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The decay $\bar{B}^0 \rightarrow \pi^+ l^- \nu$ to probe V_{bu}

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The semileptonic B decay, $\bar{B}^0 \rightarrow \pi^+ l^- \nu$, is studied in a nonrelativistic picture by treating the pion correctly as a quasi-Nambu-Goldstone boson. The branching ratio is much larger than the value predicted by a two-body bound-state picture for the pion. This decay at the $\Upsilon(4S)$ peak in e^+e^- annihilation will be one of the best suited processes to determine the Kobayashi-Maskawa mixing-matrix element V_{bu}.

The first evidence for the $b \rightarrow u$ transition has been reported by the ARGUS Collaboration[1] in the two exclusive modes: $B^+ \rightarrow p\bar{p}\pi^+$ and $B^0 \rightarrow p\bar{p}\pi^+\pi^-$. Relating the reported branching ratios

$$B(B \rightarrow p\bar{p}\pi^+) = (3.7 \pm 1.3 \pm 1.4) \times 10^{-4}$$

and

$$B(B \rightarrow p\bar{p}\pi^+\pi^-) = (6.0 \pm 2.0 \pm 2.2) \times 10^{-4}$$

to the $b \rightarrow u$ transition strength V_{bu} at the quark level involves many uncertainties. Nevertheless, it appears possible to argue that the ARGUS branching ratios suggest that the magnitude of V_{bu} is close to its upper bound set by the measurement of inclusive lepton spectrum. If so, we expect that the $b \rightarrow u$ transition ought to be observed in semileptonic B decays also. From an experimental veiwpoint, the decay $\bar{B}^0 \rightarrow \pi^+ l^- \nu$ has a special advantage because the plot of the invariant mass $(p_B - p_\pi - p_l)^2$ shows a peak at zero, the rest mass squared of the neutrino. The decay rate for $B \rightarrow \pi l \nu$ was computed previously by treating the pion as a two-body bound state[2,3] and by applying a few semiquantitative methods.[4]

We have examined the decay mode $B^0 \rightarrow \pi^+ l^- \nu$ without use of the bound-state picture for the pion. Though our approach is along the same line as one of the arguments made by the authors of Ref. 4, our final answer has come out to be much larger than their value. We have computed the decay rates for $B \rightarrow \rho l \nu$ and $B \rightarrow A_1 l \nu$ through the vector hadronic current in a similar approach. Those branching ratios are consistent, within theoretical uncertainty, with the predictions of the two-body bound-state picture for ρ and A_1. The sources of ambiguity in our calculation have been critically investigated to show how much uncertainty is involved in a theoretical estimate of the $B \rightarrow \pi l \nu$ decay rate.

Before presenting our results of computation, we would like to remark on plausible values for V_{bu} deduced from the ARGUS branching ratios. It has been known[5] that

$$B(B \rightarrow \Lambda_c X) = (7.4 \pm 2.9) \times 10^{-2}$$

and

$$B(B \rightarrow pX) = (6.1 \pm 0.8 \pm 1.0) \times 10^{-2}$$

where p stands for protons which are not decay products of Λ. These numbers allow us to deduce that the probability of the formation of a proton in the final state of decay $b \rightarrow u\bar{u}d$ is approximately 8%. The real difficult issue is how much of $B \rightarrow pX$ is the three- and four-body decays $B \rightarrow p\bar{p}\pi^+$ and $p\bar{p}\pi^+\pi^-$. The inclusive hadron spectra for π, K, and p in e^+e^- annihilation at $\sqrt{s} \simeq 5$ GeV can be nicely fitted with a statistical distribution of a common temperature $\simeq 200$ MeV.[6] It means that $\langle E_\pi \rangle \simeq 0.5$ GeV and $\langle E_p \rangle \simeq 1.3$ GeV. In a statistical picture, when N and $\bar{N}$ of typical energies are produced, energy of 2.7 GeV $(=m_B - 2 \times 1.3$ GeV) is left behind to be partitioned into pions. If we apply this picture to the B decay, it appears unlikely that $N\bar{N}\pi$ and $N\bar{N}\pi\pi$ dominate in the inclusive decay $B \rightarrow N\bar{N}X$. By assigning an equal statistical weight to each of the allowed modes, we can make the following educated guess:

$$B(B^+ \rightarrow N\bar{N}\pi) \approx 3B(B^+ \rightarrow p\bar{p}\pi^+) \ ,$$
$$B(B^+ \rightarrow N\bar{N}\pi\pi) \approx 4B(B^0 \rightarrow p\bar{p}\pi^+\pi^-) \ , \tag{1}$$

