Geometry of spin effects in proton-proton scattering

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(Received 27 July 1987)

In proton-proton elastic scattering, the rapid rise in the spin-spin correlation A_{nn} at large angles may be understood from a simple geometry of central, spin-orbit, and spin-spin interactions. A spin-orbit interaction that decreases more rapidly with the impact parameter than the spin-spin interaction guarantees a rise in A_{nn} even when both contribute negligible amounts to the forward diffraction peak. The combination of interactions also accounts for the differential cross section. Sample calculations are compared to data at 12 GeV/c.

Proton-proton scattering measurements have revealed some surprisingly large spin effects. In particular the spin-spin correlation A_{nn} at 12 GeV/c has a clear and marked rise with momentum transfer.[1] We explain this rise by the simplest geometric considerations. At the same time we reproduce the well-known fact that the proton-proton differential cross section consists of three exponential regimes.[2]

In 1975 Durand and Halzen[3] suggested that spin effects could be obtained from a central interaction and a small spin-orbit eikonal. Wakaizumi[4] extended this approach and eventually went to the full impact-parameter representation. By using many free parameters, good fits were obtained; in particular, 49 parameters were fit to reproduce the data at 12 GeV/c. Our motivation is to extract the essential features from the simplest possible considerations.

The general spin-dependent amplitude can be written as

$$M = M_0(q) + M_1(q)(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\hat{\mathbf{n}} + M_2(q)(\boldsymbol{\sigma}_1\cdot\hat{\mathbf{n}})(\boldsymbol{\sigma}_2\cdot\hat{\mathbf{n}}) + M_3(q)(\boldsymbol{\sigma}_1\cdot\hat{\mathbf{q}})(\boldsymbol{\sigma}_2\cdot\hat{\mathbf{q}}) + M_4(q)(\boldsymbol{\sigma}_1\cdot\hat{\boldsymbol{l}})(\boldsymbol{\sigma}_2\cdot\hat{\boldsymbol{l}}) \, .$$

Commonly measured observables are the differential cross section, the analyzing power A, and the spin-spin correlation transverse to the scattering plane A_{nn}. They have been measured for a wide range of angles at 12 GeV/c. They are given in terms of the amplitudes as

$$\sigma_0 = |M_0|^2 + 2|M_1|^2 + |M_2|^2 + |M_3|^2 + |M_4|^2 = \frac{p^2}{\pi}\frac{d\sigma}{dt} \, ,$$

$$A = 2\,\mathrm{Re}[(M_0+M_2)M_1^*]/\sigma_0 \, ,$$

$$A_{nn} = 2[\mathrm{Re}(M_0M_2^* - M_3M_4^*) + |M_1|^2]/\sigma_0 \, .$$

The notation is essentially that of Refs. 3 and 4.

The central amplitude M_0 provides the forward diffraction peak. It drops very rapidly with angle. M_1 is the spin-orbit amplitude and M_2, M_3, and M_4 are the spin-spin amplitudes. From the form of A_{nn} it is apparent that if M_0 and the spin-spin amplitudes decrease so rapidly with momentum transfer that they became negligible compared to M_1, A_{nn}, will approach the limit 1: i.e., if the spin-orbit amplitude dominates, A_{nn} approaches 1 (as would the observable K_{nn} and D_{nn}). The question, of course, is to what extent the dominance of M_1 at large angles can be realized.

The amplitudes can be obtained from the impact-parameter representation

$$M = \frac{ik}{2\pi}\int (1-e^{i\chi(b)})e^{-i\mathbf{q}\cdot\mathbf{b}}d^2\mathbf{b} \, ,$$

when one writes the eikonal as a sum of five terms representing the central, spin-orbit, spin-spin, and tensor interactions. The tensor couplings have been shown to contribute 5–10% to the spin observables at 6 GeV/c.[4] Motivated by that result but mainly on the grounds of simplicity, we ignore the tensor couplings and write

$$\chi(b) = \chi_C(b) + b\chi_{LS}(b)(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\hat{\mathbf{n}} + \chi_S(b)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 \, .$$

These three eikonals are sufficient for spin-spin effects. Furthermore, they offer the possibility that different geometries for each of the eikonals, i.e., different decreases with impact parameter b, will transform into a differential cross section with three exponential regimes. Specifically, the central term with a diffuse edge will yield the rapidly decreasing forward peak of the differential cross section and the other two eikonals with much less diffuse edges will produce the less rapidly decreasing cross section at larger angles.

