Radiative corrections to heavy-Higgs-scalar production and decay

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A simple formalism, based on the Goldstone-boson equivalence theorem, is described for calculating $O(g^2m_H^2/m_W^2)$ radiative corrections in the standard model. We apply this method to heavy-Higgs-boson decays and find that the dominant decay rates $\Gamma(H\to W^+W^-)$ and $\Gamma(H\to ZZ)$ are enhanced by a factor $1+(g^2/8\pi^2)(m_H^2/m_W^2)(\frac{19}{16}-\frac{3}{8}\sqrt{3}\pi+\frac{5}{48}\pi^2)$. The same enhancement applies to heavy-Higgs-boson production from W^+W^- and ZZ fusion. Corrections of $O(g^2\ln m_H)$ are also briefly discussed.

The standard $SU(2)_L\times U(1)$ model of electroweak interactions predicts the existence of a neutral spin-zero particle H, called the Higgs scalar. It is a necessary remnant of the symmetry-breaking mechanism responsible for generating the $W^\pm$, Z, quark, and lepton masses. As such, its discovery is crucial for final confirmation of the standard model.

During the last few years, considerable attention has focused on the possibility of a very heavy Higgs scalar, $m_H \gg m_W$, and the physics associated with it.[1–12] Indeed, such a scenario has been a primary goal of SSC (Superconducting Super Collider) studies.[13] It is generally accepted that for $m_H \gtrsim 1$ TeV, one enters a strong-coupling domain which cannot be reliably described perturbatively. However, at somewhat smaller scales, perturbation theory should still be reliable. To study the perturbative domain and the approach to strong coupling, we have initiated a systematic analysis of $O(g^2m_H^2/m_W^2)$ radiative corrections in the standard model.

In this paper we describe a simple method for calculating $O(g^2m_H^2/m_W^2)$ radiative corrections. We then apply this formalism to the various Higgs-scalar decay rates, which in lowest order are given by[2,14]

$$\Gamma(H\to W^+W^-)=\frac{g^2}{64\pi}\frac{m_H^3}{m_W^2}\left[1-\frac{4m_W^2}{m_H^2}\right]^{1/2}\times\left[1-4\frac{m_W^2}{m_H^2}+12\frac{m_W^4}{m_H^4}\right], \tag{1}$$

$$\Gamma(H\to ZZ)=\frac{g^2}{128\pi}\frac{m_H^3}{m_W^2}\left[1-\frac{4m_Z^2}{m_H^2}\right]^{1/2}\times\left[1-4\frac{m_Z^2}{m_H^2}+12\frac{m_Z^4}{m_H^4}\right], \tag{2}$$

$$\Gamma(H\to f\bar{f})=(3)\frac{g^2}{32\pi}\frac{m_f^2m_H}{m_W^2}\left[1-\frac{4m_f^2}{m_H^2}\right]^{3/2}, \tag{3}$$

where g is the $SU(2)_L$ gauge coupling and the (3) in Eq. (3) is a color factor appropriate only when the fermion f is a quark. Since we are interested in the limit $m_H \gg m_W$, where transverse-vector-boson decay modes are relatively suppressed, we shall concentrate on the corrections to the longitudinal-vector-boson decay modes. However, we shall also calculate the $O(g^2m_H^2/m_W^2)$ corrections to the transverse-vector-boson and fermion-antifermion decay modes. In the case of the vector-boson decay modes, we compare our results with a full $O(g^2)$ calculation by Fleischer and Jegerlehner.[15]

The basis of our computational scheme is an equivalence theorem originally due to Cornwall, Levin, and Tiktopoulos[16] and further developed by others.[2,6,17] This theorem states that at high energies ($s \gg m_W^2$), S-matrix amplitudes involving external longitudinal components of $W^\pm$ and Z are equivalent, up to $O(m_W/\sqrt{s})$, to the corresponding amplitudes in the Higgs-Goldstone scalar theory with the Goldstone bosons[18] $w^\pm$, z replacing $W_L^\pm$, Z_L (L denotes longitudinal components). Since the longitudinal components dominate high-energy amplitudes, and calculations in the equivalent scalar theory are quite easy, the leading high-energy behavior of $W^\pm$ and Z physics is considerably simplified by this theorem. As such, it has been widely employed at the tree level in SSC studies[19] as well as in analyses of the large-m_H limit.[2,6,11,12] Here, on the basis of an all orders proof of the equivalence theorem by Chanowitz and Gaillard,[6] we expand its use to $O(g^2m_H^2/m_W^2)$ radiative corrections. In that regard, our results can be viewed as an extension of early pioneering work by Veltman[3] which also explored the large-m_H limit of radiative corrections.

