Highly accurate solution of the neutron-antineutron-transition problem in an external oscillating magnetic field

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A nonperturbative method is used to find the solution of neutron-antineutron transitions in an external magnetic field. The convergency of the method is very high, and allows one to obtain a practically exact solution of the problem for all physical values of the parameters of applied magnetic field. In the resonant region of frequencies the eigenstate and initial-value problems are solved exactly, with the shift and the width of the resonance calculated to second order in the ratio of coupling and applied frequencies. The obtained results go over smoothly into the first-order perturbation results for frequencies far out of resonance.

I. INTRODUCTION

The possibility of the conversion of free neutrons into antineutrons, guessed in the early 1980s from a purely phenomenological point of view, has gotten a great deal of attention recently in the modern context of gauge theories.[1] As an alternative to theories predicting the proton decay, a class of left-right-symmetry models[2] predicts the $n\leftrightarrow\bar{n}$ transitions.

Several experiments were undertaken and some are underway to search for those very slow baryon-nonconserving transitions.[3] Since neutrons do have internal structure and therefore an anomalous magnetic moment, the experiments face a technical difficulty of degaussing the Earth's magnetic field. If not properly degaussed, that field pushes the neutron and the antineutron states away from each other and reduces the transitions by a few orders of magnitude.

It is then a natural idea to try to restore these oscillations by driving them with additional oscillatory magnetic field. The free parameters of the field, of course, have to be tuned so as to optimize the growth of the antineutron probability.

Several authors attacked that problem by trying to solve either numerically or analytically the underlying system of linear differential equations with periodic coefficients:[4]

$$\frac{dn(t)}{dt}=-i\omega_B(t)n(t)-i\omega_m\bar{n}(t)\ , \tag{1.1a}$$

$$\frac{d\bar{n}(t)}{dt}=-i\omega_m n(t)+i\omega_B(t)\bar{n}(t)\ , \tag{1.1b}$$

where units $\hbar=c=1$ are used, ω_m is the frequency characterizing the fundamental baryon mixing force, $\omega_B(t)$ is the time-dependent external-magnetic-field coupling frequency, while n and $\bar{n}$ are the neutron and the antineutron wave functions, respectively.

This system of equations is equivalent to a Hill equation and an exact solution, of course, cannot be expected. The existing solutions[4–8] might to a certain extent meet the purpose from a practical point of view; however, they are not fully satisfactory from a mathematical point of view. Being approximate, it is hard to understand some of their properties; for instance, why the constant-field limit is not properly reproduced, how large the uncertainties are, etc.

We believe that the solution we present here is an essential improvement as it throws light on previous solutions as special cases.

The applied-field frequency is assumed to have the general form

$$\omega_B(t)=\omega_0-A\sin\omega t+B\cos\omega t$$
$$\equiv\omega_0+W\cos(\omega t+\phi)\ . \tag{1.2}$$

Obviously $W=(A^2+B^2)^{1/2}$ and $\phi=\arctan(A/B)$. Since, according to estimates by other authors,[2] $\omega_m\sim10^{-4}\ \mathrm{s}^{-1}$ and $\omega_0\sim10^{4}\ \mathrm{s}^{-1}$, we note that $\omega_m\ll\omega_0$.

The plan of the article is the following. In Sec. II the eigenvalue problem is solved for the total Hamiltonian, with the magnetic field (1.2) present. This is used in Sec. III to define and solve the initial-value problem for neutron-antineutron oscillations. The transition probabilities are found in closed form. Our conclusions are given in Sec. IV.

II. EIGENVALUE PROBLEM

It is well known[9,10] (although in different context) that the interaction of a quantum system with a time-dependent oscillating field can be defined with a time-independent Hamiltonian, represented by an infinite matrix. Therefore, the problem can be studied in extended Hilbert space[10] of "steady states" with quasienergies which are defined up to mod($n\omega$). The problem of transitions between bound states of a system in oscillating field can be then treated by time-independent perturbation theory,[10] or, in case of resonances, by degenerate time-independent perturbation theory.[9]

To obtain the "steady states" and quasienergies for the system described by Eqs. (1.1) and (1.2) it is convenient to define the Schrödinger equation which corresponds to the coupled equations (1.1):

$$\left[i\frac{\partial}{\partial t}-H_0(t)-V\right]\Psi=0 \tag{2.1}$$

with

$$\Psi=n(t)\,|\,1\rangle+\bar{n}(t)\,|\,2\rangle\ , \tag{2.2}$$

where $|\,1\rangle$ and $|\,2\rangle$ are (orthonormalized) neutron and antineutron states, respectively, and n, $\bar{n}$ are interpreted as the time-dependent amplitudes of population of the respective states. According to Eq. (1.1), H_0 and V in Eq. (2.1) are defined by relations

$$H_0\,|\,1\rangle=\omega_B\,|\,1\rangle,\quad H_0\,|\,2\rangle=-\omega_B\,|\,2\rangle\ , \tag{2.3a}$$

$$V\,|\,1\rangle=\omega_m\,|\,2\rangle,\quad V\,|\,2\rangle=\omega_m\,|\,1\rangle\ . \tag{2.3b}$$

