Evaluation of the conformal anomaly of Polyakov's string theory by the stochastic quantization method

Jin Woo Jun* and Jae Kwan Kim
Physics Department, Korea Advanced Institute of Science and Technology, P.O. Box 150, Cheongryang, Seoul, Korea
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The stochastic quantization method is applied to the calculation of the conformal anomaly of Polyakov's string theory in Fujikawa's string path-integral formalism. The calculation is based on the stochastic quantization of Bose fields and Fermi fields. As a result, the standard dimension $D=26$ is obtained once again.

The string field theory has received much attention recently, in view of rising prospects for a unified theory with gravity.[1] When one deals with the quantum string field theory, however, some of its fundamental symmetries are spoiled by the quantization procedure (i.e., various anomalies appear), as in the case of ordinary quantum field theory.[2] In the case of the string path-integral formalism, the integration measures lose some of their classical symmetries.[3,4] In this paper, we calculate the conformal anomaly of Polyakov's bosonic string theory[5] in Fujikawa's path-integral formalism[3] using the stochastic quantization method.[6] Since it was first proposed, the stochastic quantization method has been applied by many authors to derive the chiral anomaly[7,8] and to clarify the quantum origin of the chiral anomaly.[9,10] The stochastic quantization method was applied recently to calculate the conformal anomaly of Polyakov's string theory by Koh and Zhang,[11] and they used the bosonization technique to treat the ghost fields. However, the stochastic quantization method can be more simply applied to Fujikawa's Becchi-Rouet-Stora- (BRS-) invariant string path-integral formalism,[3] because the space-time manifold is effectively flat in this formalism. In this paper, we directly apply the stochastic quantization method for the Fermi fields[12] and the Bose fields,[13] respectively, to the ghost fields and the weighted string variables that appear in Fujikawa's string path-integral formalism. The solutions of the corresponding Langevin equations are then used to calculate the conformal anomaly.

Let us briefly review the conformal anomaly in Fujikawa's string path-integral formalism.[3] We start with the Polyakov string Lagrangian[5] (our conventions are those of Ref. 3):

$$\mathcal{L}=-\tfrac{1}{2}\sqrt{g}\,g^{\mu\nu}\partial_\mu X^a(x)\partial_\nu X^a(x) \tag{1}$$

with the string variables $X^a(x)$, $a=1,\ldots,D$, the two-dimensional parameter x^μ, and the two-dimensional gravitational field $g^{\mu\nu}(x)$. Here

$$g=\det g_{\mu\nu}\ . \tag{2}$$

Taking the conformally Euclidean gauge

$$g_{\mu\nu}(x)=\rho(x)\delta_{\mu\nu}\ , \tag{3}$$

and following the conventional Faddeev-Popov prescription, Fujikawa obtained the partition function

$$Z=\int \mathcal{D}\sqrt{\rho(x)}\mathcal{D}\tilde{X}^a(x)\mathcal{D}\xi(x)\mathcal{D}\tilde{\eta}(x)\times\exp\left\{\int dX\left[-\tfrac{1}{2}\partial_\mu\left[\frac{\tilde{X}^a}{\sqrt{\rho}}\right]\partial_\mu\left[\frac{\tilde{X}^a}{\sqrt{\rho}}\right]+\xi\left[\sqrt{\rho}\,\partial\!\!\!/\frac{1}{\rho}\right]\tilde{\eta}\right]\right\} \tag{4}$$

with the Faddeev-Popov ghost and antighost

$$\tilde{\eta}=\begin{bmatrix}\rho & \eta_1\\ \rho & \eta_2\end{bmatrix},\quad \xi=\begin{bmatrix}\xi_1\\ \xi_2\end{bmatrix} \tag{5}$$

and

$$\partial\!\!\!/\equiv\sigma^1\partial_1+\sigma^3\partial_2 \tag{6}$$

and

$$\tilde{X}^a(x)=\sqrt{\rho(x)}X^a(x)\ . \tag{7}$$

Let us ignore $\mathcal{D}\sqrt{\rho(x)}$ for the moment and regard $\rho(x)$ as the background field. Then the partition function becomes

