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Astrophysical constraints on axion and Majoron couplings

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We discuss in detail photon propagation within a medium. The results are then used to calculate the cross section of the photoproduction of pseudoscalars within the stellar medium in the most general case when the pseudoscalar has an appreciable mass and the photon is also massive due to matter effects. We discuss the Compton-type and the Primakoff-type processes, including the interference of them. Finally, we use the cross section to estimate the solar energy leakage due to axions. This produces bounds on the axion parameters which are discussed.

I. INTRODUCTION

It is generally agreed that the most elegant solution of the strong *CP* problem is through the spontaneous breaking of a new global chiral symmetry.[1] The Weinberg-Wilczek axion,[2] which assumes that the Peccei-Quinn symmetry breaks at the weak scale, has been ruled out experimentally. If one raises the symmetry-breaking scale to a much higher value[3] the resulting axion is extremely weakly coupled to the ordinary matter. It is then effectively invisible in all laboratory experiments and only astrophysical considerations can constrain its parameters.

If axions exist, stars will lose energy by producing axions which subsequently come out of the stellar medium. For the main-sequence and red-giant stars, the mechanisms mostly responsible for this energy loss are the Compton-type process shown in Fig. 1 and the Primakoff-type one shown in Fig. 2. Only at higher temperatures and densities other axion production processes[4,5] such as the annihilation channel $e^+ + e^- \rightarrow \gamma + a$, plasmon decay[6] $\gamma_{\rm pl} \rightarrow \gamma + a$, or more exotic axion bremsstrahlung processes such as $e + (Z, A) \rightarrow e + (Z, A) + a$, or $n + n \rightarrow n + n + a$, etc., come into play where a is an axion, (Z, A) a nucleus, and n is a neutron.

In this paper, we calculate the energy loss of the Sun owing to the photoproduction of axions through the Compton- and the Primakoff-type processes. We determine the cross section due to the processes in the most general case when the axion has an appreciable mass and the photon also acquires a mass due to the plasma effects inside the stellar core. In doing this, we also include the interference between the two processes, an effect not discussed by previous authors.[4,5] We show in detail how to take into account the matter effects in the stellar core in determining the cross section.

We organize the paper as follows. In Sec. II we analyze the plasma effects due to the interaction among photons and a dense electron gas in the stellar core and show how only the transverse photons acquire a mass given by the plasma frequency and that the longitudinal-photon contributions are suppressed in the case of interest, i.e., when the temperature of the heat bath is much larger than the plasma frequency. In Sec. III we show how to take into account the possibility of producing a real, longitudinal photon in the intermediate state of the Primakoff-type process. Then in Sec. IV we compute the photoproduction cross section of any pseudoscalar particle in stellar core. We find the cross section of the pro-

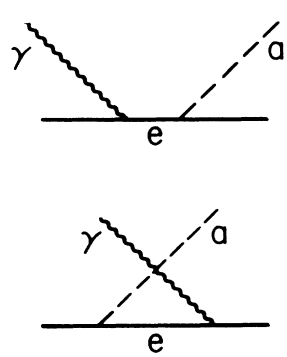

FIG. 1. The Compton-type diagrams for photoproduction of pseudoscalars.

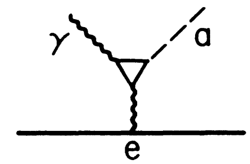

FIG. 2. The Primakoff-type diagram for photoproduction of pseudoscalars.

cess without any assumption about the smallness of the mass of pseudoscalar. Applying these formulas to the case of light axions, we compute the solar energy loss due to axion emission in Sec. V. Demanding that this energy loss is not larger than the observed solar luminosity, we obtain a general bound on a combination of the axion parameters. In Sec. VI we discuss the implications of these bounds on axion models. We also discuss the bounds on models involving Majorons[7,8] in that context.

II. PHOTON PROPAGATOR WITHIN A MEDIUM

In a vacuum, the Primakoff term of the photoproduction cross section has a logarithmic singularity in the limit in which the axion mass m_a is neglected. This effect is important since m_a is very small compared to the other mass scales involved. However, as emphasized by Fukugita, Watamura, and Yoshimura,[4] if the plasma effects in the stellar interior are included, the photon effectively obtains a mass and the logarithm will be cut off at m_γ^2. This produces important corrections, as we will see. The conventional wisdom is to include the plasma effects by introducing the effective photon mass in the propagator. Since the discussion of plasma effects in luminosity calculations appears to be scattered in the literature,[6,9,10] in this section we include a careful discussion of the same for completeness.

Let q^μ be momentum of the propagating photon and u^μ the four-velocity of the center of mass of the medium in which the propagation takes place. In the covariant α gauge, the tree-level photon propagator is given by $i\Delta_{\mu\nu}$, where

$$\Delta_{\mu\nu}(q)=\frac{1}{q^2}\left[-g_{\mu\nu}+\alpha\frac{q_\mu q_\nu}{q^2}\right] . \tag{2.1}$$

The quantum corrections induce a self-energy $\pi_{\mu\nu}$ for the photons. It is transverse because of gauge invariance:

$$q^\mu\pi_{\mu\nu}(q)=q^\nu\pi_{\mu\nu}(q)=0 . \tag{2.2}$$

It is convenient to define the following tensors[9] which are orthogonal to q^μ:

$$\tilde{g}_{\mu\nu}\equiv g_{\mu\nu}-\frac{q_\mu q_\nu}{q^2} ,$$
$$Q_{\mu\nu}\equiv\frac{\left[u_\mu-\dfrac{(q\cdot u)q_\mu}{q^2}\right]\left[u_\nu-\dfrac{(q\cdot u)q_\nu}{q^2}\right]}{1-\dfrac{(q\cdot u)^2}{q^2}} \tag{2.3}$$

