PHYSICAL REVIEW D

PARTICLES AND FIELDS

THIRD SERIES, VOLUME 37, NUMBER 2 15 JANUARY 1988

Fields due to kinky, cuspless, cosmic loops

David Garfinkle

Physics Department, University of Florida, Gainesville, Florida 32611

Tanmay Vachaspati*

Bartol Research Foundation, University of Delaware, Newark, Delaware 19716

(Received 17 September 1987)

We study the gravitational field of a kinky, cuspless loop of cosmic string. The gravitational field does not have a repulsive component as in the case of loops with cusps and so the gravitational field produced by cuspless loops is spherically symmetric and Newtonian when we go far away from the loop. We also find the electromagnetic field due to kinks on a current-carrying superconducting cosmic loop and the Goldstone field produced by a loop of global string. For the case of the Goldstone field we also consider loops with cusps.

I. INTRODUCTION

Loops of cosmic strings are a possible mechanism for seeding galaxies. They can also be a source of many other astrophysical phenomena. An excellent review on cosmic strings may be found in Ref. 1.

In order to understand what effects cosmic strings might have on surrounding matter, it is necessary to study the fields produced by the loops of cosmic string. The relevant field of a loop depends on the kind of particle physics that produces the cosmic string. If the string is produced by the breaking of a gauge ("local") symmetry,[2] the string's gravitational field is of interest. If, on the other hand, the symmetry was a global symmetry, then the Goldstone field will be of interest. In some situations, the strings are superconducting and can carry currents.[3] For such strings, the electromagnetic field produced by the loop is the relevant field.

The field of a loop is largely determined by the motion of the loop. In most of the work so far, attention was focused on loops with cusps. A cusp is a point on the loop that reaches the speed of light at some instant during the loop's period of oscillation. In Ref. 4, however, it was realized that cusps are not generic features of a loop. We introduced a new class of loops and called them "kinky" loops because these loops have kinks on them. Kinks are discontinuities in the tangent to the loop and are expected to be very prevalent in the system of cosmic strings. It is also argued in Ref. 4 that the presence of kinks inhibits cusps.[5] So it is very important to study the properties of loops with kinks and without cusps.

A first step in this direction was taken when we looked at the radiation from kinky, cuspless loops.[4] The cases of local strings which emit gravitational radiation, global strings which emit Goldstone bosons, and current-carrying superconducting strings which emit electromagnetic radiation were all considered. Here we take the next step and find the fields produced by local, global, and superconducting, kinky, cuspless loops.

The results of these calculations are less dramatic than the results for the loops with cusps. For example, in the case of the local string loop with cusps, it was found that the gravitational field could repel particles away from the cusp.[6] Such behavior is absent when we look at cuspless loops. However, this very absence of "interesting" behavior leads to some very interesting speculations.[7]

After finding the gravitational field in Sec. III we shall compare the motion of particles near a kinky, cuspless loop with the motion of particles near a loop with cusps in Sec. IV. The electromagnetic field due to a kink on the loop is analyzed in Sec. V. In Sec. VI the Goldstone field due to a kink on a loop of global string is discussed. In the Appendix we calculate the Goldstone field produced by global string loops with cusps. This calculation is done for completeness.

II. PRELIMINARIES

The position f^{μ} of a string is labeled by two parameters: σ and τ. (For our purpose we can think of σ being along the length of the string.) Since we treat closed loops, we choose σ so that $0 \leq \sigma \leq L$ with $\sigma = 0$ and $\sigma = L$ identified. The constant L is called the length of the loop. The action for the string is just the Nambu action

$$S=-\mu\int d\tau\, d\sigma[(\dot f^{\mu}f'_{\mu})^2-\dot f^{\mu}\dot f_{\mu}f'^{\nu}f'_{\nu}]^{1/2}\ , \qquad (2.1)$$

where the overdots and primes refer to differentiation with respect to τ and σ, respectively, and μ is the energy per unit length of a straight string. Then the equation of motion for the string in a flat nonexpanding background is

$$\ddot f^{\mu}-f''^{\mu}=0\ . \qquad (2.2)$$

Together with this we also have the constraint equations[8]

$$\dot f^{\mu}f'_{\mu}=0\ , \qquad (2.3)$$

$$\dot f^{\mu}\dot f_{\mu}+f'^{\mu}f'_{\mu}=0\ . \qquad (2.4)$$

The solution to this set of equations can be written as[8]

$$f^0=\tau\ , \qquad (2.5)$$

$$\mathbf{f}(\sigma,\tau)=\frac{\mathbf{a}(\sigma-\tau)+\mathbf{b}(\sigma+\tau)}{2}\ , \qquad (2.6)$$

$$|\mathbf{a}'|^2=|\mathbf{b}'|^2=1\ . \qquad (2.7)$$

Here the primes refer to differentiation with respect to the argument of the function. From now on we shall take $\tau=t$.

