Reexamining the Fritzsch-Stech SO(10) model of quarks and leptons

M. Gronau
Physics Department, Technion, 32000 Haifa, Israel

R. Johnson, S. Ranfone, and J. Schechter
Physics Department, Syracuse University, Syracuse, New York 13244-1130
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In a rather predictive model of the quark and lepton mass matrices we discuss restrictions on the quark and lepton mixings implied by two different phenomena: (1) the Mikheyev-Smirnov-Wolfenstein explanation of the solar-neutrino problem and (2) the recent ARGUS measurements of $B\bar{B}$ mixing and of charmless B decays involving baryons. We conclude that the model allows for neutrino masses and mixings which may account for matter oscillations of either of the two types ν_e to ν_μ or ν_e to ν_τ. In the quark sector the phenomenology of the model is identical to that of the Fritzsch quark matrix. Within our scheme the simplest interpretation of the recent ARGUS results suggests a large value of m_t around 80 GeV. The model predicts $|U_{ub}|$ to be around 0.0032.

I. INTRODUCTION

It was earlier shown[1,2] that combining the Fritzsch[3] and Stech[4] *Ansätze* for the up- and down-quark mass matrices enabled one to completely predict the quark mixing matrix in terms of physical quark masses. An extension[5–7] to include leptons is reasonably made in an SO(10) framework. In order to fit the charged-lepton masses this model requires a **126** Higgs field to be present. The new term in the quark mass matrix has the effect of allowing a greater range for the top-quark mass (up to around 95 GeV) compared to the original model (which essentially contained a **10** and a **120** Higgs field).

The generalized model coincides with the most general Fritzsch model for the quark sector, but then enables one to compute (assuming, say, the "seesaw" mechanism) the lepton mixing matrix and neutrino mass ratios. The "Stech" ingredient is the assumption of Hermitian mass matrices in an appropriate basis. The only new parameter required (apart from the known charged-lepton masses) is the (in principle, calculable) renormalization parameter r specifying the evolution from the grand unified theory (GUT) to the ordinary scale of the down-quark/charged-lepton mass ratio.

We should remark that the addition of the **126** to the Stech model was first proposed[4] by Stech himself in order to explain the lepton sector. The generalized model discussed here, based closely on the earlier model of Ref. 1, was treated by Bottino *et al.*[5] and by Johnson *et al.*[6,7] Bottino *et al.*[5] did not employ the seesaw mechanism (preferring a direct Majorana mass term to be dominant) and did not fully allow for the variation of the input parameters within the expected range. The last point was discussed in detail by Johnson *et al.*[7] and is now rather topical since the simplest interpretation of the recently reported[8] large value of the B_d-$\bar{B}_d$ mixing parameter tends to push the allowed range of parameters to an extreme corner characterized by a large top-quark mass and a relatively small strange-quark mass. A top-quark mass larger than about 45 GeV is not compatible with the original form of the model in which the **126** is absent. This has been fully discussed in the recent literature.[2,7,9]

We shall, in Sec. II, examine once more the predictions of the lepton mixing matrix K and the neutrino mass ratios. The possibility of a large top-quark mass will, as in our previous treatment,[7] be included in the discussion. A more detailed study of the effect of varying the renormalization quantity r will be given. We shall contrast the results of our model to an often used[10] and not *a priori* implausible assumption which relates K to the Cabibbo-Kobayashi-Maskawa matrix U as $K=U^\dagger$ and which, furthermore, predicts the neutrino masses to be in the ratio $m(\nu_e):m(\nu_\mu):m(\nu_\tau)=m_u^2:m_c^2:m_t^2$. We find that our predictions differ in a number of important ways from those of this model as we shall explain later. These differences will affect details of the Mikheyev-Smirnov-Wolfenstein (MSW) explanation[11,12] of the solar-neutrino paradox. We note that laboratory bounds on ν_μ-ν_τ vacuum oscillations provide useful restrictions[7,13] on the relevant parameters. For relatively small r (around 3) the ν_μ-ν_τ data eliminate a sizable portion of the allowed range for ν_e-ν_μ MSW oscillation. Thus if r is small, ν_e-ν_τ MSW oscillation is more likely. On the other hand, for larger r (around 4) the ν_e-ν_τ mixing angle is just on the borderline for allowing the MSW oscillations, so in this case ν_e-ν_μ MSW oscillation is more likely.

