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Dynamical composite models of electroweak bosons

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It is demonstrated explicitly in a $1/N$ expansion of cutoff field theory that tightly bound vector particles made of fermion pairs behave exactly like gauge bosons as the binding becomes infinitely strong. Two examples of composite models for electroweak gauge bosons are constructed by breaking global electroweak SU(2) symmetry explicitly. The low-energy effective Lagrangian of these models is identical to that of the standard theory based on spontaneously broken gauge symmetry with the physical Higgs-boson mass let to infinity. A small deviation from gauge symmetry due to compositeness is computed for three-point vertices to illustrate how the compositeness effect manifests itself.

I. INTRODUCTION

When we attempt to build a composite model for gauge particles with or without spontaneous symmetry breaking, a gauge symmetry is implemented by a dynamical requirement. It has been known for a long time, independently of compositeness, that gauge couplings are obtained by requiring nonsingular high-energy behavior for tree diagrams of scattering.[1] A connection between gauge coupling and vector-meson dominance was noticed in the late 1960s.[2] A more recent way to achieve gauge symmetry is to saturate the current algebra of a relevant global symmetry with light vector bosons.[3] Though it may appear somewhat different, truncating an infinite sum of intermediate states in current algebra at the lowest one-particle states is closely related to the requirement of nonsingular high-energy behavior. Another recent approach, called the "strongly coupled standard model" by Claudson *et al.*,[4] has made a step further along this line to justify good high-energy behavior by a large-mass scale and weak coupling of the "exotic" sector in the model. They have shown quantitatively that compositeness corrections to electroweak gauge symmetry can actually remain small enough to be irrelevant for a wide range of values of the compositeness scale.

In parallel to these attempts, many people have speculated that when masses of tightly bound vector states approach zero, their interactions become equal to appropriate gauge interactions by some dynamical consistency.[5] This line of argument does not refer directly to high-energy behavior although good high-energy behavior should come out eventually in successful composite models. In this paper, I would like to study the relation between tight binding of composite particles and gauge symmetry. In order to focus on this specific aspect of the dynamical issue, I will choose a field-theory model of composite W and Z bosons which is solvable in a $1/N$ expansion though unrenormalizable and unconfining. I hope that unrenormalizability and nonconfinement of the force is not essential to the mechanism of interplay between tight binding and gauge symmetry. Our model Lagrangian consists of fermionic preons with a global symmetry. In the limit of an infinitely strong binding force, vector bound states become massless and all of their dimension-four interactions approach gauge coupling. When preons carry electric charges and a photon is introduced as an elementary particle, the photon mixes with the neutral component of the composite gauge bosons to turn it into the observed Z boson, as usual. The resulting effective Lagrangian at low energies is identical with that of the standard theory with an infinite Higgs-boson mass.

I will elaborate the SU(n) model in Sec. II. In Sec. III two models are presented for composite W and Z bosons with γ being either composite or elementary. In Sec. IV deviations from the gauge limit due to compositeness will be discussed for the three-body coupling in detail and then for general vertices. In Sec. V our findings and proposals are summarized, and then speculative remarks are made in connection with our proposals. Throughout this paper I do not attempt to construct a composite theory of light chiral fermions, namely, quarks and leptons, since problems involved in light fermions are quite different in nature from composite electroweak bosons.

II. DYNAMICAL GAUGE SYMMETRY IN TIGHT-BINDING LIMIT

My purpose is to demonstrate in solvable models that spin-one composite states approach gauge particles as a globally symmetric binding force becomes strong. A local symmetry emerges from a global symmetry in the tight-binding limit. In these models, gauge symmetries are generated not by the requirement of good high-energy behavior of tree diagrams, but by the smallness of composite-vector-boson mass relative to the preon mass. Although cancellation of singular diagrams characteristic of gauge theories does arise, it is a consequence of tight binding, not an input. Plausibility and necessity arguments have been put forth by many theorists, for instance, by Veltman and Mandelstam[5] among others. Here I present a class of models in which such a dynamical mechanism is clearly visible. The basic dynamical features of these models have been observed and discussed intermittently in the literature over many years.[6–9] I incorporate its non-Abelian version in these models with a slightly different interpretation and a little more rigor in some aspects.

We must solve for tightly bound states in field theory. Since there is no renormalizable model solvable for tightly bound states, we give up renormalizability. Furthermore, we adopt models for which a $1/N$ expansion is allowed, since otherwise computational complexity is beyond our capabilities. In the simplest version of our models, N families of fermion multiplets are introduced as preons in a fundamental representation of SU(n). Their interaction Lagrangian is given by

$$L_{\text{int}}=\sum_{ij} L^{(ij)}\ , \tag{2.1}$$

$$L^{(ij)}=-(\tfrac{1}{2}G/N)(\bar{\psi}^{(i)}\gamma_\mu \tfrac{1}{2}\lambda_a \psi^{(i)})(\bar{\psi}^{(j)}\gamma^\mu \tfrac{1}{2}\lambda_a \psi^{(j)})\ , \tag{2.2}$$

where i and j are family indices running from 1 to N, and $\frac{1}{2}\lambda_a$'s are the $n\times n$ matrices of the adjoint representation of SU(n). When the force is attractive ($G>0$), vector bosons of the adjoint representation are formed from preon-antipreon pairs through iteration of bubble diagrams in the leading $1/N$ order. It is straightforward to compute poles and residues of multipreon scattering amplitudes and to identify them with coupling constants of composite bosons. We compute couplings of composite vector bosons at zero momenta rather than on their mass shells because no infrared singularity exists in the leading order of the present models. We will find that these coupling constants approach the gauge coupling constant in the leading $1/N$ order as $G\to\infty$, at which the vector-boson mass goes to zero. This statement will be proven not only for logarithmically divergent terms but also for all finite terms of loop integrals.

