3.14b}$$

The quantity λ_D is the Debye screening length. If a point charge Q is placed in the medium, then the modified Eq. (3.14a) has a solution[13]

$$A_0=(4\pi r)^{-1}Q\exp(-r/\lambda_D)\ . \tag{3.15}$$

Thus, the $(\text{mass})^2$ or $\partial^2V/\partial A_3^2$ is related to the Debye screening length in the corresponding thermal problem. In the case of a charged spinor field, one uses antiperiodic boundary conditions in the corresponding thermal problem. The $(\text{mass})^2$ in that case is similarly related to the Debye screening length. The effective potential for the thermal plasma problem with a non-Abelian gauge field was calculated by Gross, Pisarski, and Yaffe[14] and was applied to $S^1\times R^n$ by Hosotani,[9] who showed that a non-Abelian gauge field with massless fermions having SU(n) symmetry can acquire a nonzero VEV in the stable vacuum state.

IV. THE DEFINITION OF THE EFFECTIVE COUPLING CONSTANTS

QED in $S\times R^3$ spacetime is renormalizable at least up to one-loop order. To dispose of ultraviolet (UV) divergences one may use the same renormalized coupling constant as in Minkowski spacetime. The renormalized coupling constant e_R is related to the bare coupling constant e_B by $e_R^2=Z_3e_B^2$, where Z_3 is the on-shell wave-function renormalization constant for A_M in Minkowski spacetime.[15]

In $S^1\times R^3$ spacetime physical quantities such as cross sections depend on the size of the compactified dimension L. Although the renormalization of the coupling constant described above is enough to eliminate UV divergences, it is expected that some portion of the L dependence of physical quantities will be expressed in terms of

L-dependent effective coupling constants, which are analogous to running coupling constants introduced when a large momentum transfer is involved.[16] (These are analogous to the curvature-dependent coupling constants studied in Refs. 1 and 2.) We will define such L-dependent effective coupling constants in this section and calculate their explicit L dependence in the following two sections to one-loop order.

To define the effective coupling constants we consider photon emission by a classical current $J^M(x)$. Thus we introduce the following term in the Lagrangian:

$$\mathcal{L}_I = e_B A_B^M J_M \ , \tag{4.1}$$

where the subscript B indicates bare quantities. We require that J_M be conserved, i.e., $\partial^M J_M = 0$. Then in Minkowski spacetime the production probability of a photon with momentum in the range $\mathbf{k}$ to $\mathbf{k}+d\mathbf{k}$ and with polarization ϵ^M (with $\epsilon^M \epsilon_M = -1$) is given to the lowest order by

$$dP = e_B^2 \frac{d^3\mathbf{k}}{2k^0(2\pi)^3} \, | \epsilon^M J_M(k^0,\mathbf{k}) |^2 \ , \tag{4.2}$$

where

$$J_M(k^0,\mathbf{k}) = \int d^4x \, e^{ik^0x^0 - i\mathbf{k}\cdot\mathbf{x}} J_M(x) \tag{4.3}$$

and $k^0 = |\mathbf{k}|$. At one-loop level one finds that e_B^2 in (4.2) is replaced by the renormalized coupling constant e_R^2. In fact it is possible, theoretically, to define e_R through this process. Thus we are led to define the effect coupling constants for QED in $S^1 \times R^3$ spacetime by using this process.

In the process considered above, the photon is "on-shell," so that any L dependence of the coupling constant in $S^1 \times R^3$ must come only from the size of the spacetime. Also the coupling constant approaches the Minkowski value e_R in the limit $L \to \infty$. Thus we find this definition natural. However, it should be kept in mind that other definitions are also possible and may be appropriate in some cases (an example will be given later in this section).

Before proceeding to give the formulas for the effective coupling constants, we need to clarify the particle content (from the 3+1 Kaluza-Klein point of view) of QED in $S^1 \times R^3$ spacetime. To this end we study the free theory given by

$$\mathcal{L}_0 = -\tfrac{1}{4} F_{MN} F^{MN} \ . \tag{4.4}$$

Let us expand the gauge potential $A_N(x^M)$ as follows:

$$A_N(x^M) = \sum_{n=-\infty}^{\infty} [A_N^{(+,n)}(x^\mu) e^{i(2\pi n/L)x^3} + A_N^{(-,n)}(x^\mu) e^{i(2\pi n/L)x^3}] \ , \tag{4.5}$$

where $A_N^{(+,n)}$ and $A_N^{(-,n)}$ consist of positive- and negative-frequency modes, respectively. First we notice that $A_3^{(\pm,n)}(x^M)$ with $n \neq 0$ can be gauged away without changing the boundary conditions, as follows. Define the new field $A'_N(x^M)$ by

$$A'_N(x^M) = A_N(x^M) - \partial_N \Lambda(x^M) \ , \tag{4.6}$$

where

$$\Lambda(x^M) = \sum_{\substack{n=-\infty \\ (n\neq 0)}}^{\infty} \frac{L}{2\pi i n} [A_3^{(+,n)}(x^\mu) e^{i(2\pi n/L)x^3} - A_3^{(-,n)}(x^\mu) e^{i(2\pi n/L)x^3}] \ . \tag{4.7}$$

Then we find

$$A'_3(x^M) = A_3^{(+,0)}(x^\mu) + A_3^{(-,0)}(x^\mu) \ . \tag{4.8}$$

The Lagrangian becomes

$$\mathcal{L} = \sum_{n=-\infty}^{\infty} \left[-\frac{1}{4} F'^{(n)}_{\mu\nu}(x^\alpha) F'^{(n)\mu\nu}(x^\alpha) + \frac{1}{2} \left[\frac{2\pi n}{L} \right]^2 A'^{(n)}_\mu(x^\alpha) A'^{(n)\mu}(x^\alpha) \right] + \tfrac{1}{2} \partial_\mu A'^{(0)}_3(x^\alpha) \partial^\mu A'^{(0)}_3(x^\alpha) \ , \tag{4.9}$$

where

$$A'^{(n)}_N(x^\alpha) \equiv A'^{(+,n)}_N(x^\alpha) + A'^{(-,n)}_N(x^\alpha) \ , \tag{4.10a}$$

$$F^{(n)}_{\mu\nu}(x^\alpha) = \partial_\mu A'^{(n)}_\nu(x^\alpha) - \partial_\nu A'^{(n)}_\mu(x^\alpha) \ . \tag{4.10b}$$

In Eq. (4.9), terms which do not contribute to $\int d^4x \, \mathcal{L}$ have been dropped. Thus this theory consists of one massless vector and one massless scalar field ($n=0$) and an infinite number of massive vector fields with

$$(\text{mass})^2 = \left[\frac{2\pi n}{L} \right]^2 \quad (n = \pm 1, \pm 2, \ldots)$$

at the tree level when regarded as a three-dimensional field theory in the spacetime with coordinates x^0, x^1, x^2.

Lorentz invariance guarantees that the on-shell renormalized coupling constant is independent of the momentum and the polarization of the emitted photon in Minkowski spacetime. In $S^1 \times R^3$ spacetime this is no longer the case. The effective coupling constant is expected to depend on n (since the number of nodes is invariant) as well as on L. Moreover, for $n=0$, the effective coupling constant for the scalar modes is expected to be different from that for the vector modes.

The effective coupling constant for the nth Fourier component with $n \neq 0$ will be denoted by $e_n(L)$. That for the vector modes of the zeroth Fourier component will be denoted by $e_v(L)$, and that for the scalar modes by $e_s(L)$. We define these coupling constants in such a way that the production probability dP for photons with momenta in the range $(k^1, k^2, 2\pi n/L)$ to $(k^1+dk^1, k^2+dk^2, 2\pi n/L)$ and with polarization ϵ^M is

$$dP = e_n^2(L) \frac{1}{L} \frac{d^2\mathbf{k}}{(2\pi)^2} \, | \epsilon^M J_M(k^\alpha, n) |^2 \ , \tag{4.11}$$

where

$$J_M(k^\alpha, n) = \int d^4x \exp \left[i k^\alpha x_\alpha - i \frac{2\pi n}{L} x^3 \right] J_M(x) \ . \tag{4.12}$$

(n is replaced in this definition by v or s for the zeroth Fourier component.)

To calculate $e_n(L)$, or to find the relationship between $e_n(L)$ and e_R (the measured coupling constant in Minkowski spacetime) at one-loop level, one needs the photon propagator. The photon self-energy due to coupling to charged scalars or spinors may be written in a matrix form as

$$\pi^{MN}(q^2,n)=\begin{bmatrix}\pi^{\mu\nu}(q^2,n) & \pi^{\mu 3}(q^2,n)\\ \pi^{3\nu}(q^2,n) & \pi^{33}(q^2,n)\end{bmatrix}, \tag{4.13}$$

where $q^2=\eta_{\mu\nu}q^\mu q^\nu$. Let

$$e_B^2\pi_\infty^{MN}(q^2,n)=(Z_3^{-1}-1)(\Delta_{(0)}^{-1})^{MN}(q^2,n), \tag{4.14}$$

where Z_3 is the on-shell renormalization constant for the gauge potential in Minkowski spacetime and $(\Delta_{(0)}^{-1})^{MN}(q^2,n)$ is the lowest-order inverse photon propagator (minus the gauge-fixing term) given by

$$(\Delta_{(0)}^{-1})^{MN}(q^2,n)=\begin{bmatrix} q^\mu q^\nu-\left[q^2-\left[\frac{2\pi n}{L}\right]^2\right]\eta^{\mu\nu} & \frac{2\pi n}{L}q^\mu\\ \frac{2\pi n}{L}q^\nu & q^2\end{bmatrix}. \tag{4.15}$$

(π_∞^{MN} is the self-energy in Minkowski spacetime near $q^2=0$.)

