Thermal fluctuations in new inflation

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The effects of thermal fluctuations on the evolution of a weakly coupled scalar field in the preinflationary phase of a model of new inflation are studied. Limits on the coupling constants are derived below which the effects of thermal fluctuations are negligible. The stochastic approach to inflation is extended to cover a situation in which spatial gradient terms and acceleration terms in the equation of motion of the scalar field dominate.

I. INTRODUCTION

The inflationary universe scenario has become an attractive scenario which can explain the observed flatness, homogeneity, and isotropy of the observed part of the Universe.[1,2] The first successful model that has been studied in detail was the so-called new inflationary universe,[3] which is based on a particle physics containing a scalar field ϕ in a double-well effective potential $V(\phi)$ that has global minima at $\phi=\pm\sigma$ (see Fig. 1). It was assumed that after a phase transition at a critical temperature $T_c \sim \sigma$ the scalar field $\phi(\mathbf{x})$ is localized at the origin $\phi=0$ at all points of space. The field ϕ then starts to roll towards $\phi=\pm\sigma$, the motion being initiated by quantum fluctuations. However, in order that the energy density of the fluctuations produced during inflation does not exceed the observational bounds, ϕ has to be very weakly coupled. Hence it is very unlikely that $\phi(\mathbf{x})$ will be confined to $\phi \sim 0$ at high temperatures.[4,5]

Given a scalar field that is very weakly coupled to itself and to other fields, one expects large spatial fluctuations at early times.[4] The equation of state of the scalar field will be dominated by spatial-gradient and kinetic terms and thus will not give rise to inflation. This has led to a development of a new mechanism[6] by which new inflation can be realized. We assume that the scalar field is not the only matter contribution to the energy-momentum tensor $T_\mu{}^\nu$. We assume that there is a thermal radiation bath, which contributes to the energy-momentum tensor; thus we have

$$T_\mu{}^\nu = T_\mu{}^\nu(\phi) + T_\mu{}^\nu(\mathrm{rad}) \ . \tag{1}$$

The first term on the right-hand side is the contribution from the scalar field, the second is that from the radiation bath. It should be stressed, however, that, with our assumptions, homogeneity, isotropy, and flatness are prerequisites, not consequences. At the initial time, which we take to be the Planck time t_{Pl}, $T_\mu{}^\nu$ is dominated by $T_\mu{}^\nu$ (rad) and hence the Universe is expanding like a radiation-dominated Friedmann-Robertson-Walker (FRW) space-time. The metric for this particular Universe is

$$ds^2 = dt^2 - a^2(t)(dx^2+dy^2+dz^2) \ , \tag{2}$$

where $a(t)$ is the scale factor and it varies as $a(t) \sim t^{1/2}$ for a radiation-dominated Universe.

We can now consider the evolution of a scalar field configuration $\phi(\mathbf{x})$ with large initial fluctuations under the assumption that ϕ is coupled to the thermal bath. In this case it has been demonstrated both analytically[6] and numerically[7] that, provided ϕ is weakly self-coupled, $\phi(\mathbf{x})$ will uniformly relax to its initial spatial average $\langle\phi\rangle$, which in general will be zero by symmetry. Thus new inflation can be dynamically realized in the sense that, at the critical temperature T_c that occurs when the poten-

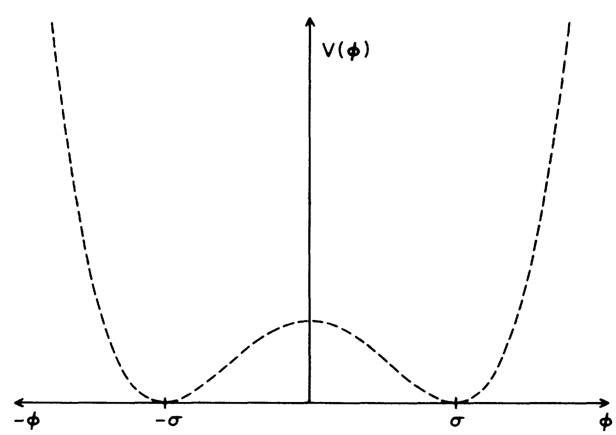

FIG. 1. A double-well potential $V(\phi)$.

tial energy density $V(0)$ equals the energy density $\rho(\mathrm{rad})$ of the radiation bath, $\phi(\mathbf{x})$ is confined to a region close to $\phi=0$, thus reproducing the usual initial conditions for new inflation.

