Comparison of the vector- and axial-vector-meson exchange contributions to the decay rate of the short-lived neutral kaon into two photons

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(Received 20 November 1987)

The contributions of the charged axial-vector mesons $b_1(1235)$ and $a_1(1270)$ as exchange particles in the decay mode $K_S \to \gamma\gamma$ are compared with those of the vector mesons $\rho(770)$, $\omega(783)$, $\phi(1020)$, and $\psi(3097)$. The range of the rates of decay obtained for several choices of the cutoff mass falls within the limits of the value $\Gamma(K_S \to 2\gamma)/\Gamma(K_S \to \text{all}) = (2.4 \pm 1.2) \times 10^{-6}$ obtained recently by the NA31 experimental group at the CERN SPS.

The purposes of this Brief Report are to evaluate the contributions of the charged axial-vector mesons $b_1(1235)$ and $a_1(1270)$ to the decay rate of the short-lived neutral kaon into two photons and to compare them with the updated contributions of the vector mesons $\rho(770)$, $\omega(783)$, $\phi(1020)$, and $\psi(3097)$ (Ref. 1).

The decay rate of $K_S \to \gamma\gamma$ has been measured at the CERN SPS by the CERN Dortmund-Edinburgh-Mainz-Orsay-Pisa-Siegen Collaboration.[2] The result of this first observation of the decay mode $K_S \to \gamma\gamma$ is that

$$\frac{\Gamma(K_S \to \gamma\gamma)}{\Gamma(K_S \to \text{all})} = (2.4 \pm 1.2) \times 10^{-6} . \tag{1}$$

Using the above result and the current data on the K_S mean life,[3] which is 0.892×10^{-10} sec, one can infer that

$$\Gamma(K_S \to \gamma\gamma) = (2.69 \pm 1.34) \times 10^4 \ \text{sec}^{-1} \tag{2}$$

with an upper limit of $4.03 \times 10^4 \ \text{sec}^{-1}$ and a lower limit of $1.35 \times 10^4 \ \text{sec}^{-1}$.

The above result is in agreement with previous theoretical calculations made on the assumption that the decay $K_S \to \gamma\gamma$ is dominated by the 2π intermediate state.[1,4] In particular, in the model where the exchange particle is the pion, the computed decay rate[1] is $2.26 \times 10^4 \ \text{sec}^{-1}$. The result of the present computations shows that, with reasonable choices of the cutoff masses, the inclusion of the vector mesons and axial-vector mesons to the pion as exchange particles gives a range of decay rates that fall between $4.03 \times 10^4 \ \text{sec}^{-1}$ and $1.35 \times 10^4 \ \text{sec}^{-1}$.

A brief outline of the method used in deriving the result follows. *CP* conservation is assumed in this paper; hence, the short-lived neutral kaon will be indicated by K_1.

In a previous paper, the formalism for the pion and vector-meson exchange contributions had been written down.[1] In this paper, we add the effects of the axial-vector-meson contributions. Previous results that are relevant for comparisons will be reproduced for the sake of clarity.

The Feynman diagrams for the decay mode $K_1 \to \gamma\gamma$ are in Fig. 1. The figure shows the pion π, axial-vector meson A, and vector meson V, which are the exchange particles considered in this paper.

The Lorentz-invariant matrix element M which satisfies gauge invariance and Bose statistics can be written as

$$M = H(s)[(\epsilon_1 \cdot \epsilon_2)(k_1 \cdot k_2) - (\epsilon_1 \cdot k_2)(\epsilon_2 \cdot k_1)] , \tag{3}$$

where ϵ_i and k_i refer to the polarization and momentum of the ith photon. $H(s)$, in the rest frame of the K_1, is a function of the square of the kaon mass, $s = M_K^2$. In terms of H, the decay rate of $K_1 \to \gamma\gamma$ is