A cusp on the loop is a point (σ,t) where $-\mathbf{a}'=\mathbf{b}'$. It can easily be checked that the cusp moves with the speed of light in the direction of $-\mathbf{a}'$. Note that the cusp occurs at an instant during the loop's period of oscillation.

A kink is a point σ on the loop where there is a discontinuity in either of the functions $\mathbf{a}'$ or $\mathbf{b}'$. For definiteness we shall always consider the discontinuity to be in the function $\mathbf{a}'$. The kink is not an instantaneous occurrence, but persists throughout the period of oscillation of the loop. Assume that the kink occurs at $\sigma-t=0$. Then the velocity of the kink, $\mathbf{v}_{\rm kink}$, is to be found by differentiating Eq. (2.6), with respect to t but holding $\sigma-t=0$. The result is $\mathbf{v}_{\rm kink}=\mathbf{b}'(2\sigma)$. The magnitude of the velocity is one. Hence the kink in the $\mathbf{a}'$ trajectory moves with the speed of light in the $\mathbf{b}'$ direction. However, in contrast with the case of a cusp, this is a "phase velocity" and no part of the string actually attains this velocity.

We can find the energy-momentum tensor for the string[9] by varying the action with respect to the background metric.[10] The result is

$$T^{\mu\nu}=\mu\int_0^L d\sigma(\dot f^{\mu}\dot f^{\nu}-f'^{\mu}f'^{\nu})\delta^{(3)}(\mathbf{x}-\mathbf{f}(\sigma,t))\ . \qquad (2.8)$$

Let the gravitational field of the loop be written as $\eta_{\mu\nu}+h_{\mu\nu}$, where $\eta_{\mu\nu}=(-,+,+,+)$. Then, using the weak-field limit of Einstein's equation, $h_{\mu\nu}$ satisfies the following equation:

$$\partial_\sigma\partial^\sigma h_{\mu\nu}=-16\pi G S_{\mu\nu}\ , \qquad (2.9)$$

where

$$S_{\mu\nu}\equiv T_{\mu\nu}-\tfrac{1}{2}\eta_{\mu\nu}T^{\alpha}{}_{\alpha}\ . \qquad (2.10)$$

Here we have also used the harmonic coordinate condition $g^{\mu\nu}\Gamma^{\lambda}{}_{\mu\nu}=0$, where $\Gamma^{\lambda}{}_{\mu\nu}$ is the Christoffel symbol.

The solution to Eq. (2.9) is

$$h_{\mu\nu}(\mathbf{x},t)=4G\int d^3x'\frac{S_{\mu\nu}(\mathbf{x}',\tau)}{|\mathbf{x}-\mathbf{x}'|}\ , \qquad (2.11)$$

where $\tau=t-|\mathbf{x}-\mathbf{x}'|$ is the retarded time. We can now use Eqs. (2.10) and (2.8) to write

$$h_{\mu\nu}(\mathbf{x},t)=4G\mu\int_0^L d\sigma\frac{F_{\mu\nu}(\sigma,\tau)}{|\mathbf{x}-\mathbf{f}(\sigma,\tau)|}\frac{1}{1-\boldsymbol{\epsilon}\cdot\dot{\mathbf{f}}(\sigma,\tau)}\ , \qquad (2.12)$$

where

$$F_{\mu\nu}=\dot f_{\mu}\dot f_{\nu}-f'_{\mu}f'_{\nu}-\eta_{\mu\nu}\dot f^{\alpha}\dot f_{\alpha} \qquad (2.13)$$

and

$$\boldsymbol{\epsilon}=\frac{\mathbf{x}-\mathbf{f}(\sigma,\tau)}{|\mathbf{x}-\mathbf{f}(\sigma,\tau)|}\ . \qquad (2.14)$$

A kink in the loop means that there is a discontinuity in $\mathbf{a}'$ or $\mathbf{b}'$. If the loop is cuspless, it means that $|\dot{\mathbf{f}}|$ is never unity and the integrand in Eq. (2.12) is always finite. However, because of the kink, the field $h_{\mu\nu}$ will have a discontinuity and its derivatives may diverge. The motion of a test particle depends on the derivatives of $h_{\mu\nu}$ (see below) and so we might hope that there will still be an interesting effect. In Sec. III we will find the behavior of the derivatives of $h_{\mu\nu}$ due to a kink on the loop.