The subtracted self-energy $\pi_R^{MN}(q^2,n)$ can in general be written in the form

$$\pi_R^{MN}(q^2,n)=\pi^{MN}(q^2,n)-\pi_\infty^{MN}(q^2,n)$$
$$=\begin{bmatrix}A_n\eta^{\mu\nu}+B_nq^\mu q^\nu & C_nq^\mu\\ C_nq^\nu & D_n\end{bmatrix}, \tag{4.16}$$

where A_n, B_n, C_n, and D_n are functions of q^2 and are UV finite. Now recall that

$$\pi^{MN}(q^2,n)=i\int d^4x\langle 0\,|\,T[j^M(x)j^N(0)]\,|\,0\rangle \times\exp\left[iq^\alpha x_\alpha-i\frac{2\pi n}{L}x^3\right], \tag{4.17}$$

where $j^M(x)$ is the conserved electromagnetic current operator in the theory. The conservation of $j^M(x)$, i.e., $\partial_M j^M(x)=0$ implies

$$q_\nu\pi^{M\nu}(q^2,n)-\frac{2\pi n}{L}\pi^{M3}(q^2,n)=0\ . \tag{4.18}$$

Hence we have (since $q_N\pi_\infty^{MN}=0$)

$$A_n+B_nq^2-\frac{2\pi n}{L}C_n=0\ , \tag{4.19a}$$

$$C_nq^2-\frac{2\pi n}{L}D_n=0\ . \tag{4.19b}$$

(These relations are valid to all orders.)

To discuss the propagator one needs to introduce a gauge-fixing term into the Lagrangian. We choose it to be

$$\mathcal{L}_{\rm gf}=-\frac{1}{2\alpha}(\partial_\mu A^\mu)^2\ . \tag{4.20}$$

Notice that this is different from the usual gauge-fixing term taken in Minkowski spacetime, which is

$$\mathcal{L}'_{\rm gf}=-\frac{1}{2\alpha}(\partial_\mu A^\mu+\partial_3A^3)^2\ . \tag{4.21}$$

We find the gauge-fixing term (4.20) more convenient especially in discussing gauge symmetry breaking in Sec. VII (Ref. 17).

For $n=0$, Eq. (4.19) becomes

$$A_0=-q^2B_0\ , \tag{4.22a}$$

$$C_0=0\ . \tag{4.22b}$$

To dispose of UV divergences we perform the renormalization

$$A_R^M=Z_3^{-1/2}A_B^M\ . \tag{4.23}$$

By substituting Eq. (4.22) in (4.16) we find the bare inverse propagator for $n=0$ with the gauge-fixing term (4.20) as

$$(\Delta_B^{-1})^{MN}(q^2,0)=\begin{bmatrix}(Z_3^{-1}+e_B^2B_0)(q^\mu q^\nu-q^2\eta^{\mu\nu})-\frac{1}{\alpha}q^\mu q^\nu & 0\\ 0 & Z_3^{-1}q^2+e_B^2D_0\end{bmatrix}. \tag{4.24a}$$

Then the renormalized propagator for $n=0$, $\Delta_{MN}(q^2,0)$, is obtained by inverting this matrix and dividing the result by Z_3. Thus

$$\Delta_{MN}(q^2,0)=\begin{bmatrix}\frac{1}{1+e_R^2B_0}\left[-\eta_{\mu\nu}\left[1-\alpha_R(1+e_R^2B_0)\frac{q_\mu q_\nu}{q^2}\right]\right]\Big/q^2 & 0\\ 0 & \frac{1}{q^2+e_R^2D_0}\end{bmatrix}, \tag{4.24b}$$

where $\alpha_R = Z_3^{-1}\alpha$.

In order to identify the effective coupling constants, note that the squared S-matrix element appearing in (4.11) is $|\langle 1 \text{ photon,out} | 0\text{in}\rangle|^2$. Using the Lehmann-Symanzik-Zimmermann (LSZ) reduction technique[15,18] at one loop, one finds that the effective coupling constant defined by (4.11) is $e_{\text{eff}}^2 = (\text{residue of pole of bare propagator at } q^2=0)e_B^2$. Then from (4.24b) with $\Delta_{MN} = Z_3^{-1}(\Delta_B)_{MN}$ we obtain

$$e_v^2(L) = \frac{e_R^2}{1+e_R^2 B_0^0} , \tag{4.25a}$$

$$e_s^2(L) = \frac{e_R^2}{1+e_R^2 D_0^1} , \tag{4.25b}$$

where

$$B_0 = B_0^0 + B_0^1 q^2 + \cdots , \tag{4.26a}$$

$$D_0 = D_0^0 + D_0^1 q^2 + \cdots . \tag{4.26b}$$

Recall that we have found in the previous section that a finite mass is generated for the scalar modes at one-loop level by studying the effective potential. Thus D_0^0 is nonzero and the (mass)^2 is given by $-e_R^2 D_0^0$.

For $n \neq 0$, C_n and D_n can be expressed in terms of A_n and B_n in the self-energy (4.16) by using (4.19). Thus we have

$$\pi_R^{MN}(q^2,n) = \begin{bmatrix} A_n \eta^{\mu\nu} + B_n q^\mu q^\nu & \frac{L}{2\pi n}(A_n + B_n q^2) q^\mu \\ \frac{L}{2\pi n}(A_n + B_n q^2) q^\nu & D_n \end{bmatrix} , \tag{4.27}$$

where

$$D_n = \left[\frac{L}{2\pi n}\right]^2 q^2 (A_n + B_n q^2) \quad (n \neq 0) . \tag{4.28}$$

Then using the same methods as before, the renormalized propagator is calculated to be

$$\Delta_{\mu\nu}(q^2,n) = -\left[\eta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right] \Big/ \left[q^2 - \left[\frac{2\pi n}{L}\right]^2 - e_R^2 A_n\right] - \alpha_R \frac{q_\mu q_\nu}{(q^2)^2} , \tag{4.29a}$$

$$\Delta_{3\nu}(q^2,n) = \alpha_R \left[\frac{2\pi n}{L}\right] q_\nu \Big/ (q^2)^2 , \tag{4.29b}$$

$$\Delta_{33}(q^2,n) = \frac{1}{q^2 + e_R^2 D_n} - \alpha_R \left[\frac{2\pi n}{L}\right]^2 \Big/ (q^2)^2 . \tag{4.29c}$$

By taking the Landau gauge $\alpha_R = 0$, we find that the propagator $\Delta_{\mu\nu}(q^2,n)$ corresponds to a massive vector field for each n in three dimensions which has two polarization states for each momentum. The physical polarization vectors are $\epsilon_M^{(i)}$ $(i=1,2)$ and satisfy $\epsilon_\mu^{(i)} q^\mu = 0$, $\epsilon_3^{(i)} = 0$, and $\epsilon_\mu^{(i)} \epsilon^{(i)\mu} = -1$ (not summed over i). This agrees with the analysis at the tree level earlier in this section. Unlike the $n=0$ case, the propagator $\Delta_{33}(q^2,n)$ does not correspond to a physical scalar particle in three dimensions.[19]

When the rate of production of real photons by a classical current is calculated using the reduction formula, the relevant part of the propagator is

$$\epsilon^{(i)M} \Delta_{MN}(q^2,n) J^N(q^\alpha,n) = -\frac{\epsilon_\nu^{(i)} J^\nu(q^\alpha,n)}{q^2 - \left[\frac{2\pi n}{L}\right]^2 - e_R^2 A_n} . \tag{4.30}$$

Near the mass-shell this becomes to one-loop order

$$\epsilon^{(i)M} \Delta_{MN}(q^2,n) J^N(q^\alpha,n) \approx -\frac{\epsilon_\nu^{(i)} J^\nu(q^\alpha,n)}{(1 - e_R^2 A_n^1)\left[q^2 - \left[\frac{2\pi n}{L}\right]^2 - e_R^2 A_n^0\right]} , \tag{4.31}$$

where

$$A_n \equiv A_n^0 + A_n^1 Q_n^2 + \cdots , \tag{4.32a}$$

$$Q_n^2 \equiv q^2 - \left[\frac{2\pi n}{L}\right]^2 . \tag{4.32b}$$

In the same way as in the case with $n=0$, we find

$$e_n^2(L) = \frac{e_R^2}{1 - e_R^2 A_n^1} . \tag{4.33}$$

The shift of the (mass)^2, ΔM^2, is given by

$$\Delta M^2 = e_R^2 A_n^0 \ , \tag{4.34}$$

which will turn out to be nonzero in general.

As we have mentioned before, other definitions of the effective coupling constants are possible. Let us give an example below for $n \neq 0$. First note that by letting $A_M \to 1/e_B A_M$ we have

$$\mathcal{L} = -\frac{1}{4e_B^2} F_{MN} F^{MN} + \cdots \ . \tag{4.35}$$

Then the lowest-order inverse propagator (minus the gauge-fixing term) is

$$(\Delta'^{-1}_{(0)})^{MN}(q^2,n) = \frac{1}{e_B^2}\begin{bmatrix} q^\mu q^\nu - \eta^{\mu\nu} q^2 & 0 \\ 0 & 0 \end{bmatrix} + \frac{1}{e_B^2}\begin{bmatrix} \left[\frac{2\pi n}{L}\right]^2 \eta^{\mu\nu} & \frac{2\pi n}{L} q^\mu \\ \frac{2\pi n}{L} q^\nu & q^2 \end{bmatrix} . \tag{4.36}$$