$$Z_0\equiv\int \mathcal{D}\tilde{X}^a(x)\mathcal{D}\xi(x)\mathcal{D}\tilde{\eta}(x)\times\exp\left\{\int dx\left[-\tfrac{1}{2}\partial_\mu\left[\frac{\tilde{X}^a}{\sqrt{\rho}}\right]\partial_\mu\left[\frac{\tilde{X}^a}{\sqrt{\rho}}\right]+\xi\left[\sqrt{\rho}\,\partial\!\!\!/\frac{1}{\rho}\right]\tilde{\eta}\right]\right\}\equiv\ln W[\rho]\ . \tag{8}$$

The conformal anomaly is defined for the variations

$$\begin{aligned}&\tilde{X}^a(x)\to\exp[\tfrac{1}{2}\alpha(x)]\tilde{X}^a(x)\ ,\\&\xi(x)\to\exp[-\tfrac{1}{2}\alpha(x)]\xi(x)\ ,\\&\tilde{\eta}(x)\to\exp[\alpha(x)]\tilde{\eta}(x)\ ,\\&\rho(x)\to\exp[\alpha(x)]\rho(x)\ .\end{aligned} \tag{9}$$

The action $\tilde{S}$ in (8) is invariant under (9). Therefore, if the integration measure of (8) were also invariant under (9), we should obtain

$$W[e^\alpha\rho]=W[\rho]\ . \tag{10}$$

The existence of a conformal anomaly means that this relation is invalid, and the functional $W[\rho(x)]$ is not conformally invariant. Therefore, we intend to calculate the variation of W by an infinitesimal conformal transformation:

$$\delta W=\int dx_1\alpha(x_1)\rho(x_1)\frac{\delta W[\rho]}{\delta[\alpha(x_1)\rho(x_1)]}=\left\langle\int dx_1\rho(x_1)\frac{\delta\tilde{S}}{\delta\rho(x_1)}\right\rangle$$
$$=\left\langle\int dx\left[-\tfrac{1}{2}\partial_\mu\partial_\mu\left[\frac{\tilde{X}^a}{\sqrt{\rho}}\right]\frac{\tilde{X}^a}{\sqrt{\rho}}+\frac{1}{2}\xi\left[\sqrt{\rho}\not{\partial}\frac{1}{\rho}\right]\bar{\eta}+\bar{\xi}\sqrt{\rho}\overleftarrow{\not{\partial}}\frac{1}{\rho}\bar{\eta}\right]\right\rangle$$
$$=\delta W_1+\delta W_2\ , \tag{11}$$

where

$$\delta W_1=\frac{\int\mathcal{D}\tilde{X}^a(x)\int dx\left[-\tfrac{1}{2}\partial_\mu\partial_\mu\left[\frac{\tilde{X}^a}{\sqrt{\rho}}\right]\frac{\tilde{X}^a}{\sqrt{\rho}}\right]\exp\left\{\int dx\left[-\tfrac{1}{2}\partial_\mu\left[\frac{\tilde{X}^a}{\sqrt{\rho}}\right]\partial_\mu\left[\frac{\tilde{X}^a}{\sqrt{\rho}}\right]\right]\right\}}{\int\mathcal{D}\tilde{X}^a(x)\exp\left\{\int dx\left[-\tfrac{1}{2}\partial_\mu\left[\frac{\tilde{X}^a}{\sqrt{\rho}}\right]\partial_\mu\left[\frac{\tilde{X}^a}{\sqrt{\rho}}\right]\right]\right\}} \tag{12}$$

and

$$\delta W_2=\frac{\int\mathcal{D}\xi(x)\mathcal{D}\bar{\eta}(x)\int dx\left[\tfrac{1}{2}\xi\left[\sqrt{\rho}\not{\partial}\frac{1}{\rho}\right]\bar{\eta}+\xi\sqrt{\rho}\overleftarrow{\not{\partial}}\frac{1}{\rho}\bar{\eta}\right]\exp\left[\int dx\,\xi\sqrt{\rho}\not{\partial}\frac{1}{\rho}\bar{\eta}\right]}{\int\mathcal{D}\xi(x)\mathcal{D}\bar{\eta}(x)\exp\left[\int dx\,\xi\sqrt{\rho}\not{\partial}\frac{1}{\rho}\bar{\eta}\right]}\ . \tag{13}$$