Because of the smallness of ω_m, the reasonable choice of the unperturbed basis states are the eigenstates of $(H_0-i\partial/\partial t)$, that is

$$\left[H_0-i\frac{\partial}{\partial t}\right]u_1(t)\,|\,1\rangle=\epsilon_1 u_1(t)\,|\,1\rangle\ , \tag{2.4a}$$

$$\left[H_0-i\frac{\partial}{\partial t}\right]u_2(t)\,|\,2\rangle=\epsilon_2 u_2(t)\,|\,2\rangle\ . \tag{2.4b}$$

According to the Floquet theorem,[11] $u_i(t)$, $i=1,2$ are periodic functions of time of the period $T=2\pi/\omega$, where ω is the frequency of oscillation of the applied magnetic field. From Eqs. (2.4) it is clear that

$$u_{1,2}=\exp\left[i\left[\epsilon_{1,2}t\mp\omega_0 t-\frac{w}{\omega}\sin(\omega t+\phi)\right]\right] \tag{2.5}$$

and the condition of periodicity yields the sets of eigenvalues of the problems (2.4):

$$\epsilon_{1,2}^{(l)}=\pm\omega_0+lw\ , \tag{2.6}$$

where l is an integer. An unperturbed eigenstate $u_{1,2}^{(l)}$, given by Eq. (2.5), corresponds to each $\epsilon_{1,2}^{(l)}$.

The eigenstates defined by Eqs. (2.5) and (2.6) have the following properties:

$$\langle u_i^{(l)}\,|\,u_i^{(l')}\rangle=\delta_{l,l'},\quad i=1,2\ , \tag{2.7a}$$

$$\langle u_1^{(l)}\,|\,u_2^{(l')}\rangle=\langle u_2^{(l')}\,|\,u_1^{(l)}\rangle^* =J_{l-l'}(x)e^{i(l-l')\phi}\ , \tag{2.7b}$$

$$x=\frac{2W}{\omega}\ , \tag{2.7c}$$

where the brackets in Eqs. (2.7) mean

$$\langle a\,|\,b\rangle=\frac{1}{T}\int_0^T a^*b\,dt\ , \tag{2.7d}$$

and $J_{l-l'}(x)$ is the Bessel function. From the frequency spectrum of the basis states (2.6) (Fig. 1), it is clear that the perturbation ω_m strongly couples $u_1^{(l)}$ and $u_2^{(l')}$ in the region of frequencies ω where $\epsilon_1^{(l)}$ and $\epsilon_2^{(l')}$ are nearly or completely degenerate (resonant region). Obviously, $\epsilon_1^{(l)}=\epsilon_2^{(l')}$ for

$$\omega=\omega_N^{(0)}\equiv\frac{2\omega_0}{N}\ , \tag{2.8a}$$

where

$$N=l-l',\quad l,l'\ \text{integers}\ . \tag{2.8b}$$

It should be noted that for each resonant frequency $\omega_N^{(0)}$ there are infinitely many resonant regions (crossings of the $\epsilon_1^{(l)}$ and $\epsilon_2^{(l')}$ in Fig. 1) defined by Eq. (2.8).

Far enough from the crossings, the coupling between the states $u_{1,2}^{(l)}$ can be treated perturbatively. In order to develop a theory that will simultaneously account for the strong coupling of the states $u_{1,2}^{(l)}$ in vicinity of resonance and the nonresonant weak coupling, it is convenient to use the Feshbach[12] projection operator formalism in the