The motion of particles around a loop can be found by using the geodesic equations[10]

$$\frac{d^2x^{\mu}}{d\lambda^2}+\Gamma^{\mu}{}_{\alpha\beta}\frac{dx^{\alpha}}{d\lambda}\frac{dx^{\beta}}{d\lambda}=0\ , \qquad (2.15)$$

where λ is the proper time of the particle. For nonrelativistic particles this equation reduces to

$$\frac{d^2x^i}{dt^2}+\Gamma^{i}{}_{00}=0\ . \qquad (2.16)$$

So the acceleration a^i of a particle is just $-\Gamma^{i}{}_{00}$. Then in the weak-field limit we have

$$a^i=-h_{0i,0}+\tfrac{1}{2}h_{00,i}\ . \qquad (2.17)$$

In the analytic work of the next section we shall only deal with nonrelativistic particles. There we will find the behavior of the acceleration of a particle using Eq. (2.17).

The electromagnetic field due to a current-carrying superconducting loop is also given by an inhomogenous wave equation[11]

$$\partial_\sigma\partial^\sigma A_{\mu}=4\pi j_{\mu}\ . \qquad (2.18)$$

Here the current j^{μ} is given by

$$j^{\mu}=-e\int_0^L d\sigma(\dot f^{\mu}\phi'-f'^{\mu}\dot\phi)\delta^{(3)}(\mathbf{x}-\mathbf{f}(\sigma,t))\ . \qquad (2.19)$$

Here $\phi=\phi(\sigma,t)$ is a field that lives on the string and, for our purposes, obeys the homogenous wave equation[11]

$$-\ddot{\phi}+\phi''=0\ . \tag{2.20}$$

In this paper we shall only look at the case when there is a constant current i on the loop. Hence, $\dot{\phi}=i/e$ and $\phi'=0$.

In the above analysis we have assumed that the current in the loop is small enough so that its effects on the motion of the loop can be ignored. We have also considered a loop that is much smaller than the horizon so that the effects of Hubble expansion can be ignored. Equations (2.2)–(2.20) are valid only under these assumptions.

For completeness we have also found the Goldstone field in the vicinity of a loop of global string. From Ref. 12 we see that the Goldstone field is equivalent to an antisymmetric field $G_{\mu\nu}$. The equation that this field obeys is once again the inhomogenous wave equation

$$\partial_\sigma\partial^\sigma G_{\mu\nu}=4\pi j_{\mu\nu} \tag{2.21}$$

with the current given by

$$j^{\mu\nu}=\tfrac{1}{2}\eta\int_0^L d\sigma(\dot{f}^{\,\mu}f'^{\nu}-f'^{\mu}\dot{f}^{\,\nu})\delta^{(3)}(\mathbf{x}-\mathbf{f}(\sigma,t))\ . \tag{2.22}$$

Here η is the scale of symmetry breaking that produced the global string.[12]

It is clear from the above equations that the only equation that needs to be solved is the inhomogenous wave equation. In the case of the gravitational field of a loop, this is already done in Eq. (2.12). So we shall consider the gravitational case in some detail. For the other cases we shall simply have to change the form of the source.

III. THE GRAVITATIONAL FIELD OF A KINKY, CUSPLESS LOOP

For convenience we will assume that at $t=0$ the loop has a kink at $x=0=y=z$ and also choose the coordinate axes such that the velocity of the kink is along the positive z axis. As shown in Sec. II, the velocity of the kink at $t=0$ is $\mathbf{b}'(0)=\mathbf{e}_z$.