Each of the five eikonals, and each of our truncated set of three, appears in each of the amplitudes M_0–M_4.[4] The temptation to treat the spin-orbit and spin-spin eikonals to first order must be avoided when dealing with large momentum transfers. The transforms of second- and higher-order terms become very significant at large momentum transfer. This result is particularly easy to demonstrate when Gaussian forms are used for the eikonals. The spin-orbit amplitude will therefore not completely dominate the others at large angles and A_{nn} is unlikely to reach the limit 1. However, the experimental value for A_{nn} at 12 GeV/c is 0.6. Our calculations confirm that if χ_{LS} decreases sufficiently rapidly with b compared to χ_S, then at large angles M_1 has not fallen as fast as the other amplitudes and large values of

A_{nn} are easily attained.

We parametrize the central profile function as

$$1-e^{i\chi_C}=(A_{CR}+iA_{CI})e^{-b^2/\beta_C^2} ,$$

where the strength has real and imaginary parts A_{CR} and A_{CI}. The spin-orbit and spin-spin profiles are treated similarly but in order to reduce the number of parameters the strengths are assumed to be real; accordingly

$$\cos\chi_{LS}=1-A_{LS}be^{-b^2/\beta_L^2}$$

and

$$\cos\chi_S=1-A_S e^{-b^2/\beta_S^2} .$$

The sine function of these eikonals is written in terms of the cosine function using the positive square root. Gaussian forms are chosen for simplicity since they require only one distance parameter.

The profile functions can readily be translated into expressions for the eikonals but we want to emphasize that we parametrize the profile functions and thus calculate to all orders of our eikonals. In other words, our model is an impact-parameter representation with the approximations that tensor interactions have been ignored and that the remaining interactions have been given the very simple forms indicated above.

Our calculations show that the spin observables and the differential cross section at large angles are very sensitive to the strengths and diffusenesses of the spin-spin and spin-orbit interactions. More specifically they show that large values of A_{nn} are readily produced when $\beta_C > \beta_S > \beta_L$ and $A_L < A_S < |A_C|$.

Figures 1–3 show the results obtained after some adjustment of the parameters. These are not fits but rather calculations that indicate what our simple model might contain. The height and slope of the forward peak essentially determine A_C and β_C. An indication of the sensitivity is that when A_L is reduced to 0.0006 for the case shown in Fig. 3, the bottom halves of the error bars of the last three data points in A_{nn} will be intercepted; i.e., the calculated result moves slightly to the right and down. In general, A_{nn} will cross the axis at larger values of $|t|$, as the ratio A_L/A_S decreases; it will rise more rapidly as the ratio β_L/β_S decreases.

A study of the energy dependence is the next task but some brief comments can be made now. A weak energy dependence in β_C could change the slope parameter of the forward peak in the known manner.[2] Furthermore, because the spin observables are so sensitive to the strength and diffuseness parameters, a modest energy dependence in them could have dramatic effects—which

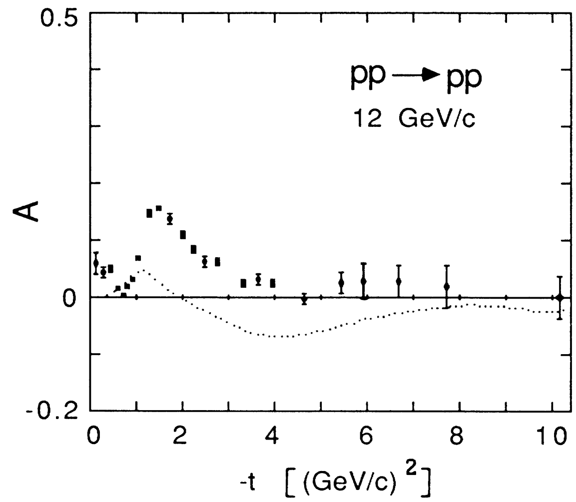

FIG. 2. Analyzing power. The dotted curve is calculated from the model with parameters as in Fig. 1.