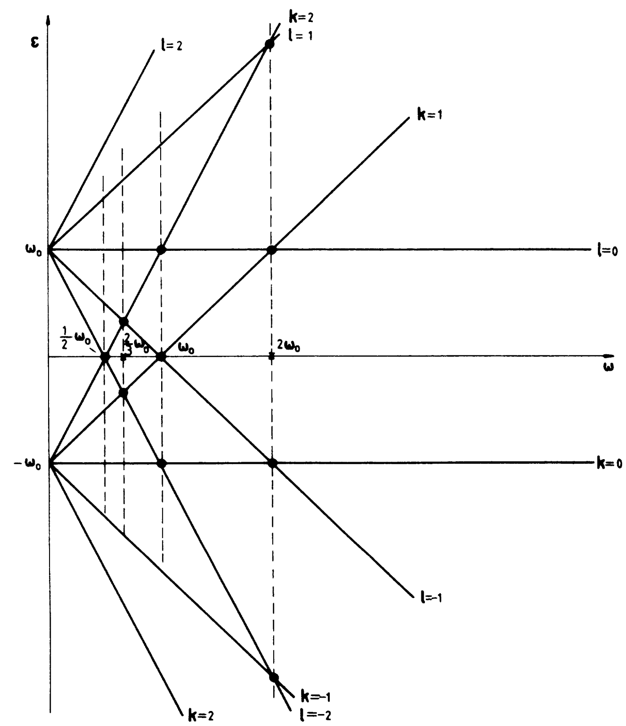

FIG. 1. Frequency spectrum of eigenvalues of unperturbed Hamiltonian $H_0(t)$.

form appropriate for the time-dependent problems.[13]

Define a projection operator

$$P_{l,N}=|1\rangle|u_1^{(l)}\rangle\langle u_1^{(l)}|\langle 1| + |2\rangle|u_2^{(l+N)}\rangle\langle u_2^{(l+N)}|\langle 2| \qquad (2.9)$$

with the properties

$$P_{l,N}^2=P_{l,N},\quad Q_{l,N}^2=Q_{l,N},\quad P_{l,N}Q_{l,N}=0\ , \qquad (2.10)$$

where $Q_{l,N}$ is the complement of $P_{l,N}$ in the whole Hilbert space of the problem

$$Q_{l,N}=1-P_{l,N}\ . \qquad (2.11)$$

From the Schrödinger equation (2.1), the eigenvalue problem for the total Hamiltonian is

$$\left[H_0+V-i\frac{\partial}{\partial t}\right]u=\epsilon u\ . \qquad (2.12)$$

With the use of Eqs. (2.10) and (2.11), Eq. (2.12) is decomposed into a set of two coupled equations:

$$P_{l,N}\left[i\frac{\partial}{\partial t}+\epsilon-H_0-V\right]P_{l,N}u=P_{l,N}VQ_{l,N}u\ , \qquad (2.13a)$$

$$Q_{l,N}\left[i\frac{\partial}{\partial t}+\epsilon-H_0-V\right]Q_{l,N}u=Q_{l,N}VP_{l,N}u\ . \qquad (2.13b)$$

Defining the Green's function

$$Q_{l,N}\left[i\frac{\partial}{\partial t}+\epsilon-H_0-V\right]Q_{l,N}G=Q_{l,N}\ , \qquad (2.14)$$

the formal solution for $Q_{l,N}u$ is obtained in the form

$$Q_{l,N}u=Q_{l,N}GQ_{l,N}P_{l,N}u\ . \qquad (2.15)$$

This is substituted back into Eq. (2.13a), to produce the equation for $P_{l,N}u$:

$$P_{l,N}\left[i\frac{\partial}{\partial t}+\epsilon-H_0-V-VQ_{l,N}GQ_{l,N}V\right]P_{l,N}u=0\ . \qquad (2.16)$$

Although this equation is exact, it presumes the knowledge of the exact Green's function of the problem. Fortunately, $Q_{l,N}$ excludes the resonant states l and $l+N$ form G which enables one to use the perturbation expansion for $Q_{l,N}GQ_{l,N}$. Therefore, one has

$$Q_{l,N}GQ_{l,N}\approx Q_{l,N}G_0Q_{l,N}+O\left[\frac{\omega_m}{\omega}\right]\ , \qquad (2.17a)$$

where

$$Q_{l,N}G_0Q_{l,N}=\sum_{j\neq l}\left[\frac{|1\rangle|u_1^{(j)}\rangle\langle u_1^{(j)}|\langle 1|}{\epsilon-\epsilon_1^{(j)}}+\frac{|2\rangle|u_2^{(j+N)}\rangle\langle u_2^{(j+N)}|\langle 2|}{\epsilon-\epsilon_2^{(j+N)}}\right]\ . \qquad (2.17b)$$