δW_1 and δW_2 will be calculated using the stochastic quantization method for Bose fields[13] and Fermi fields,[12] respectively. In calculating the quantum average, δW_1 and δW_2, we use the ultraviolet regularization scheme introduced by Breit, Gupta, and Zaks,[14] and the infrared regularization scheme,[7,8] which assumes finite, nonzero eigenvalues for the involved operators.

We calculate δW_1 first. Remembering that the index a runs from 1 to D, δW_1 becomes

$$\delta W_1=D\frac{\int\mathcal{D}\tilde{X}(x)\int dx\left[-\tfrac{1}{2}\partial_\mu\partial_\mu\left[\frac{\tilde{X}}{\sqrt{\rho}}\right]\frac{\tilde{X}}{\sqrt{\rho}}\right]\exp\left\{\int dx\left[-\tfrac{1}{2}\partial_\mu\left[\frac{\tilde{X}}{\sqrt{\rho}}\right]\partial_\mu\left[\frac{\tilde{X}}{\sqrt{\rho}}\right]\right]\right\}}{\int\mathcal{D}\tilde{X}(x)\exp\left\{\int dx\left[-\tfrac{1}{2}\partial_\mu\left[\frac{\tilde{X}}{\sqrt{\rho}}\right]\partial_\mu\left[\frac{\tilde{X}}{\sqrt{\rho}}\right]\right]\right\}}\ . \tag{14}$$

Then our action for $\tilde{X}$ is

$$S=\int dx\,\tfrac{1}{2}\partial_\mu\left[\frac{\tilde{X}}{\sqrt{\rho}}\right]\partial_\mu\left[\frac{\tilde{X}}{\sqrt{\rho}}\right]\ . \tag{15}$$

The Langevin equation is given by[13]

$$\frac{\partial\tilde{X}(x,\tau)}{\partial\tau}=\frac{\delta S}{\delta\tilde{X}}+\zeta(x,\tau)=\Box\tilde{X}+\zeta(x,\tau)\ , \tag{16}$$

where

$$\Box\tilde{X}\equiv\frac{1}{\sqrt{\rho}}\partial_\mu\partial_\mu\left[\frac{\tilde{X}}{\sqrt{\rho}}\right]\ . \tag{17}$$

Here τ denotes the fictitious time, and the Gaussian random variable $\zeta(x,\tau)$ satisfies the relations

$$\langle\zeta(x,\tau)\rangle_\zeta=0\ ,$$
$$\langle\zeta(x,\tau)\zeta(x',\tau')\rangle_\zeta=2\delta(x-x')a_\Lambda(\tau-\tau')\ . \tag{18}$$

The angular brackets denote the white-noise average with respect to the random variable $\zeta(x,\tau)$, and the regulator function $a_\Lambda(\tau-\tau')$ introduced by Breit, Gupta, and Zaks[14] has the properties

$$a_\Lambda(\tau)=a_\Lambda(-\tau),\quad\int d\tau' a_\Lambda(\tau-\tau')=1\ ,$$
$$\lim_{\Lambda\to\infty}a_\Lambda(\tau-\tau')=\delta(\tau-\tau')\ . \tag{19}$$

The limit $\Lambda\to\infty$ will be performed after all the calculations have been done. The solution of the Langevin equation (16) reads

$$\tilde{X}_\zeta(x,\tau)=\int_0^\tau e^{\Box(\tau-\tau_1)}\zeta(x,\tau_1)d\tau_1\ . \tag{20}$$

According to the prescription of the stochastic quantization method for Bose fields,

$$\delta W_1=D\lim_{\tau\to\infty}\left\langle\int dx\left[-\tfrac{1}{2}\partial_\mu\partial_\mu\left[\frac{\tilde{X}_\zeta(x,\tau)}{\sqrt{\rho}}\right]\frac{\tilde{X}_\zeta(x,\tau)}{\sqrt{\rho}}\right]\right\rangle_\zeta\ . \tag{21}$$

