Number of neutrinos from W,Z hadroproduction: Last count

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We determine the number of light neutrinos directly from data on (i) the production of W and Z bosons in $p\bar{p}$ collisions and (ii) deep-inelastic scattering of leptons on hydrogen and deuterium. Ambiguities related to the use of structure functions are avoided. We conclude that at 95% confidence level, $N_\nu \leq 3$ and $m(\text{top}) \leq 60$ GeV. $N_\nu = 4$ is allowed if the fourth neutrino is accompanied by a heavy charged lepton lighter than M_W.

The experimentally observed ratio of $W \to e\nu$ and $Z \to ee$ events in $p\bar{p}$ collisions contains information on the number of neutrinos[1-7] and the mass of the top quark.[5-7] The information is extracted from the identity

$$R = \frac{N(W \to e\nu)}{N(Z \to ee)} = \frac{\sigma_W}{\sigma_Z} \frac{\Gamma(W \to e\nu)}{\Gamma(Z \to ee)} \frac{\Gamma_Z}{\Gamma_W} . \tag{1}$$

Within the standard model with three generations, Eq. (1) allows for a high-precision test of the theory using hadron colliders. At present, Eq. (1) can be viewed as an experimental determination of Γ_Z/Γ_W and, therefore, a measurement of the mass of the top quark, which is the only remaining unknown parameter in the standard-model value of Γ_Z/Γ_W. Alternatively, given the top-quark mass, Eq. (1) determines the number of light neutrinos, as any new neutrino beyond $N_\nu = 3$ adds 167 ± 9 MeV to Γ_Z and thus modifies the detailed balance present in this equation.

Historically, this powerful method has been plagued by the fact that the standard-model value of σ_W/σ_Z is calculated via quark structure functions. This has two unfortunate consequences: (i) the spread of values for σ_W/σ_Z is rather large, forcing one to assign a large error to this factor, relating the experimental value of R to Γ_Z/Γ_W in Eq. (1) and (ii) this error cannot be given a precise statistical meaning. This clumsy procedure can clearly be avoided. Heuristically, the valence-quark flux factors appearing in the calculation of σ_W/σ_Z are of the form

$$(u_V \bar{d}_V + \bar{u}_V d_V)/(u_V \bar{u}_V + d_V \bar{d}_V) ;$$

hence, the calculation depends only on the ratio u_V/d_V rather than on u_V and d_V individually. It is well known that this ratio can be calculated from data on F_2^p and F_2^n measured in leptoproduction on H and D targets. The ratio u_V/d_V can also be extracted from data on ν and $\bar{\nu}$ scattering on H and D targets.

In this Brief Report we extract information on σ_W/σ_Z directly from data. We perform the calculations using recent F_2^n/F_2^p measurements of the Bolgona-CERN-Dubna-Munich-Saclay (BCDMS) muon collaboration at CERN (Ref. 8). Our method has not only the advantage that there are no longer large ambiguities associated with structure functions, but the errors for the standard-model value of R are well defined in terms of the errors in $\sin^2\theta_W$, M_W (or M_Z), Λ_{QCD}, and the BCDMS data shown in Fig. 1. Our results are shown in Fig. 2(a) for $N_\nu = 3$, 4, 5 and a range of values of the top-quark mass m_t above its present lower limit of 23 GeV. R(theory) can be compared to the present experimental value obtained from the combined UA1 and UA2 experiments[9,10] of $8.4^{+1.1}_{-0.9}$. As both errors in R(theory) and R(expt) are well defined we can calculate their overlap integral for each value of N_ν and m_t. The results are shown in Fig. 2(b). We conclude that at 95% confidence level (C.L.) $N_\nu = 3$ and $m_t < 60$ GeV. The extent to which $N_\nu = 4$ is excluded can be reduced by adding a heavy charged lepton, as the decay channel $W \to L\nu_4$ reduces the value of R(theory) by about 0.5 for $m_L = 41$ GeV, which is the present UA1 lower limit.[11]

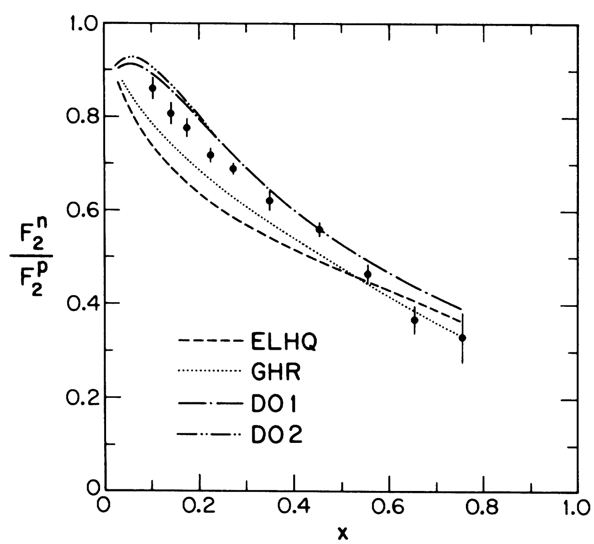

FIG. 1. Data of the BCDMS experiment, used as input in the present analysis, compared to calculations using various structure functions.