In Sec. III we briefly discuss the consequences for the generalized Fritzsch-Stech model of the large value of the $B_d\bar{B}_d$ mixing parameter found by the ARGUS Collaboration.[8] Using the simplest interpretation, this requires m_t around 80 GeV and m_s around 120 MeV. All other parameters are predicted. We point out some caveats for this simple interpretation based on the dominance of top-quark exchange. At present the ARGUS result seems suggestive rather than conclusive for a large top-quark mass. We also note that the very recent observation also by the ARGUS Collaboration,[14] of decays of the B meson into noncharmed final states, suggests a lower bound for the matrix element $|U_{ub}|$ which is roughly consistent with (though perhaps slightly higher than) the value predicted here.

II. LEPTON MIXING MATRIX AND SOLAR NEUTRINOS

In the model of Ref. 7 (to which we also refer for notation) the lepton mixing matrix K is obtained in a manner consistent with all the known information about the charged fermions as well as the quark mixing matrix U. It seems useful to contrast this model with what is generally considered[10] as a reasonable first guess:

$$K = U^\dagger \ . \tag{2.1}$$

Equation (2.1) may be derived from the seesaw-mechanism formula for the effective 3×3 neutrino mass matrix, M_ν^{eff}:

$$M_\nu^{\text{eff}} = -M_{\nu D}^T M_{\nu R}^{-1} M_{\nu D} \tag{2.2}$$

(where $M_{\nu D}$ is the "Dirac" piece and $M_{\nu R}$ is heavy Majorana piece), when one assumes

$$M_{\nu R} = 1 m_H \ , \tag{2.3a}$$

$$M_{\nu D} = M_{\nu D}^T \propto M_u \ , \tag{2.3b}$$

$$M_e \propto M_d \ , \tag{2.3c}$$

where m_H is a large mass and M_u, M_d, M_e are the charged-quark and -lepton mass matrices. Bringing the various matrices to diagonal form is easily seen to result in (2.1). However, these assumptions are subject to criticism. Regardless of the model, (2.3c) implies the eigenvalue ratio $m_\mu/m_e = 210$ to be the same as $m_s/m_d \approx 20$ and furthermore requires $m_b/m_s \approx 30$ to be the same as $m_\tau/m_\mu = 17$. The assumption (2.3a) that $M_{\nu R}$ is proportional to a unit matrix is, for example, very far from the situation in typical grand unified models, e.g., the SO(10) model under present consideration. Finally, while (2.3b) is not very unreasonable, it also does not always accurately hold in SO(10) models. For these reasons we feel that the form for K to be discussed now is more realistic than (2.1).

In the generalized Fritzsch-Stech model[5–7] the 3×3 mass matrix subblocks needed for the seesaw mechanism description in (2.2) are taken to be

$$M_{\nu R} = \gamma S' \ , \tag{2.4a}$$

$$r' M_{\nu D} = S - 3\epsilon S' \ , \tag{2.4b}$$

$$r M_e = \alpha S - 3S' + \delta A \ , \tag{2.4c}$$

where S and S' are real symmetric matrices and A is an imaginary antisymmetric matrix, corresponding to the **10**, **126**, and **120** representations, respectively. The "Stech" ingredient in Eqs. (2.4) is that S, S', and A are taken to be Hermitian. A "Fritzsch" structure is imposed, in addition, for S, S', and A. As stressed previously,[1] the mass matrices need not be purely symmetric as in the original Fritzsch *Ansatz*. r and r' are renormalization factors appropriate to comparing masses at a low energy, rather than at a grand unified, scale. Traditionally people have estimated[7,10,15]

$$r \approx 3\text{–}4.5, \quad r' \approx 3.3\text{–}5 \ . \tag{2.5}$$

Furthermore, α, δ, ϵ, and γ are real constants. To compare (2.4) with (2.3) we note that the expressions for the "up-" and "down-" quark mass matrices are

$$M_u = S + \epsilon S' \ , \tag{2.6a}$$

$$M_d = \alpha S + S' + A \ . \tag{2.6b}$$

The (small) contribution of the **120** has been neglected in (2.6a) as well as in (2.4b). Rather than (2.3c) we now have

$$M_d - r M_e = 4S' + (1-\delta) A \ , \tag{2.7}$$

so the right-hand side acts as an important correction,[16] which enables us to fit all three down-quark as well as charged-lepton masses. Rather than the unit matrix for $M_{\nu R}$ in (2.3a) we now have the matrix S' in (2.4a), which turns out to be of Fritzsch form without the complete Fritzsch hierarchy. In this model the eleven parameters (not counting the large-scale parameter γ) α, δ, ϵ, S_{12}, S_{23}, S_{33}, S'_{12}, S'_{23}, S'_{33}, A_{12}, A_{23} are fit in terms of the nine charged-fermion masses and the two Kobayashi-Maskawa (KM) matrix elements $|U_{us}|$ and $|U_{cb}|$. Thus, assuming a value for the t-quark mass, the complete KM matrix, the complete lepton mixing matrix as well as all neutrino mass ratios are determined.

