W-Z interference in ν-nucleus scattering

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The creation of muon pairs by (anti)neutrinos in the Coulomb field of the nucleus provides a direct test of the interference between the intermediate-vector-boson amplitudes, as predicted by the weak-interaction theory. This paper summarizes the main features of the above process and discusses the feasibility of measuring the W-Z interference by searching for recoilless dimuon events using fine-grained counter neutrino detectors. The result from an earlier experiment which searched for this process is discussed in the context of the present calculation.

The cross section for the reaction

$$\overset{(-)}{\nu} + N \rightarrow \overset{(-)}{\nu} + \mu^+ + \mu^- + N \tag{1}$$

can be estimated, rather accurately, by applying Feynman rules to Fig. 1 and using simple dimensional analysis. The calculation will be done in two steps and is based upon the neglect of the boson propagator.

First one computes the muon-pair production cross section in the neutrino collision with a real photon. The amplitude for this process is proportional to the Fermi coupling constant G, so that the cross section must be of the form

$$\sigma_{\nu\gamma} \sim G^2 s \ ,$$

where s, the square of the center-of-mass energy, is a Lorentz-invariant variable which has the required dimension

$$s = (k+q)^2 \ .$$

Here q and k are the photon and the neutrino four-momenta, respectively. This cross section has to be multiplied by the fine-structure constant $4\pi\alpha$ (the γ-μ vertex contributes a factor proportional to the electric charge: $e^2 = 4\pi\alpha$) and, in analogy with the extreme-relativistic case of Compton scattering, by an s-dependent factor $\sim \ln(s/s_{\min})$. Including the phase-space factor for a three-body final state, which is proportional to $1/\pi^3$, it follows that

$$\sigma_{\nu\gamma} \approx \frac{\alpha G^2 s}{\pi^2} \ln\left[\frac{s}{s_{\min}}\right] , \tag{2}$$

where $s_{\min} = (2m_\mu)^2$, since there are two muons in the final state. The exact calculation, which is described in Appendix A, produces the same result, apart from a factor of $\frac{1}{9}$.

The next step is to compute the probability for creating a virtual photon in the Coulomb field of a nucleus (charge Ze), with such a four-momentum q that the center-of-mass energy squared is s. As shown in Appendix B [Eq. (B8)], this probability is

$$P(q^2, s) = \frac{Z^2\alpha}{\pi} \frac{dq^2}{q^2} \frac{ds}{s} \ . \tag{3}$$

The cross section for reaction (1) is therefore

$$\sigma = \frac{Z^2\alpha^2 G^2}{9\pi^3} \int_{s_{\min}}^{s_{\max}} ds \ln\left[\frac{s}{4m_\mu^2}\right] \int_{Q^2=(s/2E)^2}^{Q^2=Q^2_{\max}} \frac{dq^2}{q^2} \ . \tag{4}$$

In this expression $Q_{\max}$ is the maximum momentum that can be transferred to the nucleus without causing it to disintegrate. Obviously, $1/Q_{\max}$ has to be proportional to the size of the nucleus. Since the strong-interaction coupling constant is ~ 1, there is only one fundamental length for the hadrons, which can be taken to be the Compton wavelength of the pion. Therefore,

$$\frac{1}{Q_{\max}} \sim R_{\text{nucleus}} \sim \frac{A^{1/3}}{m_\pi}$$

(A is the total number of nucleons in the nucleus; m_π is the pion mass). $Q_{\min}$ is given by the threshold for the interaction of the neutrino with the virtual photon:

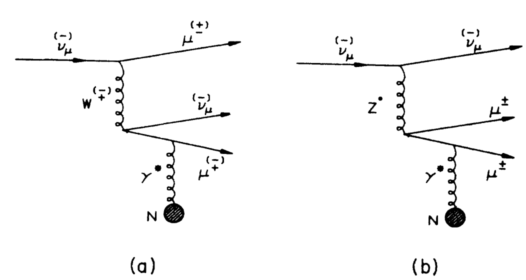

FIG. 1. Diagrams representing (a) charged- and (b) neutral-current "trident" production processes.

