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Effects of d-state quarks on the nucleon electric form factors

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Considering the d orbital excitation of a quark in the bag, we calculate the nucleon electric form factors in the cloudy bag model. In these calculations, we have taken into account the πNN, $\pi\Delta N$, and $\pi\gamma$ form factors though neglecting the c.m. correction. It turns out that the neutron charge form factor is very sensitive to the d-state quark admixture in the overall region of the momentum transfer but the proton charge form factor remains unchanged. Taking the d-state quark admixture in the intermediate-state baryons, we can obtain the nucleon rms radii in remarkable agreement with the experimental values. We also investigate the roles of Δ particles in the nucleon charge form factors.

I. INTRODUCTION

Although a number of calculations have been presented on baryon structures on the basis of quark models, there seems to be no theory sufficiently successful to describe even the static properties of nucleons.

The intrinsic deformations of the nucleon ground state, which were initially speculated on by Glashow[1] to evaluate the nucleon axial-vector coupling constant g_A, have quantitatively been discussed by Ma and Wambach[2] and Navarro and Vento.[3] These authors[2,3] investigated the effect of the d-state quark admixture, respectively, by the linearized chiral bag model with pions expelled from the bag interior and the potential quark model with a pion cloud. Through their calculations, Ma and Wambach found that the isobar was deformed although the nucleon remained spherical if all quarks occupied the lowest deformed single-particle state. However, they could not draw any final quantitative conclusions on deformations. On the other hand, Navarro and Vento[3] investigated the baryon magnetic moments and the nucleon axial-vector coupling constant by a potential quark model with the pion field penetrating inside the quark confinement region. They found that the d-state quark admixtures for the nucleon and isobar, $P_D^N \geq 2\%$ and $P_D^\Delta \geq 7\%$, were necessary to obtain reasonable results. These calculations are restricted only to the magnetic moments and the axial-vector coupling constant and never applied to the other physical quantities such as electromagnetic form factors.

Tegen and Weise[4] investigated the electromagnetic form factors of nucleons by the quark confining-potential model. Their results for the neutron electromagnetic form factor were totally unsatisfactory. Similar calculations by Barik and Dash[5] were also regrettably disappointing. However, these unsuccessful results may not mean an inadequacy of the quark picture for the baryon.

Recently, Theberge and Thomas[6] proposed the cloudy bag model (CBM) to describe successfully the intrinsic structure of baryons. Although the CBM may be better than the usual chiral bag model[7] in a sense that the pion is not explicitly excluded from the bag volume, it is still not entirely free from any objection, because it is difficult to believe that the spherical bag boundary remains static and unperturbed by creation of a pion. Nevertheless, this model is successful to a certain extent in reproducing overall magnetic moments of the nucleon octet.[6]

In this context, it is interesting to examine the CBM by including the d-state quark admixture into the nucleon charge form factors. We will take into account the πNN, $\pi\Delta N$, and $\pi\gamma$ form factors which have been so far neglected in most calculations.

II. THEORY

In the CBM, the interaction Hamiltonian can be obtained by keeping only the linear terms of pion field in the Lagrangian density,[6] and the quark wave functions are derived by solving the Dirac equation inside the bag. The nucleon and Δ wave functions can then be obtained by antisymmetrizing these quarks in the usual way.[8] With these wave functions, we are easily able to derive the nucleon charge form factors.

All these calculations will be carried out under the requirement that the free nucleon consists of all three s-state quarks, but the intermediate-state baryons have a variety of s- and d-state quarks.