and therefore

$$B(B^+ \rightarrow N\bar{N}\pi + N\bar{N}\pi\pi) \approx 3.5 \times 10^{-3} \ , \tag{2}$$

with experimental errors from the ARGUS data.[1] Consequently, $|V_{bu}|^2$ is to be extracted from

$$(8 \times 10^{-2})(r_2 + r_3) 2.2 |V_{bu}/V_{bc}|^2 \simeq 3.5 \times 10^{-3} \ , \tag{3}$$

where r_n is the fraction of $N\bar{N}n\pi$ in $N\bar{N}X$ and 2.2 comes from a phase-space correction for $b \rightarrow u\bar{u}d$ relative to

$b \to c\bar{u}d$. A very conservative value, $r_2+r_3 \simeq 0.5$, leads us to

$$|V_{bu}/V_{bc}| \simeq 0.2 \ , \tag{4}$$

which is almost coincident with the upper bound set by the inclusive lepton spectrum.[7] Considering the large uncertainty in the experiment and especially in the theoretical deduction, we can take this value, Eq. (4), only as a hint. Nevertheless, the ARGUS experiment kindles a hope to observe the $b \to u$ transition in exclusive semileptonic decay modes.

The hadronic matrix elements of semileptonic $b \to u$ transitions were computed in the two-body bound-state picture for B and a final hadron X at a vanishing spatial momentum transfer, namely, for B and X both at rest.[2-4] For a moving X, the matrix element $\langle X|\bar{u}\gamma_\lambda(1-\gamma_5)b|\bar{B}\rangle$ obtained at $q^2=(p_B-p_X)^2=(m_B-m_X)^2$ can be extrapolated with form factors dominated with B^* and B_A^*. When X is a pion, a quasi-Nambu-Goldstone boson, the two-body bound-state picture is a big suspect. Our main purpose is to compute $\langle \pi|\bar{u}\gamma_\lambda(1-\gamma_5)b|\bar{B}\rangle$ with the two-body bound-state picture only for B and B^*, not for π. The pion is treated correctly as a Nambu-Goldstone boson, namely, by PCAC (partial conservation of axial-vector current). We need to assume B^* dominance in the vector form factor at least near $q^2=(m_B-m_\pi)^2$.

Our picture of the decay $B \to \pi l \nu$ is drawn in Fig. 1. Only the vector current contributes to $B \to \pi$. In the bound-state picture for B and π, the matrix element was evaluated with B and π on mass shell. By contrast, we can evaluate it for a soft pion with B and B^* on mass shell and then extrapolate it back to the physical region. This method avoids the two-body bound-state picture for the pion and incorporates the Nambu-Goldstone nature of the pion. The success of PCAC and soft-pion calculations in the past makes us feel more confident in this method. Computation is straightforward. B^* dominance gives us

$$\langle \pi^+|\bar{u}\gamma_\lambda b|\bar{B}^0\rangle = F_+(q^2)(p_B+p_\pi)_\lambda + F_-(q^2)(p_B-p_\pi)_\lambda \ , \tag{5}$$

$$F_+(q^2) = 2f_{B^*}g_{B^*\bar{B}\pi}/(m_B^2-q^2) \ , \tag{6}$$

where $q=p_B-p_\pi$, $\sqrt{2}g_{B^*\bar{B}\pi}$ is the strong coupling of $B^{*+} \to B^0\pi^+$, and f_{B^*} is defined by

$$\langle 0|\bar{u}\gamma_\lambda b|B^{*-}\rangle = \sqrt{2}f_{B^*}\epsilon(p_B)_\lambda \ . \tag{7}$$

The constant f_{B^*} is related to the B decay constant f_B,

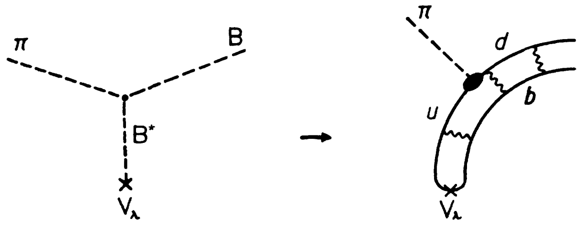

FIG. 1. The B-to-B^* transition through the SU(2) chiral current to which the pion field is related by PCAC.