A. Yukawa coupling to preons

In the leading $1/N$ order, bound states are formed in the preon-antipreon channels of $J^P=1^-$ and the adjoint representation through an infinite series of bubble diagrams depicted in Fig. 1(a). Our computation can be reproduced entirely by the functional method,[9] but we choose to use the diagrammatic language since physics is more clearly visible in diagrams. The preon-antipreon amplitude is given by

$$T^{\mu\nu}_{ab}(q)=\delta_{ab}g^{\mu\nu}T^{(2)}(q^2)\ , \tag{2.3}$$

$$T^{(2)}(q^2)=-(G/N)/[1+G\Pi(q^2)]\ , \tag{2.4}$$

where μ and ν are Lorentz indices, a and b are SU(n) indices, and $\Pi(q^2)$ is the Lorentz-scalar function of the vector vacuum polarization $\Pi^{\mu\nu}(q)=(g^{\mu\nu}-q^\mu q^\nu/q^2)\Pi(q^2)$. In Eq. (2.3) and thereafter, preon amplitudes are defined with external preon wave functions $\bar{v}\gamma_\mu\frac{1}{2}\lambda_a u$ removed from each end.

When M is large, the denominator of $T^{(2)}(q^2)$ is expanded around $q^2=0$ and a vector-boson pole can be located in the approximation of $m \ll M$, where m and M are masses of bound states and preons, respectively. It should be noted that $\Pi(0)=0$ must hold by vector-current conservation because of global SU(n) symmetry. Therefore, however strong the force may be, the bound state mass can never reach zero in our models. In his pioneering work,[6] Bjorken found a massless photon pole in a nonperturbative self-consistent solution triggered by a Lorentz-noncovariant term added to a Lagrangian

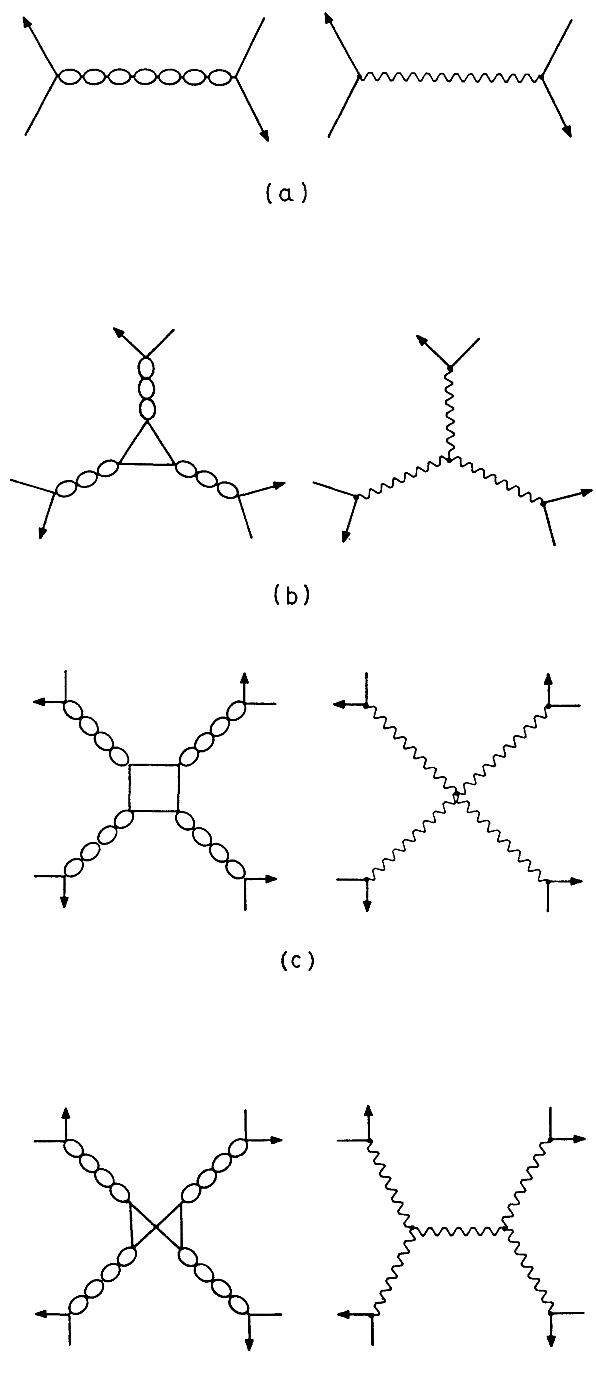

FIG. 1. Preon diagrams for (a) a gauge boson, (b) three-body coupling, and (c) four-body coupling. The diagram (d) is to be included in a one-boson-exchange process.

which is switched off subsequently. In the succeeding works,[8,9] the existence of a pole at $q^2=0$ is simply assumed either following Bjorken or else in an *ad hoc* manner. We take the most straightforward interpretation of the solution with neither degenerate vacua nor abnormal solutions. In our models, global current conservation thus forbids a pole at $q^2=0$ for any finite value of G (Ref. 10).