We begin by writing down the interaction Lagrangian for the Higgs-Goldstone scalar theory

$$\mathcal{L}=-\lambda_0(w^+w+\tfrac{1}{2}z^2+\tfrac{1}{2}H^2+v_0H+\tfrac{1}{2}v_0^2-\mu_0^2/2\lambda_0)^2\ , \tag{4}$$

where $w^\pm$ and z are the Goldstone bosons, λ_0 is the bare coupling of the underlying $\lambda_0\phi^4$ theory, and v_0 is the vacuum expectation value which gives rise to spontaneous symmetry breaking. H is the physical Higgs scalar with zero vacuum expectation value. The last two terms, which cancel at the tree level, yield a tadpole counterterm, as we will discuss.

From the H^2 term, one finds the Higgs-scalar bare mass

$$(m_H^0)^2 = 2\lambda_0 v_0^2 \tag{5}$$

and, from the gauge-boson sector (not discussed here), the $W^\pm$ bare mass

$$m_W^0 = g_0 v_0 / 2 \;, \tag{6}$$

where g_0 is the bare $SU(2)_L$ gauge coupling. [Note that we have chosen not to absorb an extra Higgs-boson mass counterterm[20] $-\lambda_0(v_0^2 - \mu_0^2/\lambda_0)$ that follows from Eq. (4) in our definition of $(m_H^0)^2$.] Combining Eqs. (5) and (6) leads to

$$\lambda_0 = g_0^2 \frac{(m_H^0)^2}{8(m_W^0)^2} \;, \tag{7}$$

which illustrates why the large-m_H limit corresponds to a strong-coupling domain. Using Eqs. (5)–(7), we can rewrite the Lagrangian in Eq. (4) as

$$\mathcal{L} = -g_0^2 \frac{(m_H^0)^2}{8(m_W^0)^2} \left[w^+ w + \tfrac{1}{2} z^2 + \tfrac{1}{2} H^2 + \frac{2m_W^0}{g_0} H + \frac{2\delta T}{g_0 (m_H^0)^2 / m_W^0} \right]^2 , \tag{8}$$

where $\delta T = \lambda_0 v_0 (v_0^2 - \mu_0^2/\lambda_0)$ is a counterterm generated by the incomplete cancellation of v_0^2 and μ_0^2/λ_0 beyond the tree level. This counterterm is constructed to exactly cancel tadpole loop corrections order by order in perturbation theory. An analysis by Taylor[21] has shown that δT is related to the Goldstone boson $(w^\pm, z)$ self-energies at zero momentum transfer, $\Pi(0)$, by

$$\delta T = -v_0 \Pi(0) = -\frac{2m_W^0}{g_0} \Pi(0) \;. \tag{9}$$

Therefore, a simple computational strategy is to ignore all tadpole diagrams and the tadpole counterterms associated with δT (since they exactly cancel) and subtract the zero-momentum Goldstone-boson self-energy $\Pi(0)$ from all scalar self-energies, including the physical Higgs scalar H. This subtraction follows from the existence of effective mass counterterms

$$\delta\mathcal{L} = \Pi(0)(w^+ w + \tfrac{1}{2} z^2 + \tfrac{1}{2} H^2) \tag{10}$$

generated by the δT term in Eq. (8). As previously noted, we have chosen not to absorb these counterterms into our definition of bare quantities such as $(m_H^0)^2$.