with the normalization of $f_\pi=93$ MeV, by

$$f_{B^*}=(m_{B^*}m_B)^{1/2}f_B \ , \tag{8}$$

in the nonrelativistic model of B and B^*. The numerical values of f_B from different theoretical estimates[8] and from the B^*-B mass difference[9] give the f_B/f_π ratio in the range of

$$f_B/f_\pi = 0.7^{+0.2}_{-0.1} \ . \tag{9}$$

B^* dominance in the F_+ form factor is expected to be accurate near the kinematical boundary $q^2=(m_B-m_\pi)^2$ but less certain as q^2 decreases. The strong-interaction coupling $g_{B^*\bar{B}\pi}$ is obtained by use of the PCAC relation for a massless pion combined with the nonrelativistic bound-state picture for B and B^*,

$$g_{B^*\bar{B}\pi}=(m_{B^*}m_B)^{1/2}/2f_\pi \ . \tag{10}$$

Then the differential decay rate for $B^0 \to \pi^+ l^- \nu$ is given by

$$\frac{d\Gamma}{dE_\pi}=\frac{G^2}{12\pi^3}m_B|V_{bu}|^2\left(\frac{m_B}{m_{B^*}}\right)^2\left(\frac{f_B}{f_\pi}\right)^2\frac{p_\pi^3}{(1-q^2/m_{B^*}^2)^2} \ , \tag{11}$$

where the pion energy E_π is equal to $(m_B^2+m_\pi^2-q^2)/2m_B$. When we integrate Eq. (11) in E_π with the B^*-dominated form factor over the entire phase space, we obtain

$$\Gamma(\bar{B}^0 \to \pi^+ l^- \nu)/\Gamma(b \to u l^- \nu) = 0.48(f_B/f_\pi)^2 \ . \tag{12}$$

With $f_B=60$ MeV, the right-hand side of Eq. (12) is 0.20, in contrast with the previous prediction 0.02 of Ref. 3. The authors of Ref. 4 derived a formula equivalent to our Eq. (11), but they concluded that their numerical answer is 0.03 and consistent with the prediction of Ref. 3. This discrepancy with our answer originates largely in the unacceptably small value for f_B chosen by them, which explains a factor of ~ 4. The remainder is unclear.

We would like to advocate our method and to clarify a degree of uncertainty involved in it. The first comment is on the F_+ form factor in Eq. (6). For the K_{l3} form factors, the current-algebra relation

$$F_+^{(K)}(m_K^2)+F_-^{(K)}(m_K^2)=f_K/f_\pi \ ,$$

supplemented with the experimental values[10] $F_+^{(K)}(0)=0.961$, $f_K/f_\pi=1.22$, and the slope parameter $\lambda_+=0.030$, gives us $F_-^{(K)}(m_K^2) \simeq -0.11$. If we treated π and K as two-body bound states, the overlap of their wave functions would be given by

$$\langle \psi_K|\psi_\pi\rangle = \tfrac{1}{2}(m_\pi m_K)^{-1/2}[F_+((m_K-m_\pi)^2)(m_K+m_\pi) + F_-((m_K-m_\pi)^2)(m_K-m_\pi)] . \tag{13}$$

Extrapolating $F_+^{(K)}$ with $\lambda_+=0.030$ and $F_-^{(K)}$ with a flat q^2 dependence as evidenced in experiment, one finds that the right-hand side of Eq. (13) is 1.32. This means that K and π wave functions "overlap more than 100%." This is one evidence that the two-body bound-state picture does not work for the pion.[11] Our value of F_+ for the $B \to \pi$ transition translates to "more than 100% overlap" in the two-body bound-state calculation for π, but we are not disturbed with it at all because of this reason. The second remark is on the possibility of testing our technique in heavy bosons other than B. The charmed-meson version of our crucial relation Eq. (10) reads

$$g_{D^*\bar{D}\pi} = (m_{D^*} m_D)^{1/2}/2f_\pi \ . \tag{14}$$

The relation can be tested in principle in D^* decay if we know how to compute radiative D^* decay rates. If we adopt the nonrelativistic $M1$ transition picture with $m_{u,d} = 0.3$ GeV and $m_c = 1.7$ GeV, we obtain $B(D^{*+} \to D^+\gamma) \simeq 1.3\%$ and $B(D^{*0} \to D^0\gamma) \simeq 25\%$ as compared with the world-average values[12] of data, $(17 \pm 11)\%$ and $(48.5 \pm 7.6)\%$, respectively. However, the calculated branching ratios are highly sensitive to the tiny phase space $m_{D^*} - m_D - m_\pi$, and there is still too large experimental uncertainty in $B(D^{*+} \to D^+\gamma)$, which used to be quoted as $(8 \pm 8)\%$. Therefore, Eq. (14) cannot be tested reliably in D^* decay.