Substituting (20) into (21), δW_1 becomes

$$\delta W_1 = -\tfrac{1}{2}D \lim_{\tau\to\infty} \left\langle \int dx \int_0^\tau d\tau_1 \int_0^\tau d\tau_2 e^{\Box(\tau-\tau_2)}\zeta(x,\tau_2)\Box e^{\Box(\tau-\tau_1)}\zeta(x,\tau_1)\right\rangle_\zeta . \tag{22}$$

Performing the ζ average

$$\delta W_1 = -D \lim_{\tau\to\infty}\lim_{\Lambda\to\infty} \int dx \int_0^\tau d\tau_1 \int_0^\tau d\tau_2 a_\Lambda(\tau_1-\tau_2)\langle x \,|\, e^{\Box(2\tau-\tau_1-\tau_2)}\Box \,|\, x\rangle . \tag{23}$$

At this point, it is advantageous to introduce new integration variables

$$t=\tau_1-\tau_2, \quad T=\tfrac{1}{2}(\tau_1+\tau_2) . \tag{24}$$

Thus, one gets

$$\delta W_1 = -D \lim_{\tau\to\infty}\lim_{\Lambda\to\infty} \int dx \left[\int_0^{\tau/2} dT \int_{-2T}^{2T} dt + \int_{\tau/2}^{\tau} dT \int_{-2(\tau-T)}^{2(\tau-T)} dt\right] a_\Lambda(t)\langle x \,|\, e^{\Box 2(\tau-T)}\Box \,|\, x\rangle . \tag{25}$$

Taking the properties of the regulator function a_Λ into account, it is easy to evaluate the t integration for $\Lambda\to\infty$:

$$\int_{-2T}^{2T} dt\, a_\Lambda(t) = \Theta\left[T-\frac{1}{2\Lambda^2}\right] + O\left[\frac{1}{\Lambda^2}\right], \quad \int_{-2(\tau-T)}^{2(\tau-T)} dt\, a_\Lambda(t) = \Theta\left[\tau-T-\frac{1}{2\Lambda^2}\right] + O\left[\frac{1}{\Lambda^2}\right] . \tag{26}$$

Hence we end up with

$$\begin{aligned}\delta W_1 &= -D \lim_{\tau\to\infty}\lim_{\Lambda\to\infty} \int dx \int_{1/2\Lambda^2}^{\tau-1/2\Lambda^2} dT \langle x \,|\, e^{2\Box(\tau-T)}\Box \,|\, x\rangle \\ &= \tfrac{1}{2}D \lim_{\tau\to\infty}\lim_{\Lambda\to\infty} \int dx (\langle x \,|\, e^{2\Box(1/2\Lambda^2)} \,|\, x\rangle - \langle x \,|\, e^{2\Box(\tau-1/2\Lambda^2)} \,|\, x\rangle) .\end{aligned} \tag{27}$$

Let us assume that $\Box$ always has finite negative eigenvalues. Thus we can take the limit $\tau\to\infty$, while $\Box$ is finite. Then (27) gives

$$\delta W_1 = \tfrac{1}{2}D \lim_{\Lambda\to\infty} \int dx \langle x \,|\, e^{\Box(1/\Lambda^2)} \,|\, x\rangle . \tag{28}$$

Since we are only interested in the short-distance behavior, we can use the plane-wave representation,[15] although in curved space-time a global definition of the momentum representation is not available:

$$\delta W_1 = \tfrac{1}{2}D \lim_{\Lambda\to\infty}\lim_{y\to x} \int dx \int \frac{d^2k}{(2\pi)^2} e^{-ikx} \exp\left[\left[\frac{1}{\sqrt{\rho}}\partial_\mu\partial_\mu\frac{1}{\sqrt{\rho}}\right]\frac{1}{\Lambda^2}\right] e^{iky} . \tag{29}$$