Expanding $T^{(2)}$ around $q^2=0$, we obtain

$$T^{(2)}(q^2)=\{[-N\Pi'(0)]^{-1}/(q^2-m^2)\}[1+O(q^2/M^2)]\ , \tag{2.5}$$

$$m^2=[-G\Pi'(0)]^{-1}\ , \tag{2.6}$$

where $\Pi'(0)=d\Pi(q^2)/dq^2\,|_{q^2=0}$ can be evaluated from the vector bubble diagram. We would like to be particularly careful about convergent terms in the logarithmically divergent integral which appears in $\Pi(q^2)$. To define finite terms unambiguously, we make a dimensional regularization for the integral and identify $(2-\frac{1}{2}D)^{-1}+\ln 4\pi-\gamma_E$ with a logarithmic divergence $\ln\overline{\Lambda}^2$. Then we obtain, by an explicit calculation,

$$\Pi'(0)=-\frac{1}{24\pi^2}\ln(\overline{\Lambda}^2/M^2)\ , \tag{2.7}$$

which gives $m^2>0$ for $G>0$. The vector-boson coupling to a preon is obtained by comparing Eq. (2.5) with

$$T^{(2)}(q^2)=g^2/(q^2-m^2)\ , \tag{2.8}$$

where the Yukawa coupling constant g is defined by $g\bar{\psi}\gamma_\mu\frac{1}{2}\lambda_a\psi A_a^\mu$. Therefore, the coupling g which is to be identified with the SU(n) gauge coupling in the $m\to 0$ limit is given by

$$g^2=[-N\Pi'(0)]^{-1}>0\ . \tag{2.9}$$

Even when G is large and the binding is strong, g^2 can be a small number. When the external preons are taken off the mass shell in Fig. 1(a), we can read off the term proportional to $q^\mu q^\nu$ of the vector-boson propagator. The propagator obtained by the series of preon bubbles is that of the unitary gauge: $\Delta^{\mu\nu}(q)=(-g^{\mu\nu}+q^\mu q^\nu/m^2)/(q^2-m^2)$.

B. Trilinear self-coupling

Trilinear self-couplings of vector bosons are generated in diagrams with six external preons where three bosons meet at the center, as shown in Fig. 1(b). In the leading $1/N$ order, the three series of bubbles interact among themselves through a triangular preon loop for which we have two distinct diagrams of different orderings. A straightforward computation gives us

$$T^{\mu\nu\rho}_{abc}(q_a,q_b,q_c)=if_{abc}[g^{\mu\nu}(q_2-q_1)^\rho+g^{\nu\rho}(q_3-q_2)^\mu+g^{\rho\mu}(q_1-q_3)^\nu]T^{(3)}\ , \tag{2.10}$$

$$T^{(3)}=N(-G/N)^3\Gamma_3(q_1^2,q_2^2,q_3^2)/\prod_i[1+G\Pi(q_i^2)]\ , \tag{2.11}$$

where Γ_3 is the scalar part of the triangular-preon-loop diagrams. As in the case of $\Pi(q^2)$, we compute $\Gamma_3(q_1^2,q_2^2,q_3^2)$ by dimensional regularization with identification $\ln\overline{\Lambda}^2=(2-\frac{1}{2}D)^{-1}+\ln 4\pi-\gamma_E$. The result of computation at $q_i^2\ll M^2$ ($i=1,2,3$) is

$$\begin{aligned}\Gamma_3(q_1^2,q_2^2,q_3^2)&=\frac{1}{24\pi^2}\ln(\overline{\Lambda}^2/M^2)+O(q_i^2/M^2)\\&=-\Pi'(0)+O(q_i^2/M^2)\ .\end{aligned} \tag{2.12}$$

Here we have used Eq. (2.7) in the second line. The equality $\Gamma_3(0,0,0)=-\Pi'(0)$ is understood as a Ward-Takahashi-type identity. We can isolate the three-point vertex of vector bosons at zero momenta as

$$\begin{aligned}T^{(3)}=&\{N\Gamma_3(0,0,0)[-N\Pi'(0)]^{-3}/q_1^2q_2^2q_3^2\}\\&\times[1+O(q_i^2/M^2)]\\=&\{[-N\Pi'(0)]^{-2}/q_1^2q_2^2q_3^2\}[1+O(q_i^2/M^2)]\end{aligned} \tag{2.13}$$

and rewrite it by use of Eqs. (2.9) and (2.12) into

$$T^{(3)}=(g^4/q_1^2q_2^2q_3^2)[1+O(q_i^2/M^2)]\ . \tag{2.14}$$

With g^3 removed, $T^{(3)}$ is equal to what we ought to obtain for the trilinear self-coupling vertex of gauge bosons. It is worth noting that the correct trilinear couplings have been obtained here including the finite terms of the triangular-loop diagrams because the finite terms have been computed in a regularization method compatible with global current conservation. A naive four-momentum cutoff would give $\Gamma_3(0,0,0)=-\Pi'(0)+\frac{1}{2}+O(q_i^2/M^2)$ instead of $\Pi'(0)+O(q_i^2/M^2)$.

C. Quartic self-coupling

Examine diagrams with eight external preon lines in which vector bound states merge at the center through a preon square loop as depicted in Fig. 1(c). Diagrams where the four vector states merge through two triangular loops are of the same order in $1/N$, but they are actually the first terms of the series for the reducible one-boson-exchange processes [Fig. 1(d)]. There exist six square-loop diagrams with different vertex orderings which enter the center of the left diagram in Fig. 1(c). Adding up the six diagrams, we obtain, for the matrix element of Fig. 1(c),

$$\begin{aligned}T^{\mu\nu\rho\sigma}_{abcd}(q_i)=-[&f_{abe}f_{cde}(g^{\mu\rho}g^{\nu\sigma}-g^{\mu\sigma}g^{\nu\rho})\\&+f_{ace}f_{bde}(g^{\mu\nu}g^{\rho\sigma}-g^{\mu\sigma}g^{\nu\rho})\\&+f_{ade}f_{bce}(g^{\mu\nu}g^{\rho\sigma}-g^{\mu\rho}g^{\nu\sigma})]T^{(4)}\ ,\end{aligned} \tag{2.15}$$

$$T^{(4)}=N(G/N)^4\Gamma_4(q_1,q_2,q_3,q_4)/\prod_i[1+G\Pi(q_i^2)]\ , \tag{2.16}$$

where Γ_4 is the scalar part of the preon square-loop diagrams. Again by the same dimensional regularization as before, we find, including all finite terms that survive at