To get a feeling for the allowed ranges of the input parameters in light of the lack of a precise knowledge of m_t, m_s and experimental uncertainties in $|U_{us}|$ and $|U_{cb}|$ one may consult Fig. 2 of Ref. 7. It is probably most reasonable to focus on the central diagram[17] there with $|U_{us}| = 0.225$ and $|U_{cb}| = 0.05$. For convenience this is reproduced as Fig. 1 of the present paper. It is crucial to note that for relatively large m_t [m_t (1 GeV) > 60 GeV corresponding to the physical $m_t(m_t) > 37$ GeV] the predictions[18] for the KM matrix U, the lepton mixing matrix K, and the neutrino mass ratios vary smoothly with changes in m_t and m_s. As an example let us consider m_t roughly as large as possible. Fig. 1 shows this situation to be around $m_t(1\text{ GeV}) = 130$ GeV [$m_t(m_t) \approx 78$ GeV] and $m_s = 120$ MeV. With $r = 4$, we have the KM matrix

$$U = \begin{bmatrix} 0.97 & 0.225 & 0.0032e^{i101^\circ} \\ -0.225 & 0.97 & 0.050 \\ 0.012e^{i14^\circ} & -0.049e^{-i1^\circ} & 1.00 \end{bmatrix} \tag{2.8}$$

and the lepton mixing matrix

$$K = \begin{bmatrix} 1.00 & 0.061e^{-i166^\circ} & 0.007e^{-i108^\circ} \\ 0.061e^{-i13^\circ} & 0.99 & 0.085e^{i103^\circ} \\ 0.005e^{-i22^\circ} & 0.085e^{i77^\circ} & 1.00 \end{bmatrix} . \tag{2.9}$$

The predicted neutrino mass ratios in this example are

$$m_2/m_1 = 382, \quad m_3/m_2 = 44 \ . \tag{2.10}$$

It is very interesting to compare the K in this model to

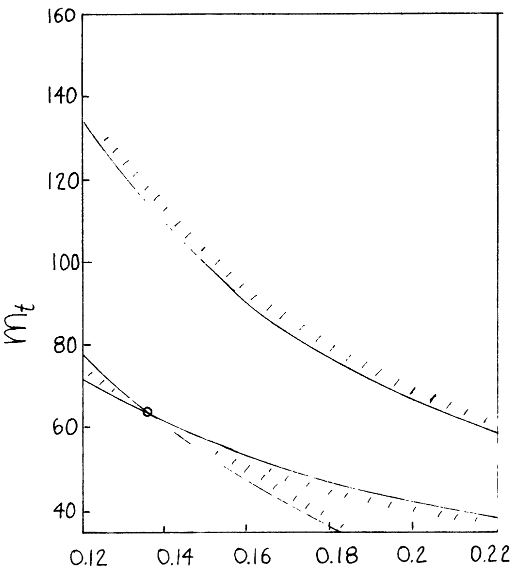

FIG. 1. Allowed (unshaded) region in the m_s-m_t plane for $|U_{us}|=0.225$ and $|U_{cb}|=0.05$. The axes are labeled by the running masses at 1 GeV. This is a reproduction of the central plot in Fig. 2 of Ref. 7. The circle shows the unique point corresponding to the original Fritzsch-Stech model for these mixing angles.

$K=U^\dagger$, formula (2.1).

(i) $|K_{12}|\approx|K_{21}|$ in the present model is about one-third $|K_{12}|$ in the model of (2.1). This feature is independent of parameter choices in the model and is of great importance for possible ν_e-ν_μ oscillations.

(ii) $|K_{23}|\approx|K_{32}|$ in the present model is somewhat larger than $|K_{23}|$ in the model of (2.1). The effect is magnified for r closer to the "magic" value[6] of about 3. Then the leading term on the LHS of (2.7) gets very small. For this range of r one may apply experimental information on ν_μ-ν_τ mixing in a useful way.

(iii) In the present model $|K_{13}|$ is approximately the same as $|K_{31}|$. However, in the model of (2.1) $|K_{31}|$ is only about one-third $|K_{13}|$ (assuming that U has a form consistent with a Fritzsch structure). This suggests that a simple 2×2 approximation for ν_e-ν_τ oscillations in the model of (2.1) is not *a priori* obviously correct.