In absence of any medium, u^μ and hence $Q^{\mu\nu}$ cannot be defined, and so the vacuum polarization is proportional to $\tilde{g}_{\mu\nu}$, which is why the photon is massless in the vacuum. Within a medium, however, the most general form of $\pi_{\mu\nu}(q)$ satisfying Eq. (2.2) is given by[9]

$$\pi_{\mu\nu}(q)=\pi_T R_{\mu\nu}+\pi_L Q_{\mu\nu} , \tag{2.4}$$

where $R_{\mu\nu}=\tilde{g}_{\mu\nu}-Q_{\mu\nu}$ and π_T and π_L are functions of the Lorentz-invariant quantities q^2 and $q\cdot u$. The full photon propagator is $iD_{\mu\nu}$, where

$$D_{\mu\nu}=(\Delta_{\mu\nu}^{-1}+\pi_{\mu\nu})^{-1}$$
$$=-\frac{R_{\mu\nu}}{q^2-\pi_T}-\frac{Q_{\mu\nu}}{q^2-\pi_L}-\frac{(1-\alpha)q_\mu q_\nu}{q^4} . \tag{2.5}$$

The quantum correction to the free Lagrangian of the electromagnetic field is given by $\frac{1}{2}A^\mu\pi_{\mu\nu}A^\nu$. Using Eq. (2.4) and comparing with the momentum-space Lagrangian in a medium, we can identify[9] the dielectric and the magnetic permeabilities of the medium by

$$\epsilon=1-\frac{\pi_L}{q^2},\quad \frac{1}{\mu}=1+\frac{\pi_T-\pi_L\dfrac{(q\cdot u)^2}{q^2}}{(q\cdot u)-q^2} . \tag{2.6}$$

In the astrophysics literature, it is customary to denote ε by ε_l and define

$$\epsilon_t=1-\frac{\pi_T}{(q\cdot u)^2} . \tag{2.7}$$

Using these as the independent objects rather than ε and μ makes the equations look much simpler, as we will see below.

For convenience, from now on we go to the rest frame of the heat bath so that $u^\mu=(1,\mathbf{0})$ and $q\cdot u=q_0$. The poles of $D_{\mu\nu}$ of Eq. (2.5) occur when either π_T or π_L equals q^2. In the notation of Eqs. (2.6) and (2.7), these conditions are

$$q_0^2\epsilon_t=\mathbf{q}^2 \quad \text{transverse modes} ,$$
$$\epsilon=0 \quad \text{longitudinal modes} . \tag{2.8}$$

This equation gives the dispersion relations for the transverse and the longitudinal photons, which we write down more explicitly in what follows.

For a nonrelativistic gas,

$$\epsilon_t=\epsilon=1-\frac{\omega_P^2}{q_0^2} \tag{2.9}$$

so that $\mu=1$. Here ω_P is the *plasma frequency*. For a nondegenerate gas of electrons, the value of ω_P can be calculated by simple considerations:[11]

$$\omega_P^2=\frac{n_e e^2}{m_e} . \tag{2.10}$$

Substituting Eq. (2.9) in Eq. (2.8), we obtain the following dispersion relations for transverse and longitudinal modes:

$$\omega_T^2(\mathbf{q})=\omega_P^2+\mathbf{q}^2,\quad \omega_L^2(\mathbf{q})=\omega_P^2 . \tag{2.11}$$

Thus the transverse modes have a particlelike dispersion with the effective mass given by

$$m_\gamma^2=\omega_P^2 . \tag{2.12}$$

The longitudinal photon, on the other hand, does not have a particlelike dispersion at all. However, although it looks strange, it is well known that for propagation of waves in a medium, one gets in general dispersion relations which are not particlelike. For example, for phonons in a crystal lattice, the so-called optical branch has the energy almost independent of the momentum,[12] just like the second equation in Eq. (2.11).

Using Eq. (2.12) in conjunction with Eqs. (2.7) and (2.9), we can now write down the propagator in Eq. (2.5) in terms of the photon mass:

$$D_{\mu\nu}(q)=-\frac{1}{q^2-m_\gamma^2}(g_{\mu\nu}+u_\mu u_\nu \mathcal{D}) , \tag{2.13}$$

where

$$\mathcal{D}=\frac{m_\gamma^2}{q_0^2-m_\gamma^2} . \tag{2.14}$$

In Eq. (2.13) we have dropped the terms involving q_μ or q_ν. This amounts to choosing the Feynman gauge. It is easy to see that such terms do not contribute to the matrix element for photoproduction of pseudoscalars since the propagator connects to conserved currents in Fig. 2.