$M\to\infty$,

$$\Gamma_4(q_1,q_2,q_3,q_4)=\frac{1}{24\pi^2}\ln(\overline{\Lambda}^2/M^2)+O(q_i\cdot q_j/M^2)$$
$$=-\Pi'(0)+O(q_i^2/M^2)\ , \qquad (2.17)$$

where Eq. (2.7) has been used in the second line. Equation (2.17) is another Ward-Takahashi-type identity. The quartic coupling of the vector bound states is now obtained from Eqs. (2.16) and (2.17) as

$$T^{(4)}=\left[N\Gamma_4(0,0,0,0)[-N\Pi'(0)]^{-4}/\prod_i q_i^2\right]$$
$$\times[1+O(q_i\cdot q_j/M^2)]$$
$$=\left[[-N\Pi'(0)]^{-3}/\prod_i q_i^2\right][1+O(q_i\cdot q_j/M^2)]\ . \qquad (2.18)$$

This last line can be expressed with Eq. (2.9) as

$$T^{(4)}=g^6/\prod_i q_i^2$$

near $q_i^2=0$ $(i=1,2,3,4)$, which is identical with the quartic vertex of the gauge bosons after g^4 has been assigned to the preon vertices. Evaluation of loops by dimensional regularization is even more important for Γ_4; if $\Gamma_4(0,0,0,0)$ is computed with naive four-momentum cutoff, not only would it be different from $-\Pi'(0)$ by a finite amount, but also could the sum of the six square loops not be cast into the Lorentz and SU(n) structures of the quartic gauge coupling.

We have thus shown in our composite model that tightly bound bosons of fermionic preon pairs automatically obey local gauge symmetries in the massless limit. The relative strength of the self-couplings to the Yukawa coupling to preons conforms to the gauge principle. Once this universality of couplings has been established, any composite state of preons obeys the correct gauge-coupling rule required by local SU(n) symmetry. In order to claim dynamical generation of perfect gauge symmetry, the mass of vector bosons must go to zero. When they remain massive, their couplings defined on the mass shell deviate from respective gauge limits by $O(m^2/M^2)$. These deviations are distinct from radiative corrections for elementary vector bosons which acquire mass by spontaneous symmetry breaking. We will come back to this subject in a separate section.

The dynamical generation of gauge symmetries obtained here relies on the existence of global symmetries which require current conservation and warrant smallness of vector bound-state masses by the constraint $\Pi(0)=0$. If currents are not conserved, the natural magnitude of composite-vector-boson mass is of the order of M unless a fine-tuning is made. We expect conversely that if composite vector bosons are much lighter than preons, they imply global current conservation associated with each channel of SU(n) quantum numbers. It is obvious that a global symmetry implanted in the preon Lagrangian need not be SU(n). The dynamical structure of the interaction Lagrangian, Eqs. (2.1) and (2.2) can be modified without changing our conclusions. We therefore suggest that an approximate gauge symmetry is a dynamical consequence of formation of light composite vector bound states, and vice versa.

Before concluding this section, we remark on Abelian symmetry. If an interaction such as

$$L^{(ij)}=-(\tfrac{1}{2}G'/N)(\overline{\psi}^{(i)}\gamma_\mu\psi^{(i)})(\overline{\psi}^{(j)}\gamma^\mu\psi^{(j)}) \qquad (2.19)$$

is added to our Lagrangian, and the coupling G' is large and positive, a light vector bound state of SU(n) singlet is formed. This singlet boson does not have a three-body nor a four-body coupling with itself nor with SU(n)-adjoint composite vector bosons which we have studied above. The reason exists in the global-symmetry structure of trilinear and quartic couplings, Eqs. (2.10) and (2.15); because of the Lorentz structure of the triangle and square loops, every group index of boson is antisymmetrized with another, making all singlet couplings vanish. Therefore, this SU(n)-singlet vector boson can be identified with an Abelian gauge boson associated with preon number conservation. In this way, extension to semisimple group is easily made. Addition of an Abelian gauge boson is almost trivial if one introduces new families of singlet preons to form it.

III. SU(2)×U(1) MODELS OF ELECTROWEAK BOSONS

One can introduce scalar bound states in addition to vector bound states by adding a new four-fermion interaction of scalar type to the preceding model Lagrangian. If one follows either Bjorken's argument[6] or takes the strong-coupling limit to make the vector bound states exactly massless, the effective low-energy Lagrangian of such a system would be identical with that of the standard theory prior to spontaneous symmetry breaking. If the potential of the scalar composite fields makes a perturbative vacuum unstable, the rest of the scenario duplicates the standard theory with spontaneous symmetry breaking.[11] In the absence of a constraint such as current conservation, a natural scale of scalar bound-state masses determined by a quadratically divergent scalar vacuum-polarization part is of the order of the compositeness scale M. There is not much new to be said in this scenario. We will not pursue this line. Instead, we present two examples of composite models for electroweak bosons with explicit soft or hard breaking of the global electroweak symmetries. In either model, the mass relation between W and Z is automatically satisfied.

A. Model with infinitely strong coupling

The first model, hereafter referred to as model A, starts with a composite theory of massless SU(2)×U(1) gauge bosons by taking the infinitely strong limit of binding, $G\to\infty$. Taking the $G\to\infty$ limit is a little uncomfortable not only mathematically but also physically. By this reason we prefer the second model to be presented later. Nevertheless, we describe it since it contains many interesting features.