It is also interesting to compare the neutrino mass ratios (2.10) with the expectations $m_2/m_1=(m_c/m_u)^2\approx7\times10^4$ and $m_3/m_2=(m_t/m_c)^2\approx9\times10^3$ in the model of (2.1). The "hierarchy" in the present case is suppressed due to a partial hierarchy present in $M_{\nu R}$ [denominator of (2.2)]. In the present model the neutrino generation heirarchy is roughly similar to that of the charged fermions.

Some aspects of the form of (2.9) can be readily understood by writing $K=\Omega^t V$, where Ω^t diagonalizes M_e and V diagonalizes M_2^{eff}. Neglecting CP violations an Ω^t of the Fritzsch form would be

$$\begin{bmatrix} 1 & -\sqrt{e/\mu} & +\sqrt{e/\tau} \\ \sqrt{e/\mu} & 1 & -\sqrt{\mu/\tau} \\ 0 & +\sqrt{\mu/\tau} & 1 \end{bmatrix} \approx \begin{bmatrix} 1 & -0.07 & +0.017 \\ 0.07 & 1 & -0.25 \\ 0 & +0.25 & 1 \end{bmatrix} . \quad (2.11)$$

The (12) elements of (2.9) and (2.11) are very close while the (23) elements are only qualitatively similar. At smaller r the (23) elements do become quite comparable though. The asymmetry between $|K_{13}|$ and $|K_{31}|$ in (2.11) is modified by V.

A more detailed description of the dependence of the lepton parameters K_{ab}, m_2/m_1, m_3/m_2 on the quantity r can be quickly obtained by comparing Table I (for $r=3$) with Table II (for $r=4$), wherein the quantities $|K_{12}|$, $|K_{23}|$, $|K_{13}|$, m_2/m_1, m_3/m_2 are displayed for allowed values of the quark masses m_s and m_t. First note that for relatively large m_t (corresponding to the upper allowed region in Fig. 1) these parameters do not really change in a drastic way with m_s and m_t. There is a more important variation as r deviates from about 3.1, since the dominant 33 component on the LHS of (2.7) vanishes there. This means that S' will play a more and more important role in determining neutrino properties [see (2.4b)] as one goes away from "magic" r. The following is a summary of these tables, as r decreases from 4 to 3: (i) $|K_{12}|$ remains about constant; (ii) $|K_{23}|$ increases from about 0.08 to 0.16; (iii) $|K_{13}|$ increases from 0.007 to 0.014; (iv) m_2/m_1 decreases from about 400 to about 100; (v) m_3/m_2 increases from about 40 to about 400. The variations of these quantities as one increases r from 4 to 4.5 is not very great. One notices that $|K_{13}|$ decreases to about 0.006 at $r=4.5$.

Experimental constraints on vacuum $\nu_\mu\to\nu_\tau$ oscillations can, because $|K_{23}|$ is relatively sizable, be used to provide[7,13] interesting upper mass bounds for the neutrinos. The data[19,20] are shown in Fig. 2. In this model, of course, $m^2(\nu_\tau)-m^2(\nu_\mu)\approx(m_3)^2$. For $r\approx3$ we have $m(\nu_\tau)\lesssim1.0$ eV, $m(\nu_\mu)\lesssim2.6\times10^{-3}$ eV, and $m(\nu_e)\lesssim3\times10^{-5}$ eV. For $r=4$ we have $m(\nu_\tau)\lesssim2.2$ eV, $m(\nu_\mu)\lesssim0.05$ eV, and $m(\nu_e)\lesssim1.3\times10^{-4}$ eV.

Now we would like to show how experimental constraints on $\nu_\mu\to\nu_\tau$ oscillations can be used in the $r=3$ case to severely restrict the allowed MSW solutions[11,12] for the solar-neutrino problem. One might have resonant matter oscillations $\nu_e\to\nu'$ with either $\nu'=\nu_\mu$ or $\nu'=\nu_\tau$. One should really make a 3×3 matrix analysis,[21] but we will here simply take over the results for the 2×2 approximations. We must specify $\Delta m^2=m^2(\nu')-m^2(\nu_e)$ and the mixing angle θ, which is either $|K_{12}|$ or $|K_{13}|$ for $\nu'=\nu_\mu$ and ν_τ, respectively. There are two main branches[22] which give the appropriate suppression of detected neutrinos in the Davis ^{37}Cl detector.