$$\Gamma(K_1 \to \gamma\gamma) = \frac{M_K^3}{64} | H |^2 .$$

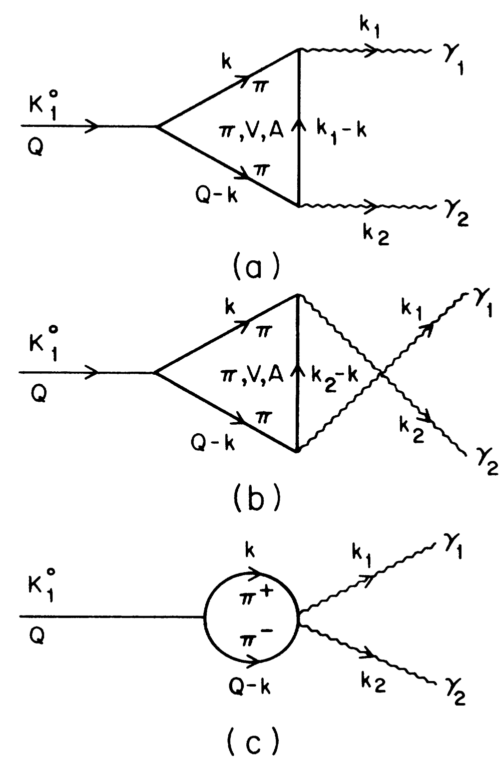

FIG. 1. The Feynman diagram for $K_1 \to \gamma\gamma$ with pion π, vector meson V, and axial-vector meson A as exchanged particles.

We will indicate by H_π, H_V, and H_A the contributions to H of the pion, vector meson, and axial-vector meson, respectively. The following interaction Lagrangians were used to determine the vertices of Fig. 1:

$$L_{K\pi\pi}=\lambda\psi_K\phi_\pi^\dagger\phi_\pi \,, \quad (4)$$

$$L_\pi=-ie\mathcal{A}_\mu(\phi_\pi^\dagger\overleftrightarrow{\partial}^\mu\phi_\pi)+e^2\mathcal{A}_\mu\mathcal{A}^\mu\phi_\pi^\dagger\phi_\pi \,, \quad (5)$$

$$L_V=\eta_V V^\mu\partial^\nu\phi_\pi^\dagger F^{\alpha\beta}\epsilon_{\mu\nu\alpha\beta}+\text{H.c.} \,, \quad (6)$$

$$L_A=\eta_A A^\mu\partial^\nu\phi_\pi F_{\mu\nu}+\text{H.c.} \,, \quad (7)$$

where $L_{K\pi\pi}$, L_π, L_V, and L_A are the $K\pi\pi$, $\pi\pi\gamma$, $V\pi\gamma$, and $A\pi\gamma$ vertices, respectively, while ψ_K, ϕ_π, $\mathcal{A}_\mu$, V_μ, and A_μ are the kaon, pion, photon, vector-meson, and axial-vector-meson fields, respectively. $F_{\mu\nu}$ is the electromagnetic field tensor $\partial_\mu\mathcal{A}_\nu-\partial_\nu\mathcal{A}_\mu$.

The $K\pi\pi$ coupling constant λ is determined from the known decay rate of K_S to two pions.[3] The coupling constants η_V and η_A are determined from the radiative decay rates $V\to\pi\gamma$ and $A\to\pi\gamma$ and the expressions for the decay rates:

$$\Gamma(V\to\pi\gamma)=\frac{1}{24\pi}\,|\eta_V|^2M_V^3\left[1-\left(\frac{\mu}{M_V}\right)^2\right]^3 , \quad (8)$$

$$\Gamma(A\to\pi\gamma)=\frac{1}{96\pi}\,|\eta_A|^2M_A^3\left[1-\left(\frac{\mu}{M_A}\right)^2\right]^3 , \quad (9)$$

where μ, M_V, and M_A are the masses of the pion, vector meson, and axial-vector meson, respectively. The known radiative rates used are[3,5–10]

$$\begin{aligned}
&\Gamma(\rho^\pm\to\pi^\pm\gamma)=70.4\text{ keV} \,,\\
&\Gamma(\omega\to\pi^0\gamma)=0.85\text{ MeV} \,,\\
&\Gamma(\phi\to\pi^0\gamma)=5.53\text{ keV} \,,\\
&\Gamma(\psi\to\pi^0\gamma)=2.52\text{ eV} \,,\\
&\Gamma(b_1^+\to\pi^+\gamma)=230\pm60\text{ keV} \,,\\
&\Gamma(a_1^+\to\pi^+\gamma)=640\pm246\text{ keV} \,.
\end{aligned} \quad (10)$$

For the $\rho^\pm\to\pi^\pm\gamma$, we have used the average of the two recent measurements by Huston *et al.*[6] and Capraro *et al.*[7] Both the radiative decay widths of a_1^+ and b_1^+ were extracted using the Primakoff technique.

