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Long-distance-physics approach to the $D\pi$ and $D^*\pi$ decays of B mesons

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A nonperturbative algebraic approach to nonleptonic weak interactions is applied to the newly observed $B\to D\pi$ and $D^*\pi$ decays. Crude estimates of the branching ratios for these decays are compared with experiments.

Recently, the ARGUS and CLEO Collaborations have started to obtain information on exclusive nonleptonic decays of B mesons.[1] The reported branching ratios seems to be smaller by more than an order of magnitude than the values predicted by the method based on the naive spectator model[2] with the factorization (or vacuum-insertion) approximation.

We have recently developed a nonperturbative algebraic approach to nonleptonic weak interactions which deals with long-distance physics in earnest but also maintains a close contact with quark-line diagrams and applied it successfully to nonleptonic weak decays of K and D mesons.[3–6] The method may be viewed as a kind of synthesis, from a new perspective, of the two once popular approaches: current algebra and broken flavor symmetry.

In this Brief Report, we discuss a crude application of the method to the $\bar{B}\to D\pi$ and $D^*\pi$ decays. The theoretical framework and method are the same as used in Refs. 3–5. See also Ref. 6 in which we have given a comprehensive and detailed review on our algebraic approach.

First, we extrapolate the amplitude of the weak, three-pseudoscalar-meson process such as $\bar{B}\to D\pi$ decays involving a π meson, $P_1(\mathbf{p}_1)\to P_2(\mathbf{p}_2)+\pi(\mathbf{q})$, to a slightly unphysical point, where we evaluate the amplitude rather easily by taking a limit $\mathbf{q}\to 0$ in the infinite-momentum frame (IMF) of the decaying particle (i.e., $\mathbf{p}_1=\mathbf{p}_2\to\infty$). This procedure effectively achieves the limit $q_\mu\to 0$ ($\mu=0,1,2,3$) without assuming the masslessness of the π meson. However, the term such as $(q\cdot p_1)$ can now remain finite in the above limiting procedure. Then we can write the amplitude approximately as $M(P_1\to P_2\pi)\simeq M_{\rm ETC}(P_1\to P_2\pi)+M_S(P_1\to P_2\pi)$. The equal-time-commutator (ETC) part

$$M_{\rm ETC}(P_1\to P_2\pi)=-(i/f_\pi)\langle P_2\,|\,[V_{\bar\pi},H_w]\,|\,P_1\rangle \tag{1}$$

is the same as in the old soft-pion extrapolation, but now has to be evaluated in the IMF. (We consider always the infinite-weak-boson-mass limit). The surface term which was dropped in the old soft-pion approximation but now survives can be expressed as

$$\begin{aligned}M_S(P_1\to P_2\pi)&\equiv \lim_{\mathbf{q}\to 0,\ \mathbf{p}_1\to\infty} i[(f_\pi)^{-1}q_\mu T_\mu]\\ &=(i/f_\pi)\Big[\sum_n[(m_2^2-m_1^2)/(m_n^2-m_1^2)]\langle P_2\,|\,A_{\bar\pi}\,|\,n\rangle\langle n\,|\,H_w\,|\,P_1\rangle\\ &\qquad+\sum_l[(m_2^2-m_1^2)/(m_l^2-m_2^2)]\langle P_2\,|\,H_w\,|\,l\rangle\langle l\,|\,A_{\bar\pi}\,|\,P_1\rangle\Big]\ .\end{aligned} \tag{2}$$

Here $T_\mu=i\int d^4x\langle P_2(p_2)\,|\,T[A_\mu^{\bar\pi}(x),H_w(0)]\,|\,P_1(p_1)\rangle e^{-iqx}$. $A_\mu^{\bar\pi}(x)$ denotes the axial-vector current which transforms like π, and f_π the decay constant of the π meson. The summation $\sum$ is extended over all the possible *on-mass-shell* single-particle hadron states. In (1), we have already used the well-known commutation relation

$$[A_\alpha,H_w]=[V_\alpha,H_w]\ . \tag{3}$$

where A_α and V_α denote the axial-vector and vector *charges* with the flavor α ($A_{\pi^-}=A_1-iA_2$, etc.).

