Comment on a paper by Pauli and Brodsky

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It is noted that a recent work by Pauli and Brodsky is in conflict with theorems obtained by the author which demonstrate that light-cone field theories lack both covariance and a conserved charged operator.

The light-cone approach to quantum field theory has received a great deal of attention in recent years in a wide variety of applications. Unfortunately, the existence of a number of theorems concerning the light-cone method casts considerable doubt upon the validity of the results which follow from this technique. In the present paper attention is focused specifically upon a recent work of Pauli and Brodsky which applies light-cone coordinates to a two-dimensional field theory. Three specific (though not entirely independent) objections to the consistency of that work are in order.

The first point is that despite the claims of Ref. 1 the model is not covariant. In fact, even for vanishing coupling the massive Dirac field in $(1+1)$-space fails the test of Lorentz invariance. Quoting results obtained earlier in a similar context[2] one finds that

$$[T^{+-}(x),J^{+-}]=-i(x^{+}\partial^{-}-x^{-}\partial^{+})T^{+-}(x)+\frac{m^2}{8\sqrt{2}}\lim_{L\to\infty}L[\psi_1(L)-\psi_1(-L)]\psi_1(x), \tag{1}$$

where $T^{+-}(x)$ is an element of the energy-momentum tensor and J^{+-} is the generator of Lorentz boosts. Using the form of the two-point function for large values of the argument it is easily verified that the extra term in (1) cannot be made to vanish and that the Poincaré algebra cannot be realized in the case $m\neq 0$.

Although it is possible formally to avoid this problem by choosing periodic boundary conditions as done in Ref. 1, this in itself introduces new complications with regard to translational invariance and the canonical commutation relations. A complete treatment would then require a careful demonstration that all noncanonical terms in the Poincaré algebra drop out in the limit of large L. This was not done in Ref. 1, nor indeed is such a program consistent with general principles of quantum field theory.[3]

The second observation is that there exists a very general argument establishing the noncovariance of the massive Dirac field which depends only upon the scale noninvariance of the theory. Indeed it can be shown[3]

$$\langle 0\,|\,[J^{+-},T^{\alpha}_{\alpha}(x)]\,|\,0\rangle=-\frac{i}{2}\int_0^\infty d\kappa^2\sigma(\kappa^2)\ ,$$

where $\sigma(\kappa^2)$ is a positive weight function as long as the trace $T^{\alpha}_{\alpha}(x)$ does not vanish. Since the mass term guarantees a nonzero form for that operator, one concludes directly the absence of a Lorentz-invariant vacuum state.

Finally, there is the assumption which is made in Ref. 1 concerning the existence of a conserved global charge operator Q. In fact, it has been shown in Ref. 4 that

$$i\langle 0\,|\,[\partial_+Q,j^{+}(x)]\,|\,0\rangle=\tfrac{1}{4}\int_0^\infty \kappa^2\rho(\kappa^2)d\kappa^2\ ,$$

where $\rho(\kappa^2)$ is a positive-definite weight function which cannot be proportional to a delta function in the variable κ^2. One thus concludes that a conserved charge operator does not exist in the two-dimensional massive light-cone theory.

In summary, it should be stressed that the difficulties inherent to the light-cone formalism are numerous and nontrivial. Aside from the question of internal consistency there exist serious legitimate questions which can be raised concerning its generally assumed equivalence to the conventional coordinate approach. It seems reasonable to suggest that for the immediate future at least, work on light-cone physics should be directed primarily to the conceptual problems which it raises rather than to calculations of questionable legitimacy.

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[3]C. R. Hagen, Phys. Rev. D **24**, 2605 (1981).
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