Branch 1:

$$m^2(\nu')-m^2(\nu_e)\approx10^{-4}\ (\text{eV})^2,\quad |\theta|>0.007 . \quad (2.12a)$$

TABLE I. The predicted values of the lepton mixing angles and neutrino mass ratios are presented for several values of top-quark and strange-quark running masses ($\mu = 1$ GeV) with the renormalization parameter fixed at $r=3$. For each value of m_s and m_t the two neutrino mass ratios m_3/m_2 and m_2/m_1 are given in the first column and the three mixing angles $|K_{13}|$, $|K_{23}|$, and $|K_{12}|$ are given in the second column as denoted schematically in the upper right corner of the table. The number in parentheses below the top-quark running mass is its corresponding physical mass. Note that quark masses are measured in GeV.

m_t \ m_s	0.12	0.14	0.16	0.18	0.20
130 (78)	0.015 380 0.204 87 0.091				$\lvert K_{13}\rvert$ m_3/m_2 $\lvert K_{23}\rvert$ m_2/m_1 $\lvert K_{12}\rvert$
110 (66)	0.014 395 0.192 108 0.079	No fit			
90 (54)	0.013 428 0.174 135 0.065	0.015 450 0.200 83 0.09	No fit	No fit	
70 (42)	No fit	0.014 590 0.192 95 0.082	0.016 815 0.219 30 0.152	0.017 1200 0.23 9 0.29	No fit
50 (31)	0.013 1060 0.173 189 0.062	0.102 730 0.152 209 0.048	No fit	0.01 20 0.15 39 000 0.068	0.44 33 0.62 11 000 0.059

TABLE II. The predicted neutrino mixing angles and mass ratios are given as in Table I, but now the renormalization parameter is fixed at $r=4$.

m_t \ m_s	0.12	0.14	0.16	0.18	0.20
130 (78)	0.007 44 0.085 382 0.061				$\lvert K_{13}\rvert$ m_3/m_2 $\lvert K_{23}\rvert$ m_2/m_1 $\lvert K_{12}\rvert$
110 (66)	0.007 49 0.093 320 0.059	No fit			
90 (54)	No fit	0.007 45 0.087 325 0.059	No fit	No fit	
70 (42)	0.005 74 0.062 1200 0.059	No fit	0.007 45 0.090 288 0.061	0.006 35 0.071 324 0.059	No fit
50 (31)	0.01 110 0.13 240 0.059	0.008 76 0.108 237 0.047	No fit	No fit	0.009 160 0.119 13 0.244

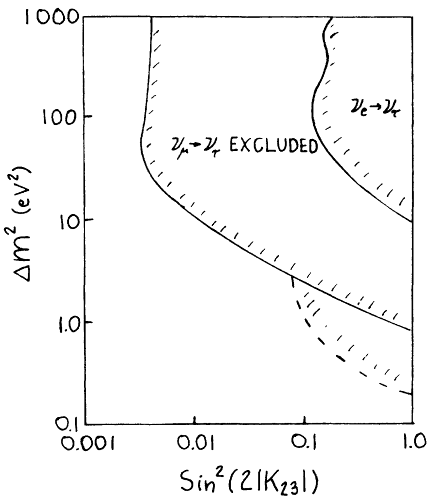

FIG. 2. Bound on $\Delta m^2 = m^2(\nu_\tau) - m^2(\nu_\mu)$ plotted against $\sin^2(2\,|K_{23}|)$ taken from Fig. 2, Ref. 19. The dashed curve gives the same bound based on interpreting $\nu_\mu \to \nu_x$ as $\nu_\mu \to \nu_\tau$. See Ref. 20.

Branch 2:

$$[m^2(\nu') - m^2(\nu_e)]\sin^2 2\theta \approx 3.2\times 10^{-8}\ (\text{eV})^2\ ,$$
$$m^2(\nu') - m^2(\nu_e) < 10^{-4}\ (\text{eV})^2\ . \tag{2.12b}$$

First notice that both branches require $\theta > 0.007$. Table I shows that this is satisfied for either $\nu' = \nu_\mu$ or $\nu' = \nu_\tau$ when $r = 3$.

Let us consider the possibility $\nu' = \nu_\mu$ for the example where $m_t(1\ \text{GeV}) = 130$ GeV in Table I. On branch 1 we must have $m(\nu_\mu) \approx 0.01$ eV and $m(\nu_\tau) \approx 3.8$ eV. Reference to Fig. 2 shows that with a mixing angle $|K_{23}| = 0.20$ as given in Table I we are in an excluded region. Thus the branch 1 solution for $\nu_e \to \nu_\mu$ matter oscillation is not allowed. This result is independent of the large-scale factor γ in (2.4a). Now consider the branch 2 solution. Using (2.12b) and $|K_{12}| = 0.09$ we see that $\nu' = \nu_\mu$ requires $m(\nu_\mu) = 0.001$ eV and hence $m(\nu_\tau) = 0.38$ eV. Reference to Fig. 2 shows that this an acceptable point. However, it is not very far from the excluded region, so a relatively small improvement of the experiment might also rule out branch 2 for the case of $\nu_e \to \nu_\mu$ matter oscillations when $r = 3$. It is easy to see that if one identifies $\nu' = \nu_\tau$ (still keeping $r = 3$), then, because the absolute value of the ν_τ mass must be appreciably smaller, the bounds of Fig. 2 are not very restrictive.