$$s_{\min}=(k+q_{\min})^2\approx 2EQ_{\min} ,$$

where E is the incident neutrino energy. Therefore,

$$Q_{\min}\approx\frac{4m_\mu^2}{2E}\ll Q_{\max} .$$

Similarly, $s_{\max}=2EQ_{\max}$. Now the total cross section becomes

$$\sigma=\frac{Z^2\alpha^2G^2}{9\pi^3}\int_{4m_\mu^2}^{2EQ_{\max}}ds\,\ln\left[\frac{s}{4m_\mu^2}\right]\ln\left[\frac{2EQ_{\max}}{s}\right]^2 .$$

A straightforward integration yields the following result (the so-called leading-logarithmic approximation):

$$\sigma\approx\frac{2Z^2\alpha^2G^2}{9\pi^3}s_{\max}\ln\left[\frac{s_{\max}}{4m_\mu^2}\right] . \tag{5}$$

[The above derivation clarifies some misprints which appear in the literature (see Refs. 1 and 2, for example).] From (5) it follows that the cross section for muon-pair production in the collision of a, say, 50-GeV neutrino with an iron nucleus is roughly

$$\sigma^{\mu\mu}\approx 2\times 10^{-40}\ \text{cm}^2 \tag{6}$$

corresponding to

$$R\equiv\left.\frac{\sigma^{\mu\mu}}{\sigma^{\text{CC}}}\right|_{E=50\ \text{GeV}}\approx 1\times 10^{-5} \tag{7}$$

"tridents" per charged-current (CC) event. The result (6) is in good agreement with the exact calculation of the cross section for the reaction (1) in the $V-A$ theory[3,4] (which yields $\sigma\approx 2.5\times 10^{-40}$ cm^2), taking into account approximations made in our calculation (such as the size of the nucleus, for example).

As shown in Fig. 1, the process (1) is actually an admixture of charged and neutral currents. Similar to the case of the neutrino-electron scattering, a Fierz transformation can be performed to relate the two amplitudes, so that the resulting amplitude has the $V-A$ form, but with different values of the vector- and axial-vector-coupling strengths C_V and C_A, respectively. (See Ref. 5 concerning the application of a Fierz transformation.) Consequently, the cross section for the reaction (1) in the Glashow-Salam-Weinberg (GSW) theory has the form

$$\sigma^{\text{GSW}}=C_A^2\sigma_{c_A^2}+C_V^2\sigma_{c_V^2}+C_VC_A\sigma_{c_Ac_V} ,$$

where

$$\begin{aligned}C_V&=\tfrac{1}{2}+2\sin^2\theta_W\approx 1,\\ C_A&=\tfrac{1}{2} ,\end{aligned}\qquad \text{GSW} \tag{8}$$

($C_V=C_A=1$ in the $V-A$ theory). According to Ref. 3, $\sigma_{c_Ac_V}$ is about 2 orders of magnitude smaller than either of the other two terms. Hence,

$$\frac{\sigma^{V-A}}{\sigma^{\text{GSW}}}\approx\frac{\sigma_{c_V^2}+\sigma_{c_A^2}}{\sigma_{c_V^2}+\sigma_{c_A^2}/4}\approx 1.7$$

for a 50-GeV incident neutrino (see Table 2 of Ref. 3). This result is almost independent of the neutrino energy, since the actual values of $\sigma_{c_V^2}$ and $\sigma_{c_A^2}$ are comparable in the energy range of interest.[6]

Therefore, the GSW theory predicts

$$R\approx 0.8\times 10^{-5} \tag{9}$$

"tridents" per charged-current event. This rate, according to (5), depends only logarithmically on the energy of the incident neutrino.