The solution for $h_{\mu\nu}$ in Eq. (2.12) can be written as

$$h_{\mu\nu}(\mathbf{x},t)=4G\mu\left[\int_0^{\sigma_{0-}}d\sigma+\int_{\sigma_{0+}}^{L}d\sigma\right]\times\frac{F_{\mu\nu}(\sigma,\tau)}{|\mathbf{x}-\mathbf{f}(\sigma,\tau)|}\frac{1}{1-\boldsymbol{\epsilon}\cdot\dot{\mathbf{f}}(\sigma,\tau)}\ , \tag{3.1}$$

where τ is the retarded time and σ_0 is the position of the kink and is determined by $\sigma_0=\tau$ (mod L), where L is the length of the loop. Note that the retarded time is a function of the field point $(t,\mathbf{x})$ and, therefore, σ_0 also depends on the field point.

The derivatives of $h_{\mu\nu}$ are to be found by differentiating Eq. (3.1). The derivative of the right-hand side of Eq. (3.1) contains two terms. The first term is obtained by differentiating the limits of the integral and the second term is obtained by differentiating the integrand. The derivative of the integrand is well behaved since we have avoided the kink by splitting the integral into two. Hence the second term is a finite integral and has no interesting divergent behavior. We shall see that the first term has a singularity and so we shall concentrate on it. Therefore we can write

$$h_{\mu\nu,\lambda}(\mathbf{x},t)\approx\frac{4G\mu}{|\mathbf{x}|}\left[\frac{F_{\mu\nu}(\sigma_{0-},\sigma_{0-})}{1-\boldsymbol{\epsilon}\cdot\dot{\mathbf{f}}(\sigma_{0-},\sigma_{0-})}-\frac{F_{\mu\nu}(\sigma_{0+},\sigma_{0+})}{1-\boldsymbol{\epsilon}\cdot\dot{\mathbf{f}}(\sigma_{0+},\sigma_{0+})}\right]\partial_\lambda\sigma_0\ , \tag{3.2}$$

where the arguments of the various functions take into account the fact that the loop is continuous at the kink, but the derivatives are discontinuous. We have also made use of the fact that the kink is at the origin of our coordinate system. Next, we need to find the derivative of σ_0.

The defining equation for σ_0 is

$$\sigma_0=\tau=t-|\mathbf{x}-\mathbf{f}(\sigma_0,\sigma_0)|\ . \tag{3.3}$$

This equation can be differentiated to give

$$\partial_\lambda\sigma_0=\frac{e_\lambda}{1-\boldsymbol{\epsilon}\cdot\mathbf{b}'(2\sigma_0)}\ , \tag{3.4}$$

where we have defined $e_\lambda\equiv(1,-\boldsymbol{\epsilon})$. We immediately notice the singularity in the derivative of σ_0 when $\boldsymbol{\epsilon}\cdot\mathbf{b}'=1$. Now we can write

$$h_{\mu\nu,\lambda}(\mathbf{x},t)\approx\frac{4G\mu}{r}\frac{e_\lambda}{1-\boldsymbol{\epsilon}\cdot\mathbf{b}'(2\sigma_0)}\Delta_{\mu\nu} \tag{3.5}$$

with

$$\Delta_{\mu\nu}\equiv\frac{F_{\mu\nu}(\sigma_{0-},\sigma_{0-})}{1-\boldsymbol{\epsilon}\cdot\dot{\mathbf{f}}(\sigma_{0-},\sigma_{0-})}-\frac{F_{\mu\nu}(\sigma_{0+},\sigma_{0+})}{1-\boldsymbol{\epsilon}\cdot\dot{\mathbf{f}}(\sigma_{0+},\sigma_{0+})}\ , \tag{3.6}$$

where, in Eq. (3.5), $r=|\mathbf{x}|$.

We choose a field point that is close to the singular direction, $b'(0)=\mathbf{e}_z$. Hence we take $\mathbf{x}=(l\cos\phi,l\sin\phi,z)$ with $l/z=\theta\ll 1$ and $z>0$. We are interested in the behavior of the gravitational field when the kink is about to affect the field point. Therefore we want to consider $t\approx r$.