The main purpose of this paper is to study the effects of coupling ϕ to the radiation bath on the evolution of $\phi(\mathbf{x})$. We find limits on the coupling constants below which the evolution of $\phi(\mathbf{x})$ outlined above remains unaffected and new inflation can be dynamically realized. This paper summarizes an approach which is mainly analytical; in a companion paper[8] one of us (H.A.F.) will discuss a numerical analysis of the problem.

The coupling to the radiation bath acts as a random-force term in the equations of motion for $\phi(\mathbf{x},t)$. There are two quite different ways to include random forces. The first is to follow a Monte Carlo simulation of the evolution of ϕ averaging over the realizations of the random force. The analysis in the second part of this paper is in the spirit of this method. An alternative approach is to derive and analyze the equation of motion for the probability distribution $P(\phi,\chi,t)$. $P(\phi_0,\chi_0,t)$ is the probability amplitude at time t for finding ϕ in the configuration ϕ_0 and χ_0, where $\chi=\dot{\phi}$. We shall refer to the second approach as the stochastic method.

The usefulness of $P(\phi,\chi,t)$ was first pointed out by Vilenkin in the context of new inflation.[9] The stochastic approach was independently developed by many authors.[10–18] In most cases,[10–13,15–18] quantum fluctuations in the scalar field ϕ itself were considered as the source of the random force. In these papers (except for Ref. 18) attention was restricted to a local patch of the Universe which was taken to be homogeneous and in which ϕ was only slowly moving so that the $\ddot{\phi}$ term in the equation of motion

$$\ddot{\phi}+3H\dot{\phi}-a^{-2}(t)\nabla^2\phi=-V'(\phi)-F_R \tag{3}$$

could be neglected. Here H is the Hubble expansion rate, $V'(\phi)=\partial V(\phi)/\partial\phi$, and F_R represents the random force. This approach is called the slow-rolling approximation. One purpose of this paper is to develop the stochastic approach to new inflation in a scenario in which the above approximations cannot be made and the full second-order differential equation must be considered. This problem has been addressed by Mazenko[14] who considered an N-dimensional vector model and studied the dynamics in the large-N limit.

The stochastic approach to inflation was applied to the slow-rollover transition in the new inflationary universe[10–13,18] and to chaotic inflation.[11,15–17] We shall apply this method to the preinflationary period of the model of Ref. 6. In our case the random force is due to classical thermal fluctuations as opposed to quantum fluctuations; hence, our analysis is entirely classical. Quantum-mechanical analyses of the slow-rolling phase transition in the new inflationary model have been presented in various papers.[19–21]

We hope that our method will also be useful for analyzing the global structure of the chaotic inflationary universe.[5,22,23] Chaotic inflation is at present the only concrete model which has a chance of explaining the homogeneity, isotropy, and flatness of the observed part of the Universe without too restrictive assumptions.

One way to obtain chaotic inflation is to consider a model with a scalar field ϕ in a potential $V(\phi)$ that has small mass term and coupling constant [$V(\phi)$ need not be a double-well potential]. At the initial time, which we take to be the Planck time t_{Pl}, the only constraint on the allowed values of $\phi(\mathbf{x})$ is given by demanding that the energy density in ϕ be smaller than the Planck density

$$V(\phi(\mathbf{x}))<\frac{\pi^2}{30}m_{\mathrm{Pl}}^4 , \tag{4}$$

where m_{Pl} is the Planck mass.

We consider a sphere in space of radius twice the horizon size in which both the spatial-gradient terms and the kinetic terms are negligible compared to $V(\phi)$:

$$\tfrac{1}{2}(\nabla\phi)^2 \ll V(\phi),\quad \tfrac{1}{2}\dot{\phi}^2 \ll V(\phi) \tag{5}$$

(the second assumption can in fact be relaxed[24]).