There has been only one reported search for recoilless dimuon events.[7] A sample of 1.5×10^6 neutrino- and 1.8×10^6 antineutrino-induced charged-current events in the CHARM (CERN-Hamburg-Amsterdam-Rome-Moscow) detector yielded 1.7 ± 0.5 "trident" candidates. Taking into account (1) the fact that the mean neutrino energy in the experiment was $\sim$25 GeV, (2) an effective muon momentum cut of 7 ± 2 GeV/c, and (3) a cut on the $\mu^+\mu^-$ invariant-mass distribution of 0.8 GeV (both cuts were applied by CHARM in their data analysis), one would expect to find approximately three events. This prediction follows from a Monte Carlo calculation based on Ref. 3, where a two-parameter fit to the nuclear charge density was used. Given the limited number of "trident" events, which is mainly due to the above-mentioned muon momentum cut of 7 GeV/c, there is therefore no experimental evidence for the "trident" production process.

To conclude, the creation of muon pairs by neutrinos in the Coulomb field of the nucleus provides a direct test of the interference between the charged- and neutral-current amplitudes in the GSW theory. The predicted rate of "trident" events and their experimental signature render it quite feasible to measure the W-Z interference by searching for recoilless dimuon events using the existing fine-grained counter neutrino detectors [FMMF (Fermilab–MIT–Michigan State University–University of Florida) and CHARM 2]. For example, because of fine-grain structure of the FMMF detector (polypropylene flash chambers between sand and steel shots $X_0=12$ cm, $\lambda=90$ cm; sampling thickness is $22\% X_0$ and $3\%\ \lambda$) off-line pictures of neutrino events look like bubble-chamber photographs, making the selection of recoilless events straightforward. Possible backgrounds to trident candidates (charged pion decays and charm production with subsequent semileptonic decay) are very small due to the low-particle multiplicity (only two well-identified muons) and rather large hadronic effective mass W in the case of charm production [see Ref. 8 for a measured W distribution from a Big European Bubble Chamber (BEBC) sample of dilepton events]. As can be inferred from the predicted muon momentum distribution for "trident" events,[1] a good reconstruction of very low muon momenta (down to a few GeV/c) and a higher mean neutrino energy are essential; without them one cannot get a sufficient number of recoilless dimuon events to measure accurately the W-Z interference using fine-grained counter neutrino detectors. It is also important to use the measured form factor for a given target material to remove any remaining uncertainty in the theoretical prediction.

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APPENDIX A

The process

$$\nu+\gamma\rightarrow\nu+\mu^{+}+\mu^{-} \tag{A1}$$

is described, in the lowest order, by the sum of the two diagrams in Fig. 2. The matrix element, assuming $V-A$ coupling, has the form

$$M=M_1M_2\ ,$$

where

$$M_1=\frac{Ge}{\sqrt{2}}\bar{u}(p_2)\left[\hat{\epsilon}\frac{1}{\hat{p}_2-\hat{q}-m}\gamma_\alpha(1+\gamma_5)+\gamma_\alpha(1+\gamma_5)\frac{1}{-\hat{p}_1+\hat{q}-m}\hat{\epsilon}\right]v(p_1)$$

and

$$M_2=\bar{u}(k')\gamma_\alpha(1+\gamma_5)u(k)\ .$$

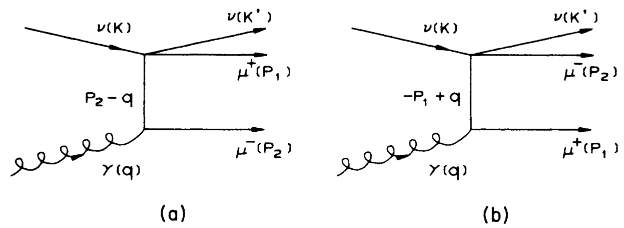

FIG. 2. Feynman diagrams for neutrino-photon scattering.

In the above expression G is the weak coupling constant, e is the electric charge, u and v are the lepton spinors, and ϵ is the polarization vector of the photon ($\hat{\epsilon}\equiv\epsilon^\alpha\gamma_\alpha$).