The higher-order corrections in ω_m/ω can be easily included in (2.17a), but for our purposes this is not necessary due to the smallness of ω_m/ω and due to the fact that the Hamiltonian of Eq. (2.16) already contains a term of order V^2. To solve Eq. (2.16), $P_{l,N}u$ is expanded as

$$P_{l,N}u=\alpha_l|u_1^{(l)}\rangle|1\rangle+\beta_{l+N}|u_2^{(l+N)}\rangle|2\rangle\ , \qquad (2.18)$$

where α_l,β_{l+N} are constants. Then, Eq. (2.16) yields two coupled equations for α_l, β_{l+N}:

$$[E_N-\omega_0-\omega_m^2A_N(E_N)]\alpha_l=\omega_mJ_{-N}(x)e^{-iN\phi}\beta_{l+N}\ , \qquad (2.19a)$$

$$[E_N+\omega_0-N\omega-\omega_m^2B_N(E_N)]\beta_{l+N}=\omega_mJ_{-N}(x)e^{iN\phi}\alpha_l\ , \qquad (2.19b)$$

where

$$A_N(E_N)=\sum_{j\neq 0}\frac{J_{N-j}^2(x)}{E_N-\epsilon_2^{(N-j)}}\ , \qquad (2.20a)$$

$$B_N(E_N)=\sum_{j\neq 0}\frac{J_{N-j}^2(x)}{E_N-\epsilon_1^{(j)}}\ , \qquad (2.20b)$$

while the quasienergies ϵ are defined by

$$\epsilon=\epsilon_{l,N}=E_N+l\omega,\quad l\ \text{an integer}\ . \qquad (2.21)$$

Since E_N, found by successive approximations from the two-state problem (2.19), does not depend upon l, the choice of a particular l in construction of the projection operator (2.9) is not important and E_N, together with relation (2.21), produces the full set of quasienergies $\epsilon_{l,N}^{(\pm)}$, valid in vicinity of the Nth-order resonance, $\omega\approx\omega_N^{(0)}$.

The result for E_N is

$$E_{N\pm}=\tfrac{1}{2}(N\omega\mp\omega_m^2a_N\pm\Omega_N)\ , \qquad (2.22)$$

where

$$\Omega_N=[\Delta_N^2+4\omega_m^2J_N^2(x)]^{1/2}\ , \qquad (2.23a)$$

$$\Delta_N=\Delta_N^{(0)}+\omega_m^2b_N\ , \qquad (2.23b)$$

$$\Delta_N^{(0)}=\epsilon_1^{(l)}-\epsilon_2^{(l+N)}=2\omega_0-N\omega\ , \qquad (2.23c)$$

and

$$a_N=\sum_{j\neq 0}\frac{J_{N-j}^2(x)\Omega_N^{(0)}}{j\omega(\Delta_N^{(0)}+j\omega)}\ , \qquad (2.24a)$$

$$b_N=\sum_{j\neq 0}\frac{J_{N-j}^2(x)(2j\omega+\Delta_N^{(0)})}{j\omega(\Delta_N^{(0)}+j\omega)}\ , \qquad (2.24b)$$

with

$$\Omega_N^{(0)}=\Omega_N(\Delta_N=\Delta_N^{(0)})\ . \qquad (2.24c)$$

Solving now for the coefficients $\alpha_l^{(\pm)}$, $\beta_{l+N}^{(\pm)}$ with each of the eigenvalues

$$\epsilon_{l,N}^{(\pm)}=E_{N\pm}+l\omega \qquad (2.25)$$

one obtains for the projected part of the wave function

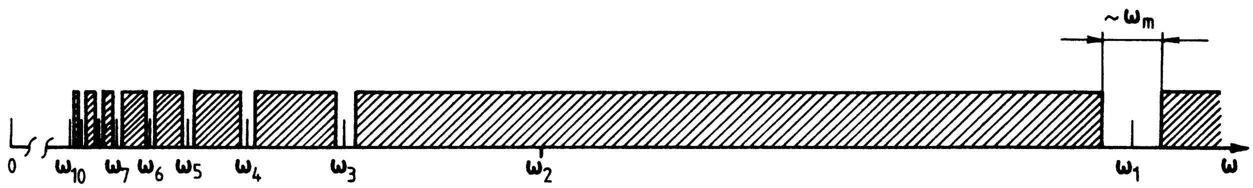

FIG. 2. The frequency domain of validity of the steady states (2.30). The shaded regions apply for $N=2$.

$$(P_{l,N}u)_{\pm}=\left[\,|u_1^{(l)}\rangle\,|1\rangle+\frac{2\omega_m J_{-N}(x)e^{iN\phi}}{\Delta_N\pm\Omega_N}\,|u_2^{(l+N)}\rangle\,|2\rangle\right] \tag{2.26a}$$

and, for the residual part of the wave function,

$$(Q_{l,N}u)_{\pm}=Q_{l,N}G_0(\epsilon_{l,N}^{(\pm)})Q_{l,N}V(P_{l,N}u)_{\pm}\ . \tag{2.26b}$$