Matrix elements for the electric form factors

The baryon electromagnetic current $\hat{j}^{\mu}(x)$ is derived by requiring the local U(1) gauge invariance after introducing the photon field into the CBM Lagrangian.[6] This current can be expressed as a sum of quark and pionic sectors:

$$\hat{j}^{\mu}(x)=\hat{j}^{\mu}_{Q}(x)+\hat{j}^{\mu}_{\pi}(x) , \tag{1}$$

where

$$\hat{j}^{\mu}_{Q}(x)=e_q\bar{q}(x)\gamma^{\mu}q(x)\theta_v \ , \tag{2}$$

$$\hat{j}^{\mu}_{\pi}(x)=ie[\phi^{\dagger}(x)\partial^{\mu}\phi(x)-\phi(x)\partial^{\mu}\phi^{\dagger}(x)] \ . \tag{3}$$

Giving the pion field in the form

$$\phi(x)=[\pi_1(x)+i\pi_2(x)]/\sqrt{2} \ , \tag{4}$$

we find the pion current as

$$j^{\mu}_{\pi}(r)=\tfrac{1}{2}ie\sum_{jj'}\epsilon_{jj'3}\int d^3k\, d^3k'(2\pi)^{-3}(w_k w_{k'})^{-1/2}k_{\mu}[a_{j'}(-\mathbf{k}')+a^{\dagger}_{j'}(\mathbf{k}')][a_j(\mathbf{k})+a^{\dagger}_j(-\mathbf{k})]e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}} \ , \tag{5}$$

where $a^{\dagger}$ and a are the pion creation and annihilation operators and w_k is the pion energy.

Taking the admixture of a d-state quark into account, we express the bare baryon wave function as

$$|B\rangle=C^{1/2}|q_{1/2}q_{1/2}q_{1/2}\rangle+(1-C)^{1/2}|q_{1/2}q_{1/2}q_{3/2}\rangle \ , \tag{6}$$

where the parameter C manifests the ratio of the d-state admixture, which was estimated as 22% by Vento, Baym, and Jackson.[9] For this case, we then find $C=0.78$. Thus, the matrix elements corresponding to Figs. 1(a) and 1(b) are calculated as

$$G^N_{EQ}(q^2)\begin{bmatrix}p\\n\end{bmatrix}=Z^N_2[E_0(q^2)+CE_1(q^2)+(1-C)E_2(q^2)] \ , \tag{7}$$

where

$$E_0(q^2)=\frac{G_0}{R^3}I_0\begin{bmatrix}1\\0\end{bmatrix} \ , \tag{8}$$

$$E_1(q^2)=\frac{3G_0}{4\pi^2R^5}\left[\frac{f^S_Q}{m_\pi}\right]^2 I_0\left[\tfrac{25}{3}I^{NN}\begin{bmatrix}1\\2\end{bmatrix}+\tfrac{32}{3}I^{\Delta\Delta}\begin{bmatrix}4\\-1\end{bmatrix}\right] \ , \tag{9}$$

$$E_2(q^2)=\frac{9G_2}{16\pi^2R^5}\left[\frac{f^D_Q}{m_\pi}\right]^2 K_0\left[\tfrac{4}{81}I^{NN}\begin{bmatrix}2\\1\end{bmatrix}+\tfrac{40}{81}I^{\Delta\Delta}\begin{bmatrix}4\\-1\end{bmatrix}\right] , \tag{10}$$

with

$$G_0=\frac{\Omega_0}{2(\Omega_0-1)j_0^2(\Omega_0)} \ , \tag{11}$$

$$G_1=-\left[\frac{\Omega_0\Omega_1}{(\Omega_0-1)(\Omega_1+2)j_0^2(\Omega_0)j_1^2(\Omega_1)}\right]^{1/2} \ , \tag{12}$$

$$G_2=\frac{\Omega_1}{2(\Omega_1+2)j_1^2(\Omega_1)} \ , \tag{13}$$

$$I_0=\int_0^R dr\, r^2\left[j_0^2\left[\frac{\Omega_0}{R}r\right]+j_1^2\left[\frac{\Omega_0}{R}r\right]\right]j_0(qr) \ , \tag{14}$$

$$K_0=\int_0^R dr\, r^2\left[j_1^2\left[\frac{\Omega_1}{R}r\right]+j_2^2\left[\frac{\Omega_1}{R}r\right]\right]j_0(qr) \ , \tag{15}$$