The problem at hand now is to find the wave functions of the transverse and longitudinal photons. Writing the wave function in the form

$$\langle 0 | A^\mu(x) | q\lambda\rangle=\sqrt{N_{q\lambda}}\epsilon^\mu(q\lambda)e^{-iq\cdot x} \tag{2.15}$$

we determine $N_{q\lambda}$ by looking at the residue of $D_{\mu\nu}$ at the poles, which are given by the dispersion relations. The polarization vectors satisfy

$$\sum_{\lambda=1}^{2}\epsilon_\mu(q\lambda)\epsilon_\nu(q\lambda)=-R_{\mu\nu} ,$$
$$\epsilon_\mu(q3)\epsilon_\nu(q3)=-Q_{\mu\nu} . \tag{2.16}$$

Starting from the definition

$$\langle 0 | T[A_\mu(x)A_\nu(y)] | 0\rangle=i\int\frac{d^4q}{(2\pi)^4}e^{-iq\cdot(x-y)}D_{\mu\nu}(q) \tag{2.17}$$

and following a standard analysis,[13] one gets the one-photon contribution as

$$D_{\mu\nu}(q)\,|_{1\text{ photon}}=\sum_{\lambda=1}^{3}N_{q\lambda}\epsilon_\mu(q\lambda)\epsilon_\nu(q\lambda)\frac{1}{q_0^2-\omega_\lambda^2+i\epsilon} , \tag{2.18}$$

where, for $\lambda=1,2$, $\omega_\lambda=\omega_T$, and for $\lambda=3$, $\omega_\lambda=\omega_L$ [see Eq. (2.11)]. The residue at $q_0=\omega_\lambda$ is then just $N_{q\lambda}/2\omega_\lambda$. On the other hand, we can start from Eq. (2.5), use Eqs. (2.6) and (2.7) to express the denominators in terms of ε and ε_t. This gives, for the denominators,

$$\frac{1}{-\mathbf{q}^2+q_0^2\epsilon_t}=\frac{1}{[q_0-\omega_T(\mathbf{q})]\left[\dfrac{\partial(q_0^2\epsilon_t)}{\partial q_0}\right]_{q_0=\omega_T(\mathbf{q})}}+\text{nonsingular terms} ,$$

$$\frac{1}{q^2\epsilon}=\frac{1}{[q_0-\omega_L(\mathbf{q})]\left[\dfrac{\partial(q^2\epsilon)}{\partial q_0}\right]_{q_0=\omega_L(\mathbf{q})}}+\text{nonsingular terms} . \tag{2.19}$$

Using Eq. (2.16) and equating the residues obtained in the two methods, we get

$$N_{q\lambda}=\frac{2\omega_T(\mathbf{q})}{\left[\dfrac{\partial(q_0^2\epsilon_t)}{\partial q_0}\right]_{q_0=\omega_T(\mathbf{q})}}=1 \quad \text{for } \lambda=1,2 ,$$
$$N_{q3}=\frac{2\omega_L(\mathbf{q})}{\left[\dfrac{\partial(q^2\epsilon)}{\partial q_0}\right]_{q_0=\omega_L(\vec{q})}}=\left[1-\frac{\mathbf{q}^2}{m_\gamma^2}\right]^{-1} . \tag{2.20}$$

The second equality in both cases is the result for a nonrelativistic gas, obtained by using Eqs. (2.9) and (2.11).

Therefore, the wave function for longitudinal photons has a multiplicative factor $[1-(\mathbf{q}^2/m_\gamma^2)]^{-1/2}$, which means that $|\mathbf{q}_L|\le m_\gamma$. For transverse photons there is no such restriction, so that $|\mathbf{q}_T|\sim T$, the temperature of the heat bath. This means that the contribution from the longitudinal photons to the axion production cross section is suppressed relative to that from the transverse photons by powers of m_γ/T. These powers turn out to be large due to the strong $|\mathbf{q}|$ dependence. Since the axions are produced mainly at the stellar core and for our cases of interest $T_{\text{core}}\gg m_\gamma$, we can neglect the contribution of the longitudinal photons altogether. We can therefore take the external photon polarization sum as

$$\sum_\gamma \epsilon_\mu(q\lambda)\epsilon_\nu(q\lambda)=-\left[g_{\mu\nu}+\frac{m_\gamma^2}{\mathbf{q}^2}u_\mu u_\nu\right] , \tag{2.21}$$

where the right-hand side is the same as that of the first equation in Eq. (2.16). We have merely expressed it in the rest frame of the heat bath and dropped the terms involving q_μ and q_ν since the coupled current is conserved.

III. THE IMAGINARY PART OF THE PHOTON PROPAGATOR

For a main-sequence star such as the Sun we may neglect axion bremsstrahlung, plasmon decay, and the e^+e^- annihilation processes.[4,5] The dominant process is photoproduction, represented in the diagrams of Figs. 1 and 2:

$$e(p)+\gamma(k)\rightarrow e(p')+a(k') , \tag{3.1}$$

a being the axion field. The interaction Lagrangian of the axion is

$$\mathcal{L}=iga\bar{e}\gamma_5+\tfrac{1}{2}\kappa\epsilon_{\mu\nu\lambda\rho}aF^{\mu\nu}F^{\lambda\rho} . \tag{3.2}$$

However, if we start calculating the amplitude of Fig. 2 by using this Lagrangian and the photon propagator of

Eq. (2.13), we immediately face a problem. This arises since the quantity $\mathcal{D}$, defined in Eq. (2.14), develops a singularity within the allowed kinematical region. Since this term owes its origin to the $Q_{\mu\nu}$ term of Eq. (2.5), the correct thing to do here is to include the imaginary part of the denominator of that term, i.e., the imaginary part of π_L, in the propagator. Physically, this means that we include the possibility that in Fig. 2, the internal line corresponds to a real, longitudinal photon. Instead of Eq. (3.1), the actual process is now described by

$$\gamma_T(k) \rightarrow a(k') + \gamma_L(q) \ , \tag{3.3}$$

followed by

$$e(p) + \gamma_L(q) \rightarrow e(p') \ , \tag{3.4}$$

where all the particles are on their mass shells. The reason that (3.4) can go through with a longitudinal photon and not with a real transverse one is to be found in the unusual dispersion relation for the longitudinal photons.

