Comparison of quantization methods for anomalous chiral models

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Based on knowledge of the $(1+1)$-dimensional chiral Schwinger model, different methods of quantizing Abelian anomalous models are analyzed by means of the Becchi-Rouet-Stora-Tyutin procedure giving indications of the corresponding physical states in $3+1$ dimensions.

I. INTRODUCTION

The quantization of anomalous chiral models might be a way to give mass to gauge bosons without need of Higgs particles. This would happen because, due to the quantum anomaly, first-class constraints would turn into second-class ones increasing the number of degrees of freedom.

Jackiw and Rajaraman[1] have shown in the $(1+1)$-dimensional chiral Schwinger model that the arbitrariness in the regularization may produce a massive mode in addition to the massless one. Faddeev and Shatashvili[2] have introduced a Wess-Zumino field to transform second-class into first-class constraints. It was shown by Babelon, Schaposnik and Viallet[3] and by Harada and Tsutsui[4] that a gauge-fixing procedure may originate, due to the regularization arbitrariness, a kinetic term for the Wess-Zumino field, and that the functional integration over fermions leads to a gauge-invariant effective theory. Rajaraman[5] has indicated that the chiral Schwinger model in $1+1$ dimensions has two degrees of freedom for the regularization parameter $a>1$ and only one for $a=1$. Similarly Falck and Kramer[6] have analyzed the model in the gauge-fixed form to see that for $a>1$ there are two first-class constraints to which two second-class ones add for $a=1$, and Boyanovsky[7] has shown that the positive-norm Wess-Zumino field cancels a massless negative-norm mode of the gauge field.

Hagen[8] and Das[9] have preferred a minimal regularization procedure which preserves as much symmetry as possible paying the price of an additional explicit mass term for the gauge bosons to have a unitary theory. Ball[10] has indicated the equivalence of the minimal regularization and the minimal renormalization which does not require new counterterms but needs the arbitrary regularization parameter, showing moreover the relation of these procedures with the cancellation of anomalies by means of very heavy fermions, a method due to D'Hoker and Farhi.[11] In the above spirit Rajeev[12] has included for the $(3+1)$-dimensional chiral electrodynamics model a kinetic part for the Wess-Zumino field corresponding to a gauge-invariant mass term, which explicitly introduces a new degree of freedom. Finally, Thompson and Zhang[13] fixed the Wess-Zumino field to the one necessary to transform the gauge field to the Lorentz condition, as done previously[14] for the quantization of massive boson fields, giving for the $(1+1)$-dimensional case a single massive mode similar to that of the normal Schwinger model.

Other contributions relevant to the subject of quantization of anomalous models have recently appeared in the literature.[15]

The purpose of this work is to analyze the physical states for the $(d+1)$-dimensional model $(d=1,3)$ by means of the Becchi-Rouet-Stora-Tyutin (BRST) method[16] applied to the treatments of Jackiw and Rajaraman (J), Faddeev and Shatashvili (F), Rajeev (R), and Thompson and Zhang (T) models. It will appear that, without solving the fermionic part, the gauge field sector will correspond to d degrees of freedom in the cases J and R and to $d-1$ degrees with the choices F and T. The possibility of obtaining massive bosons will depend therefore on the procedure of quantization of anomalous theories with F being a particular case of J (or R) and T an alternative method.

II. QUANTIZATION OF ANOMALOUS ABELIAN THEORY

We consider the Lagrangian in $d+1$ dimensions,

$$\mathcal{L}=-\tfrac{1}{4}F^2+i\bar{\psi}\partial\!\!\!/\psi+eA_\mu J_L^\mu+\frac{B^2}{2}-B\partial\cdot A+\partial^\mu\bar{c}\partial_\mu c$$
$$+m^2K(\partial_\mu\theta,A_\mu)+\lambda\theta P(\partial_\nu A_\mu)\ , \qquad (1)$$

which corresponds to an Abelian gauge theory coupled to the fermionic left current J_L^μ and quantized in the gauge $\partial\cdot A=B$ with ghosts $c,\bar{c}$. The second line of Eq. (1) is the Wess-Zumino part which should compensate the change of the fermionic measure in the functional integral to make the quantized theory gauge invariant. K is the possible kinetic term related to the gauge group variable θ and P is the Chern-Pontryagin density. The constant λ depends on e whereas m^2 may be an independent parameter.