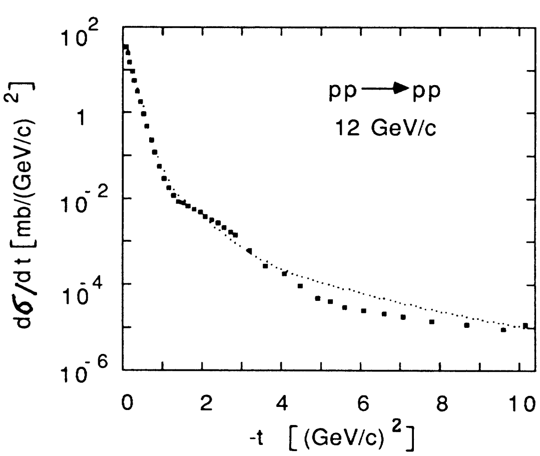

FIG. 1. Proton-proton differential cross section for $P_{\mathrm{lab}}=12$ GeV/c. The dotted curve is calculated from the profile functions given in the text with parameters $A_{CR}=0.64$, $A_{CI}=0.1$, $A_L=0.0008$ GeV/c, $A_S=0.015$, $\beta_C=4$, $\beta_L=0.8$, and $\beta_S=1.2$. The latter three are in units of (GeV/c)$^{-1}$. The data here and in the next two figures are from Ref. 1 and references contained in Ref. 4.

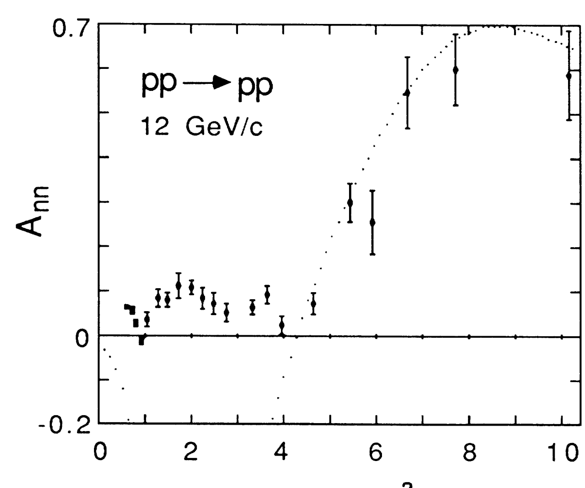

FIG. 3. Transverse spin-spin correlation A_{nn}. The dotted curve is calculated from the model with parameters as in Fig. 1.

a recent measurement of A_{nn} at 18.5 GeV/c indeed suggests.[5]

This simple model has shortcomings in the more forward directions, as in A_{nn}. For another example, the analyzing power A has been measured[6] to rise to 24% at 28 GeV/c when $P_\perp^2$ reaches 6.5 (GeV/c)^2. Although this model can readily yield such values with relatively modest changes in parameters, it cannot, at the same time, provide the structure found in A at the more forward angles. Nor can the model provide a dip in the differential cross section that is found near $|t|=1.4$ (GeV/c)^2 at higher energies, although it does readily yield a distinct shoulder. The dip in cross section probably does come about from a diffraction minimum produced by a profile with a sharper edge than a Gaussian. As is evident from the work of Wakaizumi at 12 GeV/c, the shortcomings in A and in A_{nn} at the more forward angles are remedied by including more elaborate functional forms for the profiles and by including tensor couplings. Other models, in which the phenomenology is placed in quark, parton, Pomeron, or other aspects, as well as in the impact-parameter representation, have been used to investigate spin effects in hadron-hadron scattering.[7–10] The virtue and limitation of the present approach is its extreme simplicity.

Our model shows that the rise in A_{nn} may be explained very simply by the spin-orbit interaction being less diffuse than the spin-spin interaction. At the same time the central interaction together with spin-spin and spin-orbit interactions that are insignificant in the forward directions account for the differential cross section out to large angles. To put it another way, if the large-$|t|$ falloff of the differential cross section is assumed to be governed by the spin-orbit interaction, then large values of A_{nn} are guaranteed.

This work was begun while one of use (P.A.K.) was visiting the University of Pennsylvania. We thank Professor Ralph Amado and the Department of Physics for their hospitality.

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