We apply the same technique again to the quasi-two-body decay, $P_1(\mathbf{p}_1)\to V(\mathbf{p}_2)+\pi(\mathbf{q})$, where V denotes a vector meson. Then the amplitude is written as $M(P_1\to V\pi)\simeq M_{\rm ETC}(P_1\to V\pi)+M_S(P_1\to V\pi)$, where

$$M_{\rm ETC}(P_1\to V\pi)=-(i/f_\pi)\langle V\,|\,[V_{\bar\pi},H_w]\,|\,P_1\rangle\ , \tag{4}$$

$$\begin{aligned}M_S(P_1\to V\pi)&=(i/f_\pi)\Big[\sum_n[(m_V^2-m_1^2)/(m_n^2-m_1^2)]\langle V\,|\,A_{\bar\pi}\,|\,n\rangle\langle n\,|\,H_w\,|\,P_1\rangle\\ &\qquad+\sum_l[(m_V^2-m_1^2)/(m_l^2-m_V^2)]\langle V\,|\,H_w\,|\,l\rangle\langle l\,|\,A_{\bar\pi}\,|\,P_1\rangle\Big]\ .\end{aligned} \tag{5}$$

Thus, we can describe the whole amplitudes in question in terms of *asymptotic* on-mass-shell two-particle matrix elements of H_w and A_α. Our next task is to investigate the constraints on these asymptotic matrix elements of H_w.

Constraints on *asymptotic* ground-state-meson matrix elements of H_w can be obtained from commutation relations involving A_α, V_α, and H_w. See Refs. 5 and 6 for details. Here we list only the constraints which will be used in this paper.

(i) The *asymptotic* $|\Delta \mathbf{I}|=\frac{1}{2}$ rule, its charm counterpart, and the SU(6)- and SU(8)-like *asymptotic* constraints:

$$\langle \pi^+ | H_w^S | K(K^*)^+\rangle + \sqrt{2}\langle \pi^0 | H_w^S | K(K^*)^0\rangle = 0, \text{ etc.,} \tag{6}$$

$$\langle \bar{K}^0 | H_w^C | D(D^*)^0\rangle + \langle \pi^+ | H_w^C | F(F^*)^+\rangle = 0, \text{ etc.,} \tag{7}$$

$$\begin{aligned}\langle \pi^+ | H_w^S | K^+\rangle &= \langle \rho^+ | H_w^S | K^{*+}\rangle_{\lambda=0} ,\\ \langle \pi^+ | H_w^S | K^{*+}\rangle &= \langle \rho^+ | H_w^S | K^+\rangle, \text{ etc.,}\end{aligned} \tag{8}$$

$$\begin{aligned}\langle \bar{K}^0 | H_w^C | D^0\rangle &= \langle \bar{K}^{*0} | H_w^C | D^{*0}\rangle_{\lambda=0} ,\\ \langle \bar{K}^0 | H_w^C | D^{*0}\rangle &= \langle \bar{K}^{*0} | H_w^C | D^0\rangle, \text{ etc.,}\end{aligned} \tag{9}$$

from the levelwise realization of asymptotic-flavor symmetry in chiral algebras such as

$$[[H_w, A_\alpha], A_\beta] = [[H_w, V_\alpha], V_\beta] \tag{10}$$

($\alpha=\pi^+$, $\beta=\pi^-$ and $\alpha=\pi^-$, $\beta=\pi^+$ for $H_w=H_w^S$; $\alpha=K^+$, $\beta=\pi^-$ and $\alpha=\pi^-$, $\beta=K^+$ for $H_w=H_w^C$); and