In order to evaluate the imaginary part of π_L, we first note, from Eq. (2.6), that

$$\mathrm{Im}\pi_L(q_0,\mathbf{q}) = -q^2 \mathrm{Im}\epsilon \equiv -q^2\epsilon' \ . \tag{3.5}$$

The imaginary part of the dielectric constant is responsible for the reversible energy loss in a collisionless plasma, a phenomenon known as Landau damping. It is calculated in standard textbooks[14] and is given by

$$\epsilon' = -\frac{\pi e^2 m_e}{|\mathbf{q}|^2} \left[\frac{\partial}{\partial p_\parallel} \int d^2 p_\perp f(\mathbf{r},\mathbf{p}) \right] \Bigg|_{p_\parallel = m_e q_0 / |\mathbf{q}|} \ , \tag{3.6}$$

where $f(\mathbf{r},\mathbf{p})$ is the distribution function of the electrons in phase space, $p_\parallel$ is the component of the electron momentum $\mathbf{p}$ parallel to the direction of $\mathbf{q}$, and the integral is over the components orthogonal to $p_\parallel$. Since $T \ll m_e$, the distribution function f is given by the Maxwellian form

$$f(\mathbf{r},\mathbf{p}) = n_e(\mathbf{r})(2\pi m_e T)^{-3/2} \exp(-\mathbf{p}^2/2m_e T) \ . \tag{3.7}$$

Using this, we obtain

$$\epsilon' = \left[\frac{\pi}{2} \right]^{1/2} \frac{q_0 m_\gamma^2}{|\mathbf{q}|^3} \left[\frac{m_e}{T} \right]^{3/2} \exp\left[-\frac{q_0^2 m_e}{2|\mathbf{q}|^2 T} \right] . \tag{3.8}$$

Following the same steps as in Sec. II, we can now write the photon propagator in the form of Eq. (2.13), where now we should replace Eq. (2.14) by

$$\mathcal{D} = \frac{m_\gamma^2 + i\dfrac{q^2 q_0^2}{|\mathbf{q}|^2}\epsilon'}{q_0^2 - m_\gamma^2 + i q_0^2 \epsilon'} \ . \tag{3.9}$$

We will use this modified propagator in the next section to calculate the cross section.

IV. PHOTOPRODUCTION CROSS SECTION OF PSEUDOSCALARS WITHIN A MEDIUM

We calculate the cross section in the rest frame of the heat bath, in which $u^\mu = (1,\mathbf{0})$. In that frame p^μ is not necessarily $(m_e,\mathbf{0})$, but is true to a good approximation and we use it. The usual Mandelstam variables for the process are

$$\begin{aligned} s &= (p+k)^2 = m_e^2 + 2m_e\omega + m_\gamma^2 \ , \\ t &= (k-k')^2 = q^2 \ , \\ u &= (p-k')^2 = m_e^2 - 2m_e\omega' + m_a^2 \ , \end{aligned} \tag{4.1}$$

where ω and ω' stand for the photon and the axion energies, respectively. The amplitude is written as

$$M = M_1 + M_2 \ , \tag{4.2}$$

where the terms on the right-hand side are the contributions from Figs. 1 and 2, respectively. Thus, using the Dirac equations for the spinors $u(p)$ and $u(p')$, we can write

$$M_1 = -\epsilon^\mu \bar{u}(p')(A\not{k}'\gamma_\mu - B\gamma_\mu \not{k}')\gamma_5 u(p) \tag{4.3}$$

and

$$M_2 = -iC\epsilon^\mu[\bar{u}(p')\gamma_\alpha u(p)] \times (g^{\alpha\nu} + u^\alpha u^\nu \mathcal{D})\epsilon_{\mu\nu\lambda\rho} k^\lambda k'^\rho \ , \tag{4.4}$$

where $\mathcal{D}$ has been defined in Eq. (3.9) and we have introduced the shorthand notation

$$A = \frac{eg}{s - m_e^2}, \quad B = \frac{eg}{u - m_e^2}, \quad C = \frac{4\kappa e}{t - m_\gamma^2} \ . \tag{4.5}$$

The cross section will involve $\langle |M|^2 \rangle$, where the angular brackets denote an averaging over the initial spin polarizations and summing over the final ones. Carrying out this separation, we obtain, for the pure Compton process,

$$\begin{aligned} \langle |M_1|^2 \rangle = {} & (A^2+B^2)[(s-m_e^2)(m_e^2-u) + 2m_e^2 m_a^2 + m_a^2 m_\gamma^2] + 2AB[(s-m_e^2)(m_e^2-u) + m_a^2(s+u) - m_a^4] \\ & - \frac{m_\gamma^2}{\mathbf{k}^2} \left[\frac{1}{2}(A^2+B^2)[(s-m_e^2)(m_e^2-u) - 4m_e^2 m_a^2 + m_a^2 m_\gamma^2] \right. \\ & \left. - B^2(m_a^2+m_e^2-u) \left[\frac{1}{2m_e^2}(t-2m_e^2)(m_a^2+m_e^2-u) + (s-m_e^2-m_a^2) \right] \right. \end{aligned}$$