The model A consists of $2N_1$ singlet preons Ψ and N_2

doublets of preons ψ_α which form SU(2)-singlet and -triplet of composite vector bosons, respectively. The model Lagrangian is given by

$$L=\sum_{i=1}^{2N_1}\bar{\Psi}^{(i)}(i\partial\!\!\!/-M)\Psi^{(i)}+\sum_{k=1}^{N_2}\bar{\psi}^{(k)}(i\partial\!\!\!/-M)\psi^{(k)}-\Delta M\sum_{k=1}^{N_2}(\bar{\psi}^{(k)2}\Psi^{(k)}+\bar{\Psi}^{(k)}\psi_2^{(k)})+L_{\rm int}\ , \quad (3.1)$$

$$L_{\rm int}=-(\tfrac{1}{2}G_V'/N_1)\sum_{ij}^{2N_1}(\bar{\Psi}^{(i)}\gamma_\mu\tfrac{1}{2}\Psi^{(i)})(\bar{\Psi}^{(j)}\gamma^\mu\tfrac{1}{2}\Psi^{(j)})-(\tfrac{1}{2}G_V/N_2)\sum_{kn}^{N_2}(\bar{\psi}^{(k)}\gamma_\mu\tfrac{1}{2}\tau_a\psi^{(k)})(\bar{\psi}^{(n)}\gamma^\mu\tfrac{1}{2}\tau_a\psi^{(n)})\ , \quad (3.2)$$

where the off-diagonal mass terms have been introduced between singlets and down components of doublets as SU(2)×U(1) symmetry breaking. When $\Delta M=0$, four massless vector bosons of SU(2)×U(1) arise in the limit of G_V and $G_V'\to\infty$. According to the analysis of the preceding section, they are the composite SU(2)×U(1) gauge bosons with the U(1) and SU(2) couplings given by

$$g_1^2=[-N_1\Pi_V'(0)]^{-1}\ , \quad (3.3)$$

$$g_2^2=[-N_2\Pi_V'(0)]^{-1}\ . \quad (3.4)$$

The weak hypercharges of ψ and Ψ are fixed to 0 and $-\frac{1}{2}$, respectively, in this Lagrangian. It is possible to give a nonvanishing weak hypercharge to doublet preons if a singlet interaction among doublet preons such as Eq. (2.19) is added to the Lagrangian.

The explicit symmetry breaking ΔM gives mass to W and mixes the third component of the triplet gauge bosons with the U(1) gauge boson to turn them into Z and γ [Fig. 2(a)]. To the order of $(\Delta M)^2$, the W and Z masses are found to be

$$m_W^2=[\Pi_S'(0)/\Pi_V'(0)](\Delta M)^2\ , \quad (3.5)$$

$$m_Z^2=[\Pi_S'(0)/\Pi_V'(0)][1+(N_2/N_1)](\Delta M)^2\ , \quad (3.6)$$

where $\Pi_V(q^2)$ and $\Pi_S(q^2)$ are the Lorentz-scalar functions of the vector and scalar preon bubbles, respectively. By dimensional regularization with $\ln\bar{\Lambda}^2=(2-\frac{1}{2}D)^{-1}+\ln 4\pi-\gamma_E$, $\Pi_V'(0)$ and $\Pi_S'(0)$ are given as

$$\Pi_V'(0)=-\frac{1}{24\pi^2}\ln(\bar{\Lambda}^2/M^2)\ , \quad (3.7)$$

$$\Pi_S'(0)=-\frac{1}{16\pi^2}[\ln(\bar{\Lambda}^2/M^2)-\tfrac{2}{3}]\ . \quad (3.8)$$

The photon remains massless because of a residual U(1) symmetry. The mixing angle between Z and γ is determined from the Z-γ mass matrix. The result is

$$\tan\theta_W=(N_2/N_1)^{1/2}\ . \quad (3.9)$$

By use of Eqs. (3.3) and (3.4), $\tan\theta_W$ of Eq. (3.9) can be rewritten as

$$\tan\theta_W=g_1/g_2\ . \quad (3.10)$$

Combining Eqs. (3.5), (3.6), and (3.10), we find

$$\rho\equiv m_W^2/(m_Z^2\cos^2\theta_W)=1\ , \quad (3.11)$$

as we desire.

The effect of ΔM on vector-boson coupling can be studied by computing bubble diagrams in an infinite series [Fig. 2(b)]. We find no new vertex that survives at $M\to\infty$. The leading corrections are terms of the order of $(\Delta M)^2/M^2$ for both gauge and nongauge couplings. Therefore, the effective low-energy Lagrangian is identical up to $O((\Delta M/M)^2)$ with that of the minimal standard theory with an infinite Higgs-boson mass, which means that, as energy grows, W and Z start interacting strongly[12] at the center-of-mass energy $\sim m_W/\sqrt{\alpha}\simeq 1$ TeV in this composite model. One might think of introducing complex scalar doublets by forming them as composite states with an additional four-fermion interaction $\sum_{\alpha=1,2}(\bar{\Psi}^{(k)}\psi_\alpha^{(k)})(\bar{\psi}^{(n)\alpha}\Psi^{(n)})$. However, there is little reason to call for scalar bosons in our model since the motivation for Higgs doublets is solely to give the right masses and mixing for W, Z, and γ. As was pointed out above, scalar composite mass can be kept small ($\ll M$) only by fine-tuning of a scalar binding force. If light complex scalar doublets are formed as bound states, three of them would mix with and be absorbed by the longitudinal-polarization states of W and Z as usual, without affecting the $\rho=1$ relation but leaving behind three scalar mass terms as nonpropagating modes.