$$\begin{aligned}\langle \pi^+ | H_w^S | K^+\rangle &= \pm\langle \pi^+ | H_w^S | K^{*+}\rangle ,\\ \langle \bar{K}^0 | H_w^C | D^0\rangle &= \pm\langle \bar{K}^0 | H_w^C | D^{*0}\rangle, \text{ etc.,}\end{aligned} \tag{11}$$

$$k_0 = \pm\sqrt{1/2}\langle \pi^- | A_{\pi^-} | \rho^0\rangle , \tag{12}$$

from the same level realization of

$$[H_w, A_\alpha] = [H_w, V_\alpha] , \tag{13}$$

where k_0 is the universal fraction of the ground-state-meson contribution to the left-hand side (LHS) of (13) sandwiched between two appropriate ground-state-meson states. For details see Refs. 5 and 6.

(ii) Broken-$SU_f(4)$ parametrization for the *asymptotic* matrix elements of H_w:

$$\begin{aligned}&\langle \bar{K}(\bar{K}^*)^0 | H_w^C | D(D^*)^0\rangle\\ &\quad = (U_{cs}/U_{us})\sqrt{2}\langle \pi(\rho)^0 | H_w^S | K(K^*)^0\rangle, \text{ etc.,}\end{aligned} \tag{14}$$

$$\begin{aligned}&\langle \pi(\rho)^+ | H_w^C | F(F^*)^+\rangle\\ &\quad = (U_{cs}/U_{us})\langle \pi(\rho)^+ | H_w^S | K(K^*)^+\rangle, \text{ etc.}\end{aligned} \tag{15}$$

The above relations, (14) and (15), are obtained by using asymptotic $SU_f(4)$ symmetry from the realization of the following constraint algebra involving the $SU_f(4)$ charge V_D, which is valid in the framework of QCD and electroweak theories:

$$[H_w^C, V_{D^0}] = (U_{cs}/U_{us})H_w^S . \tag{16}$$

Here U_{cs} and U_{us} denote the elements of the Kobayashi-Maskawa matrix[7] indicated by their subscripts, and all the above matrix elements of H_w^C and H_w^S are evaluated in the IMF. $H_w^S \equiv H(0,-)$ with $\Delta C=0$, $\Delta S=-1$ (strangeness changing but charm conserving), and $H_w^C \equiv H(-,-)$ with $\Delta C=\Delta S=-1$ (strangeness and charm changing). Equations (14) and (15) are valid up to the *neglect* of inter-$SU_f(4)$-multiplet mixing in the theoretical framework of asymptotic $SU_f(4)$ symmetry.

As is clear from (14) and (15), asymptotic $SU_f(4)$ symmetry can bridge, through the constraint algebra (16), the $|\Delta \mathbf{I}|=\frac{1}{2}$ rule found for the asymptotic two-particle ground-state-meson matrix elements of H_w^S such as (6) to its charm counterpart of H_w^C such as (7).

For the study of the matrix elements of $H_w^B \equiv H(\Delta B=-1,\ \Delta C=1)$ responsible for the processes $\bar{B}_d^0 \to D^+\pi^-$, $D^{*+}\pi^-$, etc., we here use the same procedure [based on asymptotic $SU_f(5)$ rotation] as used in (ii), instead of using the procedure of level realization of flavor symmetry in chiral algebras [which can be carried out within the framework of $SU_f(4)$] presented in (i). Namely, we use in place of Eq. (16) the algebra

$$[H_w^B, V_{\bar{B}_s^0}] = (U_{cb}/U_{cs})\{H_w^C\}^+ . \tag{17}$$

Here the notations concerning the B mesons are as follows: $B_u^-=(b\bar{u})$, $\bar{B}_d^0=(b\bar{d})$, $\bar{B}_s^0=(b\bar{s})$, $B_c^-=(b\bar{c})$, and their antiparticles. By sandwiching Eq. (17) between the states $\langle D(D^*)^0|$ and $|\bar{K}(\bar{K}^*)^0\rangle$ and $\langle F(F^*)^+|$ and $|\pi(\rho)^+\rangle$ with infinite momenta, we obtain constraints on the asymptotic matrix elements of H_w using asymptotic $SU_f(5)$ for the matrix elements of the $SU_f(5)$ generator $V_{B_s^0}$:

$$\begin{aligned}&\xi\langle D(D^*)^0 | H_w^B | \bar{B}_d(\bar{B}_d^*)^0\rangle\\ &\quad = (U_{cb}/U_{cs})\langle \bar{K}(\bar{K}^*)^0 | H_w^C | D(D^*)^0\rangle ,\end{aligned} \tag{18}$$

$$\begin{aligned}&\xi\langle \pi(\rho)^- | H_w^B | B_c(B_c^*)^-\rangle\\ &\quad = (U_{cb}/U_{cs})\langle \pi(\rho)^+ | H_w^C | F(F^*)^+\rangle ,\end{aligned} \tag{19}$$

where CP invariance is, of course, assumed and $H_w = H_w^{PC} + H_w^{PV}$ should be understood.

Since we are dealing with a very large symmetry breaking, although we always work only in the asymptotic limit, we have taken the following simple prescription to cope with the further possible effect of symmetry breaking, i.e., the intermultiplet mixings (i.e., mixings between the ground states under consideration and their radially excited states, etc.), which may play an appreciable role for $SU_f(N)$ with $N \geq 4$. (Actually our mixing parameters are always defined at $q^2=0$. There is thus a subtle difference from those obtained by diagonalizing the mass matrices.) For heavy mesons, the relative spacing between neighboring states becomes much narrower compared with light meson cases.

This additional effect of symmetry breaking [i.e., a possible leakage (at $q^2 \to 0$) to the excited states] through the flavor-$SU_f(5)$ charges $V_{\bar{B}_s^0}$, $V_{B_c^-}$, $V_{\bar{B}_d^0}$, and $V_{B_u^-}$ has been parametrized[8] in a simple way in (18) and (19) by a universal leakage factor ξ $(0 \le \xi \le 1)$. For simplicity, we have, however, neglected in this paper the corresponding *smaller* leakages through the $SU_f(3)$ and $SU_f(4)$ charges V_K and V_D. We have written, for the asymptotic matrix elements of V_B's, instead of their value 1 in the exact $SU_f(5)$ symmetry limit,

$$\langle \bar{B}_d^0 | V_{\bar{B}_s^0} | \bar{K}^0 \rangle = \langle B_c^- | V_{\bar{B}_s^0} | F^- \rangle = \langle \bar{B}_d^0 | V_{B_u^-} | \pi^+ \rangle = \langle B_c^- | V_{\bar{B}_d^0} | D^- \rangle = \xi \ . \tag{20}$$

By combining (7), (18), and (19), we obtain the bottom counterpart of the $|\Delta \mathbf{I}| = \frac{1}{2}$ rule for the asymptotic two-particle ground-state-meson matrix elements of H_w:

$$\langle D(D^*)^0 | H_w^B | \bar{B}_d(\bar{B}_d^*)^0 \rangle + \langle \pi(\rho)^- | H_w^B | B_c(B_c^*)^- \rangle = 0 \ . \tag{21}$$

As mentioned before, we can also derive (21) through the level realization procedure in the framework of *only* asymptotic $SU_f(4)$ symmetry.

From (7), (9), (11), (18), and (19), we also obtain a bottom counterpart of the SU(6)-type relation

$$\begin{aligned} &\langle D^0 | H_w^B | \bar{B}_d^0 \rangle = \pm \langle D^{*0} | H_w^B | \bar{B}_d^0 \rangle \ , \\ &\langle \pi^- | H_w^B | B_c^- \rangle = \pm \langle \pi^- | H^B | B_c^{*-} \rangle, \text{ etc.} \end{aligned} \tag{22}$$

Equation (21), which is the bottom counterpart of the *asymptotic* $|\Delta \mathbf{I}| = \frac{1}{2}$ rule for the two-particle matrix elements of H_w^B, can be associated with the *same* type of quark-line diagrams[9] responsible for the *asymptotic* $|\Delta \mathbf{I}| = \frac{1}{2}$ rule, Eq. (6), and its charm counterpart, Eq. (7). The diagrammatical statements of all these three constraints (the annihilation diagram) + (the W-exchange diagram) = 0.