Define the quantity α by $\alpha\equiv 1-\boldsymbol{\epsilon}\cdot\mathbf{b}'(2\sigma_0)$. Then

$$h_{\mu\nu,\lambda}(\mathbf{x},t)\approx\frac{4G\mu}{r}\frac{e_\lambda}{\alpha}\Delta_{\mu\nu}\ . \tag{3.7}$$

We would like to find an approximate expression for α as a function of the field point coordinates t, r, and θ. To do this we must first find an expression for σ_0 as a function of t, r, and θ; then we must find an expression for α as a function of σ_0. It follows from Eq. (3.3) and our conditions on the field point that σ_0 is "small." Thus we can expand Eq. (3.3) as a power series in σ_0. It turns out that in order to include all non-negligible terms, we must expand Eq. (3.3) to third order in σ_0. The resulting cubic equation can then be solved for σ_0, yielding an expression for σ_0 as a function of t, r, and θ. Similarly we can expand α in a power series in σ_0. It turns out that in order to include all non-negligible terms, we must expand α to second order in σ_0. The solution for σ_0 can then be inserted in the power series for α to yield the desired expression for α as a function of t, r, and θ. The resulting expression for α is fairly complicated. It is more illuminating to concentrate on two limiting cases. For $|t-r|/L \gg \theta^3$ we have

$$\alpha \approx 18^{1/3} |\mathbf{b}''(0)|^{2/3} (t-r)^{2/3} , \tag{3.8a}$$

and, for $\theta^3 \gg |t-r|/L$ we have

$$\alpha \approx \tfrac{1}{2}\theta^2 . \tag{3.8b}$$

Equation (3.7) with Eq. (3.8) is our final result for the gravitational field due to a kink on a cosmic loop.

In writing Eq. (3.2) we have tried to isolate the effect of the kink by treating it as a discontinuity. It might be argued that in reality the kink will not be a sharp discontinuity; instead it will be a short segment of string with a very small radius of curvature. Let us denote the radius of curvature of the string at the kink by δ. We could estimate the effect of this piece of curved string by adding another piece to the expression on the right-hand side of Eq. (3.1):

$$H_{\mu\nu}(\mathbf{x},t) = 4G\mu \int_{\sigma_0-\delta}^{\sigma_0+\delta} d\sigma \frac{F_{\mu\nu}(\sigma,\tau)}{|\mathbf{x}-\mathbf{f}(\sigma,\tau)|} \frac{1}{1-\boldsymbol{\epsilon}\cdot\dot{\mathbf{f}}(\sigma,\tau)} . \tag{3.9}$$

The derivatives of this piece, due to the finite size of the kink, must be much smaller than the expression in Eq. (3.5) for our calculations to be valid. This results in an inequality which specifies, when equation (3.5) is accurate,

$$\frac{\delta}{L} \ll \alpha^{3/2} . \tag{3.10}$$

It then follows from Eq. (3.8) that, for a given field point, if the condition $\theta \gg (\delta/L)^{1/3}$ or the condition $|t-r| \gg (\delta L)^{1/2}$ is satisfied then the finite size of the kink can be neglected. Typically we would expect δ to be of the order of the thickness of the string, and so, for loops of cosmological interest, δ/L is about 10^{-50}. Hence the finite size of the kink does not impose any interesting constraint on our expression in Eq. (3.7). If, however, radiation back reaction tends to smooth out kinks, the constraint in Eq. (3.10) can become important.

In Ref. 4 it was seen that the power emitted in gravitational radiation from the kink is finite. This must be reconciled with the fact that there is a singularity in Eqs. (3.7) and that the emitted power depends on a quadratic combination of the derivatives of the metric. We have checked that the singularities cancel out in the expression for the rate of emission of gravitational radiation and hence the total power radiated is indeed finite.

IV. PARTIAL MOTION IN THE GRAVITATIONAL FIELD OF A KINK

It has been shown in Ref. 13 that the average gravitational field that a particle experiences due to a loop is just the Newtonian field. In other words, the average gravitational acceleration of a particle is just $-G\mu L/r^2$ in the radial direction if the loop is at the origin and the particle far away from the loop. Closer to the loop the average gravitational acceleration of a particle is again given by the Newtonian acceleration, but the source appears to be the surface traced out by the loop over one oscillation period. In Ref. 6 it was found that, although these statements are valid for any loop, the particle experiences the *average* gravitational field only when it is far away from the beams of gravitational radiation emitted by the cusps. Near the beams emitted by the cusps the particles move under the instantaneous field of the loop. The motion under the instantaneous forces results in a repulsion of the particles. The question here is whether or not a similar effect holds in the gravitational field of a kink.