Recalling the well-known formula[3]

$$\lim_{\Lambda\to\infty} \mathrm{Tr} \int \frac{d^2k}{(2\pi)^2} e^{-ikx} e^{-H/\Lambda^2} e^{ikx} = \lim_{\Lambda\to\infty} 2\left[\frac{3n+1}{24\pi}(-\partial_\mu\partial_\mu \ln\rho) + \frac{\rho}{4\pi}\Lambda^2\right] , \tag{30}$$

where Tr denotes the trace in spinor space, and

$$H = -\rho^{-(n+1)/2}\partial\!\!\!/ \rho^n \partial\!\!\!/ \rho^{-(n+1)/2} , \tag{31}$$

we obtain

$$\delta W_1 = \frac{D}{48\pi} \lim_{\Lambda\to\infty} \int dx (-\partial_\mu\partial_\mu\rho + 6\rho\Lambda^2) . \tag{32}$$

Similarly, δW_2 can be calculated. From the form of δW_2, our action for $\xi(x)$ and $\bar\eta(x)$ reads

$$S = \int dx\, \xi D\!\!\!\!/\, \bar\eta , \tag{33}$$

where

$$D\!\!\!\!/ = -\sqrt{\rho}\partial\!\!\!/ \frac{1}{\rho} . \tag{34}$$

The Langevin equations are given by[12]

$$\begin{aligned}\frac{\partial}{\partial\tau}\bar\eta(x,\tau) &= -D\!\!\!\!/^\dagger D\!\!\!\!/\,\bar\eta(x,\tau) + D\!\!\!\!/^\dagger\bar\gamma(x,\tau) , \\ \frac{\partial}{\partial\tau}\xi(x,\tau) &= -\xi(x,\tau)\overleftarrow{D\!\!\!\!/}\,\overleftarrow{D\!\!\!\!/}^\dagger + \gamma(x,\tau) ,\end{aligned} \tag{35}$$

where

$$\begin{aligned}\langle \gamma_\alpha(x,\tau)\gamma_\beta(x',\tau')\rangle_\gamma &= -\langle \gamma_\beta(x',\tau')\gamma_\alpha(x,\tau)\rangle_\gamma \\ &= 2\delta_{\alpha\beta}\delta(x-x')a_\Lambda(\tau-\tau') .\end{aligned} \tag{36}$$

Equation (35) gives the evolution of the Fermi fields $\bar\eta$ and ξ with respect to the fictitious time τ as

$$\bar\eta_\gamma(x,\tau) = \int_0^\tau e^{-D\!\!\!\!/^\dagger D\!\!\!\!/(\tau-\tau_2)} D\!\!\!\!/^\dagger \bar\gamma(x,\tau_2) d\tau_2 \tag{37}$$

and

$$\xi_\gamma(x,\tau) = \int_0^\tau \gamma(x,\tau_1) e^{-\overleftarrow{D\!\!\!\!/}\,\overleftarrow{D\!\!\!\!/}^\dagger(\tau-\tau_1)} d\tau_1 . \tag{38}$$

According to the prescription of the stochastic quantization method for Fermi fields,

$$\delta W_2=\lim_{\tau\to\infty}\left\langle \int dx[\tfrac{1}{2}\xi_\gamma(x,\tau)\sqrt{\rho}\partial\!\!\!/\frac{1}{\rho}\tilde{\eta}_\gamma(x,\tau)+\xi_\gamma(x,\tau)\sqrt{\rho}\overleftarrow{\partial\!\!\!/}\frac{1}{\rho}\tilde{\eta}_\gamma(x,\tau)]\right\rangle_\gamma . \tag{39}$$