Notice that the first term corresponds to $-(1/4e_B^2)F_{\mu\nu}F^{\mu\nu}$ and the second to $-(1/4e_B^2)F_{\mu 3}F^{\mu 3}$ in the Lagrangian. The one-loop contribution to the inverse propagator is

$$\pi_{(1)}^{MN}(q^2,n) = \frac{1}{e_B^2}\left[\frac{1}{Z_3} - 1\right](\Delta'^{-1}_{(0)})^{MN}(q^2,n) + \pi_R^{MN}(q^2,n) \ , \tag{4.37a}$$

where $\pi_R^{MN}(q^2,n)$ is given by (4.16). By summing the tree and one-loop contributions we find the inverse propagator (minus the gauge-fixing term) to one-loop order as

$$(\Delta'^{-1})^{MN}(q^2,n) = \frac{1+e_R^2 B_n}{e_R^2}\begin{bmatrix} q^\mu q^\nu - \eta^{\mu\nu} q^2 & 0 \\ 0 & 0 \end{bmatrix} + \frac{1+e_R^2 E_n}{e_R^2}\begin{bmatrix} \left[\frac{2\pi n}{L}\right]^2 \eta^{\mu\nu} & \frac{2\pi n}{L} q^\mu \\ \frac{2\pi n}{L} q^\mu & q^2 \end{bmatrix} , \tag{4.37b}$$

where

$$E_n = D_n / q^2 \ . \tag{4.38}$$

Then it seems natural to define effective coupling constants as

$$e_{n(1)}^2(L) = \frac{e_R^2}{1+e_R^2 B_n^0} \ , \tag{4.39a}$$

$$e_{n(2)}^2(L) = \frac{e_R^2}{1+e_R^2 E_n^0} \ , \tag{4.39b}$$

where

$$B_n^0 = B_n \mid_{Q_n^2=0} \ , \tag{4.40a}$$

$$E_n^0 = E_n \mid_{Q_n^2=0} \ . \tag{4.40b}$$

The coupling constants $e_{n(1)}(L)$ and $e_{n(2)}(L)$ may be regarded as the on-shell effective coupling constant for $F_{\mu\nu}$ and that for $F_{\mu 3}$, respectively. [For $n=0$ $e_{0(1)}(L)$ coincides with $e_v(L)$ whereas $e_{0(2)}(L)$ cannot be defined.] We will find below that the effective couplings in (4.39a) and (4.39b) differ from the $e_n(L)$ defined earlier.

V. SCALAR-LOOP EFFECT

In this section we calculate the effective coupling constants defined in the previous section as well as the mass shift to one-loop order for the case where the electromagnetic field is coupled to a scalar field with mass M. [See Eq. (1.2).] We explicitly evaluate the self-energy π^{MN} for this theory to one-loop order, thereby obtaining the coefficients in Eq. (4.27). We then identify the L dependence of the effective coupling constants and mass shifts using Eqs. (4.25), (4.33), (4.34), and (4.39).

Let us start with the vector modes having $n=0$. $\pi^{\mu\nu}(q^2,0)$ is the sum of the two Feynman diagrams shown in Figs. 2(a) and 2(b). The contribution from the diagram in Fig. 2(a) is

$$\pi_{(a)}^{\mu\nu}(q^2,0) = \frac{2i}{L}\sum_{m=-\infty}^{\infty}\int \frac{d^2p}{(2\pi)^3} \frac{1}{p^2 - \left[\frac{2\pi m}{L}\right]^2 - M^2 + i\epsilon}\eta^{\mu\nu} \ , \tag{5.1}$$

where $d^3p = dp^0 dp^1 dp^2$. We first perform the Wick rotation $p^0 \to ip^0$. In order to use dimensional regularization[20] we change the dimensionality of the integral from 3 to $d = 3-2\epsilon$, where ϵ is an infinitesimal positive number. Then we have

$$\pi_{(a)}^{\mu\nu}(q^2,0) = \frac{2\mu^{2\epsilon}}{L}\sum_{m=-\infty}^{\infty}\int \frac{d^dp}{(2\pi)^d} \frac{1}{p^2 + \left[\frac{2\pi m}{L}\right]^2 + M^2}\eta^{\mu\nu} \ , \tag{5.2}$$

where $p^2 \equiv \eta_{\mu\nu} p^\mu p^\nu$ and a momentum scale μ has been introduced to make the dimensionality of $\pi^{\mu\nu}(q^2,n)$ independent of d. This integral can be evaluated as

$$\pi^{\mu\nu}_{(a)}(q^2,0)=\frac{2\mu^{2\epsilon}}{L}\frac{\Gamma\left[1-\frac{d}{2}\right]}{(4\pi)^{d/2}}\sum_{m=-\infty}^{\infty}\left[M^2+\left[\frac{2\pi m}{L}\right]^2\right]^{d/2-1}\eta^{\mu\nu} . \tag{5.3}$$

For some purposes it is also convenient to use Eq. (A1) to write (5.2) as

$$\pi^{\mu\nu}_{(a)}(q^2,0)=\left[\mu^{2\epsilon}\int\frac{d^dp}{(2\pi)^d\omega_p}+2\int\frac{d^3p}{(2\pi)^3\omega_p}\frac{1}{e^{L\omega_p}-1}\right]\eta^{\mu\nu} , \tag{5.4}$$

where ω_p is given by $\omega_p=\sqrt{p^2+M^2}$ and d has been set to 3 in the second term. In a similar manner the contribution from Fig. 2(b) can be evaluated for small q^2 as

$$\pi^{\mu\nu}_{(b)}(q^2,0)=-\frac{2\mu^{2\epsilon}}{L}\frac{\Gamma\left[1-\frac{d}{2}\right]}{(4\pi)^{d/2}}\sum_{m=-\infty}^{\infty}\left[M^2+\left[\frac{2\pi m}{L}\right]^2\right]^{d/2-1}\eta^{\mu\nu}$$
$$+\frac{\mu^{2\epsilon}}{3L}\frac{\Gamma\left[2-\frac{d}{2}\right]}{(4\pi)^{d/2}}\sum_{m=-\infty}^{\infty}\left[M^2+\left[\frac{2\pi m}{L}\right]^2\right]^{d/2-2}(q^\mu q^\nu-q^2\eta^{\mu\nu})+O(q^2) . \tag{5.5}$$

Hence, it follows that

$$\pi^{\mu\nu}(q^2,0)=\bar{B}^0_0(q^\mu q^\nu-q^2\eta^{\mu\nu}) , \tag{5.6}$$

where

$$\bar{B}^0_0=\frac{\mu^{2\epsilon}}{3L}\frac{\Gamma\left[2-\frac{d}{2}\right]}{(4\pi)^{d/2}}\sum_{m=-\infty}^{\infty}\left[M^2+\left[\frac{2\pi m}{L}\right]^2\right]^{d/2-2} . \tag{5.7}$$

By using Eqs. (4.14), (4.16), and (4.26a), we find

$$\bar{B}^0_0=B^0_0+(Z_3^{-1}-1)/e_B^2 . \tag{5.8}$$

To separate the second term, which is the $L\to\infty$ limit (the Minkowski-spacetime limit) of $\bar{B}^0_0$ and find B^0_0, we cast this equation back to the form involving a momentum integration as

$$\bar{B}^0_0=\frac{\mu^{2\epsilon}}{3L}\int\frac{d^dp}{(2\pi)^d}\sum_{m=-\infty}^{\infty}\frac{1}{\left[\omega_p^2+\left[\frac{2\pi m}{L}\right]^2\right]^2} . \tag{5.9}$$

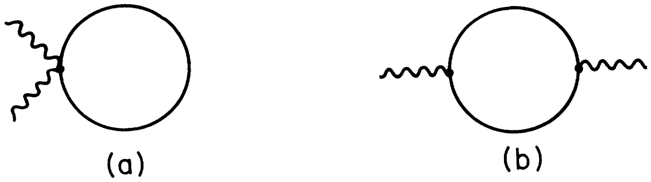

FIG. 2. (a) The Feynman diagram corresponding to $\pi^{\mu\nu}_{(a)}(q^2,0)$. (b) The Feynman diagram corresponding to $\pi^{\mu\nu}_{(b)}(q^2,0)$.

By using Eq. (A4) we find

$$\bar{B}^0_0=\frac{\mu^{2\epsilon}}{12}\int\frac{d^dp}{(2\pi)^d}\frac{1}{\omega_p^3}+\frac{1}{6}\int\frac{d^3p}{(2\pi)^3}\left[\frac{1}{\omega_p^3}\frac{1}{e^{L\omega_p}-1}+\frac{1}{4\omega_p^2}\frac{1}{\sinh^2\frac{L}{2}\omega_p}\right] . \tag{5.10}$$

Now the first term is

$$I_\infty\equiv\frac{\mu^{2\epsilon}}{12}\int\frac{d^dp}{(2\pi)^d}\frac{1}{\omega_p^3}=\frac{1}{48\pi^2}\left[\frac{1}{\epsilon}-\gamma-\ln\frac{M^2}{4\pi\mu^2}\right] , \tag{5.11}$$

where γ is Euler's constant. This is the Minkowski-spacetime limit of $\bar{B}^0_0$. Hence

$$Z_3^{-1}=1+\frac{e_B^2\mu^{-2\epsilon}}{48\pi^2}\left[\frac{1}{\epsilon}-\gamma-\ln\frac{M^2}{4\pi\mu^2}\right] \tag{5.12}$$

and the second term in Eq. (5.10) is B^0_0. By using spherical coordinates in momentum space and integrating by parts, we find

$$B^0_0=\frac{1}{24\pi^2}\int_0^\infty\frac{dy}{\sqrt{y(y+1)}}\frac{1}{e^{\alpha\sqrt{y+1}}-1}=\frac{1}{12\pi^2}\sum_{m=1}^{\infty}K_0(m\alpha) , \tag{5.13}$$

where $\alpha\equiv ML$ and $K_0(x)$ is a Bessel function of imaginary argument. Equation (A11) has been used. Now we are interested in the small-L behavior of the effective coupling constants, which would be characteristic of theories with microscopic compact dimensions. By using (A12) we have

$$B_0^0=\frac{1}{48\pi^2}\left[\frac{2\pi}{ML}+\ln\frac{M^2L^2}{(4\pi)^2}+2\gamma+O(M^2L^2)\right] \quad (5.14)$$

for small L. Hence we have, from Eq. (4.25a),

$$e_v^2(L)\approx\frac{e_R^2}{1+\frac{e_R^2}{48\pi^2}\left[\frac{2\pi}{ML}+\ln\frac{M^2L^2}{(4\pi)^2}+2\gamma\right]} , \quad (5.15)$$

where e_R^2 is given by $e_R^2=Z_3e_B^2$ with Z_3 in (5.12).[21] The logarithmic dependence of B_0^0, and hence that of $e_v^2(L)$, on L is of the form which follows by assuming the absence of logarithmic singularities in $\bar{B}_0^0$ defined by (5.6) in the $M^2\to 0$ limit (i.e., no $\ln M^2$ term). Thus the logarithmic dependence of $e_v^2(L)$ on $1/L^2$ is of the same form as that of the ordinary effective coupling constant on the (momentum)^2 in Minkowski spacetime. This logarithmic term is overwhelmed by the term which behaves like $1/L$. Also notice that B_0^0 given by (5.13) is always positive and therefore $e_v^2(L)$ is *always smaller* than e_R^2.