The energy density ρ and the pressure p of the scalar field ϕ are given by

$$\begin{aligned} \rho&=\tfrac{1}{2}\dot{\phi}^2+\tfrac{1}{2}(\nabla\phi)^2+V(\phi) ,\\ p&=\tfrac{1}{2}\dot{\phi}^2-\tfrac{1}{6}(\nabla\phi)^2-V(\phi) . \end{aligned} \tag{6}$$

Thus, with the above initial conditions, the local patch of the Universe will have $p\simeq-\rho$ and will hence inflate. Globally, however, the Universe will be chaotic, and our stochastic approach to inflation may be useful in quantifying the large-scale structure of the Universe.

The outline of this paper is as follows. In Sec. II we shall discuss the stochastic approach to inflation. We shall derive a differential equation for the probability distribution $P(\phi,\chi,t)$ and for a closed set of moments. We will solve the equations for the moments numerically and find bounds on the coupling constant between ϕ and the thermal bath below which the random force F_R due to thermal fluctuations will have a negligible effect on the dynamical evolution of ϕ. In Sec. III we will discuss a direct approach: solving the Klein-Gordon equation of motion (3) with an estimate of the effect of the random force F_R. Section IV contains our conclusions.

We shall be working in the context of a flat FRW universe with the metric given by (2). The Hubble expansion rate is $H(t)=\dot{a}(t)/a(t)$. We shall use units in which $\hbar=k_B=c=G=m_{\mathrm{Pl}}=1$.

II. TIME EVOLUTION OF THE PROBABILITY DISTRIBUTION

If a random force is allowed to influence the evolution of the scalar field ϕ, its evolution is no longer deterministic. The most we can calculate is the probability distribution $P(\phi,\chi,t)$. The goal of this section is to derive a differential equation for the time evolution of $P(\phi,\chi,t)$. We shall also derive a closed set of differential equations for a finite subset of moments of P. By solving these equations numerically we will demonstrate that the thermal fluctuations are negligible in the context of dynamical realization of new inflation, discussed in the

Introduction and in Ref. 6, provided the coupling constant describing the interaction between the scalar field ϕ and particles in the thermal bath is sufficiently small.

The starting point of the analysis is the equation of motion for ϕ, Eq. (3). In the usual approach, initial conditions are chosen such that $\phi(\mathbf{x})$ is homogeneous and slowly moving in the region of space being considered. In this case the slow rolling approximation which neglects $\ddot{\phi}$ may be self-consistent [whether or not it is depends on $V(\phi)$] and (3) reduces to a simple Langevin equation

$$\dot{\phi}=-\frac{1}{3H}V'(\phi)-\frac{1}{3H}F_R \;, \tag{7}$$

from which one can derive the Fokker-Planck equation for $P(\phi,t)$:[11-13]

$$\frac{\partial P}{\partial t}=\frac{\partial}{\partial\phi}\left[\frac{V'}{3H}P\right]+\frac{\partial^2}{\partial\phi^2}(DP) \;, \tag{8}$$

where the diffusion parameter D is determined from the random force F_R. To be more precise,

$$D=\frac{1}{2}\frac{d}{d(\delta t)}\langle\delta\phi^2(\delta t)\rangle \;, \tag{9}$$

where $\delta\phi(\delta t)$ is the perturbation of ϕ due to F_R over a time interval δt and the brackets denote expectation value with respect to the (random) measure of F_R.

We are interested in a completely different set of initial conditions for $\phi(\mathbf{x})$. Hence we also need a different approach to the stochastic analysis. We consider an initial scalar-field configuration with large spatial-gradient terms. An example is a plane-wave excitation,

$$\phi(\mathbf{x})=A\sin(\mathbf{k}\cdot\mathbf{x}) \;, \tag{10}$$

where the amplitude A is determined by the requirement that the maximal potential energy density of ϕ does not exceed the typical thermal energy density at the initial time t_i,

$$V(A)<\frac{\pi^2}{30}T_i^4 \;, \tag{11}$$

where T_i is the initial temperature.