Substituting (37) and (38) into (39), δW_2 becomes

$$\delta W_2=\lim_{\tau\to\infty}\left\langle \int dx \int_0^\tau d\tau_1\int_0^\tau d\tau_2\left[\tfrac{1}{2}\gamma(x,\tau_1)e^{-\overleftarrow{D\!\!\!\!/}\,\overleftarrow{D\!\!\!\!/}^\dagger(\tau-\tau_1)}\sqrt{\rho}\partial\!\!\!/\frac{1}{\rho}e^{-D\!\!\!\!/^\dagger D\!\!\!\!/(\tau-\tau_2)}D\!\!\!\!/^\dagger\tilde{\gamma}(x,\tau_2)\right.\right.$$
$$\left.\left.+\gamma(x,\tau_1)e^{-\overleftarrow{D\!\!\!\!/}\,\overleftarrow{D\!\!\!\!/}^\dagger(\tau-\tau_1)}\sqrt{\rho}\overleftarrow{\partial\!\!\!/}\frac{1}{\rho}e^{-D\!\!\!\!/^\dagger D\!\!\!\!/(\tau-\tau_2)}D\!\!\!\!/^\dagger\tilde{\gamma}(x,\tau_2)\right]\right\rangle_\gamma . \tag{40}$$

Performing the γ average of the noise function, we have

$$\delta W_2=\lim_{\tau\to\infty}\lim_{\Lambda-\infty}\int dx\int_0^\tau d\tau_1\int_0^\tau d\tau_2[-2a_\Lambda(\tau_1-\tau_2)]$$
$$\times\left\langle x\left|\mathrm{Tr}\left[\tfrac{1}{2}e^{-D\!\!\!\!/ D\!\!\!\!/^\dagger(\tau-\tau_1)}\sqrt{\rho}\partial\!\!\!/\frac{1}{\rho}e^{-D\!\!\!\!/^\dagger D\!\!\!\!/(\tau-\tau_2)}D\!\!\!\!/^\dagger\right.\right.\right.$$
$$\left.\left.\left.+e^{-D\!\!\!\!/ D\!\!\!\!/^\dagger(\tau-\tau_1)}\sqrt{\rho}\overleftarrow{\partial\!\!\!/}\frac{1}{\rho}e^{-D\!\!\!\!/^\dagger D\!\!\!\!/(\tau-\tau_2)}D\!\!\!\!/^\dagger\right]\right|x\right\rangle$$
$$=\lim_{\tau\to\infty}\lim_{\Lambda\to\infty}\int dx\int_0^\tau d\tau_1\int_0^\tau d\tau_2 2a_\Lambda(\tau_1-\tau_2)\left\langle x\left|\mathrm{Tr}\left[\tfrac{1}{2}e^{-D\!\!\!\!/ D\!\!\!\!/^\dagger(\tau-\tau_1)}D\!\!\!\!/ e^{-D\!\!\!\!/^\dagger D\!\!\!\!/(\tau-\tau_2)}D\!\!\!\!/^\dagger\right.\right.\right.$$
$$\left.\left.\left.-e^{-D\!\!\!\!/ D\!\!\!\!/^\dagger(\tau-\tau_1)}D\!\!\!\!/ e^{-D\!\!\!\!/^\dagger D\!\!\!\!/(\tau-\tau_2)}D\!\!\!\!/^\dagger\right]\right|x\right\rangle . \tag{41}$$

We compute the last trace

$$\mathrm{Tr}(\tfrac{1}{2}e^{-D\!\!\!\!/ D\!\!\!\!/^\dagger(\tau-\tau_1)}D\!\!\!\!/ e^{-D\!\!\!\!/^\dagger D\!\!\!\!/(\tau-\tau_2)}D\!\!\!\!/^\dagger-e^{-D\!\!\!\!/ D\!\!\!\!/^\dagger(\tau-\tau_1)}D\!\!\!\!/ e^{-D\!\!\!\!/^\dagger D\!\!\!\!/(\tau-\tau_2)}D\!\!\!\!/^\dagger)=\mathrm{Tr}(\tfrac{1}{2}e^{-D\!\!\!\!/ D\!\!\!\!/^\dagger(2\tau-\tau_1-\tau_2)}D\!\!\!\!/ D\!\!\!\!/^\dagger$$
$$-e^{-D\!\!\!\!/^\dagger D\!\!\!\!/(2\tau-\tau_1-\tau_2)}D\!\!\!\!/^\dagger D\!\!\!\!/) . \tag{42}$$