Separating Eq. (8) into renormalized and counterterm parts using $(m_H^0)^2 = m_H^2 - \delta m_H^2$, $(m_W^0)^2 = m_W^2 - \delta m_W^2$, and $g_0 = g - \delta g$, we generate Feynman rules for the effective Higgs-Goldstone theory. These rules are illustrated in Fig. 1, where we have included combinatoric factors. In the complete theory, the $w^\pm$ and z propagators are gauge

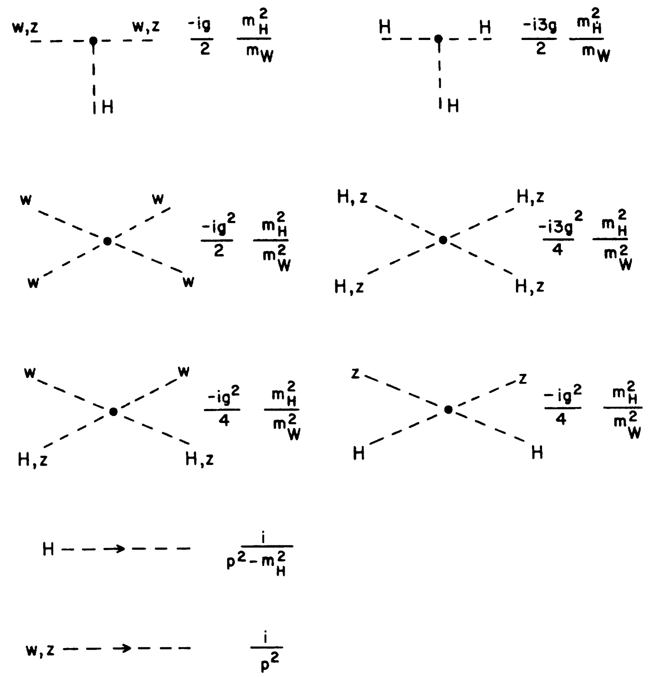

FIG. 1. Feynman rules for the Higgs-Goldstone scalar theory. Closed loops containing identical particles must be multiplied by $\frac{1}{2}$.

dependent; however we have effectively chosen to work in the Landau gauge where the $w^\pm$ and z propagators have zero mass and the $W^\pm$ and Z propagators (not explicitly dealt with) are proportional to $g^{\mu\nu} - k^\mu k^\nu / k^2$. In this way, gauge-boson–scalar mixing is avoided, since any such interaction is proportional to the gauge-boson four-momentum k^μ. Furthermore, we can neglect diagrams with internal $W^\pm$ and Z propagators, since they are suppressed by m_W^2/m_H^2 in this gauge. Thus, the Landau gauge is the simplest and most natural gauge in which to employ the Goldstone-boson equivalence theorem.

Employing the effective Higgs-Goldstone theory, one can easily calculate the $O(g^2 m_H^2/m_W^2)$ corrections to various processes. As an illustration, we describe such a calculation for the decay rates in Eqs. (1) and (2). By the SO(3) symmetry of Eq. (8), the $O(g^2 m_H^2/m_W^2)$ corrections to $\Gamma(H \to w^+ w^-)$ are the same as the corrections to $\Gamma(H \to zz)$; therefore, we need only calculate the former.

We begin by calculating the Hw^+w^- counterterm generated by expressing the bare coupling $g_0 (m_H^0)^2 / 2m_W^0$ in terms of physical parameters. To this end, we choose the physical $W^\pm$ and H masses (i.e., the real parts of the poles in the propagators) as our renormalized parameters and are at liberty to employ any of the various definitions of the renormalized coupling g in use, since its renormalization does not naturally induce $g^2 m_H^2/m_W^2$ corrections. For definiteness, we will employ a short-distance g defined from the muon decay constant[20,22]

$$\begin{aligned} g^2 &\equiv 4\sqrt{2} m_W^2 G_\mu \;, \\ G_\mu &= 1.166\,36 \pm 0.000\,02 \times 10^{-5}\ \text{GeV}^{-2} \;. \end{aligned} \tag{11}$$

The mass counterterms do generate $O(g^2 m_H^2/m_W^2)$ corrections via

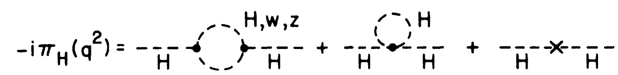

FIG. 2. Diagrams contributing to the Higgs-boson self-energy at $O(g^2 m_H^2/m_W^2)$. The counterterm corresponds to the Goldstone-boson self-energy subtraction $\Pi(0)$.