$$I^{AB}=\int_0^{\infty}dk\, k^2\frac{j_1^2(kR)}{w_k(w_{AN}+w_k)(w_{BN}+w_k)}\times f_{\pi AN}(k)f_{\pi BN}(k) \ , \tag{16}$$

where $w_{AN}=m_A-m_N$. In this case, the renormalization function is given by

$$Z^N_2=[1+C\Lambda_1+(1-C)\Lambda_2]^{-1} \ , \tag{17}$$

where

$$\Lambda_1=\left[\frac{f^S_Q}{m_\pi}\right]^2\frac{1}{12\pi^2}\left[25\int_0^{\infty}dk\frac{k^4u^2(kR)}{w_k^3}+32\int_0^{\infty}dk\frac{k^4u^2(kR)}{w_k(w_k+w_{\Delta N})^2}\right] , \tag{18}$$

$$\Lambda_2=\left[\frac{f^D_Q}{m_\pi}\right]^2\frac{1}{16\pi^2}\left[\frac{4}{9}\int_0^{\infty}dk\frac{k^4u^2(kR)}{w_k^3}+\frac{40}{9}\int_0^{\infty}dk\frac{k^4u^2(kR)}{w_k(w_k+w_{\Delta N})}\right] . \tag{19}$$

The first term of the matrix elements, Eq. (7), corresponds to that obtained when the quark baryon wave function is given by the representation **56** in SU(6). In this case, the neutron charge form factor is zero as is seen in Eq. (8). However, the pion clouds generate nonzero components. The πNN and $\pi\Delta N$ form factors are taken in the form[10]

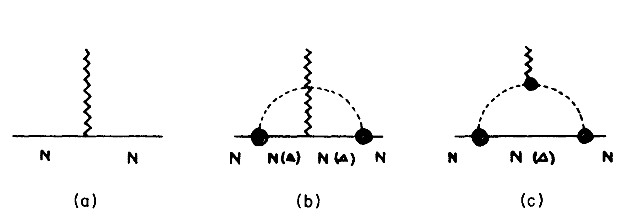

FIG. 1. Feynman diagrams of the nucleon charge form factors. (a) and (b) yield the quark contributions, and (c) shows the contribution from the pion cloud.

$$f_{\pi AN}(k)=\frac{\Lambda_A^2+m_\pi^2}{\Lambda_A^2+m_\pi^2+k^2} \tag{20}$$

with $\Lambda_N=8m_\pi$ and $\Lambda_\Delta=6m_\pi$.

Similarly, the matrix elements for the diagram Fig. 1(c) can be derived as

$$G_{E\pi}^N(q^2)\begin{bmatrix}p\\n\end{bmatrix}=[CE_3(q^2)+(1-C)E_4(q^2)]f_{\pi\gamma}(q^2) , \tag{21}$$

where

$$f_{\pi\gamma}(q^2)=\left[1+\frac{q^2}{m_\rho^2}\right]^{-1} , \tag{22}$$

$$E_3(q^2)=\frac{1}{16\pi^3R^2}\left[\frac{f_Q^S}{m_\pi}\right]^2(800J^{NN}-512J^{\Delta\Delta})\begin{bmatrix}1\\-1\end{bmatrix} , \tag{23}$$

$$E_4(q^2)=\frac{1}{16\pi^3R^2}\left[\frac{f_Q^D}{m_\pi}\right]^2(\tfrac{32}{3}J^{NN}-\tfrac{160}{3}J^{\Delta\Delta})\begin{bmatrix}1\\-1\end{bmatrix} , \tag{24}$$

$$J^{AB}=\int_0^\infty dr\, r^2 j_0(qr)\int_0^\infty dk\, k^2\frac{j_1(kR)j_1(kr)}{w_k+w_{AN}}f_{\pi AN}(k)\int_0^\infty dk' k'^2\frac{j_1(k'R)j_1(k'r)}{(w_{k'}+w_{BN})(w_k+w_{k'})}f_{\pi BN}(k') . \tag{25}$$