The wave number k is constrained by the requirement that the spatial gradient energy density be smaller than the initial thermal energy density

$$A^2k^2<\frac{\pi^2}{30}T_i^4 \;. \tag{12}$$

Given the above initial conditions, the slow-rolling approximation is clearly inappropriate and inconsistent. $\ddot{\phi}$ and $\nabla^2\phi$ are crucial terms in the dynamical equations. However, we are interested in weakly coupled scalar fields which interact with the thermal radiation bath. In this case we can expand $\phi(\mathbf{x})$ into Fourier modes $\phi_{\mathbf{k}}(\mathbf{x})$, neglect the mode-mode coupling, and treat each $\phi_{\mathbf{k}}(\mathbf{x})$ independently. The differential equation for

$$\phi_{\mathbf{k}}(\mathbf{x})=\phi_{\mathbf{k}}\sin(\mathbf{k}\cdot\mathbf{x}+\alpha_{\mathbf{k}}) \;, \tag{13}$$

$\phi_{\mathbf{k}}$ being the amplitude and $\alpha_{\mathbf{k}}$ the phase, is second order in time, in contrast with the case usually considered.

We can write the equation of motion for $\phi_{\mathbf{k}}$ in first-order form. We define $\chi_{\mathbf{k}}\equiv\dot{\phi}_{\mathbf{k}}$. (For notational simplicity we shall omit the subscript $\mathbf{k}$.) Since neglecting the mode-mode coupling means neglecting the nonlinear terms in $V'(\phi)$, i.e., replacing $V'(\phi)$ by $m^2\phi$, we have the following two coupled first-order-in-time differential equations:

$$\dot{\phi}=\chi,\quad \dot{\chi}=-3H\chi-[a^{-2}(t)k^2+m^2]\phi \tag{14}$$

(without thermal-fluctuation terms).

Now we shall outline the derivation of the probability distribution $P(\phi,\chi,t)$ for the kth mode. The first step is to add thermal fluctuation terms $\delta\phi$ and $\delta\chi$ to (14):

$$\begin{aligned}&\phi(t+\delta t)=\phi(t)+\chi(t)\delta t+\delta\phi(\delta t)\;,\\&\chi(t+\delta t)=\chi(t)(1-3H\delta t)-F(k,m)\phi\delta t+\delta\chi(\delta t)\;,\end{aligned} \tag{15}$$

with $F(k,m)=(a^{-1}k)^2+m^2$. $\delta\phi$ and $\delta\chi$ are random variables with measures $P_1(\delta\phi)$ and $P_2(\delta\chi)$, respectively. $P(\phi,\chi,t+\delta t)$ can be obtained from $P(\phi,\chi,t)$ by using (15) and integrating over measures P_1 and P_2:

$$\begin{aligned}P(\phi,\chi,t+\delta t)=&\int d(\delta\phi)d(\delta\chi)P_1(\delta\phi)P_2(\delta\chi)\\&\times P(\tilde{\phi},\tilde{\chi},t)\frac{\partial\tilde{\phi}}{\partial\phi}\frac{\partial\tilde{\chi}}{\partial\chi}\end{aligned} \tag{16}$$

with

$$\begin{aligned}&\tilde{\phi}=\phi-\chi\delta t-\delta\phi\;,\\&\tilde{\chi}=\chi+3H\chi\delta t+F(k,m)\phi\delta t-\delta\chi\;.\end{aligned} \tag{17}$$

After some algebra we obtain

$$\begin{aligned}P(\phi,\chi,t+\delta t)=&P(\phi,\chi,t)-\delta t\frac{\partial}{\partial\phi}(\chi P)+3H\delta t\frac{\partial}{\partial\chi}(\chi P)\\&+\delta t\frac{\partial}{\partial\chi}[F(k,m)\phi P]\\&+\frac{1}{2}\frac{\partial}{\partial\phi}\left[\langle\delta\phi^2\rangle\frac{\partial P}{\partial\phi}\right]\\&+\frac{1}{2}\frac{\partial}{\partial\chi}\left[\langle\delta\chi^2\rangle\frac{\partial P}{\partial\chi}\right]\;,\end{aligned} \tag{18}$$

where on the right-hand side P stands for $P(\phi,\chi,t)$ and

$$\begin{aligned}&\langle\delta\phi^2\rangle=\int d(\delta\phi)P_1(\delta\phi)\delta\phi^2\;,\\&\langle\delta\chi^2\rangle=\int d(\delta\chi)P_2(\delta\chi)\delta\chi^2\;.\end{aligned} \tag{19}$$