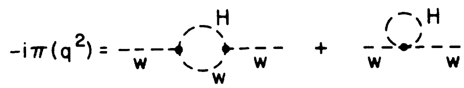

FIG. 3. Diagrams contributing to the Goldstone-boson self-energy.

$$(m_W^0)^2 = m_W^2 - \delta m_W^2 \,, \tag{12a}$$

$$(m_H^0)^2 = m_H^2 - \delta m_H^2 \,, \tag{12b}$$

$$-i\frac{g}{2}\frac{(m_H^0)^2}{m_W^0} = -i\frac{g}{2}\frac{m_H^2}{m_W}\left[1 - \frac{\delta m_H^2}{m_H^2} + \frac{1}{2}\frac{\delta m_W^2}{m_W^2}\right], \tag{12c}$$

so they must be calculated. To begin, we compute the Higgs-boson self-energy $-i\Pi_H(q^2)$ from the diagrams in Fig. 2 and subtract the Goldstone-boson self-energy obtained from Fig. 3,

$$-i\Pi(0) = -i\frac{g^2}{16\pi^2}\frac{m_H^4}{m_W^2}\frac{3}{4}\left[\frac{1}{n-4} - \ln(\mu/m_H) - \tfrac{1}{2}\right], \tag{13}$$

where n is the space-time-dimension regulator[23] and μ is a mass scale introduced to keep the coupling g dimensionless.[24] In this way, we find using

$$\delta m_H^2 = \mathrm{Re}\Pi_H(q^2)\,|_{q^2 = m_H^2} \tag{14}$$

that the Higgs-boson mass counterterm is[25]

$$\frac{\delta m_H^2}{m_H^2} = \frac{g^2}{16\pi^2}\frac{m_H^2}{m_W^2}\left[3\left[\frac{1}{n-4} - \ln(\mu/m_H)\right] - 3 + \frac{9\pi}{8\sqrt{3}}\right]. \tag{15}$$

In the case of δm_W^2, we use an existing calculation in Ref. 26 of the diagrams in Fig. 4, which gave

$$\frac{\delta m_W^2}{m_W^2} = \frac{g^2}{16\pi^2}\left[\frac{1}{8}\frac{m_H^2}{m_W^2}\right]. \tag{16}$$

Taken together, these counterterms imply

$$-i\frac{g}{2}\frac{(m_H^0)^2}{m_W^0} = -i\frac{g}{2}\frac{m_H^2}{m_W}\left\{1 - \frac{g^2}{16\pi^2}\frac{m_H^2}{m_W^2}\left[3\left[\frac{1}{n-4} - \ln(\mu/m_H)\right] - \frac{49}{16} + \frac{9\pi}{8\sqrt{3}}\right]\right\}. \tag{17}$$

Our next step is to evaluate the H and $w^{\pm}$ wave-function-renormalization factors (from Figs. 2 and 3)

$$Z_H^{1/2} = 1 + \frac{g^2}{16\pi^2}\frac{m_H^2}{m_W^2}\left[\frac{3}{4} - \frac{3\pi}{8\sqrt{3}}\right], \tag{18}$$

$$Z_w^{1/2} = 1 + \frac{g^2}{16\pi^2}\frac{m_H^2}{m_W^2}\left[-\frac{1}{16}\right]. \tag{19}$$

Finally, the real part of the proper vertex diagrams in Fig. 5 gives a vertex correction factor

$$1 + \frac{g^2}{16\pi^2}\frac{m_H^2}{m_W^2}\left[3\left[\frac{1}{n-4} - \ln(\mu/m_H)\right] - \frac{5}{2} + \frac{3\pi}{8\sqrt{3}} + \frac{5\pi^2}{48}\right]. \tag{20}$$

Multiplying the factors $Z_H^{1/2}$, Z_w (due to two external w's), and Eq. (20) gives the full loop correction

$$1 + \frac{g^2}{16\pi^2}\frac{m_H^2}{m_W^2}\left[3\left[\frac{1}{n-4} - \ln(\mu/m_H)\right] - \frac{15}{8} + \frac{5\pi^2}{48}\right]. \tag{21}$$

The divergence in Eq. (21) cancels the counterterm-induced divergence in Eq. (17) and one finds that the overall real part of the one-loop-corrected Hw^+w^- and Hzz couplings is given by

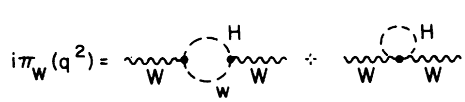

FIG. 4. Diagrams of $O(g^2 m_H^2/m_W^2)$ that contribute to the $W^{\pm}$ self-energy.