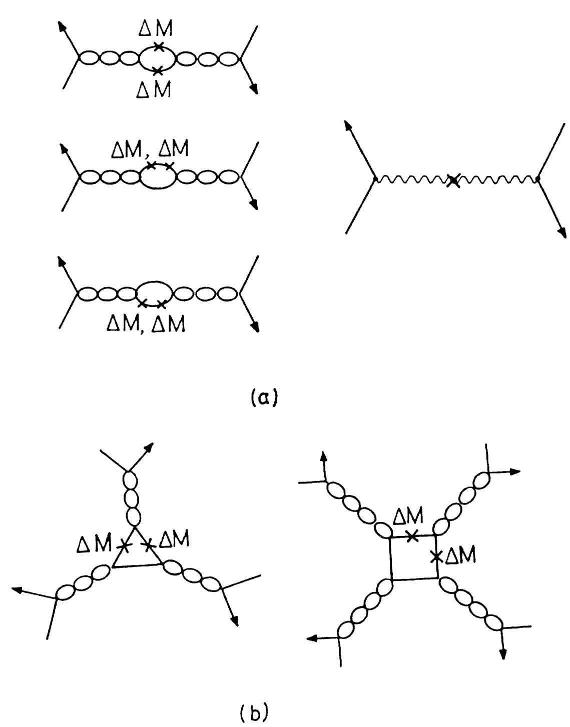

FIG. 2. (a) Preon diagrams to generate mass and mixing of the U(1) and SU(2) bosons through insertion of preon mass difference. The first diagram causes the $W^0\gamma$ mixing, while the second and third diagrams generate diagonal mass terms. (b) Preon diagrams for symmetry breaking in couplings.

B. Model without infinite coupling

An alternative approach to that of $G \to \infty$ is to keep SU(2)-triplet vector bosons, $W^\pm$ and W^0, finite in mass and mix W^0 with an elementary photon γ. Mixing occurs between W^0 and γ through preon pairs. The W^0 turns into the physical Z boson with the mass satisfying $\rho=1$. This step is by now one of the standard procedures in model building of composite weak bosons.[13] The Lagrangian of this model, referred to as model B hereafter, is the SU(2) version of Eqs. (2.1) and (2.2) with the photon field A^μ added:

$$L_{\rm int} = -e \sum_i \bar\psi^{(i)}\gamma_\mu(Y+\tfrac{1}{2}\tau_3)\psi^{(i)}A^\mu + \sum_{ij} L^{(ij)} , \quad (3.12)$$

$$L^{(ij)} = -(\tfrac{1}{2}G/N)(\bar\psi^{(i)}\gamma_\mu \tfrac{1}{2}\tau_a\psi^{(i)})(\bar\psi^{(j)}\gamma^\mu\tfrac{1}{2}\tau_a\psi^{(j)}) , \quad (3.13)$$

where Y is the weak hypercharge of the preon doublets. The SU(2) gauge coupling is given by Eq. (2.9):

$$g_2^2 = [-N\Pi'(0)]^{-1} \quad (3.14)$$

and the W mass is given by Eq. (2.6):

$$m_W^2 = [-G\Pi'(0)]^{-1} , \quad (3.15)$$

where $\Pi'(0)$ is the first derivative of the vector preon bubble defined in Eq. (2.7). The photon field A_μ mixes with the W_μ^0 field through preon pairs, as shown in Fig. 3, to form two mass eigenstates, the physical Z and the photon. Comparing the preon diagram in Fig. 3 with the composite boson diagram in Fig. 3, we find the $W^0\gamma$ transition strength as

$$-\tfrac{1}{2}(e/g_2)F_{\mu\nu}(\gamma)F^{\mu\nu}(W^0) \quad (3.16)$$

between their field strengths. Consequently, the two mass eigenstates turn out to be

$$\gamma:\ A_\mu , \quad (3.17)$$

$$Z:\ [W_\mu^0-(e/g_2)A_\mu]/[1-(e/g_2)^2]^{1/2} , \quad (3.18)$$

with eigenvalues equal to

$$m_\gamma^2 = 0 , \quad (3.19)$$

$$m_Z^2 = m_W^2/[1-(e/g_2)^2] . \quad (3.20)$$

The neutral weak current to which the physical Z couples is in the form

$$J_{\rm NC}^\mu = \{g_2/[1-(e/g_2)^2]^{1/2}\}[J_3^\mu-(e/g_2)^2 J_{\rm em}^\mu] , \quad (3.21)$$

where J_3^μ and $J_{\rm em}^\mu$ are the third component of SU(2) currents and the electromagnetic current of preons. It follows immediately that if one defines

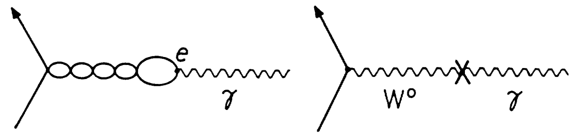

FIG. 3. Preon diagram for mixing between the elementary photon and a neutral component of the SU(2) bosons.

$$\sin\theta_W = e/g_2 , \quad (3.22)$$

Eqs. (3.17)–(3.21) turn into the familiar relations written with θ_W in the standard theory.

No fine-tuning or stretching of numbers is needed in this model. We do not have to argue for how the mixing between W^0 and γ can be made large.[13,14] With g_2 given by Eq. (3.14), g_2 can be easily as small as one wishes, so that the experimental value $\simeq 0.23$ for $\sin^2\theta_W=(e/g_2)^2$ can be realized without difficulty. In model B, again, Higgs particles are not called for. Light scalar bound states are simply unnatural unless some dynamical mechanism such as supersymmetry controls scalar masses.

We have presented here two models of composite electroweak bosons. We do not attempt to extend our models to light, chiral composite fermions, namely, quarks and leptons, since there are so many obstacles in building a realistic model for them.[15] In the following section we will study how compositeness reveals itself in the self-coupling of the electroweak bosons in the two models presented here.