The strategy of bringing B^* on shell and extrapolating a final meson to zero energy is applicable to the transitions $B \to \rho$ and $B \to A_1$ as well, if one adopts the usual assumption that isovector vector and axial-vector currents are dominated by ρ and A_1, respectively. Treating B and B^* as s-wave bound states, we obtain by this method the matrix elements

$$\begin{aligned} &\langle \rho^+ | \bar{u}\gamma_\lambda b | \bar{B}^0 \rangle = i\epsilon_{\lambda\mu\nu\kappa}\epsilon^\kappa(\rho) p_\rho^\mu p_B^\nu G(q^2) \ , \\ &G(q^2) = m_B (f_B/f_\rho)(m_\rho^2/m_{u,d})/(m_{B^*}^2 - q^2) \ , \end{aligned} \tag{15}$$

and

$$\begin{aligned} &\langle A_1^+ | \bar{u}\gamma_\lambda b | \bar{B}^0 \rangle = \epsilon_\lambda(A_1) H(q^2) \ , \\ &H(q^2) = 2(m_B m_{B^*})(f_B/f_{A_1}) m_{A_1}^2/(m_{B^*}^2 - q^2) \ , \end{aligned} \tag{16}$$

where f_ρ and f_{A_1} are defined as

$$\begin{aligned} &\langle 0 | \bar{u}\gamma_\lambda d | \rho^- \rangle = \sqrt{2} f_\rho \epsilon_\lambda(\rho) \ , \\ &\langle 0 | \bar{u}\gamma_\lambda\gamma_5 d | A_1^- \rangle = \sqrt{2} f_{A_1} \epsilon_\lambda(A_1) \ . \end{aligned} \tag{17}$$

We know that $f_{A_1} \simeq f_\rho$ (Ref. 13) and $f_\rho^2 = 2m_\rho^2 f_\pi^2$.[14] The matrix elements of Eqs. (15) and (16) give us with $f_B = 60$ MeV

$$\Gamma(\bar{B}^0 \underset{V}{\to} \rho^+ l^- \nu)/\Gamma(b \to ul^-\nu) \simeq 0.08 \ , \tag{18}$$

$$\Gamma(\bar{B}^0 \underset{V}{\to} A_1^+ l^- \nu)/\Gamma(b \to ul^-\nu) \simeq 0.12 \ . \tag{19}$$

In our approach, the axial-vector current $\bar{u}\gamma_\lambda\gamma_5 b$ does not lead dominantly to one-hadron final states if $\bar{u}\gamma_\lambda\gamma_5 b$ is dominated with B_A^* of $J^P = 1^+$. The reason is that B cannot make a transition to a p wave bound state B_A^*, after π, ρ, and A_1 fields are expressed in terms of generators of SU(6). Therefore, the axial-vector current $\bar{u}\gamma_\lambda\gamma_5 b$ tends to generate multibody final states in the leading nonrelativistic order. Keeping this fact in mind, we find that our rates, Eqs. (18) and (19), are not in marked disagreement with the prediction by the bound-state model for ρ and A_1. When we combine our numbers, Eqs. (12), (18), and (19), for $\bar{B}^0 \to \pi^+ l^- \nu$, $\rho^+ l^- \nu$, and $A_1^+ l^- \nu$, the sum of these rates saturates about 80% of the rate for $b \to ul^-\nu$ through the weak vector current, $\Gamma(b \underset{V}{\to} ul^-\nu) = \frac{1}{2}\Gamma(b \to ul^-\nu)$. We would like to argue that this saturation by π, ρ, and A_1 is not absurd.

In the spectator model of semileptonic B decay, the invariant mass of the spectator $\bar{d}$ and the produced u is given by $m_h = (m_d^2 + m_u^2 + 2m_d E_u)^{1/2}$ with $m_u \le E_u \lesssim \frac{1}{2} m_B$. Here, for simplicity, we have replaced Fermi motion of the spectator $\bar{d}$ by the constituent mass m_d of $\bar{d}$ at rest. For $m_d \simeq 0.3$ GeV, m_h is no larger than about 1.3 GeV ($\approx m_{A_1}$). Although actual Fermi motion creates a high-mass tail, saturating $u\bar{d}$ pair states below 1.5 GeV with π, ρ, and A_1 is not out of line from our experiences in current-algebra sum rules.