Using the integral variable t and T and the properties of the regulator function a_Λ as before, we obtain

$$\delta W_2=\lim_{\tau\to\infty}\lim_{\Lambda\to\infty}\int dx\left[\int_0^{\tau/2}dT\int_{-2T}^{2T}dt+\int_{\tau_2}^{\tau}dT\int_{-2(\tau-T)}^{2(\tau-T)}dt\right]2a_\Lambda(t)\langle x\mid \mathrm{Tr}(\tfrac{1}{2}e^{-D\!\!\!\!/ D\!\!\!\!/^\dagger 2(\tau-T)}D\!\!\!\!/ D\!\!\!\!/^\dagger$$
$$-e^{-D\!\!\!\!/^\dagger D\!\!\!\!/ 2(\tau-T)}D\!\!\!\!/^\dagger D\!\!\!\!/)\mid x\rangle$$
$$=2\lim_{\tau\to\infty}\lim_{\Lambda\to\infty}\int dx\int_{\Lambda^2/2}^{\tau-\Lambda^2/2}dT\langle x\mid \mathrm{Tr}(\tfrac{1}{2}e^{-D\!\!\!\!/ D\!\!\!\!/^\dagger 2(\tau-T)}D\!\!\!\!/ D\!\!\!\!/^\dagger-e^{-D\!\!\!\!/^\dagger D\!\!\!\!/ 2(\tau-T)}D\!\!\!\!/^\dagger D\!\!\!\!/)\mid x\rangle$$
$$=\lim_{\tau\to\infty}\lim_{\Lambda\to\infty}\int dx[\langle x\mid \mathrm{Tr}(\tfrac{1}{2}e^{-D\!\!\!\!/ D\!\!\!\!/^\dagger(1/\Lambda^2)}-e^{-D\!\!\!\!/^\dagger D\!\!\!\!/(1/\Lambda^2)})\mid x\rangle$$
$$-\langle x\mid \mathrm{Tr}(e^{-D\!\!\!\!/ D\!\!\!\!/^\dagger 2(\tau-1/2\Lambda^2)}-e^{-D\!\!\!\!/^\dagger D\!\!\!\!/ 2(\tau-1/2\Lambda^2)})\mid x\rangle] . \tag{43}$$

Let us assume that $D\!\!\!\!/ D\!\!\!\!/^\dagger$ and $D\!\!\!\!/^\dagger D\!\!\!\!/$ always have finite positive eigenvalues. Thus we can take the limit $\tau\to\infty$, while $D\!\!\!\!/ D\!\!\!\!/^\dagger$ and $D\!\!\!\!/^\dagger D\!\!\!\!/$ are finite. Then (43) gives

$$\delta W_2=\lim_{\Lambda\to\infty}\int dx\langle x\mid \mathrm{Tr}(\tfrac{1}{2}e^{-D\!\!\!\!/ D\!\!\!\!/^\dagger(1/\Lambda^2)}-e^{-D\!\!\!\!/^\dagger D\!\!\!\!/(1/\Lambda^2)})\mid x\rangle . \tag{44}$$

Using the plane-wave basis[15] same as before and Eqs. (30) and (31),

$$\delta W_2=\lim_{\Lambda\to\infty}\lim_{y\to x}\int dx\int\frac{d^2k}{(2\pi)^2}e^{-ikx}\mathrm{Tr}\left\{\tfrac{1}{2}\exp\left[\left[\sqrt{\rho}\partial\!\!\!/\frac{1}{\rho^2}\partial\!\!\!/\sqrt{\rho}\right]\frac{1}{\Lambda^2}\right]-\exp\left[\left[\frac{1}{\rho}\partial\!\!\!/\rho\partial\!\!\!/\frac{1}{\rho}\right]\frac{1}{\Lambda^2}\right]\right\}e^{iky}$$
$$=\lim_{\Lambda\to\infty}\int dx\left[\tfrac{1}{2}\times 2\left[\frac{-5}{24\pi}(-\partial_\mu\partial_\mu\ln\rho)+\frac{\rho}{4\pi}\Lambda^2\right]-2\left[\frac{4}{24\pi}(-\partial_\mu\partial_\mu\ln\rho)+\frac{\rho}{4\pi}\Lambda^2\right]\right]$$
$$=\frac{-26}{48\pi}\lim_{\Lambda\to\infty}\int dx(-\partial_\mu\partial_\mu\ln\rho+\tfrac{6}{13}\rho\Lambda^2) . \tag{45}$$