The origin of this singular term can be explained as follows. When L is very small, from Eq. (4.9) one can see that the modes of the scalar field with $n\neq 0$ become very massive and can be dropped from the effective low-energy Lagrangian, which is then independent of x^3. Then the theory reduces to $(2+1)$-dimensional QED with $e/\sqrt{L}$ as the coupling constant, with $A_3^{\prime(0)}$ acting as a scalar field. The dimensionless constant that characterizes the theory is e^2/ML. Therefore the n-loop contribution to the kinetic term of the gauge field in the effective Lagrangian behaves like $(e^2/ML)^n$. This also implies that the one-loop result cannot be trusted if L is so small that e^2/ML is of order 1.[22]

The quantities D_0^0 and D_0^1, which are necessary to find $e_s^2(L)$ and the mass shift for the scalar modes, can be obtained by calculating $\pi^{33}(q^2,0)$. After some calculation, one finds

$$D_0^0=-\frac{L}{2}\int\frac{d^3p}{(2\pi)^3}\frac{1}{\sinh^2\frac{L}{2}\omega_p} , \quad (5.16a)$$

$$D_0^1=B_0^0-\frac{L}{48\pi^2}\int_0^\infty\frac{dp}{\sinh^2\frac{L}{2}\omega_p}$$
$$=B_0^0-\frac{1}{24\pi}\frac{1}{ML}+O(M^2L^2) , \quad (5.16b)$$

where Eq. (A20) has been used. Hence by using (4.25b) we have

$$e_s^2(L)=\frac{e_R^2}{1+\frac{e_R^2}{48\pi^2}\left[\ln\frac{M^2L^2}{(4\pi)^2}+2\gamma\right]}+0(M^2L^2) . \quad (5.17)$$

Notice that the term which behaves like $1/L$ is absent and that the logarithmic dependence of $e_s^2(L)$ on L is the same as that of $e_v^2(L)$. The (mass)^2 obtained from D_0^0 agrees, of course, with the one from the effective potential.

Let us go on to calculate $e_n^2(L)$ with $n=\pm 1,\pm 2,\ldots$. By calculating $\pi^{\mu\nu}(q^2,n)$ one has

$$A_n=\int\frac{d^dp}{(2\pi)^d}\frac{1}{\omega_p}+2\int\frac{d^3p}{(2\pi)^3\omega_p}\frac{1}{e^{L\omega_p}-1}$$
$$-\frac{2}{L}\frac{\Gamma\left[1-\frac{d}{2}\right]}{(4\pi)^{d/2}}\sum_{m=-\infty}^{\infty}\int_0^1 dx\left[M^2+\left[\frac{2\pi}{L}\right]^2(m-xn)^2-x(1-x)Q_n^2\right]^{d/2-1}+I_\infty Q_n^2 , \quad (5.18a)$$

$$B_n=\frac{1}{L}\frac{\Gamma\left[2-\frac{d}{2}\right]}{(4\pi)^{d/2}}\sum_{m=-\infty}^{\infty}\int_0^1 dx\,(1-2x)^2\left[M^2+\left[\frac{2\pi}{L}\right]^2(m-xn)^2-x(1-x)Q_n^2\right]^{d/2-2}-I_\infty , \quad (5.18b)$$

where

$$Q_n^2=q^2-\left[\frac{2\pi n}{L}\right]^2 . \quad (5.19)$$

First using the obvious formula

$$\sum_{m=-\infty}^{\infty}\int_0^1 dx\,f(xn-m)=\int_{-\infty}^{\infty}f(y)dy , \quad (5.20)$$

which is valid for an arbitrary function $f(x)$, we find

$$A_n^0=2\int\frac{d^3p}{(2\pi)^3\omega_p}\frac{1}{e^{L\omega_p}-1} , \quad (5.21)$$

where A_n^i $(i=0,1,\ldots)$ is defined in Eq. (4.32a). Then the mass shift ΔM^2 follows from (4.34) as

$$\Delta M^2=2e_R^2\int\frac{d^3p}{(2\pi)^3\omega_p}\frac{1}{e^{L\omega_p}-1} . \quad (5.22)$$

Notice that this mass shift is different from that for the scalar with $n=0$ but that it is independent of n (for $n\geq 1$). The positivity of ΔM^2 ensures that these modes are not tachyonic. That is, the *four*-momenta $(q^\mu,2\pi n/L)$ of these modes are timelike.

Next, we find, for $n\neq 0$,

$$A_n^1=\frac{1}{4\pi^4n^2}\sum_{m=1}^{\infty}\frac{1}{m^2}K_0(m\alpha) , \quad (5.23)$$

where $\alpha = ML$ and we have used Eq. (A13). We also have

$$B_n^0 = 2A_n^1 = \frac{1}{2\pi^4 n^2} \sum_{m=1}^{\infty} \frac{1}{m^2} K_0(m\alpha) , \tag{5.24}$$

where B_n^0 is defined by Eq. (4.40a). Then by using (A18) we find, for small L,

$$e_n^2(L) \approx \frac{e_R^2}{1 + \frac{e_R^2}{48\pi^2} \frac{1}{n^2} \left[\ln \frac{M^2 L^2}{4} - \frac{6}{\pi^2} \zeta'(2) \right]} , \tag{5.25}$$

where

$$\zeta'(2) \equiv - \sum_{m=2}^{\infty} \frac{\ln m}{m^2} . \tag{5.26}$$

Notice that $e_n^2(L)$ approaches e_R^2 for $n \to \infty$. This is reasonable because the modes with large n are expected to be affected only by the local properties of spacetime.

If there were no singularities at $M^2 = 0$ in the loop integral, the logarithmic dependence of $e_n^2(L)$ on L would be the same as that of $e_v^2(L)$ and $e_s^2(L)$. There are indeed such singularities in the x integration for A_n^1 $(n \neq 1)$ as can be seen from Eq. (5.18a). These singularities reflect the fact that the intermediate scalar particles can be on shell and collinear in the limit $M^2 \to 0$.

The effective coupling constants defined differently, by means of Eqs. (4.39a) and (4.39b), are found to have the following values:

$$e_{n(1)}^2(L) \approx \frac{e_R^2}{1 - \frac{e_R^2}{24\pi^2} \frac{1}{n^2} \left[\ln \frac{M^2 L^2}{4} - \frac{6}{\pi^2} \zeta'(2) \right]} , \tag{5.27}$$

$$e_{n(2)}^2(L) \approx \frac{e_R^2}{1 - \frac{e_R^2}{24\pi^2} \frac{1}{n^2} \left[\ln \frac{M^2 L^2}{4} - \frac{6}{\pi^2} \zeta'(2) - \pi^2 \right]} . \tag{5.28}$$

These are different from $e_n^2(L)$ in an important respect. That is, while $e_n^2(L)$ is *always larger* than e_R^2, $e_{n(1)}^2(L)$, and $e_{n(2)}^2(L)$ are *always smaller* than e_R^2.

VI. SPINOR-LOOP EFFECT

In Sec. III we have seen that the spinor field obeys antiperiodic boundary conditions in the stable vacuum in the Abelian gauge field theory with one charged Dirac spinor field when the VEV $\langle A_3 \rangle$ is gauged away. We use this representation with antiperiodic boundary conditions on the spinor field from the beginning here in calculating the effective coupling constants and the mass shifts.

The spinor-loop integrals can be expressed in terms of integrals already considered in the scalar case. Let

$$P^M \equiv \left[p^\mu, \frac{2\pi}{L}(m + \tfrac{1}{2}) \right] , \tag{6.1a}$$

$$Q^M \equiv \left[q^\mu, \frac{2\pi n}{L} \right] , \tag{6.1b}$$

$$P'^M \equiv P^M + Q^M . \tag{6.1c}$$

Then we have, for the spinor self-energy,

$$\begin{aligned}
\pi^{MN}(q^2, n) &= \frac{i}{L} \sum_{m=-\infty}^{\infty} \int \frac{d^d p}{(2\pi)^d} \mathrm{Tr} \left[\frac{1}{P' - M + i\epsilon} \gamma^M \frac{1}{P - M + i\epsilon} \gamma^N \right] \\
&= \frac{2i}{L} \sum_{m=-\infty}^{\infty} \int \frac{d^d p}{(2\pi)^d} \left[\frac{(P+P')^M (P+P')^N}{(P'^2 - M^2 + i\epsilon)(P^2 - M^2 + i\epsilon)} - \frac{2\eta^{MN}}{P^2 - M^2 + i\epsilon} \right] \\
&\quad - \frac{2i}{L} (Q^M Q^N - \eta^{MN} Q^2) \sum_{m=-\infty}^{\infty} \int \frac{d^d p}{(2\pi)^d} \frac{1}{(P^2 - M^2 + i\epsilon)(P'^2 - M^2 + i\epsilon)} .
\end{aligned} \tag{6.2}$$