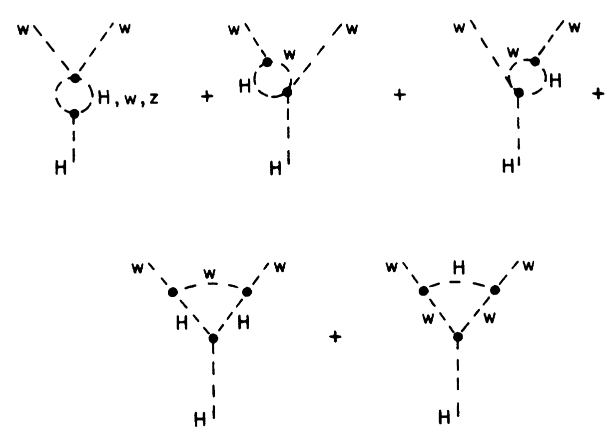

FIG. 5. Proper vertex corrections of $O(g^2 m_H^2/m_W^2)$ to the Hw^+w^- amplitude.

$$-i\frac{g}{2}\frac{m_H^2}{m_W}\left[1+\frac{g^2}{16\pi^2}\frac{m_H^2}{m_W^2}\left[\frac{19}{16}-\frac{3\sqrt{3}\,\pi}{8}+\frac{5\pi^2}{48}\right]\right]. \quad (22)$$

Note that the correction is finite and positive. It is also, in some sense, surprisingly small if we recall that a 1-TeV Higgs boson is considered to be strongly coupled to the $w^{\pm}$ and z Goldstone bosons[1,2] and the correction in Eq. (22) is only 7.5% at that scale. It is interesting to note that the three terms contained within the parentheses of Eq. (22) are each of order unity, yet they sum to only 0.175 because of cancellations.

From Eq. (22), we find that the decay rates in Eqs. (1) and (2) are enhanced by a factor

$$1+\frac{g^2}{8\pi^2}\frac{m_H^2}{m_W^2}\left[\frac{19}{16}-\frac{3\sqrt{3}\pi}{8}+\frac{5\pi^2}{48}\right]=1+0.175\frac{G_\mu m_H^2}{\sqrt{2}\pi^2} \quad (23)$$

in the large-m_H limit. [Note that there are no additional $O(g^2m_H^2/m_W^2)$ corrections from Goldstone-boson bremsstrahlung because only an even number of Goldstone bosons can be radiated. Hence, the decays $H\to 4$ Goldstone bosons will contribute corrections, but only of $O(g^4m_H^4/m_W^4)$ to the decay width.] The effect of Eq. (23) is illustrated in Fig. 6 where the decay rate sum $\Gamma(H\to W^+W^-)+\Gamma(H\to ZZ)$ is plotted as a function of m_H. The Higgs-boson width starts to exceed its mass for $m_H \gtrsim 1.3$ TeV, thereby signaling a strong-coupling domain. At this scale, the radiative correction in Eq. (23) is about 24%, but perturbation theory may no longer be valid.

The enhancement factor in Eq. (23) also leads to an increase in the heavy-Higgs-boson production cross section via W^+W^- and ZZ fusion. This process is the largest source of heavy Higgs bosons in high-energy electron-positron,[27] electron-proton,[28] and proton-proton[29] collisions. Since longitudinal vector bosons make the dominant contribution to these fusion cross sections, the enhancement factor in Eq. (23) applies.