IV. NONGAUGE COUPLINGS AND DEVIATIONS FROM GAUGE LIMIT

In the preceding models, the dynamics is identical with that of the standard theory with an infinite Higgs-boson mass, when the preon mass is let to infinity. Since all couplings of composite weak bosons automatically approach the gauge limit at $M\to\infty$, their deviations from the gauge limit are suppressed by inverse powers of M. We can actually compute these compositeness corrections in our models. Let us present here in detail the result in the case of the electromagnetic vertex of $W^\pm$. Since we are particularly interested in the momentum-dependent part of the corrections whose effect grows with energy, all three external momenta are kept off mass shell. Let us first drop all SU(2)-symmetry-breaking terms but the $W^0\gamma$ mixing. It is found in our models that the off-shell three-point function of $W^+W^-\gamma$ is described by the effective low-energy Lagrangian

$$L_{\rm int} = ief_\gamma(W_{\mu\nu}^\dagger W^\mu - W_\mu^\dagger W_\nu^\mu)A^\nu + ie\kappa_\gamma W_\mu^\dagger W_\nu A^{\mu\nu} + ie\lambda_\gamma W_{\rho\mu}^\dagger W_\nu^\mu A^{\nu\rho} + ie\mu_\gamma^{(1)}(\partial^\lambda W_\mu^\dagger \partial^\mu W_\nu \partial^\nu A_\lambda - \partial^\nu W_\mu^\dagger \partial^\lambda W_\nu \partial^\mu A_\lambda)$$
$$+ ie\mu_\gamma^{(2)}[(W_\mu^\dagger \overleftrightarrow{\partial}_\lambda W_\nu)(g^{\mu\nu}\Box - \partial^\mu\partial^\nu)A^\lambda + \text{cyclic permutations}] , \quad (4.1)$$

where f_γ, κ_γ, λ_γ, $\mu_\gamma^{(1)}$, and $\mu_\gamma^{(2)}$ are parameters, $W_{\mu\nu}=\partial_\mu W_\nu-\partial_\nu W_\mu$ and so forth, and cyclic permutation among $W_\mu^\dagger, W_\nu$ and A_λ is meant in the square brackets in the last term. After some amount of calculation to the first order in q_i^2/M^2 $(i=1,2,3)$ and m_W^2/M^2, we find that the compositeness corrections are given by

$$\begin{aligned} f_\gamma &= 1+[c_1/\ln(\overline{\Lambda}^2/M^2)](q_1^2+q_2^2+q_3^2)/M^2 ,\\ \kappa_\gamma &= 1+[c_2/\ln(\overline{\Lambda}^2/M^2)](q_1^2+q_2^2+q_3^2)/M^2 ,\\ \lambda_\gamma &= 0+[c_3/\ln(\overline{\Lambda}^2/M^2)](1/M^2) ,\\ \mu_\gamma^{(1)} &= 0+[c_4/\ln(\overline{\Lambda}^2/M^2)](1/M^2) ,\\ \mu_\gamma^{(2)} &= 0+[c_5/\ln(\overline{\Lambda}^2/M^2)](1/M^2) . \end{aligned} \tag{4.2}$$

The first terms, 1 or 0, represent the gauge symmetric limits. The numerical coefficients c_1-c_5 obey SU(2) symmetry up to the first-order $W^0\gamma$ mixing and are common in the two models presented in the preceding section:

$$\begin{aligned} &c_1=c_2=\tfrac{1}{5}, \quad c_3=-\tfrac{9}{20} ,\\ &c_4=-\tfrac{1}{5}, \quad c_5=\tfrac{1}{5} . \end{aligned} \tag{4.3}$$

The momentum-dependent correction terms grow with energy to enhance the compositeness effect in high-energy processes. The sign of c_1 and c_2 is consistent with form-factor damping in the spacelike direction. When SU(2) breaking beyond the first-order $W^0\gamma$ mixing is included, the coefficients $f-\mu_\gamma^{(2)}$ generally acquire additional corrections. These corrections are dependent on the models, reflecting a difference in dimension of the operators which break symmetry. Denoting the further corrections by $\Delta f_\gamma-\Delta\mu_\gamma^{(2)}$, we can express them to $O(1/M^2)$ as

$$\begin{aligned} \Delta f_\gamma &= [c_1'/\ln(\overline{\Lambda}^2/M^2)]\times \begin{cases} m^2/M^2 ,\\ (\alpha/4\pi)(q_i^2/M^2) ,\end{cases}\\ \Delta\kappa_\gamma &= [c_2'/\ln(\overline{\Lambda}^2/M^2)]\times \begin{cases} m^2/M^2 \\ (\alpha/4\pi)(q_i^2/M^2) ,\end{cases}\\ \Delta\lambda_\gamma &= [c_3'/\ln(\overline{\Lambda}^2/M^2)]\times \begin{cases} 0 ,\\ (\alpha/4\pi)(1/M^2) ,\end{cases}\\ \Delta\mu_\gamma^{(1)} &= [c_4'/\ln(\overline{\Lambda}^2/M^2)]\times \begin{cases} 0 ,\\ (\alpha/4\pi)(1/M^2) ,\end{cases}\\ \Delta\mu_\gamma^{(2)} &= [c_5'/\ln(\overline{\Lambda}^2/M^2)]\times \begin{cases} 0 ,\\ (\alpha/4\pi)(1/M^2) ,\end{cases} \end{aligned} \tag{4.4}$$

where the upper and lower entries following the curly brackets correspond to models A and B, respectively, and $c_1'-c_5'$ are numerical constants ~ 1 just like c_1-c_5 in Eq. (4.3), different from models A to B. Putting Eqs. (4.3) and (4.4) together, we find, for instance, that the $g-2$ of W cannot be larger than $O(m_W^2/M^2)$ in magnitude. Once SU(2) breaking is included beyond the lowest-order $W^0\gamma$ mixing, effective interactions of forms different from Eq. (4.1) can be induced with coefficients similar to those in Eq. (4.4).