Combining the results of δW_1 and δW_2 we finally obtain

$$\delta W=\delta W_1+\delta W_2$$
$$=\frac{D-26}{48\pi}\int dx(-\partial_\mu\partial_\mu \ln\rho)$$
$$+\lim_{\Lambda\to\infty}\frac{1}{8\pi}\int dx[(D-2)\rho\Lambda^2]\;. \tag{46}$$

Since the second term can be renormalized to zero by adding a bare cosmological term to the starting Lagrangian,[5] $\delta W=0$ when $D=26$.

In conclusion, we have shown by explicit calculation that the stochastic quantization method can be applied to the evaluation of the conformal anomaly of Polyakov's string theory and, as a result, we have obtained the standard dimension, $D=26$, once again. In this calculation, we directly applied the stochastic quantization method to the ghost fields without appeal to the bosonization technique. Note also that we chose a specific regularization scheme to achieve the result. In the ultraviolet regularization procedure,[14] we take the limit $\tau\to\infty$ first, keeping Λ finite. Then the limit $\Lambda\to\infty$ is taken. This limiting process is physically plausible, since the τ limit is the equilibrium limit that is necessary to obtain the Euclidean Green's function and the Λ limit is the regularization limit. Our assumption for the infrared regularization of ghost fields is equivalent to that used by Tzani[8] and Alfaro and Gavela[7] for the infrared regularization of the Fermi fields. In this paper we have not clarified the quantum origin of the conformal anomaly in the context of the stochastic quantization formalism, but we have shown how to apply the stochastic quantization method to the calculation of the conformal anomaly expressed in terms of the string path-integral formalism. The quantum origin of the conformal anomaly in the context of the stochastic quantization formalism and the generalization of this approach to the fermionic string theory require further investigations.

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*Present address: Department of Physics, Inje College, Kimhae, Kyungnam, Korea.

[1]M. B. Green and J. H. Schwarz, Phys. Lett. **149B**, 117 (1984).

[2]S. L. Adler, in *Lectures in Elementary Particles and Quantum Theory,* edited by S. Deser *et al.* (MIT, Cambridge, MA, 1970); R. Jackiw, in *Lectures on Current Algebra and its Applications,* edited by S. Treiman *et al.* (Princeton University Press, Princeton, NJ, 1972), and references therein.

[3]K. Fujikawa, Phys. Rev. D **25**, 2584 (1982).

[4]P. Bouwknegt and P. van Nieuwenhuizen, Class. Quantum Gravit. **3**, 207 (1986).

[5]A. Polyakov, Phys. Lett. **103B**, 207 (1981).

[6]G. Parisi and Wu Yong-Shi, Sci. Sin. **24**, 484 (1981).

[7]J. Alfaro and M. B. Gavela, Phys. Lett. **158B**, 473 (1985).

[8]R. Tzani, Phys. Rev. D **33**, 1146 (1986).

[9]M. B. Gavela and N. Parga, Phys. Lett. B **174**, 319 (1986); Nucl. Phys. **B275**, 546 (1986).

[10]Mikio Namiki, Ichiro Ohba, Satoshi Tanaka, and Danilo M. Yanga, Phys. Lett. B **194**, 530 (1987).

[11]I. G. Koh and R. B. Zhang, Phys. Rev. D **35**, 3906 (1987).

[12]B. Sakita, in *Lattice Gauge Theories, Supersymmetry and Grand Unification,* proceedings of the 7th Johns Hopkins Workshop, edited by G. Domokos and S. Kovesi-Domokos (World Scientific, Singapore, 1983).

[13]B. Sakita, in *World Scientific Lecture Notes in Physics* (World Scientific, Singapore, 1985), Vol. 1, p. 173.

[14]D. Breit, S. Gupta, and A. Zaks, Nucl. Phys. **B233**, 61 (1981).

[15]K. Fujikawa, Phys. Rev. D **21**, 2848 (1980).