The first term is -2 times the bosonic one-loop contribution with antiperiodic boundary conditions. It can readily be obtained from the results in the previous section by letting $\omega_p \to \omega_p + i\pi/L$. For example,

$$\coth \frac{L}{2} \omega_p \to \tanh \frac{L}{2} \omega_p , \quad \frac{1}{e^{L\omega_p} - 1} \to - \frac{1}{e^{L\omega_p} + 1} , \quad \frac{1}{\sinh^2 \frac{L}{2} \omega_p} \to - \frac{1}{\cosh^2 \frac{L}{2} \omega_p} .$$

Now define

$$J_n \equiv - \frac{2i}{L} \mu^{2\epsilon} \sum_{m=-\infty}^{\infty} \int \frac{d^d p}{(2\pi)^d} \frac{1}{(P^2 - M^2 + i\epsilon)(P'^2 - M^2 + i\epsilon)} \Bigg|_{Q^2 = 0} . \tag{6.3}$$

(Recall that there is an implicit n dependence in Q^M.) We find

$$J_0 = \frac{1}{8\pi^2} \left[\frac{1}{\epsilon} - \gamma - \ln \frac{M^2}{4\pi\mu^2} \right] - \frac{1}{2\pi^2} \sum_{m=1}^{\infty} (-1)^{m+1} K_0(m\alpha) , \tag{6.4a}$$

$$J_n = \frac{1}{8\pi^2} \left[\frac{1}{\epsilon} - \gamma - \ln \frac{M^2}{4\pi\mu^2} \right] \quad (n \neq 0) , \tag{6.4b}$$

where $\alpha \equiv ML$.

Now let us proceed to find the effective coupling constants. For $n = 0$ one has

$$\pi^{\mu\nu}(q^2,0)=\left[\frac{1}{12\pi^2}\left[\frac{1}{\epsilon}-\gamma-\ln\frac{M^2}{4\pi\mu^2}\right]-\frac{1}{3\pi^2}\sum_{m=1}^{\infty}(-1)^{m+1}K_0(m\alpha)\right]\times(q^\mu q^\nu-\eta^{\mu\nu}q^2)+O(q^4)\ . \tag{6.5}$$

Hence we find

$$B_0^0=-\frac{1}{3\pi^2}\sum_{m=1}^{\infty}(-1)^{m+1}K_0(m\alpha)\ . \tag{6.6}$$

From Eq. (A17) we find, for small L,

$$B_0^0\approx\frac{1}{12\pi^2}\left[\ln\frac{M^2L^2}{(4\pi)^2}+2\gamma\right]\ . \tag{6.7}$$

Then we have

$$e_v^2(L)\approx\frac{e_R^2}{1+\frac{e_R^2}{12\pi^2}\left[\ln\frac{M^2L^2}{(4\pi)^2}+2\gamma\right]}\ . \tag{6.8}$$

There is no term proportional to $1/L$ and the logarithmic term is dominant when L is small. This is because the spinor field obeys antiperiodic boundary conditions in the gauge with $\langle A_3\rangle=0$. The logarithmic dependence of $e_v^2(L)$ on $1/L^2$ is of the same form as that of the usual effective coupling constant on the (momentum)^2 in Minkowski spacetime. In contrast with the case with a charged scalar field in the previous section, B_0^0 in (6.6) is always negative and $e_v^2(L)$ is always larger than e_R^2.

Next the calculation of $\pi^{33}(q^2,0)$ yields

$$D_0^0=L\int\frac{d^3p}{(2\pi)^3}\frac{1}{\cosh^2\frac{L}{2}\omega_p}\ , \tag{6.9a}$$

$$D_0^1=-\frac{1}{3\pi^2}\sum_{m=1}^{\infty}(-1)^{m+1}K_0(m\alpha)-\frac{L}{24\pi^2}\int_0^{\infty}\frac{dp}{\cosh^2\frac{L}{2}\omega_p}\ , \tag{6.9b}$$

where the D_0^i are defined in (4.26b). From D_0^0 we find that the mass of the scalar modes coincides with that obtained in Sec. III from the effective potential. The second term in (6.9b) approaches $-1/12\pi^2$ in the $L\rightarrow0$ limit. Hence we have, according to Eq. (4.25b),

$$e_s^2(L)\approx\frac{e_R^2}{1+\frac{e_R^2}{12\pi^2}\left[\ln\frac{M^2L^2}{(4\pi)^2}+2\gamma+1\right]}\ . \tag{6.10}$$

The effective coupling constants $e_n^2(L)$ and the mass shift ΔM^2 of Eq. (4.34) can be obtained in a similar manner. One immediately obtains the mass shift, which is n independent, by following the rules given above Eq. (6.3):

$$\Delta M^2=4e_R^2\int\frac{d^3p}{(2\pi)^3\omega_p}\frac{1}{e^{L\omega_p}+1}\ . \tag{6.11}$$

Note that this quantity is positive as in the case of scalar QED. Similarly, A_n^1 can be obtained as

$$A_n^1=\frac{1}{2\pi^4n^2}\sum_{m=1}^{\infty}\frac{(-1)^{m+1}}{m^2}K_0(m\alpha)\ . \tag{6.12}$$

For small L we find, using (A18),

$$A_n^1\approx\frac{1}{48\pi^2n^2}\left[\ln\frac{M^2L^2}{4}+2\gamma+\frac{24}{\pi^2}\lambda\right]\ , \tag{6.13}$$

where

$$\lambda\equiv\sum_{m=2}^{\infty}\frac{(-1)^{m+1}\ln m}{m^2}=-\tfrac{1}{2}[\zeta'(2)+\zeta(2)\ln2]\ . \tag{6.14}$$

Hence from Eq. (4.33) we have

$$e_n^2(L)\approx\frac{e_R^2}{1+\frac{e_R^2}{48\pi^2n^2}\left[\ln\frac{M^2L^2}{4}+2\gamma+\frac{24}{\pi^2}\lambda\right]}\ . \tag{6.15}$$

The qualitative features of $e_n^2(L)$ are the same as those in scalar QED.

Finally, let us give $e_{n(1)}^2(L)$ and $e_{n(2)}^2(L)$, defined by (4.39a) and (4.39b), in this model. The formula $B_n^0=2A_n^1$ is valid here as well. Hence we have

$$e_{n(1)}^2(L)\approx\frac{e_R^2}{1-\frac{e_R^2}{24\pi^2n^2}\left[\ln\frac{M^2L^2}{4}+2\gamma+\frac{24}{\pi^2}\lambda\right]}\ . \tag{6.16}$$

A_n^0 approaches $1/6L^2$ when L becomes small. Thus,

$$e_{n(2)}^2(L)\approx\frac{e_R^2}{1-\frac{e_R^2}{24\pi^2n^2}\left[\ln\frac{M^2L^2}{4}+2\gamma+\frac{24}{\pi^2}\lambda-\pi^2\right]}\ . \tag{6.17}$$

These effective coupling constants are quite different from $e_n^2(L)$ in L dependence as in the case of scalar QED.

VII. GAUGE SYMMETRY BREAKING

In Sec. III, we showed that if periodic boundary conditions are imposed on the charged spinor field, then the gauge potential A_μ in $S^1\times R^3$ spacetime develops a VEV. If this VEV is gauged away, then the spinor field obeys antiperiodic boundary conditions.

In this section, we discuss a similar phenomenon in non-Abelian gauge field theories. Aspects of this phenomenon were first discussed by Hosotani[9] for SU(n) gauge theories. Here, in contrast with electrodynamics, we show that it leads to spontaneous breakdown of local gauge symmetry, in the sense that the originally massless modes of the gauge field become massive in some cases if we initially impose periodic boundary conditions on charged spinor fields. (This possibility was not discussed in Hosotani's work.) We study this phenomenon in $S^1\times R^4$ spacetime with its possible relevance to Kaluza-Klein-type grand unified theories in mind.

The theories we study here are given by Lagrangians of the following form:

$$\mathcal{L}=-\frac{1}{4}F_{MN}F^{MN}+i\bar{\psi}_i(\not{D}\psi)^i-M\bar{\psi}_i\psi^i-\frac{1}{2\alpha}(\partial_\mu A^{a\mu})^2-i\partial^\mu\bar{c}^{\,a}(D_\mu c)^a , \tag{7.1}$$

where

$$(D_M\psi)^i=\partial_M\psi^i-ie(T^a)^i_j A^a_M\psi^j , \tag{7.2a}$$

$$(D_\mu c)^a=\partial_\mu c^a+ef^{abc}A^b_\mu c^c , \tag{7.2b}$$

$$F^a_{MN}=\partial_M A^a_N-\partial_N A^a_M+f^{abc}A^b_M A^c_N , \tag{7.2c}$$

and $M,N=0,1,2,3$ or 5 and μ,ν run from 0 to 3. The field c^a and $\bar{c}^{\,a}$ are the Faddeev-Popov ghosts. The structure constants f^{abc} of the gauge group and the generators $(T^a)^i_j$ satisfy

$$[T^a,T^b]=if^{abc}T^c . \tag{7.3}$$

The spacetime point (x^μ,x^5+L) $(\mu=0,1,2,3)$ is identified with the point (x^μ,x^5) and both A^a_M and ψ^i satisfy periodic boundary conditions.

Let us define a matrix A_M by

$$A_M\equiv iT^aA^a_M . \tag{7.4}$$

Then the Lagrangian (7.1) is invariant under the following gauge transformation:

$$A'_M=gA_Mg^{-1}-\frac{1}{e}g\partial_Mg^{-1} , \tag{7.5a}$$

$$\psi'=g\psi , \tag{7.5b}$$

where the spacetime-dependent matrix g is an element of the gauge group and ψ is a column vector having components ψ^i in this representation.