We now discuss the rates of the $\bar{B} \to D\pi$ and $D^*\pi$ decays in comparison with the $K_S^0 \to \pi^+\pi^-$ decay, since this route seems least ambiguous at present. For the $K_S^0 \to \pi^+\pi^-$ decay, we have obtained a simple result:[3,6]

$$M(K_S^0 \to \pi^+\pi^-) \simeq i(2f_\pi)^{-1}\sqrt{2}\langle \pi^+ | H_w^S | K^+ \rangle \times (1 + 0.2 + \cdots) \ . \tag{23}$$

In deriving (23), we have chosen the positive sign in (11). The ellipsis represents the small contribution of the neglected excited states, which can involve the $|\Delta \mathbf{I}| = \frac{1}{2}$ rule-violating term. The term 0.2 expresses the fact that even the most important ground-state-meson contribution to the surface term M_S is only 20% relative to $M_{\rm ETC}$ and the excited-state contribution can be neglected safely.

In the present case of $\bar{B}_{d(u)} \to D\pi$ and $D^*\pi$, the contribution of excited states will be estimated crudely to be at most 30% later. Under the simplifying approximation in which the contribution of excited states is neglected, $M_{\rm ETC}$ and M_S for the $B_u^- \to D^0\pi^-$ decay are given by, through (1) and (2),

$$M_{\rm ETC}(B_u^- \to D^0\pi^-) = -(i/f_\pi)\langle D^0 | H_w^B | \bar{B}_d^0 \rangle \tag{24}$$

and

$$M_S^{(L=0)}(B_u^- \to D^0\pi^-) = (i/f_\pi)(m_D^2 - m_{B_u}^2)/(m_{B_d^*}^2 - m_\pi^2) \times \langle D^0 | H_w^B | \bar{B}_d^{*0} \rangle \times \langle \bar{B}_d^{*0} | A_{\pi^+} | B_u^- \rangle \ . \tag{25}$$

In (25), the asymptotic matrix elements of A_{π^+}, $\langle \bar{B}_d^{*0} | A_{\pi^+} | B_u^- \rangle$ can be evaluated, by realizing the constraint algebra $[V_{\bar{B}_d^0}, A_{\pi^+}] = 0$ using asymptotic $SU_f(5)$ symmetry, to be

$$\sqrt{2}\langle \bar{B}_d^{*0} | A_{\pi^+} | B_u^- \rangle = \langle \pi^+ | A_{\pi^+} | \rho^0 \rangle = -H \ . \tag{26}$$

We expect that this is a reliable result, since the constraint algebras used involve the SU(2) axial-vector charges A_π, which apparently have small asymptotic matrix elements between the ground-state mesons and their radially excited states. Indeed, the constraint algebras, $[\dot{V}_{\bar{B}_d^0}, A_{\pi^+}] = [V_{\bar{B}_d^0}, A_{\pi^+}] = 0$, produce a surprisingly well-satisfied mass formula, $m_{B^*}^2 - m_B^2 = m_\rho^2 - m_\pi^2$, using asymptotic $SU_f(5)$.

With the bottom counterpart of the $|\Delta \mathbf{I}| = \frac{1}{2}$ rule, Eq. (21), asymptotic $SU_f(5)$ parametrization, Eq. (26), and the approximations

$$m_{B_c^*}^2 \simeq m_{B_d^*}^2 \simeq m_{B_d}^2 \simeq m_{B_u}^2 \gg m_{D^*}^2 \simeq m_D^2 \gg m_\pi^2 \ , \tag{27}$$

we obtain

$$M(B_u^- \to D^0\pi^-) \simeq (i/f_\pi)\langle \pi^- | H_w^B | B_c^- \rangle (1 - H/\sqrt{2}) \ , \tag{28}$$

where we have chosen the positive sign in (22).