At this point, we compare our result in Eq. (23) with a complete $O(g^2)$ analysis of radiative corrections to $H\to W^+W^-$ and $H\to ZZ$ by Fleischer and Jegerlehner.[15] These authors give an analytic expression for the radiative corrections to $H\to ZZ$ in the large-m_H limit [see Eqs. (5.4), (7.4), and (7.5) in their paper].[15] If we assume that the $\mathrm{Re}c_{2V}$ term in their Eq. (5.4) has an incorrect sign, then their result agrees with our Eq. (23).

We have also calculated the $O(g^2m_H^2/m_W^2)$ radiative correction to the Higgs-scalar decay to transverse W or Z bosons, for which one cannot employ the Goldstone-boson equivalence theorem. Nevertheless, the calculation is rather simple if performed in the Landau gauge. One need only calculate the corrections to the proper vertex due to Goldstone-boson loops, the Higgs-boson wave-function renormalization (18), and the W mass counterterm (16). There are no $O(g^2m_H^2/m_W^2)$ corrections to the gauge-boson wave functions. One finds the real part of the one-loop-corrected HZ_TZ_T coupling to be

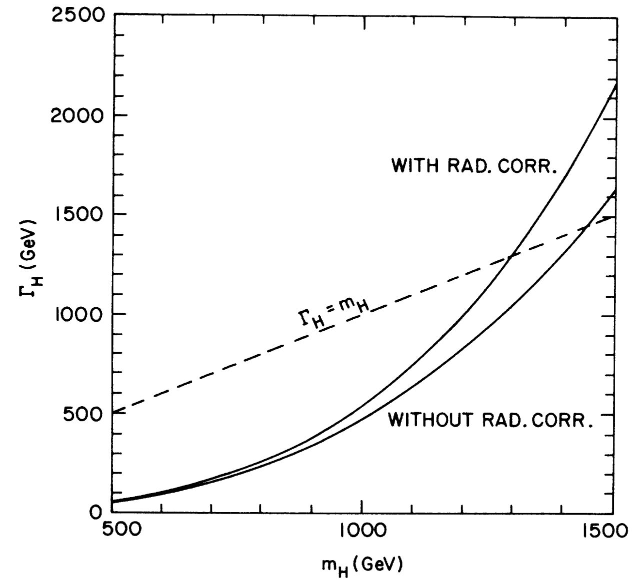

FIG. 6. The decay width of H into gauge bosons $\Gamma_H=\Gamma(H\to W^+W^-)+\Gamma(H\to ZZ)$ as a function of m_H with and without the $O(g^2m_H^2/m_W^2)$ radiative corrections. The dashed line represents equality between m_H and Γ_H.

$$igm_W\left[1+\frac{g^2}{16\pi^2}\frac{m_H^2}{m_W^2}\left[\frac{15}{16}+\frac{\sqrt{3}\pi}{8}-\frac{\pi^2}{8}-\sin^2\theta_W\cos^2\theta_W\right]\right], \quad (24)$$

which is comparable to the correction to the HZ_LZ_L coupling [Eq. (22)]. This agrees with the result of Fleischer and Jegerlehner,[15] see Eqs. (4.23) and (7.4) in their paper. The correction to $HW_T^+W_T^-$ is given by Eq. (24) with the last term set to zero. We do not include these corrections in our analysis of the large-m_H limit of the Higgs-boson width since the transverse partial decay width is suppressed by (m_W/m_H) (Ref. 4).

We next comment on radiative corrections of $O(g^2\ln m_H)$. In principle, such effects can be as large as the $g^2m_H^2/m_W^2$ corrections for some range of m_H values, especially since the latter are only about $+4\%$ for $m_H\simeq 500$ GeV. Fortunately, much can be discerned about the logarithmic corrections without doing any new calculations. First note that we have chosen to define the renormalized g^2 in terms of $G_\mu m_W^2$ [see Eq. (11)]. Therefore, it is effectively defined at the short-distance scale m_W, and employing this g^2 in the lowest-order formulas of Eqs. (1)–(3) will not lead to any $g^2\ln m_W/m_f$ corrections[30] for $m_f<m_W$. If we had instead employed a long-distance coupling such as $g_R^2\equiv 4\pi\alpha/\sin^2\theta_W$, $\alpha=\frac{1}{137}$, as was done in Ref. 15, then additional $g^2\ln m_W/m_f$ corrections would be present and would constitute an additional $+7\%$ correction[30] at the decay-rate level. The difference corresponds to the running of α from its long-distance value of $\frac{1}{137}$ to its short-distance value of $\alpha(m_W)\simeq\frac{1}{128}$.