Combining Eqs. (2.26), and normalizing the resulting eigenstates, the steady states of the total Hamiltonian of the problem are obtained in the form

$$\begin{aligned} u_{l\pm}^{(N)}&=C_{0\pm}^{(N)}(\alpha_N^{(\pm)}\,|1\rangle+\beta_N^{(\pm)}\,|2\rangle)e^{il\omega t}\\ &=u_{0\pm}^{(N)}e^{il\omega t}\ , \end{aligned} \tag{2.27}$$

where

$$\alpha_N^{(\pm)}=|u_1^{(0)}\rangle+\omega_m s_N^{(\pm)}\sum_{j\neq 0} r_j^{(\pm)}(N)\,|u_1^{(j)}\rangle\ , \tag{2.28a}$$

$$\beta_N^{(\pm)}=s_N^{(\pm)}\,|u_2^{(N)}\rangle+\omega_M\sum_{j\neq N} q_j^{(\pm)}(N)\,|u_2^{(j)}\rangle\ , \tag{2.28b}$$

$$C_{0\pm}^{(N)}=\left[1+\frac{4\omega_m^2 J_N^2(x)}{(\Delta_N\pm\Omega_N)^2}+\omega_m^2\sum_{j\neq N}\frac{J_j^2(x)}{(E_{N\pm}-\epsilon_2^{(j)})^2}+O\left[\frac{\omega_m^4}{\omega^4}\right]\right]^{-1/2}, \tag{2.28c}$$

and

$$s_N^{(\pm)}=\frac{2\omega_m J_{-N}(x)e^{iN\phi}}{\Delta_N\pm\Omega_N}\ , \tag{2.29a}$$

$$r_j^{(\pm)}(N)=\frac{J_{j-N}(x)e^{i(j-N)\phi}}{E_{N\pm}-\epsilon_1^{(j)}}\ ,$$

$$q_j^{(\pm)}(N)=\frac{J_{-j}(x)e^{ij\phi}}{E_{N\pm}-\epsilon_2^{(j)}}\ . \tag{2.29b}$$

Since the whole l dependence of the eigenstates (2.27) is in the exponential factor $e^{il\omega t}$, using the relation (2.25) it follows that the eigenstates with different l are physically equivalent and, therefore, there are only two physically resolvable steady states in the considered problem. These are given by

$$\psi_{\pm}^{(N)}=u_{l\pm}^{(N)}e^{-i\epsilon_{l,N}^{(\pm)}t}=u_0^{(N)}e^{-iE_{N\pm}t}\ , \tag{2.30}$$

$\psi_{\pm}^{(N)}$ form an orthonormal set of states. It is important to determine the range of validity of the "steady states" (2.30). These states are derived for the vicinity of the resonance $\omega_N^{(0)}=2\omega_0/N$, where the coupling of the eigenstates $u_1^{(l)}$, $u_2^{(l')}$, $l'-l=N$, is treated exactly, while the coupling of these states to all other eigenstates, is treated perturbatively. Therefore, the conditions of validity of the steady states $\psi_{\pm}^{(N)}$ (Fig. 2) is that the frequency ω of the applied magnetic field is far enough from all other resonances, except ω_N, that is, for $\kappa\neq 0$,

$$\delta=\frac{\omega_m}{2\omega_0-(N\pm\kappa)\omega}\ll 1,\quad \kappa \text{ an integer}\ . \tag{2.31}$$

At the exact resonance, $\Delta_N=0$, from Eq. (2.27) it follows that

$$u_{0\pm}^{(N)}=\frac{1}{\sqrt{2}}(\,|u_1^{(0)}\rangle\,|1\rangle\pm e^{iN\phi}\,|u_2^{(N)}\rangle\,|2\rangle) \tag{2.32}$$

and, therefore, the neutron and antineutron states form two mixed states which are split in energy by

$$E_{N+}-E_{N-}=2\omega_m\,|J_N(x)|\ . \tag{2.33}$$

This splitting introduces "avoided crossings" in Fig. 1 at the resonant frequencies.

On the other hand, if the relation (2.31) is satisfied for each κ, then the steady states become those that would follow directly from the first-order perturbation theory in the small parameter δ:

$$\psi_{+}^{(N)}\to\left[\,|u_1^{(0)}\rangle\,|1\rangle+\omega_m\sum_j\frac{J_{-j}(x)e^{ij\phi}}{2\omega_0-j\omega}\,|u_2^{(j)}\rangle\,|2\rangle\right]e^{-i(\omega_0+\delta_s)t}\ , \tag{2.34a}$$

$$\psi_{-}^{(N)}\to\left[\omega_m\sum_j\frac{J_{-j}(x)e^{-ij\phi}}{2\omega_0-j\omega}\,|u_1^{(-j)}\rangle\,|1\rangle-|u_2^{(0)}\rangle\,|2\rangle\right]e^{+i(\omega_0+\delta_s)t+iN\phi}\ , \tag{2.34b}$$