Writing the invariant matrix M for each of the exchange particles is straightforward.[11] Using unitarity and dispersion-relations technique,[1,4] one gets for the imaginary and real parts of the amplitudes H_π, H_V, and H_A the following:

$$\text{Im}H_\pi=-\frac{2\lambda}{\pi}\alpha\left[\frac{\mu}{s}\right]^2\pi\ln\left[\frac{1+\beta}{1-\beta}\right] , \quad (11)$$

$$\text{Re}H_\pi=-\frac{2\lambda}{\pi}\alpha\frac{1}{2s}\left\{\frac{\mu^2}{s}\left[\pi^2-\ln^2\left[\frac{1+\beta}{1-\beta}\right]\right]-1\right\} , \quad (12)$$

$$\text{Im}H_V=-\lambda\left[\frac{\eta_V^2}{4\pi}\right]\left[\frac{M_V^2}{s}Z_V-\beta\right] , \quad (13)$$

$$\text{Re}H_V=-\lambda\left[\frac{\eta_V^2}{4\pi}\right]\frac{1}{\pi}\left[\frac{M_V^2}{s}\mathcal{P}(K_V)-\mathcal{P}(F_V)\right] , \quad (14)$$

$$\text{Im}H_A=-\frac{\lambda}{4}\left[\frac{\eta_A^2}{4\pi}\right]\left[\frac{M_A^2}{s}Z_A-\beta\right] , \quad (15)$$

$$\text{Re}H_A=-\frac{\lambda}{4}\left[\frac{\eta_A^2}{4\pi}\right]\frac{1}{\pi}\left[\frac{M_A^2}{s}\mathcal{P}(K_A)-\mathcal{P}(F_A)\right] , \quad (16)$$

$$\begin{aligned}
\mathcal{P}(K_V)&=\mathcal{P}\int_{4\mu^2}^{\infty}ds'\frac{1}{(s'-s)}\frac{\pi}{s'}Z_V(s')\\
&=\frac{\pi}{s}\left[\frac{1}{6}\pi^2-\frac{1}{2}[\ln(V)]^2-\frac{1}{2}[\ln(w_1)]^2\right.\\
&\quad-\frac{1}{2}[\ln(w_2)]^2-\text{Li}\left[\frac{1}{w_1}\right]-\text{Li}\left[\frac{1}{w_2}\right]\\
&\quad\left.-\text{Li}(-w_3)-\text{Li}(-w_4)\right] ,
\end{aligned} \quad (17)$$

$$\begin{aligned}
\mathcal{P}(F_V)&=\mathcal{P}\int_{4\mu^2}^{4\Lambda_V^2}ds'\frac{1}{(s'-s)}\beta(s')\\
&=\ln\left[\frac{1+L_V}{1-L_V}\right]-\beta\ln(V) ,
\end{aligned} \quad (18)$$

$$a=\frac{e^2}{4\pi},\quad \beta=\left[1-\frac{4\mu^2}{s}\right]^{1/2},\quad V=\frac{1+\beta}{1-\beta} , \quad (19)$$

$$\sigma_V=\frac{M_V^2-\mu^2}{2},\quad \delta_V=1+\frac{\mu^2}{\sigma_V},\quad b_V=\frac{s\beta}{s+4\sigma_V} , \quad (20)$$

$$w_1=\frac{\delta_V-\beta}{1-\beta},\quad w_2=\frac{\delta_V+\beta}{1+\beta} , \quad (21)$$