$$+AB[m_a^2(t-2m_e^2)+(m_a^2+m_e^2-u)^2-2m_a^2m_e^2]\Bigg] . \tag{4.6}$$

The interference term is given by

$$\langle M_1^* M_2 + \text{c.c.}\rangle = -2m_e C(A+B)\Bigg[2m_a^2m_\gamma^2-\tfrac{1}{2}(m_\gamma^2+m_a^2-t)^2 + |\mathbf{k}\times\mathbf{k}'|^2\left\{\text{Re}\mathcal{D}+\frac{4m_e^2m_\gamma^2}{(s-m_e^2-m_\gamma^2)^2-4m_e^2m_\gamma^2}\right\}\Bigg] , \tag{4.7}$$

and finally the Primakoff term is

$$\langle |M_2|^2\rangle = C^2\Bigg[t[m_\gamma^2m_a^2-\tfrac{1}{4}(m_\gamma^2+m_a^2-t)^2] +2m_e^2|\mathbf{k}\times\mathbf{k}'|^2\left\{1+2\,\text{Re}\mathcal{D}+|\mathcal{D}|^2+\frac{tm_\gamma^2}{(s-m_e^2-m_\gamma^2)^2-4m_e^2m_\gamma^2}\right\}\Bigg] . \tag{4.8}$$

Since

$$\mathbf{k}^2=\frac{(s-m_e^2-m_\gamma^2)^2}{4m_e^2}-m_\gamma^2 , \tag{4.9}$$

$$|\mathbf{k}\times\mathbf{k}'|^2=m_a^2m_\gamma^2-\tfrac{1}{4}(t-m_a^2-m_\gamma^2)^2-\frac{t}{4m_e^2}[(s-m_e^2)(m_e^2-u)+m_\gamma^2m_a^2] , \tag{4.10}$$

and in the definition of $\mathcal{D}$, we can substitute $q_0=-t/2m_e$ and $|\mathbf{q}|^2=(t^2/4m_e^2)-t$, Eqs. (4.6)–(4.8) give $\langle |M|^2\rangle$ in terms of the Mandelstam variables.

For the sake of convenience, we now define the dimensionless variable x as follows:

$$m_a^2+m_\gamma^2-t=x(s-m_e^2) , \quad m_e^2-u=(1-x)(s-m_e^2) . \tag{4.11}$$

The differential cross section is then given by

$$\frac{d\sigma}{dx}=\frac{s-m_e^2}{16\pi I^2}\langle |M|^2\rangle , \tag{4.12}$$

where

$$I^2=[s-(m_e+m_\gamma)^2][s-(m_e-m_\gamma)^2] . \tag{4.13}$$

This gives the differential cross section for the most general case where both the axion mass and the photon mass are nonzero.[15]

To obtain the total cross section σ, one needs to integrate Eq. (4.12) over x. The limits of this integration, found from kinematics, are given by

$$x_1\le x\le x_2 \tag{4.14}$$

with

$$x_{1,2}=\frac{1}{2s(s-m_e^2)}\{(s-m_e^2+m_\gamma^2)(s-m_e^2-m_a^2)\mp[(s-m_e^2+m_\gamma^2)^2-4sm_\gamma^2]^{1/2}[(s-m_e^2+m_a^2)^2-4sm_a^2]^{1/2}\} . \tag{4.15}$$

So far, we did not make any assumption about the mass of the axion. For the rest of the calculation, we assume $m_a \ll m_e, m_\gamma$. Notice that the couplings g and κ, and hence the quantities A, B, and C defined in Eq. (4.5), depend implicitly on the scale of the Peccei-Quinn symmetry breaking and therefore on the axion mass. Thus, to obtain the leading contributions in the present limit, we can set $m_a=0$ after factoring out the coupling constants. We thus obtain for the pure Compton term

$$\langle |M_1|^2\rangle=(eg)^2\left[\frac{x^2}{1-x}-\frac{4m_e^2m_\gamma^2}{(s-m_e^2-m_\gamma^2)^2-4m_e^2m_\gamma^2}\left\{-\frac{x^2}{2(1-x)}+\frac{x(s-m_e^2)-m_\gamma^2}{2m_e^2}\right\}\right] , \tag{4.16}$$

whereas the interference term is

$$\langle M_1^*M_2+\text{c.c.}\rangle=8(eg)^2\chi\frac{1}{1-x}\left[\tfrac{1}{2}x^2-\frac{|\mathbf{k}\times\mathbf{k}'|^2}{(s-m_e^2)^2}\left\{\text{Re}\mathcal{D}+\frac{4m_e^2m_\gamma^2}{(s-m_e^2-m_\gamma^2)^2-4m_e^2m_\gamma^2}\right\}\right] \tag{4.17}$$

and the Primakoff term

$$\langle |M_2|^2\rangle = 32(eg)^2\chi^2\left[-\frac{t}{8m_e^2}+\frac{|\mathbf{k}\times\mathbf{k}'|^2}{(t-m_\gamma^2)^2}\left[1+2\,\mathrm{Re}\mathcal{D}+|\mathcal{D}|^2+\frac{2tm_\gamma^2}{(s-m_e^2-m_\gamma^2)^2-4m_e^2m_\gamma^2}\right]\right], \quad (4.18)$$

where we have introduced the dimensionless ratio

$$\chi=\frac{\kappa m_e}{g} . \quad (4.19)$$

In the next section, we apply these formulas to calculate the energy loss of the Sun owing to axion photoproduction.