The dynamical variables, apart from fermions, will be in general A_μ, c, $\bar{c}$, and θ, with the canonical momenta given by

$$\pi_{A^i}=\frac{\partial\mathcal{L}}{\partial\dot{A}^i}=-F_{0i}+\lambda\theta\frac{\partial P}{\partial\dot{A}^i}\ ,$$
$$\pi_{A^0}=\frac{\partial\mathcal{L}}{\partial\dot{A}^0}=-B\ ,\quad \pi_{\bar{c}}=\frac{\partial\mathcal{L}}{\partial\dot{\bar{c}}}=\dot{c}\ ,\quad \pi_c=\frac{\partial\mathcal{L}}{\partial\dot{c}}=\dot{\bar{c}}\ , \tag{2}$$
$$\pi_\theta=\frac{\partial\mathcal{L}}{\partial\dot{\theta}}=m^2\frac{\partial K}{\partial\dot{\theta}}\ .$$

The functional integration is done over A_μ, B, c, $\bar{c}$, θ, ψ, and $\bar{\psi}$, where B has no conjugate momentum. The classical equations of motion for A_ν and θ are

$$-\partial_\mu F^{\mu\nu}-eJ_L^\nu-\partial^\nu B-m^2\frac{\partial K}{\partial A_\nu}+\lambda\partial_\mu\theta\frac{\partial P}{\partial\partial_\mu A_\nu}=0\ ,$$
$$m^2\partial_\mu\frac{\partial K}{\partial\partial_\mu\theta}-\lambda P=0\ , \tag{3}$$

apart from the gauge condition $\partial\cdot A=B$, and the decoupled ghost equations $\Box c=\Box\bar{c}=0$. From Eq. (3) it follows, using $\Box B=0$ [necessary to cancel the anomaly with the Wess-Zumino part of Eq. (1)], that,

$$-e\partial\cdot J_L+\lambda P-m^2\partial_\nu\left[\frac{\partial K}{\partial A_\nu}+\frac{\partial K}{\partial\partial_\nu\theta}\right]=0 \tag{4}$$

which means that $e\partial\cdot J_L=\lambda P$ if K is gauge invariant. Defining the BRST current through the transformations of A_μ, ψ, $\bar{\psi}$, $\bar{c}$, and θ,

$$J_B^\nu=F^{\nu\mu}\partial_\mu c-\lambda\theta\frac{\partial P}{\partial\partial_\nu A_\mu}\partial_\mu c+eJ_L^\nu c$$
$$+B\partial^\nu c-m^2\frac{\partial K}{\partial\partial_\nu\theta}\delta\theta\ , \tag{5}$$

its divergence is

$$\partial\cdot J_B=e\partial\cdot J_L c-\lambda P\delta\theta-m^2\left[\frac{\partial K}{\partial A_\nu}\partial_\nu c+\frac{\partial K}{\partial\partial_\nu\theta}\partial_\nu\delta\theta\right]\ ,$$

so that if the BRST change of θ is $\delta\theta=c$ and K is gauge invariant, J_B^ν is conserved. The BRST charge

$$Q_B=\int d^dx\left[F^{0i}\partial_i c+eJ_L^0c+B\partial_0 c-m^2\frac{\partial K}{\partial\partial_0\theta}\delta\theta-\lambda\theta\frac{\partial P}{\partial\partial_0 A_i}\partial_i c\right] \tag{6}$$

generates the appropriate transformations of $\mathbf{A}$, A_0, ψ, $\bar{\psi}$, $\bar{c}$, and θ. Note that the last term of Eq. (6) does not contain momenta being therefore ineffective for this transformation.

We shall analyze different choices for K which will be referred to as those of Faddeev and Shatashvili (F), Jackiw and Rajaraman (J), Rajeev (R), and Thompson and Zhang (T).

(i) F: $K=0$ so that $\delta\theta=0$. The functional integral over θ leads to $P=0$.

(ii) J: $K=\partial_\mu\theta(\frac{1}{2}\partial^\mu\theta-A^\mu)$, $\delta\theta=c$.

This expression of K, which is inspired by the regularization arbitrariness of the (1 + 1)-dimensional case,[3,4] satisfies the one-cocycle property but is not gauge invariant. Therefore, from Eqs. (4) and (5),

$$-e\partial\cdot J_L+\lambda P+m^2\partial\cdot A=0\quad\text{and}\quad\partial\cdot J_B=m^2\partial_\nu(cA^\nu)\ .$$

The constant charge is thus $Q_B-m^2\int d^dx\, cA^0$ which does not modify the above BRST transformations since the additional term does not contain momenta.

(iii) R: $K=\frac{1}{2}(\partial_\mu\theta-A_\mu)^2$, $\delta\theta=c$.

K is gauge invariant but does not verify the one-cocycle property so that it must be interpreted as a mass term for the gauge boson in the original Lagrangian.

(iv) T: $K=\partial_\mu D(\partial^\mu\theta-A^\mu)$, $\delta\theta=c$.

K is again gauge invariant and does not satisfy the one-cocycle property. In the functional integral there is an additional integral over the scalar field D which produces the constraint $\Box\theta-\partial\cdot A=0$ indicating that the original functional integral, before the introduction of the Faddeev-Popov identity, included the Lorentz condition as an anticipation of the possible appearance of mass for the boson field.