If our conclusions turn out to be false we learn that (i) some of the input data are wrong [UA1 (Ref. 9), UA2 (Ref. 10), or BCDMS (Ref. 8); the precise values of $\sin^2\theta_W$, M_W (or M_Z), and $\Lambda_{\rm QCD}$ are not crucial] or (ii) some new physics leads to additional decay channels of the W and Z bosons. We do not believe that large QCD corrections contribute differently to W and Z boson production. This difference has been calculated to order α_s^2 and is very small.[12] Finally, it is important to point out that the overlap of the errors in R(theory) and R(expt) is only one of many ways to define C.L. with which an N_ν, m_t assumption is compatible with the input data. Also, the 95%-C.L. cut is arbitrary and is supposed to be a guideline only.

In the rest of this Brief Report we discuss some technical details of the computation of the results in Fig. 2.

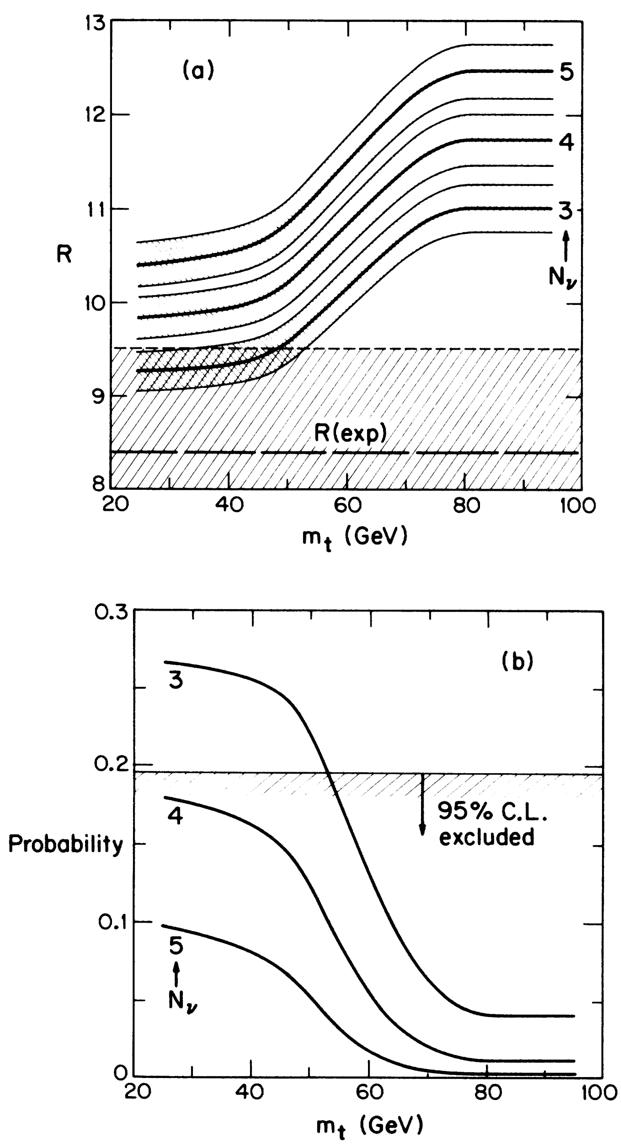

FIG. 2. (a) R, defined by Eq. (1), from experimental data and calculated within the standard model for several values of N_ν and m_t; (b) overlap probability of R(theory) and R(expt) from (a).

In practice, previous analyses of Eq. (1) have relied on structure functions for compelling reasons. Data[13] on F_2^p and F_2^n were only available for low Q^2 values (although this is no excuse, as we will see further on), but, more importantly, the data on $(F_2^p - F_2^n) \sim (u_V - d_V)$ were simply inconsistent with the quark-parton description of the nucleon. We will come back to this point later. Higher-Q^2 European Muon Collaboration (EMC) data also seem to defy[8] a detailed fit to QCD-evolved structure functions. The neutrino information is relatively poor in statistics and extracted u_V/d_V ratios vary between experiments.[14] As a result of these inconsistencies one is forced to rely on a smorgasbord of structure functions[15–18] to evaluate σ_W/σ_Z. In fact, none of these structure functions could claim to provide a satisfactory description of the available experimental information on the u_V/d_V ratio, which is their most important aspect for the problem under consideration.[5]