Exactly the same procedure as used in deriving (28) yields

$$M(\bar{B}_d^0 \to D^+\pi^-) \simeq (i/f_\pi)\langle D^0 | H_w^B | \bar{B}_d^0 \rangle (1 - H/\sqrt{2}) \ . \tag{29}$$

The relative size of the surface term to the ETC term, $r \equiv M_S/M_{\rm ETC}$, is thus estimated to be $r^{(L=0)}(\bar{B}_d^0 \to D^+\pi^-) \simeq -\sqrt{1/2}H \simeq 0.7$, in the approximation in which only the ground-state contribution is retained. Here we have chosen the positive sign in (22) and used the value $H \equiv \langle \pi^- | A_{\pi^-} | \rho^0 \rangle \simeq -1.0$ obtained from the value $\Gamma(\rho \to 2\pi)_{\rm expt} \simeq 160$ MeV, using PCAC (partial conservation of axial-vector current) with hard-pion extrapolation.[6]

As a matter of fact, $r^{(L=0)}(\bar{B}_d^0 \to D^+\pi^-)$ given above coincides approximately with the value of k_0 given by (12). Therefore, the contribution of excited states to M_S $(\bar{B}_d^0 \to D^+\pi^-)$ amounts roughly to [using the general formula (2)] $|r^{(\text{excited states})}(\bar{B}_d^0 \to D^+\pi^-)| \lesssim 1 - |k_0|$

$\simeq 0.3$, since the masses of bottom mesons are much larger than the bottomless meson masses. Therefore, the error caused by the neglect of excited meson contribution to M_S will not be very crucial (at most, of the order of 60% errors in the rate). Thus, by using (6), (14), and (18), we finally get $|M(\bar{B}^0_d \to D^+\pi^-)/M(K^0_S \to \pi^+\pi^-)| \simeq 2|U_{cb}/U_{us}|/\xi$. In order to compare our result with experiment, we use the present estimates[10] $|U_{cb}| \simeq 0.059$ and $|U_{us}| \simeq 0.225$ and also $\Gamma(K^0_S \to \pi^+\pi^-)_{\text{expt}} \simeq 0.77 \times 10^{10}\ \text{sec}^{-1}$. The lifetime of $\bar{B}^0_d$ meson $\tau(\bar{B}^0_d)$ is not yet precisely known. Recent measurements[11] show that it is of the same order of magnitude as the b-quark lifetime[1] $\tau(b)_{\text{expt}} = (1.21 \pm 0.15) \times 10^{-12}$ sec measured in the B-meson semileptonic decays. The value of the "leakage" factor ξ is also still unknown. For a reasonable value of ξ, $\xi \simeq 0.5$, we obtain $B(\bar{B}^0_d \to D^+\pi^-) \simeq 1 \times 10^{-3}$, which is compatible with experiment.[1] For a crude estimate of $\xi = 0.5$, see Ref. 12.

Comparing (28) and (29) and using (21), we have $|M(B^-_u \to D^0\pi^-)/M(\bar{B}^0_d \to D^+\pi^-)| \simeq 1$ in the present approximation. Neglecting the small difference of phase-space volumes between the two decays, we then obtain $\Gamma(B^-_u \to D^0\pi^-) \simeq \Gamma(\bar{B}^0_d \to D^+\pi^-)$.