Thus, the nucleon electric form factors are given by a sum of the quark sector $G_{EQ}^N(q^2)$ and the pion contribution $G_{E\pi}^N(q^2)$ as

$$G_E^N(q^2)=\alpha_1 G_{EQ}^N(q^2)+\alpha_2 G_E^N(q^2) . \tag{26}$$

The weight coefficients α_1 and α_2 can be determined by the charge-conservation condition:

$$\begin{aligned}&\alpha_1 G_{EQ}^p(0)+\alpha_2 G_{E\pi}^p(0)=1 ,\\ &\alpha_1 G_{EQ}^n(0)+\alpha_2 G_{E\pi}^n(0)=0 .\end{aligned} \tag{27}$$

III. NUMERICAL RESULTS AND CONCLUSION

The results of numerical calculations are shown in Figs. 2–6. The nucleon charge form factors are very much dependent on the bag radius R. This can be seen in Figs. 2 and 3 where we have taken the $\pi\gamma$ and πAN form factors into account. In these figures one can see the trend that for a larger bag radius, the charge form factors decreases versus the square of momentum transfer more rapidly. Such a trend is due to the fact that the overlap integrals, (14) and (15), give smaller values for larger R.

Figure 4 shows that our calculation for the proton charge form factor with $R=0.9$ fm is in remarkable agreement with the data.[11–13] The $\pi\gamma$ and πAN form factors have been taken into account in these calculations. The dot-dashed curve shows the quark contributions, i.e., Figs. 1(a) and 1(b), while the dotted curve is the pion contribution, i.e., Fig. 1(c). Although the pion contribution is smaller by one order of magnitude than the quark contributions, its importance can be seen, particularly, in the lower-q region. And the theoretical curve decreases slightly too fast in the region $q^2>15$ fm^{-2}. This might be improved by taking the higher-order diagrams into account.

On the other hand, the pion sector contributes to the neutron charge form factor almost in the same order of magnitude but in an opposite sign compared with the quark sectors. Particularly, cancellation of the contributions from the pion and quark sectors at $q^2=0$ makes the neutron electrically neutral as a whole (see Fig. 5).

The fact that the pion sector contributes conclusively to the nucleon charge form factors in the low-q^2 region implies that the pion cloud has its own territory mainly in the outer region of the bag. This is consistent with the picture of CBM in which the bag is surrounded by a pion cloud coupled to the quarks at the bag surface.

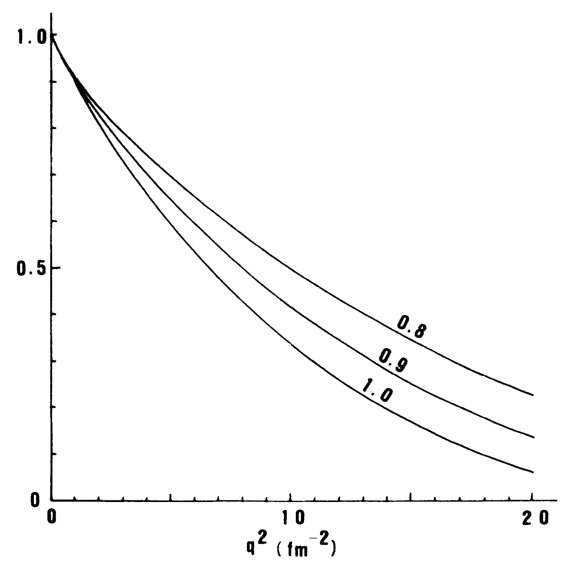

FIG. 2. The proton charge form factors for various values of R. The $\pi\gamma$ and πAN form factors are taken into account.