$$w_3=\frac{\delta_V+\beta}{1-\beta},\quad w_4=\frac{\delta_V-\beta}{1+\beta} , \quad (22)$$

$$Z_V=\ln\left[\frac{1+b_V}{1-b_V}\right],\quad L_V=1-\frac{4\mu^2}{4\Lambda_V^2} , \quad (23)$$

where $\mathcal{P}(K_A)$ and $\mathcal{P}(F_A)$ are identical to $\mathcal{P}(K_V)$ and $\mathcal{P}(F_V)$, respectively, with the mass of the vector meson replaced by the axial-vector-meson mass. $\mathcal{P}$ indicates the principal value; Li is the dilogarithm function;[12] Λ_V is the cutoff mass.

In Eq. (23), L is the dimensionless cutoff parameter corresponding to the cutoff mass Λ. We have computed the decay rates for three values of L: μ/M_x, $2\mu/M_x$, and M_K/M_x. M_x is the mass of the exchanged particle, and μ is the pion mass and M_K the kaon mass. These choices were made based on the conjecture that the value of L can depend on the ratio of the masses involved in the exchange process.

This author has previously obtained the values[1]

$$\text{Im}H_\pi=-0.217(\hbar c)^2\ \text{cm}^{-1}\text{MeV}^{-2} , \quad (24)$$

$$\text{Re}H_\pi=+0.213(\hbar c)^2\ \text{cm}^{-1}\text{MeV}^{-2} , \quad (25)$$

$$\Gamma_\pi(K_1\to\gamma\gamma)=2.26\times10^4\ \text{sec}^{-1} \quad (26)$$

TABLE I. The values of the decay rate of $K_1 \to \gamma\gamma$ for the indicated exchanged particles and for $L = \mu/M_x$, $2\mu/M_x$, and M_K/M_x, where μ, M_K, and M_x are the masses of the pion, kaon, and the exchanged particle. L is the cutoff parameter.

Exchanged particles	ImH	ReH_1 ($\hbar^2c^2$ cm^{-1} MeV^{-2})	ReH_2	ReH_3	$\Gamma_1(K_1 \to \gamma\gamma)$	$\Gamma_2(K_1 \to \gamma\gamma)$ (10^4 sec^{-1})	$\Gamma_3(K_1 \to \gamma\gamma)$
$\pi^\pm,\omega,\phi,\psi,\rho^\pm$	−0.188	−0.181	−0.153	−0.075	2.48	2.13	1.49
$\pi^\pm,b_1^\pm,a_1^\pm$	−0.214	+0.038	+0.041	+0.048	1.72	1.73	1.75
$\pi^\pm,\omega,\phi,\psi,\rho^\pm,b_1^\pm,a_1^\pm$	−0.186	−0.267	−0.234	−0.168	3.83	3.25	2.28
$\pi^\pm,\rho^\pm,b_1^\pm,a_1^\pm$	−0.212	+0.012	+0.019	+0.030	1.63	1.64	1.66

for the imaginary and real parts of H_π and the corresponding decay rate when the pion is the only exchange particle. In Table I we have tabulated the imaginary and real parts of the total amplitude H and the decay rates for different combinations of the exchange particles and different cutoff masses. The first column indicates the exchange particles included. We have considered four different cases: when only the pion and vector mesons are included, when only the pion and axial-vector mesons are included, when the pion, vector mesons, and axial-vector mesons are included, and when only the charged particles $\pi^\pm$, $\rho^\pm$, $b_1^\pm$, and $a_1^\pm$ are included. The second column is the imaginary part of H, which is the sum of the H's due to the exchange particles. The third, fourth, and fifth column are the real parts of H corresponding to the three values of the cutoff parameter L: μ/M_x, $2\mu/M_x$, and M_K/M_x. The last three columns are the decay rates corresponding to these three values of L. All of these decay rates are within the upper and lower limits of the measured value $(2.69 \pm 1.34)\times 10^4$ sec^{-1}.

The author acknowledges the hospitality of the Department of Physics of the University of Pennsylvania where parts of this work were done and the support of the Faculty Grants Committee of Millersville University.

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