V. SOLAR ENERGY LEAKAGE DUE TO PSEUDOSCALAR EMISSION

Assuming spherical symmetry, the solar energy leakage due to axion emission is given by

$$L=4\pi\int_0^{R_\odot} dr\, r^2 n_e(r) \times\int_{m_\gamma}^{\infty} d\omega\frac{dn_\gamma}{d\omega}\left[1-\frac{m_\gamma^2}{\omega^2}\right]^{1/2}\int_{x_1}^{x_2}dx\frac{d\sigma}{dx}\omega' , \quad (5.1)$$

where the differential cross section $d\sigma/dx$ has been expressed as a function of x and ω in Sec. IV. The axion energy is given by

$$\omega'=(1-x)\left[\omega+\frac{m_\gamma^2}{2m_e}\right] \quad (5.2)$$

using Eq. (4.1). The expression under the square root in Eq. (5.1) is the velocity of the photon, which need not be unity in the solar medium. The photon spectrum, $dn_\gamma/d\omega$, is given by the Planck distribution formula

$$\frac{dn_\gamma}{d\omega}=\frac{\omega^2}{\pi^2}\frac{1}{e^{\omega/T}-1} . \quad (5.3)$$

To obtain the electron density distribution function, we just assume that the Sun is composed solely of hydrogen and helium. Then

$$n_e(r)=n_{\mathrm{proton}}(r)=\frac{\rho(r)}{2m_p}[1+X_H(r)] , \quad (5.4)$$

where the hydrogen abundance function $X_H(r)$ and the density profile function $\rho(r)$ are found in standard tables.[16]

We can now carry out the integration in Eq. (5.1). However, before doing so, there is one point worth noting. There are protons in the Sun, and the axions can also be photoproduced off them. The amplitude for the Compton-type term involving the protons will be suppressed by the proton propagator. The Primakoff-type term, on the other hand, involves the photon propagator. Its contribution therefore is almost the same no matter whether the fermion line in Fig. 2 is an electron for a proton. Therefore, after performing the numerical integration in Eq. (5.1) using the formulas in Sec. IV, we just double the pure Primakoff term to take account of the protons. Demanding that the resulting energy loss L be less than the observed solar luminosity, we obtain

$$g^2(1.3\times10^{20}+6.9\times10^{20}\chi+1.4\times10^{25}\chi^2)\le 1 . \quad (5.5)$$

This puts conservative bounds on the parameters g and χ and therefore on the axion models, which we discuss in the next section.

VI. BOUNDS ON AXION PARAMETERS

In a general axion model, the parameters g and χ are independent and only the combination given in Eq. (5.5) is bounded. In the literature, often some extra assumption is introduced[4,5] before obtaining the upper limits for each of them. While the resulting bounds are true in specific axion models, it is conceivable that one can arrange models which avoid the bounds. Therefore, we try to keep our discussion as model independent as possible.

Hermiticity of the axion-electron and the axion-photon interaction terms in the Lagrangian demand reality of both g and χ. From Eq. (5.5), which involves an expression quadratic in χ, it is easy to deduce the condition for the reality of χ to be[17]

$$g^2\le 7.7\times10^{-21} . \quad (6.1)$$

Similarly, we can rewrite Eq. (5.5) in terms of κm_e and χ only, eliminating g using Eq. (4.19). Demanding the reality of χ from that expression, we obtain

$$(\kappa m_e)^2\le 7.1\times10^{-26} . \quad (6.2)$$

However, by adjusting the sign of the real field a in Eq. (3.2), we can always choose g to be positive. The sign of κ (and therefore of χ) is then undetermined. In Fig. 3 we show which region of the range of the coupling parameters is ruled out by the constraint in Eq. (5.5). If one wants to find the bounds on the scale of symmetry breaking, one will have to invoke the details of a model connecting the couplings with the scale.

For example, if we consider the emission of a Majoron[7,8] from the Sun, we put $\kappa=0$ since the Majoron arises out of the spontaneous breaking of a nonanomalous symmetry. In this case, the only bound is the vertical asymptote of Fig. 3 which almost coincides with the bound in Eq. (6.1). If, in addition, we assume that g is given by the naive estimate m_e/v this implies that $v\ge 5.8\times10^6$ GeV for the scale v of the symmetry breaking that produces the Majoron.

While this bound holds for the simplest version of the singlet Majoron model,[7] it is easy to construct models which violate this naive estimate. For example, in the so-called triplet Majoron model,[8] the global symmetry breaks at a very low scale v but the Majoron coupling to the electron is given by $g=m_e v/(250\text{ GeV})^2$. The bound on g thus implies an upper bound $v<10$ MeV. For a Majoron model in the context of left-right-symmetric models of electroweak interaction,[18] there are also extra suppressions to the naive coupling so that the scale v can be much lower than the estimate given above.

We now consider the other extreme of the hadronic axion models.[19] In these models, $g \to 0$ but κ is finite. The relevant bound is now given by the horizontal asymptote in Fig. 3. With the bound in Eq. (5.5), this coincides with the bound given in Eq. (6.2). Once again, if we assume the naive estimate of $\kappa \simeq e^2/16\pi^2 v$ and $m_a v = f_\pi m_\pi$, we obtain $v \gtrsim 1.1\times 10^6$ GeV for the scale of the Peccei-Quinn symmetry breaking and $m_a \lesssim 12$ eV for the axion mass.