where

$$\delta_s=\omega_m^2 \sum_j \frac{J_j^2(x)}{2\omega_0-j\omega} \tag{2.35}$$

is the second-order perturbation theory shift of the "bare state" energies. In derivation of Eqs. (2.34) small terms of order of $(\omega_m/\omega)^2$ have been neglected. The constant phase $iN\phi$ in (2.34b) does not influence the transition probability (Sec. III) and can be omitted. Since the first-order perturbation theory in δ (when $\delta \ll 1$) and the two-state strong-coupling theory (when $\delta \approx 1$) are adequate descriptions of the respective physical situations we conclude that for arbitrary values of the parameters of the applied oscillatory magnetic field, the "steady states" (2.30) are adequate description of the eigenfunctions for the total Hamiltonian of the problem.

When the amplitude W of the applied magnetic field tends to zero, the resulting out-of-resonance situation would be described well by the corresponding limit $W \to 0$ in Eqs. (2.34). Really, from (2.34),

$$\psi_+^{(N)} \xrightarrow[W\to 0]{} \left[|1\rangle + \frac{\omega_m}{2\omega_0} |2\rangle \right] \exp\left[-i \left[\omega_0 + \frac{\omega_m^2}{2\omega_0} \right] t \right] , \tag{2.36a}$$

$$\psi_-^{(N)} \xrightarrow[W\to 0]{} \left[\frac{\omega_m}{2\omega_0} |1\rangle - |2\rangle \right] \times \exp\left[i \left[\omega_0 + \frac{\omega_m^2}{2\omega_0} \right] t + iN\phi \right] , \tag{2.36b}$$

which is the well-known first-order perturbation-theory result (in small ω_m/ω_0) when $\omega_B=\omega_0$.

The exact steady states in the static-field limit can be obtained if one starts from Eqs. (2.30), instead of from the perturbation expansion (2.34) of the steady states, since the former states account for the coupling of the projected states $P_{l,N}u$ exactly. But, when the applied field is zero, the only two unperturbed eigenstates (Fig. 1) that interact are the states with energies $\pm\omega_0$ (which correspond to indices $l=0$, $l'=0$ in Fig. 1). Therefore $P_{l,N}$ projects to the states with $N=l-l'=0$, and together with the limit $W \to 0$ one must set $N=0$ in Eq. (2.30). Then it follows

$$\psi_\pm^{(N)} \xrightarrow[W\to 0]{} \frac{(\epsilon_0 \pm \omega_0)^{1/2}}{(2\epsilon_0)^{1/2}} \left[|1\rangle \pm \frac{\omega_m}{\epsilon_0 \pm \omega_0} |2\rangle \right] e^{\pm i\epsilon_0 t} , \tag{2.37a}$$

where

$$\epsilon_0=(\omega_0^2+\omega_m^2)^{1/2} , \tag{2.37b}$$

which can be easily shown to be the exact eigenstates for the neutron-antineutron oscillation problem in the static magnetic field ω_0.

According to Eq. (2.17a), our steady states are not valid for those frequencies ω for which $\omega_m/\omega \sim 1$. Therefore $N \ll 2\omega_0/\omega_m \sim 10^8$. Still, even at $\omega \sim 1$ Hz, $\omega_m/\omega \sim 10^{-4} \ll 1$. Since no conditions are set to the magnetic-field amplitude W and initial phase ϕ one can say that the steady states are the quasiexact eigenfunctions of the total Hamiltonian for any realistic range of the applied-field parameters.

III. THE INITIAL-VALUE PROBLEM AND TRANSITION PROBABILITIES

With the states (2.30), used as a basis, the initial-value problem can be formulated, expanding the total wave function

$$\psi(t)=D_+^{(N)}\psi_+^{(N)}+D_-^{(N)}\psi_-^{(N)} . \tag{3.1}$$

The amplitudes $n(t)$ and $\bar{n}(t)$ of the neutron and antineutron states, respectively, are

$$\begin{aligned} n(t)&=\langle 1 | \psi(t)\rangle = D_+^{(N)}\langle 1 | \psi_+^{(N)}\rangle + D_-^{(N)}\langle 1 | \psi_-^{(N)}\rangle \\ &= D_+^{(N)}C_{0+}^{(N)}\alpha_N^{(+)}(t)e^{-E_{N+}t} \\ &\quad + D_-^{(N)}C_{0-}^{(N)}\alpha_N^{(-)}(t)e^{-iE_{N-}t} , \end{aligned} \tag{3.2a}$$

$$\bar{n}(t)=D_+^{(N)}C_{0+}^{(N)}\beta_N^{(+)}(t)e^{-iE_{N+}t}+D_-^{(N)}C_{0-}^{(N)}\beta_N^{(-)}(t)e^{-iE_{N-}t} . \tag{3.2b}$$