We next study the $\bar{B}^0_d \to D^{*+}\pi^-$ and $B^-_u \to D^{*0}\pi^-$ decays. By using the bottom counterpart of the $|\Delta \mathbf{I}| = \frac{1}{2}$ rule in (4) and (5), we obtain

$$M_{\text{ETC}}(\bar{B}^0_d \to D^{*+}\pi^-) = (i/f_\pi)\langle D^{*0}|H^B_w|\bar{B}^0_d\rangle \ , \tag{30}$$

$$M_S^{(L=0)}(\bar{B}^0_d \to D^{*+}\pi^-) = (i/f_\pi)[(m^2_{D^*} - m^2_{B_d})/(m^2_D - m^2_{B_d})]\langle D^{*+}|A_{\pi^+}|D^0\rangle\langle D^0|H^B_w|\bar{B}^0_d\rangle \ , \tag{31}$$

and a similar result for the $B^-_u \to D^{*0}\pi^-$ decay.

Again using the same procedure as the $B \to D\pi$ decays, we obtain

$$M(\bar{B}^0_d \to D^{*+}\pi^-) \simeq M(B^-_u \to D^{*0}\pi^-) \simeq (i/f_\pi)\langle D^{*0}|H^B_w|\bar{B}^0_d\rangle(1 - H/\sqrt{2}), \tag{32}$$

choosing the positive sign in (22).

Here, we compare the $\bar{B}^0_d \to D^{*+}\pi^-$ decay with the $\bar{B}^0_d \to D^+\pi^-$. From (29) and (32) we can predict in our approximation $B(\bar{B}^0_d \to D^{*+}\pi^-) \simeq B(\bar{B}^0_d \to D^+\pi^-)$, *independently* of the factor ξ, neglecting the small phase-space volume difference. Therefore, we find within a factor 2 or so

$$B(\bar{B}^0_d \to D^+\pi^-) \simeq B(B^-_u \to D^0\pi^-) \simeq (\bar{B}^0_d \to D^{*+}\pi^-) \simeq B(B^-_u \to D^{*+}\pi^-) \ , \tag{33}$$

when we assume $\tau(\bar{B}^0_d) \simeq \tau(B^-_u)$. The result on $B(\bar{B}^0_d \to D^{*+}\pi^-)$ seems to be compatible with the new data[1] obtained by the CLEO group,[1] $(0.35 \pm 0.14 \pm 0.11)\%$, and the value of ARGUS group $(0.27 \pm 0.14 \pm 0.10)\%$.

In conclusion, we have presented a perhaps feasible scenario for the $B \to D\pi$ and $D^*\pi$ decays from a long-distance-physics approach.

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[8]If $|\langle\text{ground state}|H_w|\text{ground state}\rangle| \gg |\langle\text{ground state}|H_w|\text{radially excited state}\rangle|$, our prescription will be quite accurate.

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[12]The value $\xi \simeq 0.5$ was inferred as follows. From $F^{K\pi}_+(0) \simeq 1$ in the $K \to \pi e^+\nu$ decays, the leakage in $SU_f(3)$ is certainly small. From $f^{DK}_+(0) \simeq 0.73 \pm 0.05$ in the $D \to Ke^+\nu$ decays [see, for example, Y. Koide and S. Oneda, Phys. Rev. D **36**, 2867 (1987)], the leakage in $SU_f(4)$ could amount to $\simeq(20-30)\%$ ($\xi \simeq 0.7-0.8$), and, therefore, ξ in $SU_f(5)$ may be smaller. We also made the following observation. If we choose to extrapolate both $\bar{B}$ and π mesons for $\bar{B} \to D^*\pi$ decay, we then obtain, instead of Eqs. (30) and (31),

$$M(\bar{B}^0_d \to D^{*+}\pi^-) = M_{\text{ETC}}(\bar{B}^0_d \to D^{*+}\pi^-) \times [1 - (H/\sqrt{2})f_{B_d}(\xi f_\pi + f_{B_d})^{-1}] \ ,$$

where f_{B_d} is the decay constant of B_d. If we use the value of f_{B_d} ($f_\pi/f_{B_d} \simeq 2.5$), which has been predicted recently [M. Suzuki, Phys. Lett. **162B**, 392 (1985)], the two extrapolations are found compatible with $\xi \simeq 0.5$.