III. (1+1)-DIMENSIONAL CHIRAL SCHWINGER MODEL

Since this is a model that can be exactly solved, we will use it to show the solutions corresponding to the above choices as well as the insight which may be obtained before performing the functional integration over the fermion fields.

Exploiting the fact that the fermionic determinant can be exactly expressed, and using a nonminimal regularization procedure in terms of an arbitrary parameter a, the Lagrangian of Eq. (1) (for the choice J above) may be rewritten as[4]

$$\mathcal{L}=-\tfrac{1}{4}F^2+i\bar{\psi}\partial\!\!\!/\psi+eA_\mu J_L^\mu+\frac{B^2}{2}-B\partial\cdot A+\partial^\mu\bar{c}\partial_\mu c$$
$$+\frac{e^2}{8\pi}[(a-1)\partial_\mu\theta(\partial^\mu\theta-2A^\mu)-2\theta\epsilon^{\mu\nu}\partial_\mu A_\nu]\ . \tag{7}$$

The integration over fermions[1] leads to an effective Lagrangian

$$\mathcal{L}_{\text{eff}}=-\tfrac{1}{4}F^2+\frac{e^2}{8\pi}A_\mu\left[ag^{\mu\nu}-(g^{\mu\alpha}+\epsilon^{\mu\alpha})\frac{\partial_\alpha\partial_\beta}{\Box}\times(g^{\beta\nu}-\epsilon^{\beta\nu})\right]A_\nu$$
$$+\frac{B^2}{2}-B\partial\cdot A+\partial^\mu\bar{c}\partial_\mu c$$
$$+\frac{e^2}{8\pi}[(a-1)\partial_\mu\theta(\partial^\mu\theta-2A^\mu)-2\theta\epsilon^{\mu\nu}\partial_\mu A_\nu] \tag{8}$$

which, taking $A_\mu=\partial_\mu\eta-\epsilon_{\mu\nu}\partial_\nu\rho$, can be expressed as[3]

$$\mathcal{L}_{\text{eff}}=-\tfrac{1}{4}F^2+\frac{e^2}{8\pi}[(1+a)\rho\Box\rho+(1-a)\eta\Box\eta-2\eta\Box\rho]+\frac{B^2}{2}-B\Box\eta+\partial^\mu\bar{c}\partial_\mu c+\frac{e^2}{8\pi}[(1-a)(\theta\Box\theta-\theta\Box\eta-\eta\Box\theta)+2\theta\Box\rho] \tag{9a}$$

$$=-\tfrac{1}{4}F^2+\frac{e^2}{8\pi}[(1+a)\rho\Box\rho+(1-a)(\eta-\theta)\Box(\eta-\theta)-2(\eta-\theta)\Box\rho]+\frac{B^2}{2}-B\Box\eta+\partial^\mu\bar{c}\partial_\mu c \tag{9b}$$

which is clearly BRST invariant.

In Eq. (9a) we have kept distinct the $O(e^2)$ term coming from fermions and the one corresponding to the Wess-Zumino part. The shift in the variable of integration $\theta\to\theta+\eta+\rho/(a-1)$ allows us to rewrite Eq. (9b) as

$$\mathcal{L}_{\text{eff}}=-\tfrac{1}{4}F^2+\frac{e^2}{8\pi}\left[\frac{a^2}{a-1}\rho\Box\rho+(1-a)\theta\Box\theta\right]+\frac{B^2}{2}-B\Box\eta+\partial^\mu\bar{c}\partial_\mu c\ , \tag{10}$$

from which it is clear that for $a>1$, ρ corresponds to a massive mode

$$m^2=\frac{e^2}{4\pi}\frac{a^2}{a-1}$$

and θ to a massless mode, whereas η, $\bar{c}$, and c refer to gauge fixing and ghosts. For the particular case $a=1$, corresponding to the choice F above, Eq. (9b) shows that the shift $\theta\to\theta+\eta$ transforms θ into a Lagrange multiplier which fixes $\Box\rho=0$ so that ρ remains as the only (massless) mode.

If one takes $\theta=\eta$, which corresponds to choice T, Eq. (9b) shows that ρ is the single (massive) mode. Finally, the choice R implies adding in the square brackets of the last line of Eq. (9a) a term $(a-1)(\rho\Box\rho-\eta\Box\eta)$, which simply means an additional contribution (positive for $a>1$) to the (mass)^2 of the mode ρ of case J without changing the massless mode.