The initial conditions

$$n(0)=1, \quad \bar{n}(0)=0 , \tag{3.3}$$

yield

$$D_+^{(N)}=\frac{\beta_N^{(-)}(0)}{C_{0+}^{(N)}D_0^{(N)}}, \quad D_-^{(N)}=-\frac{\beta_N^{(+)}(0)}{C_{0-}^{(N)}D_0^{(N)}} , \tag{3.4}$$

where

$$\begin{aligned} D_0^{(N)}&=\alpha_N^{(+)}(0)\beta_N^{(-)}(0)-\alpha_N^{(-)}(0)\beta_N^{(+)}(0) \\ &= -\frac{\Omega_N e^{iN\phi}}{\omega_m J_{-N}(x)} \left[1+O\left[\left[\frac{\omega_m}{\omega} \right]^2 \right] \right] . \end{aligned} \tag{3.5}$$

From Eq. (3.2) it follows that

$$\bar{n}(t)=e^{i\bar{\phi}_N(t)}(\bar{N}_1 \cos\xi_N t - i\bar{N}_2 \sin\xi_N t) , \tag{3.6a}$$

$$n(t)=e^{i\phi_N(t)}(N_1 \cos\xi_N t - iN_2 \sin\xi_N t) , \tag{3.6b}$$

where

$$\bar{N}_1=\omega_m f_N^{(-)} , \tag{3.7a}$$

$$\bar{N}_2=\frac{\omega_m}{\Omega_N}[2J_{-N}(x)+\Delta_N f_N^{(+)}] , \tag{3.7b}$$

$$N_1=1+O\left[\frac{\omega_m^2}{\omega^2} \right] , \tag{3.7c}$$

$$N_2=\frac{\Delta_N}{\Omega_N} , \tag{3.7d}$$

$$\bar{\phi}_N=2x[\sin\phi+\sin(\omega t+\phi)]+N\left[\frac{\omega t}{2}+\phi \right] , \tag{3.7e}$$

$$\phi_N=2x[\sin\phi-\sin(\omega t+\phi)]-iN\frac{\omega t}{2} , \tag{3.7f}$$

$$\xi_N=\tfrac{1}{2}(\Omega_N-\omega_m^2 a_N)\ , \tag{3.7g}$$

and

$$f_N^{(\pm)}=\sum_{j\neq 0}\frac{J_{j-N}(x)e^{-ij\phi}}{j\omega+\Delta_N}(e^{-ij\omega t}\pm 1)\ . \tag{3.8}$$

The probability of the transition from neutron to antineutron states is then

$$\begin{aligned}P_{21}(t)&=|\bar{n}(t)|^2\\&=|\bar{N}_1|^2\cos^2\xi_N t+|\bar{N}_2|^2\sin^2\xi_N t\\&\quad-2\frac{\omega_m^2}{\Omega_N}[J_{-N}(x)\,\mathrm{Im}(f_N^{(-)})+\Delta_N\,\mathrm{Im}(f_N^{(-)}f_N^{(+)*})]\\&\quad\times\sin(2\xi_N t)\ ,\end{aligned} \tag{3.9}$$

where

$$|\bar{N}_1|^2=\omega_m^2|f_N^{(-)}|^2\ , \tag{3.10a}$$

$$\begin{aligned}|\bar{N}_2|^2=\frac{\omega_m^2}{\Omega_N^2}[4J_N^2(x)+\Delta_N^2|f_N^{(+)}|^2\\+4\Delta_N J_{-N}(x)\mathrm{Re}(f_N^{(+)})]\ .\end{aligned} \tag{3.10b}$$

In the vicinity of resonance ($\delta\approx 1$), P_{21} reduces to

$$P_{21}=\frac{\Gamma_N^2}{(\omega-\omega_N)^2+\Gamma_N^2}\sin^2\left\{\frac{N}{2}[(\omega-\omega_N)^2+\Gamma_N^2]^{1/2}\times t\left[1+O\left[\frac{\omega_m}{\omega}\right]\right]\right\}, \tag{3.11}$$

where Γ_N is the width of the Lorentzian resonant peak

$$\Gamma_N=\frac{2\omega_m}{N}|J_N(x)|\left[1+O\left[\frac{\omega_m}{\omega}\right]\right] \tag{3.12a}$$

and ω_N is the shifted position of the resonance peak

$$\omega_N=\frac{2\omega_0}{N}+\frac{\omega_m^2}{N}b_N\left[1+O\left[\frac{\omega_m}{\omega}\right]\right]\ . \tag{3.12b}$$

At $\omega=\omega_N$,

$$P_{21}^{(R)}=\sin^2[\omega_m J_N(x)t]\left[1+O\left[\frac{\omega_m}{\omega}\right]\right]\ . \tag{3.13}$$

It is interesting to note that at particular values of the amplitude W of the applied magnetic field at which, for the fixed ω, $J_N(2W/\omega)=0$, the transition probability drops to the off-resonance value, even at $\omega=\omega_N$.