Finally, we should examine critically how much we can trust our numerical answer for $B(\bar{B}^0 \to \pi^+ l^- \nu)$. Extrapolation of $F_+(q^2)$ by B^* dominance can be questioned when q^2 is far away from m_B^{*2}. However, experimenters can always select data samples from a large-q^2 region and compare them with the differential decay rate of Eq. (11) to extract V_{bu}. The uncertainty related to B^* dominance can be largely circumvented in this way in data analysis. The cornerstone of our calculation is the value of $g_{B^*\bar{B}\pi}$ given by Eq. (10). How reliable is it? The same formula for K^*, $g_{K^*\bar{K}\pi} = (m_K m_{K^*})^{1/2}/2f_\pi$, gives a K^* width of 61 MeV, as compared with the experimental value of 51 MeV. This is a 20% overestimate.[15] If we trust the world-average values[12] of $m(D^{*0}) - m(D^0)$ and $B(D^{*0} \to D^0\gamma)$ data and the nonrelativistic $M1$ transition calculation for $D^{*0} \to D^0\gamma$, $g_{D^*\bar{D}\pi} = (m_{D^*} m_D)^{1/2}/2f_\pi$ overestimates the value of $g_{D^*\bar{D}\pi}$ by about a factor of 1.7. In general, deviation from the static quark limit reduces the magnitude of $g_{B^*\bar{B}\pi}$. This reduction is caused by relativistic mixing between spin and orbital wave functions, which used to be called "leakage" in sum rules. Therefore, it is not surprising if an experimental value of $\Gamma(\bar{B}^0 \to \pi^+ l^- \nu)$ comes out to be a little smaller than our prediction. However, we find no reason to expect a reduction in rate to be much more than a factor of 2 near the B^* pole.

To conclude, we present a final form of our prediction on the decay rate for $\bar{B}^0 \to \pi^+ l^- \nu$ from PCAC and the nonrelativistic picture of B and B^*:

$$d\Gamma/dE_\pi = (0.80^{+0.20}_{-0.45}) \times \text{Eq. (11)} \ . \tag{20}$$

If one integrates in E_π over the entire kinematical region with the B^*-dominated form factor and takes account of the experimental value[12] for $B(B \to lX)$ and a phase-space correction factor $\Gamma(b \to ul^-\nu)/\Gamma(b \to cl^-\nu) \simeq 2.2$, the branching ratio comes out to be

$$B(\bar{B}^0 \to \pi^+ l^- \nu) \simeq (0.098^{+0.025}_{-0.055})(f_B/f_\pi)^2 |V_{bu}/V_{bc}|^2 \ . \tag{21}$$

The upper limits correspond to no leakage, Eq. (12), the central values are obtained with the reduction factor taken from the K^* width, and the lower limits come from the most prudent values based on the current world averages of the D^{*0} decay parameters. Some readers may be disappointed with rather large theoretical uncertainty, in particular, in the lower limits. Note, however, that the errors are not statistical and that even the lower limit is still more than three times larger than the prediction of Ref. 3. Claiming any better theoretical reliability would simply be unfounded at the present moment. With $f_B = 60$ MeV, where f_π is normalized to 93 MeV ($=131$ MeV$/\sqrt{2}$), and $|V_{bu}/V_{bc}|^2 \simeq 0.05$, for instance, the central value of the right-hand side of Eq. (21) is 2×10^{-3}, which is within reach of current experiments. For the purpose of extracting a value of V_{bu}/V_{bc}, it will be more reliable if one selects data toward the large end of q^2, namely, low-energy pions for analysis. It is also interesting to look for the decay $D^0 \rightarrow \pi^- l^+ \nu$ at the $\psi(3S)$ peak in e^+e^- annihilation. Our prediction based on the same set of assumptions is

$$B(D^0 \rightarrow \pi^- l^+ \nu) \simeq (0.071^{+0.017}_{-0.040})(f_D/f_\pi)^2 |V_{cd}|^2 , \quad (22)$$

where $B(D^0 \rightarrow l^+ X) \simeq 7\%$ has been used and the upper and lower bounds have been obtained in the same way as in Eqs. (20) and (21). Since the value of $|V_{cd}/V_{cs}|$ is better known, experiment in $D^0 \rightarrow \pi^- l^+ \nu$ will be able to narrow the theoretical uncertainty.

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