It is clear that if a particle is not influenced by the field of a kink, then there is no strong gravitational field and the particle will experience the average gravitational acceleration. A particle near the loop will see the surface traced out by the loop during one period of oscillation. The surface density of this surface is $|\dot{\mathbf{f}}|/|\mathbf{f}'|$ and the total mass of the surface is the mass of the loop. The presence of a kink means that the surface has a sharp edge to it (something like the edge of a box) but is not characterized by a divergence in the surface density as in the case of the cusp.[6] We will now look at the situation when a particle *is* influenced by the field of the kink. We can use Eq. (3.7) in Eq. (2.17) to obtain the acceleration of a nonrelativistic test particle in the field of a kink. Let us look at the acceleration in the radial direction only. (An identical analysis can be done for the other components.) When $\theta=0$ the result is

$$a_r \approx \left[\frac{32}{9}\right]^{1/3} \frac{G\mu}{r|\mathbf{b}''(0)|^{2/3}(t-r)^{2/3}} \left[\frac{(\mathbf{a}'_+ - \mathbf{a}'_-)\cdot\mathbf{b}'}{(1+\mathbf{a}'_-\cdot\mathbf{b}')(1+\mathbf{a}'_+\cdot\mathbf{b}')}\right] . \tag{4.1}$$

In the above equation, $\mathbf{a}'_-$ denotes $\mathbf{a}'$ on the side of the kink where $\sigma < t$ and $\mathbf{a}'_+$ denotes $\mathbf{a}'$ on the other side of the kink.

It is clear that the acceleration becomes very large as t approaches r. This means that the acceleration is singular in the directions that the vector $\mathbf{b}'$ sweeps out over one period of the loop. Furthermore, the sign of this acceleration is determined by the sign of the expression in large parentheses. This means that in some directions swept out by $\mathbf{b}'$ the singular acceleration is attractive and in other directions it is repulsive.

This very interesting behavior of the acceleration, however, does not lead to any interesting motion of the surrounding particles simply because the impulse on any particle is finite. So, if we want to find the trajectory of a particle near a kinky, cuspless loop, we can just use the average gravitational field of the loop. This will be the Newtonian force as described earlier.

We have checked that the worst singular behavior of the derivatives of the metric cancel out of the Riemann tensor. However, we have not checked if the Riemann tensor has a weaker singularity or not. This may not be of much interest as far as the scenario of galaxy formation goes, since the singularity in the metric does not lead to any unusual motion of test particles.

It is instructive to contrast the gravitational field of a kink with that of a cusp. The gravitational acceleration that a particle experiences due to the cusp has been found in Ref. 6. The time dependence is

$$a_z \approx -\mathrm{sgn}(t-z)/(t-z)^{4/3} .$$

Here, if it were not for the sign change, the impulse would be divergent. It is the sign change that is crucial for keeping the impulse finite. However, if the impulse is calculated over a time interval on only one side of the singularity, it can be arbitrarily large. The impulse is positive and very large at first. This causes repulsion. By the time the large negative impulse is turned on, the particle's position has changed and the attractive impulse felt by the particle is less. Hence the net effect of the cusp is to repel.

The kink never exerts an enormous impulse on the particle, and the impulse over any time interval is finite. So the particle will feel the time-averaged field of the loop. A consequence of this is that, far away from the loop, the gravitational field of a cuspless loop is spherically symmetric and the accretion of matter is well approximated by a Newtonian gravitational field. This fact, together with the observation that loops with cusps do not produce a spherically symmetric gravitational field, could be of some astrophysical interest as described in Ref. 7.

V. ELECTROMAGNETIC FIELD DUE TO A KINK

As we pointed out at the end of Sec. II, the calculation of the electromagnetic field due to a current-carrying superconducting cuspless, kinky loop is almost identical to the calculation of the gravitational field done in Sec. III. The only difference is in the source. Hence we can directly write the analogues of Eq. (3.7) for the electromagnetic field strength $F_{\mu\nu}$:

$$F_{\mu\nu}(\mathbf{x},t) \approx \frac{i}{r\alpha}(e_\nu\Delta_\mu - e_\mu\Delta_\nu) , \tag{5.1}$$

where α is given, in the limiting cases, by Eq. (3.8) and the function Δ_μ is

$$\Delta_\mu \equiv \frac{f'_\mu(\sigma_{0-},\sigma_{0-})}{1-\boldsymbol{\epsilon}\cdot\dot{\mathbf{f}}(\sigma_{0-},\sigma_{0-})} - \frac{f'_\mu(\sigma_{0+},\sigma_{0+})}{1-\boldsymbol{\epsilon}\cdot\dot{\mathbf{f}}(\sigma_{0+},\sigma_{0+})} . \tag{5.2}$$

It is seen from these equations that the electric and magnetic fields are transverse and the Poynting vector is directed radially outward. This implies that the drift velocity of any charge in this field will be radially outward and hence the field of the kink repels charges. It may be mentioned that this is consistent with the interpretation of the field as the electromagnetic radiation emitted from the kink.