For the three-body vertex of Z, we can define corresponding couplings, $f_Z-\mu_Z^{(2)}$ by replacing A_μ and $A_{\mu\nu}$ by Z_μ and $Z_{\mu\nu}$, respectively, and e by $g_2\cos\theta_W$. The result of the calculation shows that the gauge-symmetric terms and the SU(2)-symmetric terms of $f_Z-\mu_Z^{(2)}$ are identical with those of $f_\gamma-\mu_\gamma^{(2)}$, respectively. The symmetry-breaking terms $\Delta f_Z-\Delta\mu_Z^{(2)}$ are different from $\Delta f_\gamma-\Delta\mu_\gamma^{(2)}$ of the photon coupling.

Characteristics observed in the three-point functions above emerge in all other multiboson vertices. In the case of four-point functions, couplings of dimension four receive SU(2)-symmetric corrections of $O(p^2/M^2)$, where p^2 represents a properly symmetrized Lorentz-scalar variable made of four external momenta squared, q_i^2 $(i=1-4)$, and the Mandelstam variables, s, t, and u. They are momentum dependences of Lorentz-scalar functions. In addition, corrections of $O(m^2/M^2)$ appear generally as explicit SU(2) breaking beyond $W^0\gamma$ mixing. Couplings of interaction operators with dimension D (>4) are of the order of $(1/M^2)^{D/2-2}$ with SU(2)-symmetric coefficients up to $W^0\gamma$ mixing. Symmetry-breaking effects are further down by another factor of $(\Delta M/M)^2$ in model A, while in model B, photon loops generate SU(2)-breaking corrections of the order of $(\alpha/4\pi)(1/M^2)^{D/2-2}$ to SU(2)-symmetric terms.

When a deviation from a gauge symmetry is embedded in a loop diagram, its effect would be very singular by lack of gauge symmetric cancellation among gauge-related diagrams. Such a compositeness effect is expected to be largest in the coefficients of the operators having the lowest dimension: namely, self-energies of W and Z. The contribution of the nonstandard gyromagnetic ratio of W, $\Delta\kappa_\gamma\equiv\kappa_\gamma-1$, was previously studied in the ρ parameter[16] and the anomalous magnetic moment of a muon.[17,18] Our composite models predict that $\Delta\kappa_\gamma$ is suppressed by inverse powers of M^2. Nonetheless, other deviations, in particular, the fourth and fifth terms of the dimension-six interactions in Eq. (4.1) give rise to corrections of $O((g_2^2/16\pi^2)(m_\mu/m_W)^2)$ through their momentum dependence. Therefore, the compositeness correction to $(g-2)_\mu$ is comparable with the standard theory correction itself.[19] In our models with $1/N$ expansion, however, loop diagrams of physical W and Z are in the next leading order down by $1/N$ relative to the leading $1/N$ order. Consequently, computation does not close with W and Z loops alone, but has to include all other diagrams in the same $1/N$ order, if we want to obtain quantitative answers. Because of computational complexity, it is very difficult to obtain precise results in a nonleading order of $1/N$ expansion. A detailed analysis of radiative corrections in our models is deferred to a separate paper.

V. SUMMARY AND SPECULATIONS

We have shown explicitly in the solvable cutoff field-theory models that non-Abelian gauge symmetry is an inevitable consequence of the formation of light composite vector bosons in the strong limit of binding forces which obey global symmetry. No reference has been made to the high-energy behavior of amplitudes for physical bound states. The origin of non-Abelian symmetry in our models can be compared with the field-current identity by Kroll, Lee, and Zumino.[2] They showed that if massive vector fields are proportional to conserved currents and if their three-body couplings are proportional to $f_{abc}(\partial_\mu A_{a\nu}-\partial_\nu A_{a\mu})\times A_b^\mu A_c^\nu$, their four-body self-couplings obey the rule of non-Abelian gauge theory. In our models, before an explicit symmetry breaking is switched in, a triplet of composite vector particles is formed from the conserved current-current interactions of four-fermion form which obey global SU(2) symmetry. In this sense, our composite vector fields, if they are defined, are closely related to the conserved global current.[20] One might say that the observation by Kroll, Lee, and Zumino has been tacitly incorporated in our binding forces. The essence of the field-current identity was put in under the name of the superconductor-type model or the mean-field approximation in the non-Abelian extension of Bjorken's work.[8,9] One dynamical issue is whether the formation of light composite vector bosons by arbitrary binding forces implies the existence of conserved currents. We have argued to advocate this viewpoint in Sec. II, but no general proof has been offered because we have no tractable method in field theory to handle tightly bound states. The two-body approximation in the Bethe-Salpeter equation is known to have problems for strong forces.[21] Computing to higher order in $1/N$ in our models is a formidable task. Extending our models to more realistic ones, for instance, with confining binding forces is not easy unless exact solvability is given up. Despite this limitation, we would like to assert this line of logic here again: The very existence of light composite vector particles requires conserved currents having the quantum numbers of vector particles, or equivalently a global symmetry generated by those charges. Without a global symmetry, a natural scale of vector-boson masses would be of the order of preon masses. When a global symmetry exists, it is guaranteed by the dynamical mechanism shown in Sec. II that all the couplings of vector bound states obey the rule of non-Abelian gauge field theory.

In order to build electroweak models, we have turned on explicit symmetry breaking through preon mass splitting or electromagnetic interaction mediated by an elementary photon. The mass splitting between W and Z is realized by an explicit symmetry breaking. No Higgs scalar particles are called for to generate and split W and Z masses. Symmetry breaking by the preon mass splitting ΔM does not manifest itself at low energies by a dimensional reason, except in the W-Z mass parameters. Symmetry breaking by electromagnetic interaction can affect low-energy phenomena through QED loop corrections, which could be different from those in the standard theory. This difference can be sizable when the preon mass scale M is only several times larger than m_W. By contrast, symmetry breaking by the preon mass splitting resembles more closely the standard theory in this respect. In either case, radiative corrections to physical parameters attributable to compositeness turn out to be smaller than one might have feared.

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