The situation may be rather different for larger r. Table II shows that when $r = 4$, $|K_{13}| \approx 0.007$, which is the minimum allowed for solution of the solar-neutrino problem. At $r = 4.5$, $|K_{13}| \approx 0.006$ so the solution with $\nu_e \to \nu_\tau$ is problematical. On the other hand, there are no present restrictions on the $\nu_e \to \nu_\mu$ matter oscillation from $\nu_\mu \to \nu_\tau$ laboratory measurements for $r = 4$ since the ratio m_3/m_2 has decreased by a factor of 10.

We should stress that the above analysis is independent of the precise choice of the grand unified scale except insofar as it (slightly) modifies r. This is an advantage since one does not really know the precise vacuum values hidden in γ in (2.4a). From (2.2) and (2.3b) one sees that

$$m(\nu_T) = O(10^4/\gamma)\ \text{GeV} \tag{2.13a}$$

and furthermore [see (4.1) of Ref. 7]

$$\gamma = O(Z/V_-^{(126)}) \tag{2.13b}$$

where Z is a vacuum value of GUT size and $V_-^{(126)}$ is a vacuum value of ordinary size. The most conventional assumptions would be $Z = O(10^{16}\ \text{GeV})$ and $V_-^{(126)} = O(100\ \text{GeV})$. This would give $m(\nu_\tau) = O(0.1\ \text{eV})$ and $m(\nu_\mu) = O(10^{-3}\ \text{eV})$, which is evidently consistent with ν_e-ν_μ MSW oscillations. A larger GUT scale or a smaller scale for $V_-^{(126)}$ could accommodate ν_e-ν_τ MSW oscillations.

We have seen that if $r \approx 3$, the $\nu_e \to \nu_\tau$ MSW oscillation is more probable as a consistent solution for the solar-neutrino paradox. On the other hand, if $r \approx 4$ the $\nu_e \to \nu_\mu$ MSW oscillation is more probable. Strictly speaking though, neither is excluded. With the present data, evaluation of r by the one-loop formula [see (2.4) of Ref. 7, for example] favors r around 4.

III. QUARK MIXING MATRIX: B_d-$\bar{B}_d$ MIXING AND U_{ub}

A very interesting aspect of the quark mixing matrix, as determined from (2.6a) and (2.6b), is that it varies only slightly as the values of m_s and m_t are varied over the complete allowed range (holding $|U_{us}|$ and $|U_{cb}|$ fixed).

The typical situation is given in Eq. (2.8), while the variation of $|U_{ub}|$ and the invariant phase Φ with m_s and m_t is shown in Table III (taking $|U_{cb}| = 0.05$). Recall that the KM matrix may approximately be written as[23]

$$U \approx \begin{bmatrix} 1 & |U_{us}| & |U_{ub}|e^{i\Phi} \\ -|U_{us}| & 1 & |U_{cb}| \\ -|U_{ub}|e^{-i\Phi} + |U_{us}U_{cb}| & -|U_{cb}| & 1 \end{bmatrix} . \tag{3.1}$$

Insofar as one is predicting U it does not make much difference whether one uses the original[1] Stech-Fritzsch model [$S' \to 0$ in (2.6a) and (2.6b)] or the generalized model.[5–7] What is different in the two models is essentially the allowed ranges of m_t and m_s. We should stress that, for computing U, the generalized Fritzsch-Stech model is *identical* to the Fritzsch model. This may be seen by noting that, as discussed in Ref. 1, the general Fritzsch *Ansatz* may be written as

$$M_u = \mathcal{S}_u,\quad M_d = P\mathcal{S}_d P^{-1}\ ,$$

where $\mathcal{S}_u$ and $\mathcal{S}_d$ are real symmetric matrices while $P = \text{diag}(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3})$ with $\sum \phi_i = 0$. There are evidently eight inputs required which are conveniently taken to be the six quark masses and $|U_{us}|$ and $|U_{cb}|$. The above is the same as (2.6a) and (2.6b) when we identify

$$\mathcal{S}_u = S + \epsilon S' ,$$

$$(\mathcal{S}_d)_{ij} \cos(\phi_i - \phi_j) = (\alpha S + S')_{ij} ,$$

$$i(\mathcal{S}_d)_{ij} \sin(\phi_i - \phi_j) = A_{ij} .$$

In our analysis we used the same eight inputs for the quark sector, so the resulting predictions must be identical to those of the general Fritzsch model. From the quark sector evidently $(S+\epsilon S')$, $(\alpha S+S')$, and A are determined. The three parameters needed to completely specify the model may be taken to be α, ϵ, and δ [see Eq. (2.4c)]. These are found by specifying the three charged-lepton masses. Note that for $\alpha\epsilon=1$ the combinations $(S+\epsilon S')$ and $(\alpha S+S')$ are proportional to each other and we regain the original Fritzsch-Stech model.