A natural initial choice of boundary conditions is that ψ and A_M be single valued (i.e., periodic in x^5 with period L). Then under a gauge transformation (with K^a constant)

$$g(K,x^5)=\exp(-ieT^aK^ax^5) , \tag{7.6}$$

the gauge transformation of A_M is

$$A'_\mu(x)=g(K,x^5)A_\mu(x)g^{-1}(K,x^5) \quad (\mu=0-3) , \tag{7.7a}$$

$$A'_5(x)=g(K,x^5)A_5(x)g^{-1}(K,x^5)-iT^aK^a . \tag{7.7b}$$

The boundary conditions of the new fields A'_M and ψ' become

$$A'_M(x^5+L)=g(K,L)A'_M(x^5)g^{-1}(K,L) , \tag{7.8a}$$

$$\psi'(x^5+L)=g(K,L)\psi'(x^5) , \tag{7.8b}$$

where the coordinates x^μ are suppressed.

If $K^a\equiv K^a_+$ is such that $g(K_+,L)=1$, the boundary conditions are unchanged while A_5 changes in accordance with Eq. (7.8a) as

$$A'_5=A_5-iT^aK^a_+ . \tag{7.9}$$

Hence the gauge-invariant effective potential is single-valued under these changes in A_5. It follows that the effective potential must be a function of

$$g(A_5,L)=\exp(-ieT^aA^a_5L)$$

since this has the desired single-valuedness and any function having that property can be expanded as a generalized Fourier series in positive and negative powers of $g(A_5,L)$.

The next step in our argument is to show that if the gauge group is such that there exists a choice of $K^a\equiv K^a_-$ for which $g(K_-,L)=-1$, then one can make a gauge transformation from the initial periodic boundary conditions for which the minimum of the effective potential is at a nonzero value of A_5, to new boundary conditions such that the absolute minimum of the effective potential is at $A'_5=0$. These new boundary conditions are

$$A'_M(x^5+L)=A'_M(x^5) , \tag{7.10a}$$

$$\psi'(x^5+L)=-\psi'(x^5) , \tag{7.10b}$$

which are antiperiodic in the fermion field ψ'.

One can see this as follows. Consider the term in the Lagrangian having the form

$$i\bar{\psi}'\gamma^M(\partial_M-ieT^aA'^a_M)\psi' ,$$

with A'^a_M constant. The value of A'_μ $(\mu=0-3)$ can be gauged away with no physical consequences. By diagonalizing the Hermitian matrix $T^aA'^a_5$ one can write the $M=5$ term in the above expression as a sum of terms, each having the form of a fermion kinetic term with coupling to an Abelian gauge field. As shown earlier, the minimum of the effective potential with these antiperiodic boundary conditions on ψ' occurs at $A'_5=0$. It is also true, as we will see later, that the part of the one-loop Lagrangian involving only the gauge field also has its minimum at $A'_5=0$. In this case, massless modes of the gauge field A'_μ remain massless and there is no symmetry breaking.

Suppose now that there exists no choice of K^a for which $g(K,L)$ is -1. Then in the theory with initially periodic boundary conditions on the gauge and fermion fields, we can no longer state in general where the minimum of the effective potential occurs. That will depend on the gauge group and the fermion representation (we will consider examples below).

Let $\langle A^a_5\rangle$ be the VEV of the gauge field with periodic boundary conditions. One can make $\langle A'_5\rangle=0$, by means of the gauge transformation

$$g(x^5)\equiv\exp(-ieT^a\langle A^a_5\rangle x^5) . \tag{7.11}$$

Then the new boundary conditions are

$$A'_M(x^5+L)=g(L)A'_M(x^5)g^{-1}(L) , \tag{7.12a}$$

$$\psi'(x^5+L)=g(L)\psi'(x^5) . \tag{7.12b}$$

From Eq. (7.12a), we find that if $g(L)$ is not in the center of the group then the boundary condition on A'_M is not periodic, so that some massless modes of the gauge field A'_μ become massive; i.e., the gauge symmetry is spontaneously broken. Hence, if one considers the theory with periodic boundary conditions and finds that $\langle A_5 \rangle$ is nonzero at the minimum of the effective potential, then this implies symmetry breaking if and only if $g(L)$ of Eq. (7.11) is not in the center of the gauge group. We consider next two examples, one in which $g(L)$ is in the center of the gauge group which was considered earlier by Hosotani,[9] and a new one in which $g(L)$ is not in the center of the gauge group, thus implying spontaneous symmetry breakdown.

Let us take SU(2) gauge theories ($f^{abc}=\epsilon^{abc}$) with one charged spinor field. First we study the case where the spinor field is in the adjoint representation of SO(3) (for which -1 is not in the group) and then the case where the spinor field is in the fundamental representation of SU(2) (for which -1 is in the group). In the former, the gauge symmetry will be broken whereas in the latter, it will remain unbroken.

If the spinor field is in the adjoint representation, we have $(T^a)^i_j=-i\epsilon^{aij}$ in Eq. (7.2). One may assume that the VEV $\langle A_5^a \rangle$ is proportional to δ^{a3} without loss of generality because of the gauge invariance. To evaluate the effective potential to one-loop order we need only the quadratic terms in the Lagrangian with respect to the quantum fields (A_5^3 will be treated as a constant classical field since quantum effects due to A_5^3 can be neglected at one-loop level).

Now let

$$\psi^\pm \equiv \frac{1}{\sqrt{2}}(\psi^1 \mp i\psi^2) , \tag{7.13}$$

where ψ^1 and ψ^2, the first two components of the adjoint representation, are each Dirac spinors. Then the quadratic part of the fermionic Lagrangian in Eq. (7.1) becomes

$$\begin{aligned}\mathcal{L}_f\,|_{\text{quad}} = {} & i\overline{\psi^+}\gamma^\mu\partial_\mu\psi^+ + i\overline{\psi^+}\gamma^5(\partial_5+ieA_5^3)\psi^+ - M\overline{\psi^+}\psi^+ \\ & + i\overline{\psi^-}\gamma^\mu\partial_\mu\psi^- + i\overline{\psi^-}\gamma^5(\partial_5-ieA_5^3)\psi^- \\ & - M\overline{\psi^-}\psi^- + i\overline{\psi^3}\gamma^\mu\partial_\mu\psi^3 - M\overline{\psi^3}\psi^3 ,\end{aligned} \tag{7.14}$$

where γ^5 here is i times the ordinary Hermitian γ^5. The part of the effective potential due to the spinor loop can be calculated in the same way as in Sec. III. The result is

$$\begin{aligned}V_f = -\frac{4}{L}\int\frac{d^4p}{(2\pi)^4}\Bigg[& \ln\sinh\frac{L}{2}(\omega_p - ieA_5^3) \\ & + \ln\sinh\frac{L}{2}(\omega_p + ieA_5^3)\Bigg] ,\end{aligned} \tag{7.15}$$

up to an infinite constant. Here $\omega_p=\sqrt{p^2+M^2}$ with $p^2=\delta_{\mu\nu}p^\mu p^\nu$. In addition there is a contribution to the effective potential from the gauge-field loop. To calculate that contribution, define

$$\phi^\pm \equiv \frac{1}{\sqrt{2}}(A_5^1 \mp iA_5^2) , \tag{7.16a}$$

$$A_\mu^\pm \equiv \frac{1}{\sqrt{2}}(A_\mu^1 \mp iA_\mu^2) . \tag{7.16b}$$

Then the quadratic part of the Lagrangian in (7.1) relevant to our calculation is

$$\begin{aligned}\mathcal{L}_g\,|_{\text{quad}} = {} & (\partial_\mu\phi^+ - \partial_5 A_\mu^+ + ieA_5^3A_\mu^+) \\ & \times(\partial^\mu\phi^- - \partial_5 A^{-\mu} - ieA_5^3A^{-\mu}) \\ & + \tfrac{1}{2}(\partial_\mu A_\nu^+ - \partial_\nu A_\mu^+)(\partial^\mu A^{-\nu} - \partial^\nu A^{-\mu}) \\ & - \frac{1}{\alpha}\partial_\mu A^{+\mu}\partial^\nu A_\nu^- .\end{aligned} \tag{7.17}$$

Note that the Faddeev-Popov ghosts do not contribute to the effective potential because the gauge-fixing term has been chosen in such a way that the ghost Lagrangian does not have A_5^a dependence.

Now the above Lagrangian can be written in momentum space as follows:

$$A_M^+K^{MN}A_N^- \equiv (A_\mu^+,\phi^+)\begin{bmatrix} -q^2\eta^{\mu\nu} + \left[1-\frac{1}{\alpha}\right]q^\mu q^\nu + \left[\frac{2\pi n}{L}+eA_5^3\right]^2\eta^{\mu\nu} & q^\mu\left[\frac{2\pi n}{L}+eA_5^3\right] \\ q^\nu\left[\frac{2\pi n}{L}+eA_5^3\right] & q^2 \end{bmatrix}\begin{bmatrix} A_\nu \\ \phi^- \end{bmatrix} , \tag{7.18}$$

up to a factor of order unity. The determinant of the matrix K^{MN} is

$$\det K = \frac{1}{\alpha}(q^2)^2\left[-q^2+\left[\frac{2\pi n}{L}+eA_5^3\right]^2\right]^3 . \tag{7.19}$$

Hence the part of the effective potential due to the gauge-field loop, which is the sum of $\ln\det K$ for all five-momenta $(q^\mu, 2\pi n/L)$ in the Euclidean spacetime, is

$$\begin{aligned}V_g &= \frac{3}{L}\sum_{m=-\infty}^{\infty}\int\frac{d^4p}{(2\pi)^4}\ln\left[|p|^2+\left[\frac{2\pi n}{L}+eA_5^3\right]^2\right] \\ &= \frac{3}{L}\int\frac{d^4p}{(2\pi)^4}\Bigg[\ln\sinh\frac{L}{2}(|p|-ieA_5^3) \\ &\qquad + \ln\sinh\frac{L}{2}(|p|+ieA_5^3)\Bigg]\end{aligned} \tag{7.20}$$

up to an infinite constant. The effective potential V is the sum of V_f and V_g. If the mass M of the spinor field is small enough, the effective potential V takes its maximum value at $A_5^3=0$ and its minimum value at $A_5^3=\pi/eL$. In particular, if $M=0$ we have

$$V=-\frac{1}{L}\int\frac{d^4p}{(2\pi)^4}\left[\ln\sinh\frac{L}{2}(|p|-ieA_5^3)+\ln\sinh\frac{L}{2}(|p|+ieA_5^3)\right]. \quad (7.21)$$

Then the fields $\psi^{\pm}$ and $A_M^{\pm}$ obey effective antiperiodic boundary conditions, and the massless modes of $A_\mu^{\pm}$ disappear while those of A_μ^3 remain intact. That is, the local SU(2) gauge symmetry is spontaneously broken to U(1), in the sense that the gauge fields $A_\mu^{\pm}$ acquire mass. (In the present gauge, the mass is a consequence of antiperiodic boundary conditions on A_μ^a; in the original gauge with A_μ^a periodic, the mass is generated by a nonzero VEV of A_5^3.)