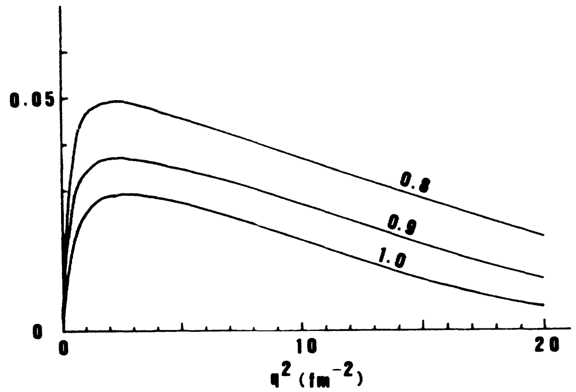

FIG. 3. The neutron charge form factors for various values of R. The $\pi\gamma$ and πAN form factors are taken into account.

Effects of the d-state quark admixtures for the neutron are shown in Fig. 6. The curves labeled with a, b, and c are the results with the d-state quark admixtures of 20%, 40%, and 50%, respectively. For the proton case, these deviations are actually insignificantly small but restore their significances only in the proton charge radius. It is interesting to see that the d-state quark admixture holds the neutron charge form factor down. This can be explained as follows: The large quantities E_1 and E_3 in Eqs. (7) and (21) are relatively suppressed by the d-state quark admixture, and G_{EQ}^N and G_E^N are then reduced. Accordingly, the neutron charge form factor decreases depending on the d-state quark admixture. However, this does not simply happen in the proton case because of the existence of the term E_0 which comes from Fig. 1(a) in-

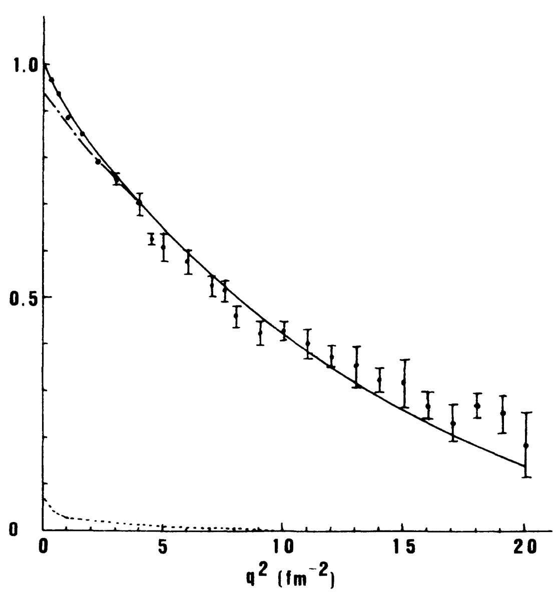

FIG. 4. The proton charge form factor obtained for $R = 0.9$ fm with the $\pi\gamma$ and πAN form factors. The d-state quark is not included. The dot-dashed and dotted curves show the quark and pion contributions, respectively. The data were taken from Refs. 11–13.

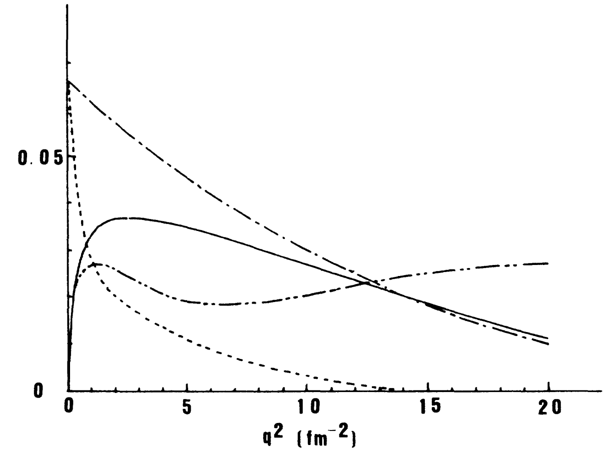

FIG. 5. The neutron charge form factor for $R = 0.9$ fm. Other explanations are same as those given in Fig. 4, except the pion contribution is shown in an opposite sign by the dotted curve. The dot-dot-dashed curve was obtained with neither the $\pi\gamma$ nor πAN form factors.

dependent of the d-state quark admixture.