In the Dine-Fischler-Srednicki model,[3] there are two different $SU(2)_L$ doublets that break the Weinberg-Salam symmetry. One of them develops a vacuum expectation value (VEV) v_u and gives mass to the up-type quarks. The other, with VEV v_d, is responsible for the masses of the charged leptons and the down-type quarks. Denoting the ratio v_u/v_d by y, one gets $g=[2y^2/(y^2+1)](m_e/v)$ and $\kappa=(e^2N/16\pi^2 v)[z/(1+z)]$, where $z=m_u\langle \bar{u}u\rangle / m_d\langle \bar{d}d\rangle$ and is usually estimated to be about 0.56. N is the number of light flavor of quarks, which is taken to be 3. On top of that, the bounds by the previous authors assume that $y=1$. This extra assumption relates g and κ and thus one can constrain the parameter v. In a more general situation, the bounds can be read from the graph in Fig. 3, as we emphasize before.

In conclusion, we have shown in great detail how the plasma effects in the stellar core affects photon propagation. Using this, we calculated the rate of solar energy loss due to photoproduction of light pseudoscalars. This yields bounds on pseudoscalar couplings as shown in Fig. 3. In specific Majoron or axion models, this produces more specific bounds, which we have also discussed.

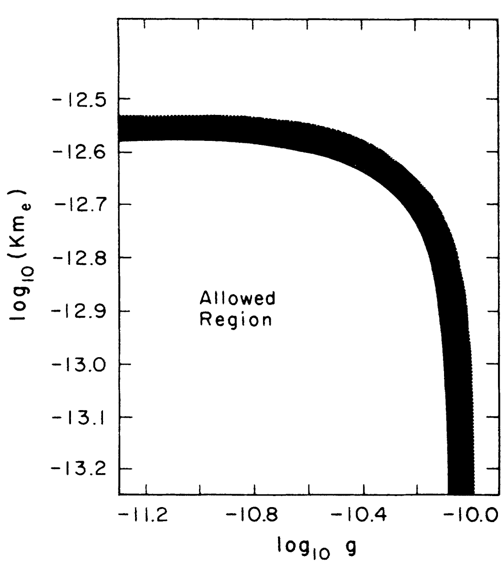

FIG. 3. The bounds on g vs κ obtained from Eq. (5.5). The bounds are a little bit different depending on the sign of κ, but that difference is unappreciable on the scale of this graph.

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APPENDIX

In this appendix we present an approximate analytic calculation of the axion luminosity. The starting point is Eq. (5.1). The formula for the cross section given by Eqs. (4.16)–(4.18) already contains the approximation $m_a \to 0$. We now make two additional ones: we calculate the x integration of Eq. (5.1) to lowest order in ω/m_e and set $m_\gamma = 0$ except in the logarithmically divergent Primakoff term.

With these approximations, the limits of x integration reduce to

$$x_1 \simeq m_\gamma^2/2m_e\omega, \quad x_2 \simeq 2\omega/m_e \ . \tag{A1}$$

Since $T \ll m_e$ everywhere in the Sun and the part of the photon spectrum that contributes most to the cross section is $\omega \sim T$, we see that $x \ll 1$ for all parts of the integration range. Thus, in Eq. (5.2), we can put $\omega' \simeq \omega$ to leading order. This gives

$$\int_{x_1}^{x_2} dx \frac{d\sigma}{dx}\omega' = \omega\sigma \ , \tag{A2}$$

where

$$\sigma = \frac{(eg)^2}{16\pi}\left[(1+4\chi)\frac{4\omega^2}{3m_e^4} + \frac{8\chi^2}{m_e^2}\left[\ln\frac{4\omega^2}{m_\gamma^2} - 1\right]\right] \ . \tag{A3}$$

Substituting Eq. (A2) into Eq. (5.1), we can write

$$L = 4\pi\int_0^{R_\odot} dr\, r^2 n_e(r) F(r) \ , \tag{A4}$$

where

$$\begin{aligned} F(r) &= \int_0^\infty d\omega \frac{dn_\gamma}{d\omega}\omega\sigma \\ &= \frac{(eg)^2}{16\pi^3}\left[(1+4\chi)\frac{4T^6}{3m_e^4}\int_0^\infty dy \frac{y^5}{e^y-1}\right. \\ &\quad + \frac{8\chi^2 T^4}{m_e^2}\int_0^\infty dy \frac{y^3}{e^y-1}(\ln y^2 - 1) \\ &\quad \left. + \frac{8\chi^2 T^4}{m_e^2}\left[\ln\frac{4T^2}{m_\gamma^2}\right]\int_0^\infty dy \frac{y^3}{e^y-1}\right] \ . \end{aligned} \tag{A5}$$

It is customary to define $\epsilon(r)$ by writing

$$L=4\pi\int_0^{R_\odot} dr\, r^2\epsilon(r)\rho(r)\ . \tag{A6}$$

Using Eq. (5.4) we thus obtain

$$\epsilon(r)=\frac{1+X_H(r)}{2m_p}F(r)\ . \tag{A7}$$

The integrals in Eq. (A5) can be done easily. The one involving the logarithm can be performed even with the help of a pocket calculator. The others are given in terms of the ζ functions and can be read from the tables. We get

$$\epsilon(r)=\tfrac{1}{2}(1+X_H)\left[a\hat{T}^6+b\hat{T}^4+c\hat{T}^4\ln\frac{\hat{T}^2}{(1+X_H)\hat{\rho}}\right], \tag{A8}$$

where $\hat{T}=T/10^6$ K, $\hat{\rho}$ is the density in units of g/cm^3, and the coefficients a, b, c are given by

$$\begin{aligned} a&=g^2(1+4\mathcal{X})(1.89\times10^{-28}\ \text{MeV})\ ,\\ b&=g^2\mathcal{X}^2(2.36\times10^{-20}\ \text{MeV})\ ,\\ c&=g^2\mathcal{X}^2(4.24\times10^{-21}\ \text{MeV})\ . \end{aligned} \tag{A9}$$

In Eq. (A9), an additional factor of 2 has been included in the terms b and c to take into account of the photoproduction of axions off protons.