We must reconcile the above equations with the divergent emission of electromagnetic radiation from the kink that was found in Ref. 4. The eletromagnetic power emitted per unit solid angle is given by

$$\frac{dP}{d\Omega} = \frac{r^2}{4\pi} F^{\mu\alpha} F^{\nu}{}_{\alpha} \nabla_\mu t \nabla_\nu r . \tag{5.3}$$

Now we substitute Eq. (5.1) in this equation together with the power-series expansion for α as a function of σ_0. To find the average power radiated by the kink in one oscillation of the loop, we first time average Eq. (5.3). The time-averaging integration is changed to an integration over the variable σ_0 using Eq. (3.4). After the time averaging, we integrate over the solid angle. The singularity in the time-averaged integrand behaves like θ^{-1}. Hence, we find that the singularity in the total power radiated by the kink is logarithmic, that is, it behaves like $\ln(\theta_{\min})$ where $\theta_{\min}$ is a lower cutoff imposed on the angular integration. This is in complete agreement with the results of Ref. 4.

We have also done the above calculation explicitly. The total power radiated by the kink turns out to be

$$P_{\text{kink}} = 8\frac{i^2}{L} \,|\, \mathbf{a}'_- - \mathbf{a}'_+ \,|^2 \ln\left[\frac{\theta_{\max}}{\theta_{\min}}\right] \times \int_0^{L/2} \frac{d\sigma}{[1+\mathbf{b}'(2\sigma)\cdot\mathbf{a}'_+][1+\mathbf{b}'(2\sigma)\cdot\mathbf{a}'_-]} , \tag{5.4}$$

where $\theta_{\min}$ is a lower cutoff and $\theta_{\max}$ is an upper cutoff on the angular integration. The lower cutoff depends on the size of the kink and is imposed so that we do not include field points which violate the condition in Eq. (3.10). The upper cutoff is imposed so that we only include the power radiated due to the kink.

The singularity in the power radiated by the kink is very weak and there are many effects that would remove this singularity. In Ref. 4 we pointed out that if we were to take the thickness of the string into account, it would impose a cutoff which would give an estimate of P_{kink}.

Another effect that would tame the singularity is the back reaction on the kink due to radiation. It has yet to be seen whether or not the back reaction will smooth out the kink. If, for a moment, we assume that the back reaction does smooth out the kink, then the analysis at the end of Sec. III becomes applicable. As the kink becomes smoother, our expression for $F_{\mu\nu}$ in Eq. (5.1) applies to a smaller region as given by Eq. (3.10).

VI. GOLDSTONE FIELD DUE TO A KINKY, CUSPLESS LOOP

The Goldstone field due to a kink can be written exactly in the way we wrote Eq. (5.1). The result is

$$G_{\mu\nu,\lambda}(\mathbf{x},t)\approx\frac{\eta}{r\alpha}e_{\lambda}\Delta_{\mu\nu}^{(Gb)}\ , \tag{6.1}$$

where α is given, in the limiting cases by Eq. (3.8) and $\Delta_{\mu\nu}^{(Gb)}$ is given by

$$\Delta_{\mu\nu}^{(Gb)}\equiv\frac{F_{\mu\nu}^{(Gb)}(\sigma_{0-},\sigma_{0-})}{1-\boldsymbol{\epsilon}\cdot\dot{\mathbf{f}}(\sigma_{0-},\sigma_{0-})}-\frac{F_{\mu\nu}^{(Gb)}(\sigma_{0+},\sigma_{0+})}{1-\boldsymbol{\epsilon}\cdot\dot{\mathbf{f}}(\sigma_{0+},\sigma_{0+})} \tag{6.2}$$

with

$$F_{\mu\nu}^{(Gb)}=\dot{f}_{\mu}f'_{\nu}-f'_{\mu}\dot{f}_{\nu}\ . \tag{6.3}$$

The Goldstone field of Eq. (6.1) will interact with the particles surrounding the loop. This interaction will depend on the details of the particle-physics model that leads to the production of the global strings. It is quite possible that since the Goldstone fields produced by the kinks and the cusps (see the Appendix) are extremely strong and long range, they could provide an interesting and observable astrophysical effect. An investigation of this subject is beyond the scope of the present paper.