Next let us consider the case where the spinor field is in the fundamental representation. Then $(T^a)^i_j$ in (7.2) is $(\tau^a)^i_j/2$ where τ^a is the Pauli spin matrix. In particular

$$T^3=\frac{1}{2}\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}. \quad (7.22)$$

Writing the spinor field as a column vector in isospin space

$$\psi\equiv\begin{bmatrix}\psi^+\\ \psi^-\end{bmatrix}, \quad (7.23)$$

we have

$$(D_5\psi)^{\pm}=\left[\partial_5\mp\frac{ie}{2}A_5^3\right]\psi^{\pm}. \quad (7.24)$$

Hence the fermionic contribution to the effective potential is

$$V_f=-\frac{4}{L}\int\frac{d^4p}{(2\pi)^4}\left[\ln\sinh\frac{L}{2}\left[\omega_p-\frac{ie}{2}A_5^3\right]+\ln\sinh\frac{L}{2}\left[\omega_p+\frac{ie}{2}A_5^3\right]\right]. \quad (7.25)$$

Then the absolute minimum of the effective potential is at $A_5^3=2\pi/eL$ since both V_f and V_g have their minima there. At this minimum the field ψ' defined by (7.8b) satisfies antiperiodic boundary conditions while the field A'_M defined by (7.8a) satisfies periodic boundary conditions. Therefore all the massless modes of the gauge field remain massless and the gauge symmetry is unbroken. Also $g(K,L)$ is -1 as expected.[23]

The VEV $\langle A_5^3\rangle=2\pi/eL$ breaks the global SU(2) symmetry corresponding to the (non-gauge-invariant) conserved current

$$j_M^a=\epsilon^{abc}(F_{MN}^bA^{cN}+\partial_\alpha A^{b\alpha}A_\mu^c\delta_M^\mu)+(\text{fermion-bilinear terms}). \quad (7.26)$$

If one considers the analogy with the Higgs mechanism, this fact would appear to be in contradiction with the absence of local gauge symmetry breaking. However, although local gauge transformations do not alter the length of a scalar Higgs field, we do have available in the present case a nonperiodic gauge transformation which takes $\langle A_5^a\rangle$ to zero. Any current obtained from j_M^a in (7.26) by a gauge transformation with a parameter depending only on x^5 is also conserved. We want the current thus obtained to satisfy periodic boundary conditions, so that the charge obtained from it is independent of which interval of length L we choose to integrate over in the x^5 direction. In the present case, the unbroken global SU(2) symmetry is generated by the charge that is obtained from Eq. (7.26) through the gauge transformation (7.5) with the matrix $g(K,x^5)$ given by Eq. (7.6).

VIII. SUMMARY AND DISCUSSIONS

In this paper we first studied the one-loop effective potential of QED in $S^1\times R^3$ spacetime. In particular, the "vacuum" state with noncausal propagation of the photon,[7] which occurs when periodic boundary conditions are imposed on the charged spinor field and $\langle A_3\rangle$ is taken to be zero, is unstable and can decay into the stable vacuum where, with a suitable gauge, $\langle A_3\rangle$ is zero and the spinor field satisfies antiperiodic boundary conditions.

We then studied the "on-shell' coupling constants for QED with one charged field in the same spacetime. Especially we found that the one-loop correction to the coupling constant for the three-dimensional vector modes of the lowest Fourier component of A_M behaves like $1/L$ (L is the size of the "third" dimension) for small L for the case with one charged scalar field. This behavior is due to the modes of the lowest Fourier component of the scalar field.

We showed that there are more than one possible definition of "on-shell" coupling constants for higher Fourier components. Two seemingly natural definitions led to different coupling constants. Their dependence on the size L of the compactified dimension was found to be logarithmic, but in each case different from the logarithmic dependence expected from the UV divergences, due to collinear singularities. Also these coupling constants were found to coincide with that in Minkowski spacetime in the limit where the number of nodes in the compactified direction is infinite.

For the lowest Fourier component in both the scalar and spinor cases, the logarithmic dependence of the effective coupling constants on L is the same as that expected from the UV divergences. In the spinor case, and for the three-dimensional scalar mode of A_M in the scalar case, this logarithmic dependence is the leading behavior for small L.

We did not extend our analysis beyond one-loop order. It will be interesting to find whether the logarithmic dependence of some of the "on-shell" coupling constants on the size of the compactified dimension is valid in the leading-logarithmic approximation.

Then we turned to non-Abelian gauge theories and pointed out that spontaneous symmetry breaking can occur with single-valued, or periodic, boundary conditions imposed on the gauge and fermion fields. If $\langle A_5 \rangle$ minimizes the effective potential, then spontaneous gauge symmetry breaking occurs if $g(L)=\exp(ieT^a\langle A_5^a \rangle L)$ is not in the center of the gauge group. We also showed that if -1 is an element of the gauge group in the representation to which the spinors belong, then $\langle A_5^a \rangle$ will be such that $g(L)=-1$ (in which case no symmetry breaking occurs).

Finally, as examples we discussed SU(2) gauge theories with charged spinor field satisfying periodic boundary conditions. We showed that if the spinor is in the adjoint representation, the gauge symmetry is spontaneously broken.

Bars and Visser[24] have proposed a mechanism for generating a small coupling between matter fields and the gauge boson which is part of the metric tensor in five-dimensional Kaluza-Klein theory. In their mechanism, another gauge field is introduced. It obeys periodic boundary conditions and is assumed to have a small VEV of the "fifth" component, which generates a small coupling between the matter field and the Kaluza-Klein gauge field. The mechanism discussed here would generate a VEV for the extra gauge field considered by Bars and Visser. This VEV would, of course, be too large for their purpose, but may be relevant, for example, for breaking SU(5) at the grand unification scale in Kaluza-Klein grand unified theories.

It is interesting that it is possible to impose a nonperiodic boundary condition

$$A_M(x^\mu,x^5+L)=g_B A_M(x^\mu,x^5)g_B^{-1} , \tag{8.1}$$

where g_B is a constant matrix in the gauge group, from the beginning. In other words, one can introduce gauge symmetry breaking discussed in Sec. VII by hand. We expect that no inconsistency will arise by imposing such a boundary condition since the Lagrangian and the boundary condition are still invariant under gauge transformations with $g(x^\mu,x^5)$ satisfying the boundary condition

$$g(x^\mu,x^5+L)=g_B g(x^\mu,x^5)g_B^{-1} . \tag{8.2}$$

This class of gauge transformations would thus replace the class of periodic gauge transformations under which periodic boundary conditions on A_M are invariant.

ACKNOWLEDGMENTS

We are grateful to Jonathan Bagger and Sidney Coleman for informing us of the paper of Hosotani, which we were not aware of in the first version of this work. We thank John L. Friedman for helpful discussions and the National Science Foundation for support under Grant No. PHY-8603173.

APPENDIX: USEFUL MATHEMATICAL FORMULAS

The reference for this section is Ref. 25. In Sec. III we used

$$\sum_{m=-\infty}^{\infty} \frac{1}{x^2+(m-a)^2} = \frac{\pi}{2x}[\coth\pi(x-ia) + \coth\pi(x+ia)] . \tag{A1}$$

This can be shown by using

$$\sum_{m=-\infty}^{\infty} \frac{1}{x^2+m^2} = \frac{\pi}{x}\coth\pi x \tag{A2}$$

and

$$\sum_{m=-\infty}^{\infty} \frac{1}{x^2+(m-a)^2} = \frac{1}{2x} \sum_{m=-\infty}^{\infty} \left[\frac{x-ia}{(x-ia)^2+m^2} + \frac{x+ia}{(x+ia)^2+m^2} \right] . \tag{A3}$$

By differentiating Eq. (A1) one obtains

$$\sum_{m=-\infty}^{\infty} \frac{1}{[x^2+(m-a)^2]^2} = \frac{\pi}{4x^2}[\coth\pi(x-ia)+\coth\pi(x+ia)] + \frac{\pi^2}{4x}\left[\frac{1}{\sinh^2\pi(x-ia)} + \frac{1}{\sinh^2\pi(x+ia)} \right] . \tag{A4}$$

Multiplying (A1) by $2x$ and integrating it from $x=0$ to x, one has

$$\sum_{m=-\infty}^{\infty} \{\ln[x^2+(m-a)^2]-\ln(m-a)^2\} = \ln\sinh\pi(x-ia)+\ln\sinh\pi(x+ia)-\ln\sin^2\pi a . \tag{A5}$$

Now

$$\sin\pi a = \pi a \prod_{k=1}^{\infty} \left[1-\frac{a^2}{k^2} \right] = \pi a \prod_{\substack{m=-\infty \\ (m\neq 0)}}^{\infty} \frac{|m-a|}{|m|} . \tag{A6}$$