Even in our renormalization prescription, there are radiative corrections of $O(g^2\ln m_H/m_W)$ which are induced by the renormalization of $g_0^2/(m_W^0)^2$ (such corrections can be viewed as due to the running of G_μ from mass scale m_W to m_H) as well as logarithms that we characterize as infrared. The latter can result from relatively light particles in loop corrections or bremsstrahlung of gauge bosons. Regarding the corrections induced by the renormalization of $g_0^2(m_H^0)^4/(m_W^0)^2$ we find from the calculations in Refs. 20 and 26 a correction factor[31] [to $\Gamma(H\to W^+W^-)$ and $\Gamma(H\to ZZ)$]

$$1-\frac{G_\mu m_W^2}{\sqrt{2}\pi^2}\left[\frac{3}{4}+\frac{3}{4\cos^2\theta_W}-\frac{3}{2}\frac{m_t^2}{m_W^2}\right]\ln\left[\frac{m_H}{m_W}\right]+\cdots\,, \tag{25}$$

where the ellipsis represents corrections not of the form $g^2\ln m_H/m_W$, $\cos^2\theta_W=0.77$, and m_t is the top-quark mass. For $m_H \lesssim 1$ TeV and $m_t \simeq 45-90$ GeV, the correction lies in the range -2% to 0% and is, therefore, not clearly larger than ordinary $O(g^2)$ corrections that we have neglected. It could of course start to grow rapidly if m_t (or some other quark or lepton mass difference) is much larger than 90 GeV. If this is the case, one would also have to include nonlogarithmic $g^2m_t^2/m_W^2$ corrections.

As mentioned before, virtual loops and bremsstrahlung involving γ, $W^\pm$, and Z bosons may also give rise to "infrared" logarithms. One expects such effects to cancel for inclusive decay rates.[32] (We have not explicitly verified that such a cancellation actually occurs.) Therefore, Eqs. (1) and (2) supplemented by the radiative correction factors in Eqs. (23) and (25) should represent a good approximation to the heavy-Higgs-scalar decay width, as long as perturbation theory remains valid.

We can use part of the above computation to also evaluate $g^2m_H^2/m_W^2$ corrections to the $H\to f\bar{f}$ decay rate in Eq. (3). In this case, one need only add the effect of W mass renormalization on the lowest-order coupling $-ig_0m_f^0/2m_W^0$ [see Eq. (16)] to the H wave-function-renormalization effect in Eq. (18). Together, they lead to an enhancement of the rate in Eq. (3) by an overall factor

$$1+\frac{g^2}{8\pi^2}\frac{m_H^2}{m_W^2}\left[\frac{13}{16}-\frac{\sqrt{3}\pi}{8}\right]=1+0.132\frac{G_\mu m_H^2}{\sqrt{2}\pi^2}\,. \tag{26}$$

This result agrees with an early calculation by Veltman.[3] Note that this enhancement factor is somewhat smaller than Eq. (23) and only significant for very large m_H. It also entails a cancellation between terms of order unity in the parentheses of Eq. (26) similar to the cancellation in Eq. (23). Radiative corrections of $O(g^2\ln m_H)$ resulting from the renormalization of g_0/m_W^0 are the same as in Eq. (25). There are additional logarithmic corrections due to the running of the fermion mass from its low-energy value to a short-distance value at m_H. This effect has been discussed somewhat in the literature,[33] so we will not review it here.

In conclusion, we have demonstrated by example how the Goldstone-boson equivalence theorem allows quick and easy calculation of $O(g^2m_H^2/m_W^2)$ radiative corrections in the standard model. This formalism may, therefore, provide insight for studies of the approach to strong coupling and consistency constraints on the Higgs-boson mass.[20,24]

Note added in proof. F. Jegerlehner has confirmed a sign error in Eq. (5.4) of Ref. 15. Consequences for their numerical analysis are under investigation.

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