This situation has now changed. The BCDMS experiment[8] has recently extracted F_2^n/F_2^p with good precision from deep-inelastic muon scattering data that can be satisfactorily described by QCD. The data is shown in Fig. 1. We use this data to extract d_V, given u_V and the smaller sea contribution s, via

$$\frac{F_2^p}{F_2^n} = \frac{\frac{1}{3}(4u_V + d_V) + 4s}{\frac{1}{3}(u_V + 4d_V) + 4s} \ . \tag{2}$$

This method is extremely powerful because it determines u_V/d_V and hence σ_W/σ_Z in a manner which is rather insensitive to what one assumes for the input functions u_V and s. This is illustrated in Table I where it is shown that $\sigma_W/\sigma_Z = 3.4$ independent of which set of u_V and s structure functions one uses as input. Table I also shows that if one uses the original values of d_V rather than d_V extracted from the BCDMS data via Eq. (2), a more than ten times larger range of σ_W/σ_Z values results. This is just one way to demonstrate our previous claim that the σ_W/σ_Z ratio depends almost entirely on u_V/d_V and that this ratio is adequately determined by the data in Fig. 1. The detailed shape of u_V and d_V individually is not important. Also, the treatment of the sea-quark structure function, which varies from set to set in Table I, does not affect the answer.

Our procedure can be illustrated in a different way. We believe that any u_V that (i) is correctly normalized to two valence quarks in a proton and (ii) reproduces the average x dependence suggested by the quark-parton

TABLE I. σ_W/σ_Z calculated from the structure functions u_V, d_V, and s of Refs. 15–17 and from a modified d_V recalculated from the data in Fig. 1 using Eq. (2).

	Original structure functions	$\frac{u_V}{d_V}$ from BCDMS data
EHLQ	2.59	3.42
GHR	2.88	3.43
DO1	3.46	3.37
DO2	3.55	3.41

analysis of the leptoproduction data will yield $\sigma_W/\sigma_Z \simeq 3.4$ as in Table I, provided the corresponding d_V is directly determined from the data in Fig. 1. For example, consider $u_V \sim (1-x)^n/\sqrt{x}$ and $s \sim (1-x)^{n'}/x$. It is well known that $n=4$ gives the best description of the data when using this simple parametrization.[18] We obtain $\sigma_W/\sigma_Z = 3.34$, independent of our choice of $n'=7$, 8, or 9 or the precise normalization of s.

We have conservatively excluded W (Z) production via $c\bar{s}$ ($c\bar{c}$) channels from the calculation. This contribution is very uncertain, and is within the quoted errors. Its inclusion would increase R(theory) and thus strengthen the limits on N_ν, m_t.

The error in the final value of σ_W/σ_Z is dominated by the error in the input data in Fig. 1. It can therefore be calculated by standard methods and has a well-defined statistical meaning unlike errors quoted in previous analyses. The result is shown in Table II.

Thus far we have ignored the fact that σ_W/σ_Z depends on the mass values of M_W and M_Z. The mass values are important as one integrates the flux factors above $x_{\min} = (M/\sqrt{s})^2$ when calculating the cross sections. Since the structure functions fall rapidly with increasing x, the cross sections are rather sensitive to $x_{\min}$. Fortunately, the cross-section ratio depends mostly on M_W/M_Z, rather than on the rather poorly determined mass values of each weak boson. In the standard model this ratio is known with high precision from the measured value of $\sin^2\theta_W = 1-(M_W^2/M_Z^2)$. These errors are propagated through the Drell-Yan calculation. The results are summarized in Table II.

We are now ready to state our result for the standard-model value of the right-hand side of Eq. (1). From Table II we see that

$$\frac{\sigma_W}{\sigma_Z} = 3.41 \pm 0.08 \ . \tag{3}$$

The other factors, $\Gamma(W \to e\nu)$, $\Gamma(Z \to ee)$, Γ_W, and Γ_Z, also depend on $\sin^2\theta_W$ and α_s [which enters as a factor $(1+\alpha_s/\pi)$ when taking into account the higher-order correction to $W,Z \to q\bar{q}'$ decay rates]. The final results are shown in Fig. 2 and have already been discussed.

TABLE II. Sources of error in σ_W/σ_Z and $B(W\to e\nu)/B(Z\to ee)$.