ACKNOWLEDGMENTS

D.G. was supported in part by the National Science Foundation under Grant No. PHY 85-13953 to Washington University. T.V. was supported by the U.S. Department of Energy under Grant No. DEAC-02-78ER-05007.

APPENDIX: GOLDSTONE FIELD DUE TO A CUSP

We closely follow the calculation of Ref. 6 where the gravitational field due to a cusp on a cosmic loop was calculated.

The Goldstone field $G_{\mu\nu}$ is given by Eq. (2.12) with $F_{\mu\nu}$ replaced by $F_{\mu\nu}^{(Gb)}$ of Eq. (6.4). The singularity in the field arises because of the $(1-\boldsymbol{\epsilon}\cdot\dot{\mathbf{f}})^{-1}$ factor in Eq. (2.12), and this singularity is the same in the case of the Goldstone field as it is in the gravitational and electromagnetic fields. However, $F_{\mu\nu}^{(Gb)}$ vanishes when evaluated at the cusp and this makes the singular behavior more like the electromagnetic case than the gravitational case. Hence we write the leading singular behavior directly from Ref. 11(a). When $\theta=0$ and $t\neq r$ we have

$$G_{\mu\nu,\lambda}(\mathbf{x},t)\sim\frac{\eta L}{r\mid t-r\mid}\ . \tag{A1}$$

In the case when $\theta\neq 0$ and $t=r$ we have

$$G_{\mu\nu,\lambda}(\mathbf{x},t)\sim\frac{\eta}{r\theta^3}\ . \tag{A2}$$

These equations hold only for the components of the Goldstone field with the worst singular behavior. The components for which the singularity is milder can be found by inspecting $F_{\mu\nu}^{(Gb)}$. If this goes to zero at the cusp quadratically in $\sigma-\tau$ and $\sigma+\tau$ then the singularity in the corresponding $G_{\mu\nu,\lambda}$ is weaker. This means that if the velocity of the cusp is in the z direction, then the only components for which Eqs. (A1) and (A2) do not apply are $G_{03,\lambda}=-G_{30,\lambda}$ and $G_{12,\lambda}=-G_{21,\lambda}$ and they also do not apply when $\lambda=1,2$. The components with $\mu=\nu$ vanish due to the antisymmetric nature of the Goldstone field.

---

*Present address: Department of Applied Mathematics and Theoretical Physics, Silver Street, University of Cambridge, Cambridge CB3 9EW, England.

[1]For a review of the subject, see A. Vilenkin, Phys. Rep. **121**, 265 (1985), and references therein.

[2]H. B. Nielsen and P. Olesen, Nucl. Phys. **B61**, 45 (1973).

[3]E. Witten, Nucl. Phys. **B249**, 557 (1985).

[4]D. Garfinkle and T. Vachaspati, Phys. Rev. D **36**, 2229 (1987).

[5]The kinks themselves may contain "small scale" cusps. However, it is argued in Ref. 4 that these small scale cusps are negligible. See also C. Thompson, Phys. Rev. D **37**, 283 (1988).

[6]T. Vachaspati, Phys. Rev. D **35**, 1767 (1987); A. Vilenkin, Tufts report (unpublished).

[7]T. Vachaspati, Bartol report (unpubished).

[8]T. W. B. Kibble and N. Turok, Phys. Lett. **116B**, 141 (1982).

[9]N. Turok, Nucl. Phys. **B242**, 520 (1984); T. Vachaspati and A. Vilenkin, Phys. Rev. D **31**, 3052 (1985).

[10]For example, S. Weinberg, *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity* (Wiley, New York, 1972).

[11](a) A. Vilenkin and T. Vachaspati, Phys. Rev. Lett. **58**, 1041 (1987); (b) D. N. Spergel, T. Piran, and J. Goodman, IAS report (unpublished).

[12]A. Vilenkin and T. Vachaspati, Phys. Rev. D **35**, 1138 (1987).

[13]N. Turok, Phys. Lett. **123B**, 387 (1983).