In order to compute the spin-averaged probability for the above process Veltman's SCHOONSCHIP algebraic computer program[9] was used. The calculation produced the following formula for the differential cross section:

$$d\sigma=\frac{4e^2G^2}{(2\pi)^5(k\cdot q)}[\ \ ]\delta^4(k+q-k'-p_1-p_2)\times\frac{d^3p_1}{E_1}\frac{d^3p_2}{E_2}\frac{d^3k'}{E'}\ , \tag{A2}$$

where

$$[\ \ ]=\frac{(p_1\cdot q)(k\cdot q)(p_2\cdot k')}{A^2}+\frac{(p_2\cdot q)(k'\cdot q)(p_1\cdot k)}{B^2}$$
$$+\frac{1}{A}\frac{1}{B}[(2p_1\cdot p_2-p_1\cdot q-p_2\cdot q)(p_1\cdot k)(p_2\cdot k')-(p_1\cdot p_2)(p_1\cdot k)(q\cdot k')$$
$$-(p_1\cdot p_2)(p_2\cdot k')(q\cdot k)+(q\cdot p_1)(k\cdot p_2)(k'\cdot p_2)+(q\cdot p_2)(p_1\cdot k)(p_1\cdot k')]$$

and $A=[(p_1-q)^2-m^2]$, $B=[(p_2-q)^2-m^2]$. In arriving at (A2) the terms proportional to m_μ^2 were neglected and $V_{\rm rel}E_1E_2=\mathbf{k}\cdot\mathbf{q}$ was used. Integrating (A2) yields the following cross section for the process (A1) (see also Ref. 1):

$$\sigma\approx\frac{\alpha G^2 s}{9\pi^2}\ln\left[\frac{s}{4m_\mu^2}\right]\ ,$$

where s is the center-of-mass energy squared and $\alpha=e^2/4\pi$.

APPENDIX B

In order to compute the probability for creating a virtual photon in the Coulomb field of the nucleus the Weizsäcker-Williams method of equivalent photons[10] will be employed. Consider a collision between the particle i and a virtual photon emitted by the particle p (charge Ze) and having a four-momentum $q=P-P'$ [Fig. 3(a)]. If the particle p is very fast, its electromagnetic field is almost transverse and hence the virtual photon is not very different from a real one, i.e., $q^2\approx0$. In that case, $P\approx P'$ and the motion of the particle may be regarded as uniform, quasiclassical motion in a straight line, so that the corresponding current is independent of the particle spin. We choose the coordinate system such that $\mathbf{q}$ is fixed, $\mathbf{P}\rightarrow\infty$, and i is at rest. The amplitude for this scattering process is

$$M\sim\frac{J_\mu}{q^2}\langle f\,|\,j_\mu\,|\,i\rangle\ , \tag{B1}$$

where $J_\mu=(P+P')_\mu$ and $P'=P+q$. The particle four-momenta are $P=(E,P_z,0,0)$ and $P'=(E',P_z-q_z,q_\perp,0)$. Hence,

$$J_0=2E-q_0\approx2P_z-q_0,\quad J_z=2P_z-q_z,\quad J_\perp=-\mathbf{q}_\perp\ .$$

Now

$$M^2=E'^2-(P_z-q_z)^2-q_\perp^2\ . \tag{B2}$$

Using $E'=E-q_0$ and $E^2-P_z^2=M^2$, (B2) yields

$$0=-2Eq_0+q_0^2+2P_zq_z-q_z^2-q_\perp^2\ . \tag{B3}$$

Since $q^2=q_0^2-|\,\mathbf{q}\,|^2$, it follows that

$$q_0-q_z=\frac{q^2}{2E}\ . \tag{B4}$$

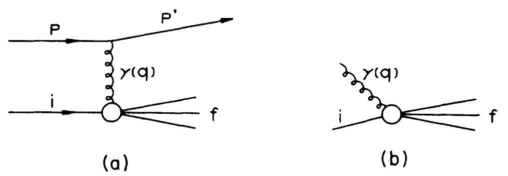

FIG. 3. Feynman diagrams for (a) electromagnetic scattering and (b) photoproduction amplitude.