In Fig. 6 we show two more curves, g and h. The former is the case of no d-state quark admixture and the latter is for the d-state quark admixture of 100%.

The importance of $\pi\gamma$ and πAN form factors can be seen in Fig. 5. The dot-dot-dashed curves show the re-

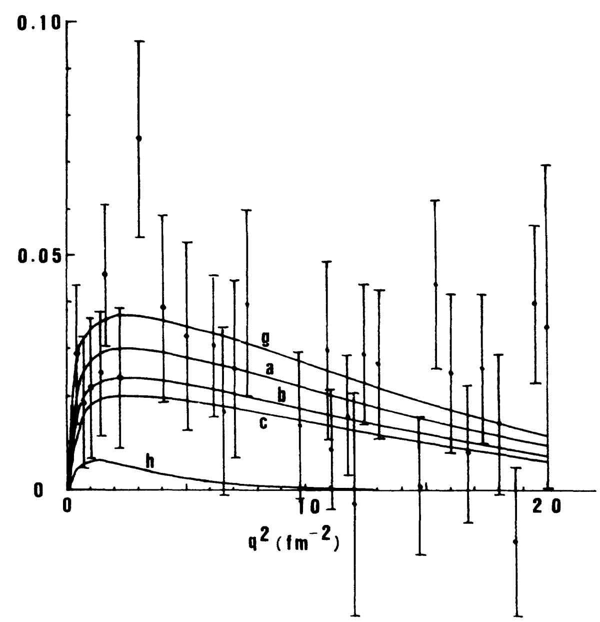

FIG. 6. The neutron charge form factors. The curves a, b, and c were obtained, respectively, with d-state quark admixtures of 20%, 40%, and 50% in the intermediate states. The curve g shows the result with all s-state quarks in the intermediate-state baryons, while the curve h is the extreme case that a quark stays in the d state and the other two quarks are in the s state. The $\pi\gamma$ and πAN form factors were taken into account. The data obtained with the McGee wave function of deuteron were taken from Ref. 18.

sults when these form factors are neglected. Although the proton charge form factor is not affected by neglecting the $\pi\gamma$ and πAN form factors, that of the neutron drastically changes its magnitude, keeping the overall shape.

As seen above, the d-state quark admixture yields negligibly small effects on the proton charge form factor, but it gives significantly large effects on that of the neutron. Therefore, only the proton charge form factor can play a key role to determine the bag radius in the present model.

The experimental data of neutron charge form factor are very much dependent on the deuteron wave function used to extract them from the cross sections of electron scattering by a proton and a deuteron. Our results with some admixtures of d-state quark are all in good agreement with the data obtained with the McGee wave function of deuteron.[14] These agreements are examined by the values of χ^2, which are indeed within 21.6–35.0 for 29 data points. Yet, the nucleon charge radii are significantly sensitive to the d-state quark admixture. When the d-state quark admixture are taken by 0%, 20%, 40%, and 50%, we find 0.89, 0.85, 0.81, and 0.79 fm for the proton charge radius and -0.60, -0.54, -0.48, and -0.44 fm for the neutron. They have been computed by applying the numerical method of differentiation[15] to the usual formula

$$\langle r_N^2 \rangle = -6 \left. \frac{\partial G_E^N(q^2)}{\partial q^2} \right|_{q^2=0} .$$

The experimental values are known as 0.84 and -0.34 fm, respectively.[12,16]

The neutron nonzero charge radius may also have some contributions from the higher components arising from a single-gluon exchange between different quarks in the bag. This was recently evaluated by Close and Horgan[17] in the framework of confined QCD perturbation theory inside a spherical cavity. However, it turns out to be very small. Therefore, we have neglected it in the present calculation.

In this stage, we should stress that the more precise and model-independent data would be required to establish importance of the d-state quark admixture. However, our present results are more plausible compared with the previous calculations, which could produce some reasonable values for $\langle r_N^2 \rangle^{1/2}$ but failed to reproduce the experimental results of the nucleon charge form factors.

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