Equation (A8) can be used to put constraints on g and $\mathcal{X}$, although the results are more model dependent than those based on the overall energetics. However, as an example, consider the function $\epsilon(r)$ at $r=0$. Using the tables[16] to find the values of $\hat{T}$, $\hat{\rho}$, and X_H at $r=0$, we obtain from Eq. (A8) the inequality

$$g^2(1+4\mathcal{X})(1.51\times10^{20})+g^2\mathcal{X}^2(7.92\times10^{25})\le 1\ . \tag{A10}$$

The departure of this formula from Eq. (5.5) is a measure of the importance of the plasma effects in the Sun. As expected, it is most prominent in the Primakoff term.

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[1]R. D. Peccei and H. Quinn, Phys. Rev. Lett. **38**, 1440 (1977).

[2]S. Weinberg, Phys. Rev. Lett. **40**, 223 (1978); F. Wilczek, *ibid.* **40**, 279 (1978).

[3]M. Dine, W. Fischler, and M. Srednicki, Phys. Lett. **104B**, 199 (1981).

[4]M. Fukugita, S. Watamura, and M. Yoshimura, Phys. Rev. D **26**, 1840 (1983).

[5]A. Pantziris and K. Kang, Phys. Rev. D 33, 3509 (1986).

[6]J. B. Adams, M. H. Ruderman, and C.-H. Woo, Phys. Rev. **129**, 1383 (1967).

[7]Y. Chikashige, R. N. Mohapatra, and R. D. Peccei, Phys. Lett. **98B**, 265 (1981).

[8]G. Gelmini and M. Roncadelli, Phys. Lett. **99B**, 411 (1981).

[9]H. A. Weldon, Phys. Rev. D **26**, 1394 (1982).

[10]V. N. Tsytovich, Zh. Eksp. Teor. Fiz. **40**, 1775 (1961) [Sov. Phys. JETP **13**, 1249 (1961)]; G. Beaudet, V. Petrosian, and E. E. Salpeter, Astrophys. J. **150**, 979 (1967); M. H. Zaidi, Nuovo Cimento **40A**, 502 (1965); J. F. Nieves, P. B. Pal, and D. G. Unger, Phys. Rev. D **28**, 908 (1983).

[11]See, e.g., N. W. Ashcroft and N. D. Mermin, *Solid State Physics* (Holt, Reinhart and Winston, Philadelphia, 1976), pp. 16–18. Their result differs from ours by a factor of 4π. This is due to the difference of the units used.

[12]See, e.g., Fig. 22.14 and related discussions in Ashcroft and Mermin, *Solid State Physics* (Ref. 11).

[13]See, e.g., J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields* (McGraw-Hill, New York, 1965).

[14]See, e.g., Eq. (30.2) of E. M. Lifshitz and L. P. Pitaevskii, *Physical Kinetics* (Landau and Lifshitz course of Theoretical Physics) (Pergamon, New York, 1981), Vol. 10. Their expression differs from ours by a factor of 4π because of the difference of the units used.

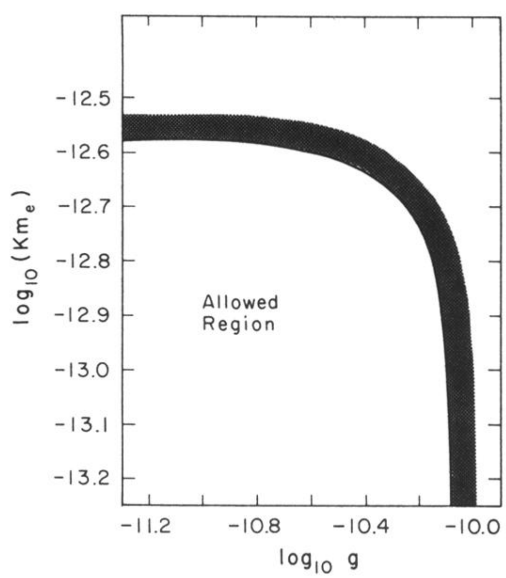
[15]In the limit $m_\gamma\to0$, our definition of x coincides with that of S. J. Brodsky, E. Mottola, I. J. Muzinich, and M. Soldate, Phys. Rev. Lett. **56**, 1763 (1986). Our final results for the interference and the Primakoff terms also agree in that limit if we identify their γ with $2\kappa m_e/g$ in our notation. In the pure Compton term, their terms proportional to the axion mass have a different sign than ours, but we believe that this is a typographical error.

[16]D. Clayton, *Principles of Stellar Evolution and Nucleosynthesis,* 2nd ed. (University of Chicago Press, Chicago, 1983). We use the model of the Sun given in Table 6-6, p. 483.

[17]Equation (5.5) can be interpreted as the statement that the left-hand side of that equation is equal to some λ which is ≤1. Now we get an equation involving λ. The condition that the roots of $\mathcal{X}$ are real is $g^2\le7.7\lambda\times10^{-21}$. Equation (6.1) merely strengthens this inequality.

[18]A. Kumar and R. N. Mohapatra, Phys. Lett. **150B**, 191 (1985); R. N. Mohapatra and P. B. Pal, University of Massachusetts report, 1987 (unpublised).

[19]D. B. Kaplan, Nucl. Phys. **B260**, 215 (1985).



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