Hence

$$\sum_{m=-\infty}^{\infty} \{\ln[x^2+(m-a)^2]-\text{“}\ln m^2\text{”}\} = \ln\sinh\pi(x-ia)+\ln\sinh\pi(x+ia) , \tag{A7}$$

where "$\ln m^2$" is defined by

$$\text{“}\ln m^2\text{”} = \ln m^2 \quad (m\neq 0) \tag{A8a}$$

$$= -2\ln\pi \quad (m=0) . \tag{A8b}$$

In Sec. V we need to evaluate the following integral:

$$f_n^-(\alpha) \equiv \int_0^1 dx\, x(1-x) \int_0^\infty dy \frac{y^{-1/2}}{\sqrt{y+1}} \frac{1}{\exp(\alpha\sqrt{y+1}-2\pi i n x)-1}$$
$$= \int_0^1 dx\, x(1-x) \int_0^\infty dy \frac{y^{-1/2}}{\sqrt{y+1}} \sum_{m=1}^{\infty} \exp(-m\alpha\sqrt{y+1}+2\pi m n x) . \tag{A9}$$

For $n=0$, we immediately find

$$f_0^-(\alpha)=\tfrac{1}{3}\sum_{m=1}^{\infty}K_0(m\alpha)\ , \tag{A10}$$

by using

$$\int_0^\infty \frac{x^{\nu-1}\exp(-\alpha\sqrt{1+x}\,)}{\sqrt{1+x}}dx=\frac{2}{\sqrt{\pi}}\left[\frac{\alpha}{2}\right]^{1/2-\nu}\Gamma(\nu)K_{1/2-\nu}(\alpha)\ . \tag{A11}$$

The infinite sum on the right-hand side may be reexpressed as

$$\sum_{m=1}^{\infty}K_0(m\alpha)=\frac{1}{2}\left[\gamma+\ln\frac{\alpha}{4\pi}\right]+\frac{\pi}{2\alpha}+\pi\sum_{l=1}^{\infty}\left[\frac{1}{[\alpha^2+(2l\pi)^2]^{1/2}}-\frac{1}{2l\pi}\right]\ . \tag{A12}$$

For $n\neq 1$ one can perform the x integration first and then the y integration in (A9). Thus one has

$$f_n^-(\alpha)=-\frac{1}{\pi^2n^2}\sum_{m=1}^{\infty}\frac{1}{m^2}K_0(m\alpha)\ . \tag{A13}$$

The integral defined by

$$f_n^+(\alpha)=\int_0^1 dx\,x(1-x)\int_0^\infty dy\frac{y^{-1/2}}{\sqrt{y+1}}\frac{1}{\exp(\alpha\sqrt{y+1}-2\pi i n x)+1} \tag{A14}$$

can be calculated in a similar manner and one has

$$f_0^+(\alpha)=\tfrac{1}{3}\sum_{m=1}^{\infty}(-1)^{m+1}K_0(m\alpha) \tag{A15}$$

and

$$f_n^+(\alpha)=-\frac{1}{\pi^2n^2}\sum_{m=1}^{\infty}\frac{(-1)^{m+1}}{m^2}K_0(m\alpha)\ . \tag{A16}$$

The right-hand side may be reexpressed by using

$$\sum_{m=1}^{\infty}(-1)^mK_0(m\alpha)=\frac{1}{2}\left[\gamma+\ln\frac{\alpha}{4\pi}\right]+\pi\sum_{l=1}^{\infty}\left[\frac{1}{\{\alpha^2+[(2l-1)\pi]^2\}^{1/2}}-\frac{1}{2l\pi}\right]. \tag{A17}$$

The Bessel function $K_0(z)$ of imaginary argument has the following expansion:

$$K_0(z)=\sum_{k=0}^{\infty}\frac{z^{2k}}{2^{2k}(k!)^2}\left[\psi(k+1)-\ln\frac{z}{2}\right]\ , \tag{A18}$$

where

$$\psi(x)=\frac{d}{dx}\ln\Gamma(x)=-\gamma-\sum_{k=0}^{\infty}\left[\frac{1}{x+k}-\frac{1}{k+1}\right]\ . \tag{A19}$$

One has the following expansion for $1/\sinh^2x$:

$$\frac{1}{\sinh^2x}=\frac{1}{x^2}+2\sum_{k=1}^{\infty}\frac{x^2-\pi^2k^2}{(x^2+\pi^2k^2)^2}\ . \tag{A20}$$

[1]L. Parker and D. J. Toms, Phys. Rev. Lett. **52**, 1269 (1984); Phys. Rev. D **29**, 1584 (1984); E. Calzetta, Ann. Phys. (N.Y.) **166**, 214 (1986); see, also, E. S. Fradkin and A. A. Tseytlin, Phys. Lett. **104B**, 377 (1981); L. Smolin, Nucl. Phys. **B208**, 439 (1982); B. Nelson and P. Panangaden, Phys. Rev. D **25**, 1019 (1982); B. L. Hu and D. J. O'Connor, *ibid.* **30**, 743 (1984).

[2]E. Calzetta, I. Jack, and L. Parker, Phys. Rev. Lett. **55**, 1241 (1985); Phys. Rev. D **33**, 953 (1986).

[3]L. Parker and D. J. Toms, Phys. Rev. D **31**, 953 (1985); I. Jack and L. Parker, *ibid.* **31**, 2439 (1985).

[4]J. Schwinger, Phys. Rev. **82**, 664 (1951); B. S. DeWitt, *Dynamical Theory of Groups and Fields* (Gordon and Breach, New York, 1965).

[5]N. K. Nielsen and B.-S. Skagerstam, Phys. Rev. D **34**, 3025 (1986).

[6]The possibility of nontrivial boundary conditions was studied in C. J. Isham, Proc. R. Soc. London **A362**, 383 (1978); **A364**, 591 (1978); S. J. Avis and C. J. Isham, Nucl. Phys. **B156**, 441 (1979).

[7]L. H. Ford, Phys. Rev. D **21**, 933 (1980).

[8]D. J. Toms, Phys. Rev. D **21**, 928 (1980); **21**, 2805 (1980).

[9]Y. Hosotani, Phys. Let. **126B**, 309 (1983).

[10]D. J. Toms, Phys. Lett. **126B**, 445 (1983).

[11]S. Coleman and E. Weinberg, Phys. Rev. D **7**, 1888 (1973).

[12]See, e.g., A. A. Abrikosov, A. A. Gorkov, and I. Ye Dzyaloskinskii, *Quantum Field Theory Methods in Statistical Physics* (Prentice-Hall, Englewood Cliffs, NJ, 1963); L. P. Kadanoff and G. Baym, *Quantum Statistical Mechanics* (Benjamin/Cummings, Reading, MA, 1962).

[13]See, e.g., J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1975).

[14]D. J. Gross, R. D. Pisarski, and L. G. Yaffe, Rev. Mod. Phys. **53**, 43 (1981).

[15]See, e.g., J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields* (McGraw-Hill, New York, 1965).

[16]E. C. G. Stueckelberg and A. Peterman, Helv. Phys. Acta **26**, 499 (1953); M. Gell-Mann and F. E. Low, Phys. Rev. **95**, 1300 (1954).

[17]The theory with the gauge-fixing term (4.20) is still invariant under the gauge transformation with a gauge parameter depending only on x^3. This residual gauge freedom corresponds to the arbitrariness of the VEV's of higher ($n\neq 0$) Fourier components of A_3, which are implicitly set equal to zero. This remark also applies to the non-Abelian case considered later.

[18]H. Lehmann, K. Symanzik, and W. Zimmermann, Nuovo Cimento **1**, 1425 (1955).

[19]This fact can be seen by coupling the propagator $\Delta_{MN}(q^2,n)$ to a conserved current j_M satisfying $q^\mu j_\mu + (2\pi n/L) j_3 = 0$. Since $n\neq 0$, j_3 is proportional to q^2 unless j_μ has a pole at $q^2=0$. (In the absence of spontaneous symmetry breaking there is no pole at $q^2=0$.) Now note that the pole of the propagator $\Delta_{33}(q^2,n)$ is at $q^2=0$ because $D_n \propto q^2$ according to Eq. (4.28). (In the case we consider, A_n and B_n have no pole at $q^2=0$.) Hence, the quantity $j^3\Delta_{33}(q^2,n)j^3$ does not have a pole at $q^2=0$, and there is no corresponding physical particle.

[20]G. 't Hooft and M. Veltman, Nucl. Phys. **B44**, 276 (1974).

[21]In dimensional regularization, which we use here, $e_R^2 = Z_3 e_B^2 \mu^{-2\epsilon}$. But the factor $\mu^{-2\epsilon}$ may be neglected because e_R^2 and $Z_3 e_B^2$ have no poles at $\epsilon=0$. See, e.g., J. C. Collins and A. J. Macfarlane, Phys. Rev. D **10**, 1201 (1974).

[22]At $M=0$ the theory takes on a different character, in that the one-loop contribution to the kinetic term behaves likes $(q^\mu q^\nu - \eta^{\mu\nu} q^2)/\sqrt{q^2}$ instead of $q^\mu q^\nu - \eta^{\mu\nu} q^2$. This is also the case in non-Abelian gauge theories. See R. Jackiw and S. Templeton, Phys. Rev. D **23**, 2291 (1981); T. Applequist and R. D. Pisarski, *ibid.* **23**, 2305 (1981).

[23]The effective potential $V=V_g+V_f$ is locally minimum at $A_5^3=0$ in the present case. Thus the vacuum with $\langle A_5^3\rangle=0$ is metastable.

[24]I. Bars and M. Visser, Phys. Rev. Lett. **57**, 25 (1986).

[25]I. S. Gradshteyn and I. M. Ryzhik, *Tables of Integrals, Series, and Products,* corrected and enlarged edition, edited by A. Jeffrey (Academic, New York, 1980).