Since the width of the resonant peak $\Gamma_{N'}$ is of the order of magnitude of ω_m, in a possible experiment the frequency ω must be controlled within the interval $\omega_m|J_N(x)|/N$; or within $\Delta\omega/\omega_N=\omega_m|J_N(x)/2\omega_0\sim(10^{-6})\%$. The applied field intensity should be chosen so that $|J_N(x)|$ has maximum, in order to increase both the resonance width and the probability [see Eq. (3.13)]. For $\omega\simeq\omega_1\simeq W\sim 10^4\ \mathrm{s}^{-1}$, and $J_1(x)\simeq 0.6$ (the value of its first maximum), and $t\sim 10^3$ s (order of magnitude of the mean neutron lifetime), $P_{21}^{(R)}$ is on the order of 10^{-3}. This value is to be compared with the value of 10^{-16} for P_{21} when ω is sufficiently far from the resonance. The above estimate holds for a broad range of variation of the field amplitude corresponding to two neighboring zeros of the J_1 Bessel function.

If $\delta\ll 1$ [Eq. (2.31)], from Eq. (3.9) it follows that the transition probability in first-order perturbation theory

$$P_{21}\xrightarrow{\delta\ll 1}\omega_m^2\left|\sum_j\frac{J_j(x)e^{-ij\phi}}{j\omega+2\omega_0}(e^{-i(j\omega+2\omega_0)t}-1)\right|^2\ , \tag{3.14}$$

which is in agreement with the discussion in the preceding section. Finally, in the static-field limit ($W\to 0$, $N=0$), Eq. (3.9) yields

$$P_{21}=\frac{\omega_m^2}{\epsilon_0^2}\sin^2\epsilon_0 t,\quad W\to 0\ , \tag{3.15}$$

which is the well-known neutron-antineutron transition probability for $\omega_B=\omega_0$.

IV. CONCLUSIONS

The quasiexact solution of the problem of neutron-antineutron oscillations in an applied oscillating magnetic field is expressed by the eigenfunctions of the problem [Eq. (2.30)] and transition probability [Eq. (3.9)] for the neutron state initially populated. The solution is valid for nearly all values of the applied field parameters (the exclusion is the frequency $\omega\sim\omega_m\approx 10^{-4}\ \mathrm{s}^{-1}$, which is unrealistic situation) and in the same time simple enough to be both physically transparent and easily programmable on a personal computer. The amplitudes (3.6) are exact in the vicinity of a resonance, and the width and the shift of the resonance peak are calculated up to second power in $\eta=\omega_m/\omega$. The former smoothly go over into the amplitudes in the off-resonance region where it is given by the first-order perturbation theory in η, and being independent on the index N of its parent resonance region this matches to the amplitudes centered in the neighboring resonant regions. The method enables one to easily improve the results and to get the shift and the width of the resonance peak up to a higher power in η, as well as the perturbative limits of the amplitudes expressed in higher-order perturbation theory expansion. This could be achieved by expanding the exact Green's function of the problem to the higher power in η and approximating $Q_{l,N}GQ_{l,N}$ in Eq. (2.17a), for example, by

$$Q_{l,N}GQ_{l,N} \simeq Q_{l,N}G_0Q_{l,N} + \omega_m \sum_{\substack{j\neq l \\ n\neq l+N}} \frac{J_{j-n}(x)}{(\epsilon-\epsilon_1^{(j)})(\epsilon-\epsilon_2^{(n)})} (| u_1^{(j)} \rangle | 1 \rangle \langle 1 | \langle u_2^{(n)} | e^{i(j-n)\phi} + u_2^{(n)} \rangle | 2 \rangle \langle 2 | \langle u_1^{(j)} | e^{-i(j-n)\phi}) + O\left[\frac{\omega_m^2}{\omega^2}\right] , \quad (4.1)$$

where $Q_{l,N}G_0Q_{l,N}$ is given by Eq. (2.17b).

The neutron-antineutron transition probability can reach values on the order of 10^{-3} provided the applied magnetic field oscillation frequency is in the region of a resonance. Note that the width of the resonance region is 10^{-4} s^{-1} and the frequency has to be kept constant within $(10^{-6})\%$, while the variation of the magnetic-field amplitude is not critical.

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