In order to make meaningful predictions it is necessary to allow for the experimental and theoretical uncertainties in the input parameters. In this way it was found in Ref. 7 that the original Fritzsch-Stech model could only tolerate a physical t-quark mass less than about 45 GeV. For a value of the K-meson bag parameter, B_k smaller than $\frac{1}{2}$, the model has difficulty in explaining the CP impurity parameter ϵ. With the present theoretical uncertainty in B_k the model cannot be ruled out on this basis alone. However, this may already suggest that the Fritzsch-Stech model be generalized as in (2.6a) and (2.6b) to allow the physical m_t to be as large[24] as about 95 GeV. In Fig. 2 of Ref. 7, the allowed ranges of m_s and m_t are displayed for various values of $|U_{us}|$ and $|U_{cb}|$ in the generalized model. The most reasonable choices of $|U_{us}|$ and $|U_{cb}|$ correspond to the central diagram—Fig. 1 of the present paper—which would then limit m_t to be less than about 80 GeV. It is seen that in order to accommodate a large m_t, as suggested by the ϵ calculation and the B_d-$\bar{B}_d$ mixing calculation to be discussed, the strange-quark mass m_s must be on the low side. For example, a physical m_t around 78 GeV [corresponding to a running mass m_t(1 GeV) of 130 GeV] requires m_s to be about 120 MeV. Very recently, a similar analysis based on approximating the Fritzsch model has been carried out by Harari and Nir[9] and the results are consistent with the *exact* results of Ref. 7.

Whether or not m_s(1 GeV) can be as low as 120 MeV is a subject of some debate. The conventional determination is $m_s = 175\pm55$ MeV, but a recent analysis[25] involving QCD sum rules suggests one take 199 ± 33 MeV.

The recent measurement by the ARGUS group[8] of a very substantial B_d-$\bar{B}_d$ mixing has, as noted by the ARGUS group themselves as well as by many theorists,[26] an important bearing on the top-quark mass if it is assumed as usual that t exchange in a box diagram provides the dominant contribution to the B_d-$\bar{B}_d$ mass mixing matrix. For example, Ellis, Hagelin, and Rudaz[26] point out that the most reasonable choice of dynamical parameters gives $m_t > 100$ GeV if one wants to fit the central value of the B_d-$\bar{B}_d$ mixing parameter. If one is willing to reasonably stretch the dynamical parameters (together with allowing the KM parameters to be as large as experimental bounds allow) and accepts the lower limit of the B_d-$\bar{B}_d$ mixing measurement one gets $m_t > 50$ GeV. It is somewhat more difficult to get a large B_d-$\bar{B}_d$ mixing parameter in the Fritzsch or generalized Fritzsch-Stech models. These models predict a KM matrix with a value of $|U_{td}|$ which is considerably lower than the upper limit obtained from unitarity.

In terms of the weak mass difference ΔM and the B_d meson width Γ, the asymmetry[8,26] r_d is given by

$$r_d = 0.21\pm0.08 \simeq \frac{x^2}{2+x^2} , \tag{3.2a}$$

$$x = \Delta M / \Gamma . \tag{3.2b}$$

For purposes of convenient discussion we repeat the main contribution[26] to x:

$$x \approx 0.30 \left| \frac{F_B B_B^{1/2}}{0.15 \text{ GeV}} \right|^2 \left| \frac{\tau_B}{10^{-12} \text{ sec}} \right| f(m_t) m_t^2 |U_{td}|^2 . \tag{3.3}$$

Here F_B and B_B are, respectively, the decay constant and the bag constant for the B_d meson, τ_B is the B lifetime (10^{-12} sec corresponds to $|U_{cb}|=0.05$), while $f(m_t)$ is a slowly varying function equal to 1 for $m_t = 55$ GeV and about 0.9 for $m_t = 80$ GeV. The number 0.15 GeV represents[27] a "central theoretical value" for $F_B B^{1/2}$. From (2.8) [or using Table III and (3.1)] we see that the Fritzsch model predicts, fairly independently of m_t, that $|U_{31}|^2 = 1.44\times10^{-4}$. Let us stretch[27] $F_B B^{1/2}$ to 0.22 GeV and take m_t at about 80 GeV, which is about the highest allowed in the Fritzsch model (for $|U_{cb}|=0.05$). Then (3.3) yields

$$x \approx 0.54 ,$$

which, from (3.2a) implies $r_d \approx 0.13$. This is around the lower experimental limit.