The amplitude (B1) reads

$$M \sim \frac{J_0\langle f \mid j_0 \mid i\rangle - J_z\langle f \mid j_z \mid i\rangle - J_\perp\langle f \mid \mathbf{j}_\perp \mid i\rangle}{q^2} ,$$

i.e.,

$$M \sim \frac{1}{q^2}[(2P_z - q_0)\langle f \mid j_0 \mid i\rangle - (2P_z - q_z)\langle f \mid j_z \mid i\rangle + \mathbf{q}_\perp\langle f \mid \mathbf{j}_\perp \mid i\rangle] .$$

Since $q_0 j_0 = q_z j_z + \mathbf{q}_\perp \cdot \mathbf{j}_\perp$ (due to current conservation) the amplitude is

$$M \sim \frac{1}{q^2}\left[2P_z\left[\frac{q_z - q_0}{q_0}\right]\langle f \mid j_z \mid i\rangle + \frac{2p_z - q_0}{q_0}\mathbf{q}_\perp\langle f \mid \mathbf{j}_\perp \mid i\rangle + \mathbf{q}_\perp\langle f \mid \mathbf{j}_\perp \mid i\rangle\right] .$$

From (B4) it follows that the first term can be neglected. The second and third terms partly cancel, so that the amplitude finally reads

$$M \sim \frac{1}{q^2}\left[\frac{2P_z}{q_0}\langle f \mid \mathbf{q}_\perp \cdot \mathbf{j}_\perp \mid i\rangle\right] . \tag{B5}$$

The differential cross section for the process in Fig. 3(a) is therefore

$$d\sigma = \frac{Z^2 e^4}{2E}\left[\frac{2P_z}{q_0}\right]^2 \frac{1}{q^4}\left|\sum_f \langle f \mid \mathbf{q}_\perp \cdot \mathbf{j}_\perp \mid i\rangle\right|^2 \times \frac{d^3P'}{(2\pi)^3 2E'}(2\pi)^4\delta^4(p_f - p_i - q) .$$

Using the definition of the photoproduction cross section σ_γ depicted in Fig. 3(b)

$$\sigma_\gamma = \frac{e^2}{2q_0}\left|\sum_f \langle f \mid j \cdot \epsilon \mid i\rangle\right|^2 (2\pi)^4\delta^4(p_f - p_i - q)$$

($j \cdot \epsilon = |\mathbf{j}_\perp|$), the differential cross section can be written as

$$d\sigma = \frac{Z^2 e^2}{2E}\left[\frac{2E}{q_0}\right]^2 \frac{q_\perp^2}{q^4}\frac{d^2q_\perp dq_0}{(2\pi)^3 2E} 2q_0\sigma_\gamma , \tag{B6}$$

where we have used $E' \approx E \approx p_z$ and $d^3p' = d^3q = d^3q_\perp dq_0$. Using $d^2q_\perp = \pi\, dq^2$ and $e^2 = 4\pi\alpha$, it follows that

$$d\sigma = \frac{Z^2\alpha}{\pi}\frac{dq^2}{q^2}\frac{dq_0}{q_0}\sigma_\gamma \tag{B7}$$

since $q^2 \approx q_\perp^2$. From the definition of the square of the center-of-mass energy s,

$$s = (k + q)^2 \approx 2k \cdot q$$

(where q and k are the photon and the neutrino four-momenta, respectively), it follows that

$$s = 2k \cdot q = 2k_0 q_0 ,$$

i.e.,

$$\frac{ds}{s} = \frac{dq_0}{q_0} .$$

Therefore,

$$d\sigma = \frac{Z^2\alpha}{\pi}\frac{dq^2}{q^2}\frac{ds}{s}\sigma_\gamma$$

and the required probability is

$$P(q^2, s) = \frac{Z^2\alpha}{\pi}\frac{dq^2}{q^2}\frac{ds}{s} . \tag{B8}$$

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