$\frac{\sigma_W}{\sigma_Z}$	
Exptl. data of BCDMS	± 0.07
M_W (80.1±2.4 GeV)	± 0.02
$\sin^2\theta_W$ (0.232±0.04)	± 0.04
$\frac{\sigma_W}{\sigma_Z} = 3.41 \pm 0.08$	
$\frac{B(W\to e\nu)}{B(Z\to ee)}$ ($m_t = 50$ GeV, $N_\nu = 3$)	
$\sin^2\theta_W$ (0.232±0.004)	∓ 0.01
Λ_{QCD} (0.4±0.2 GeV)	∓ 0.001
$\frac{B(W\to e\nu)}{B(Z\to ee)} = 2.80 \mp 0.01$	

As a final test of our procedure to evaluate R(theory) we compare our u_V/d_V values, using the data in Fig. 1 as input, with other data, and also with existing struc-

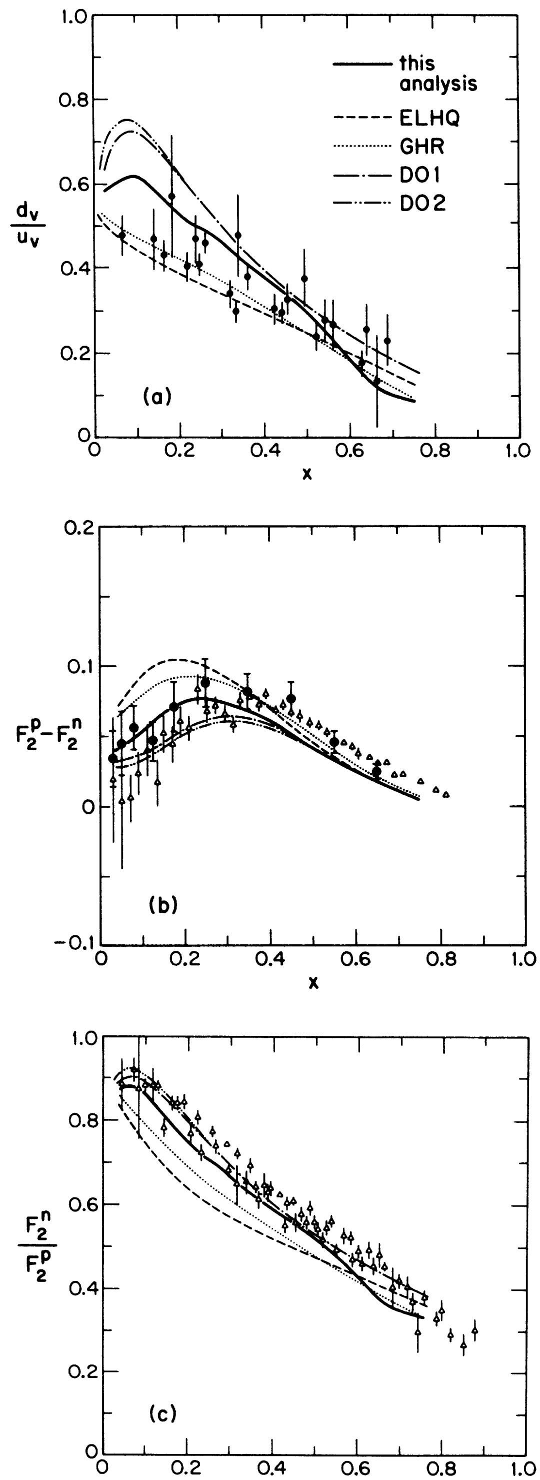

FIG. 3. Values of (a) d_V/u_V (Ref. 14), (b) $F_2^p - F_2^n$ (triangles, SLAC; circles, EMC) (Ref. 13), and (c) F_2^n/F_2^p from previous experiments are compared to the calculation in this paper and the results of various structure functions.

ture functions. The results are shown in Figs. 1 and 3. The usual structure functions describe the BCDMS data in Fig. 1 poorly, and clearly their spread in values overestimates the error on our knowledge of the u_V/d_V ratio. Figure 1 illustrates why we perform the standard-model test of Eq. (1) with improved precision. Our result is compared with a compilation of the experimental information on d_V/u_V from neutrino experiments[14] in Fig. 3(a). Although we have only determined the u_V/d_V ratio at $Q^2=M_W^2$, this ratio has a very weak Q^2 dependence and can also be confronted with the low-Q^2 SLAC data.[13] This is shown in Figs. 3(b) and 3(c), along with high-Q^2 European Muon Collaboration (EMC) data.[13] It is fair to say that overall our u_V/d_V ratio provides a better description of the data (other than the BCDMS data, which we fit by construction) than any set of structure functions. However, as illustrated in Fig. 3(b), the low-Q^2 SLAC data (triangles) is not consistent with a parton description of the nucleon.

In the near future, when higher-statistics W,Z hadroproduction data become available from CERN and Fermilab Tevatron I, one can conceive of measuring the u_V/d_V ratio with the $p\bar{p}$ colliders themselves. The most sensitive measurement is the σ_{W^+}/σ_Z (or σ_{W^-}/σ_Z) distribution as a function of rapidity. Indeed, the forward-backward asymmetry of W^+ (or W^-) production is a direct consequence of the u_V and d_V structure functions not being the same. Our prediction for the ratio of cross sections at Tevatron I ($\sqrt{s}=1.8$ TeV) is

$$\frac{\sigma_W}{\sigma_Z}=3.08\pm0.10 .$$

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