Thus we conclude that the Fritzsch model and the generalized Stech-Fritzsch scheme can accommodate the observed B_d-$\bar{B}_d$ mixing within the present experimental and theoretical uncertainties of the relevant quantities. For comparison, the original Stech model cannot give rise to

TABLE III. The two predicted parameters of the KM mixing matrix for quarks $|U_{ub}|$ and the invariant phase Φ are displayed for various values of top-quark and strange-quark masses. Note that we are fixing $|U_{us}|=0.225$ and $|U_c|=0.05$.

m_t \ m_s	0.12	0.14	0.16	0.18	0.20
130	0.0032	No			$\|U_{ub}\|$
(78)	101°	fit			Φ
110	0.0033	No			
(66)	99°	fit			
90	0.0034	0.0036	No	No	
(54)	96°	99°	fit	fit	
70	No	0.0038	0.0041	0.0043	No
(42)	fit	95°	98°	102°	fit
50	0.0035	0.0039	No	0.0046	0.005
(31)	87°	89°	fit	94°	97°

a sufficient amount of mixing,[9] basically because of the lower values allowed for m_t.

The best fit in the generalized scheme or the Fritzsch model still requires the B_d-$\bar{B}_d$ mixing to lie one σ below the central measured value. Only improvement in the latter measurement and some better estimates of the theoretical parameters involved in this study can either confirm or invalidate the generalized Stech-Fritzsch scheme.

Very recently the ARGUS Collaboration identified[14] the first charmless baryonic B decays. The ARGUS group used the data to set a lower limit of 0.07 on U_{ub}/U_{cb} using a gross estimate. A more detailed theoretical study[28] of these modes arrived at a similar estimate, although a more likely value for this ratio seems to emerge near its present upper limit of 0.20. Some suggestions were presented for a more precise determination of $|U_{ub}|/U_{cb}|$. This seems to be crucially needed for a critical test of the Fritzsch and the generalized Stech-Fritzsch models. At present the models are just about consistent with lower limit.

We would like to mention the following two caveats about the above discussion of B_d-$\bar{B}_d$.

(i) One should bear in mind that ΔM, as computed by the box diagram, is sensitive to *any* object which couples to the b quark regardless of its mass. In particular, any realistic grand unified theory will have many Higgs multiplets. This is clearly true for the present model. It has been noted[29] that the Glashow-Iliopoulos-Maiani (GIM) suppression mechanism is partially evaded for Higgs contributions to box diagrams. An experimental test for such a contribution[30] might be the observation of $\tau \rightarrow \nu\eta\pi$ at some nontrivial level (say a branching ratio between 0.1% and the present bound of around 1%).

(ii) The need for an independent confirmation of the two striking ARGUS results, of the B_d-$\bar{B}_d$ mixing and of the charmless baryonic decays of B mesons, is quite evident. The errors on both measurements are still large. The theoretical routes from these two measurements to the actual values of the mixing elements $|U_{td}|$ (correlated with m_t) and $|U_{ub}|$, respectively, are long and they involve substantial theoretical uncertainties (compare, e.g., Shifman[26] with Cudell *et al.*[26]) which deserve careful treatment.

It is interesting that while the simple interpretation of the ARGUS B_d-$\bar{B}_d$ result tends to favor relatively large m_t, the interpretation of a different experiment on neutrino counting favors a smaller m_t. The ratio of Z width to W width, Γ_Z/Γ_W, measured at UA1 and UA2 depends on top-quark mass as well as the number of light neutrinos. Assuming three families of light neutrinos, Barger *et al.*[31] have argued that at 90% confidence level one needs $m_t < 68$ GeV to fit Γ_Z/Γ_W. However, attempts to derive upper limits on m_t from UA1 and UA2 width measurements are subject to theoretical uncertainties in the gauge-boson production cross sections.[32]

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[17]Note that we have taken here $m_d/m_s=0.056$ in order to most easily fit $|U_{us}|=0.225$. By choosing an appropriate m_d/m_s for a given $|U_{us}|$ one can obtain allowed regions in the $m_s m_t$ plane similar to the central column of Fig. 2 in Ref. 7.

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