If we do not consider the fermion contribution to Eq. (9a) we are left with

$$\mathcal{L}_A=-\tfrac{1}{4}F^2+\frac{e^2}{8\pi}[(1-a)(\theta\Box\theta-2\theta\Box\eta)+2\theta\Box\rho]+\frac{B^2}{2}-B\Box\eta+\partial^\mu\bar{c}\partial_\mu c\ . \tag{11}$$

Choice F ($a=1$) leads to $\Box\rho=0$, i.e., a single massless mode which is not changed by the fermionic contribution. Choice T ($\theta=\eta$) gives, fixing $\eta=0$ for simplicity, a single massless mode which acquires mass due to the fermion part as in the normal Schwinger model for $a=1$. Choice J corresponds to the general Eq. (11) and again for $\eta=0$, a shift in θ gives for $a>1$ a massless and a massive mode whose mass is modified by the fermion contribution. Choice R corresponds to the addition of

$$\frac{e^2}{8\pi}(a-1)A^2$$

to Eq. (11) as said above, giving a further contribution to the mass of the mode ρ without altering the massless mode θ; this choice can be equivalently understood as the regularization with $a=1$ and an additional gauge-invariant mass term with arbitrary m.

IV. CONSIDERATIONS FOR THE (3+1)-DIMENSIONAL MODEL

We wish to analyze the asymptotic states of the model of Eq. (1) for $e\to 0$ and keeping m^2 and λ finite. This corresponds to the suppression of the fermionic part as discussed in 1 + 1 dimensions. The idea is that since the whole model should be BRST invariant and the integration over fermions is in general not possible, a study of the gauge-Wess-Zumino terms necessary to cancel the anomaly will give the required information on the physical states induced by the fermionic part.

We note that for 3 + 1 dimensions the choice J cannot be thought of as coming from a general regularization procedure and is included here merely as an example.

We will attempt to distinguish among dynamical fields apart from fermions [A_μ (4), c and $\bar{c}$ (2), and θ (1) in the general case], asymptotic states (those which decouple for $e\to 0$), and physical states (which are annihilated by Q_B and have positive norm). Since A_μ in the Lagrangian of Eq. (1) does not necessarily obey the Lorentz condition, we will take it as the sum of transverse A^T and longitudinal A^L parts with possible different masses.[17]

It is obvious that $|c\rangle$ and $|\bar{c}\rangle$ are massless asymptotic states since they do not interact with other fields. As $Q_B|\bar{c}\rangle=-|\partial\cdot A\rangle$, with Q_B constant of motion, A^L will correspond to a massless asymptotic state too.

For the choices J or R the dynamical field θ allows us to build another asymptotic state $|A_\mu-\partial_\mu\theta\rangle$ which is physical since it is obviously annihilated by Q_B and has positive norm. In fact, assuming that the parameter m in Eq. (1) characterizes the mass of A^T in the limit $e\to 0$, one would obtain the norms

$$\langle A^T_\mu(\mathbf{k})\,|\,A^T_\nu(\mathbf{k}')\rangle=-\left[g_{\mu\nu}-\frac{k_\mu k_\nu}{m^2}\right](2\pi)^3\times 2\sqrt{k^2+m^2}\,\delta(\mathbf{k}-\mathbf{k}')\ ,$$

$$\langle A^L_\mu(\mathbf{k})\,|\,A^L_\nu(\mathbf{k}')\rangle=-\frac{k_\mu k_\nu}{m^2}(2\pi)^3 2\sqrt{\mathbf{k}^2+\mu^2}\times\delta(\mathbf{k}-\mathbf{k}'),\quad \mu\to 0\ , \tag{12}$$

$$\langle\theta(\mathbf{k})\,|\,\theta(\mathbf{k}')\rangle=\frac{2\sqrt{\mathbf{k}^2+\mu^2}}{m^2}(2\pi)^3\delta(\mathbf{k}-\mathbf{k}'),\quad \mu\to 0\ .$$

Therefore $|\theta\rangle$ is a positive-norm state which cancels the negative norm of $|A^L\rangle$ to ensure the positivity of the norm of $| A_\mu - \partial_\mu \theta \rangle$.

For the alternative T, θ is not a dynamical variable since it is fixed by the gauge transformation from the Lorentz condition, so that the gauge field states correspond to massless asymptotic particles. We recall that in $1+1$ dimensions, the mass of the single mode was generated by integrating over the fermions.

With the choice F, θ is again not a dynamical variable and is not transformed by Q_B. Therefore the requirement $Q_B^2 \, | \bar{c} \rangle = 0$ leads to massless states.[18] We therefore see that general BRST considerations lead to the same results obtained explicitly in $1+1$ dimensions.

We have analyzed different forms of Wess-Zumino terms to quantize anomalous chiral models. The arbitrariness in the quantization leads to different degrees of freedom and consequent physical states. With the more general treatments J and R massive gauge bosons may appear. The particular choice F seems to prevent gauge bosons to acquire mass. The method T in $1+1$ dimensions gives a massive solution equivalent to that of normal Schwinger model which might be peculiar of this dimensionality. Additional criteria should be found to fix the quantization procedure to obtain definite physical predictions.

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