Equation (18) can be rewritten as a differential equation for P:

$$\begin{aligned}\frac{\partial P}{\partial t}=&-\frac{\partial}{\partial\phi}\left[\chi P\right]+3H\frac{\partial}{\partial\chi}\left[\chi P\right]+\frac{\partial}{\partial\chi}[F(k,m)\phi P]\\&+\frac{1}{2}\frac{\partial}{\partial\phi}\left[\frac{\partial}{\partial(\delta t)}\langle\delta\phi^2\rangle\frac{\partial P}{\partial\phi}\right]\\&+\frac{1}{2}\frac{\partial}{\partial\chi}\left[\frac{\partial}{\partial(\delta t)}\langle\delta\chi^2\rangle\frac{\partial P}{\partial\chi}\right]\;.\end{aligned} \tag{20}$$

This is generalization of the Fokker-Planck equation (8).

In general we are only interested in the lowest moments of P. Hence we do not proceed by solving (20) numerically (which would be a difficult task), instead we derive a closed system of equations for the lowest moments, and solve the resulting system of coupled ordinary differential equations numerically. We shall consider the three quadratic moments

$$\begin{aligned} \langle \phi^2 \rangle &= \int d\phi\, d\chi\, P(\phi,\chi,t)\phi^2 , \\ \langle \phi\chi \rangle &= \int d\phi\, d\chi\, P(\phi,\chi,t)\phi\chi , \\ \langle \chi^2 \rangle &= \int d\phi\, d\chi\, P(\phi,\chi,t)\chi^2 . \end{aligned} \tag{21}$$

If $\langle \delta\phi^2 \rangle$ and $\langle \delta\chi^2 \rangle$ are independent of ϕ and χ, we get the following system of equations:

$$\begin{aligned} \frac{d}{dt}\langle \phi^2 \rangle &= 2\langle \phi\chi \rangle + \frac{\partial}{\partial(\delta t)}\langle \delta\phi^2(\delta t) \rangle , \\ \frac{d}{dt}\langle \phi\chi \rangle &= \langle \chi^2 \rangle - 3H\langle \phi\chi \rangle - F(k,m)\langle \phi^2 \rangle , \\ \frac{d}{dt}\langle \chi^2 \rangle &= -6H\langle \chi^2 \rangle - F(k,m)\langle \phi\chi \rangle + \frac{\partial}{\partial(\delta t)}\langle \delta\chi^2(\delta t) \rangle . \end{aligned} \tag{22}$$

If the scalar field is conformally coupled to gravity, i.e., $m^2 = \frac{1}{6}R$, where R is the Ricci scalar, then the system of differential equations above can be further simplified by introducing conformal time and field coordinates.[6] The conformal time τ is given by

$$d\tau = a^{-1}(t)dt . \tag{23}$$

The conformal field f is given by

$$f = a(t)\phi . \tag{24}$$

By writing out $df/d\tau$ in terms of the original variables, we find that the system of equations for the conformal moments

$$\begin{aligned} &A = a^2\phi^2, \quad B = a^3(\phi\chi + H\phi^2) , \\ &C = a^4(H\phi + \chi)^2 , \end{aligned} \tag{25}$$

becomes very simple:

$$\begin{aligned} \frac{dA}{d\tau} &= 2B + a^3 S , \\ \frac{dB}{d\tau} &= C - k^2 A + a^4 HS , \\ \frac{dC}{d\tau} &= -2k^2 B + a^5(H^2 S + \hat{S}) , \end{aligned} \tag{26}$$

with the thermal fluctuation source terms S and $\hat{S}$ given by

$$S = \frac{\partial}{\partial(\delta t)}\langle \delta\phi^2(\delta t) \rangle, \quad \hat{S} = \frac{\partial}{\partial(\delta t)}\langle \delta\chi^2(\delta t) \rangle . \tag{27}$$

Based on Eqs. (26) we can study the effects of thermal fluctuations on the evolution of the scalar field. We know[6] that the conformal field f will oscillate in conformal time with a fixed amplitude and frequency. Hence $\phi_{\mathbf{k}}(\mathbf{x})$ will be uniformly damped in time as $a^{-1}(t)$. If the initial spatial average of ϕ vanishes as can be argued by symmetry, then $\phi_{\mathbf{k}}(\mathbf{x})$ will uniformly relax to 0. This is the key to the dynamical realization of new inflation discussed in Ref. 6. This result also emerges from (26). With vanishing S and $\hat{S}$ we get

$$\frac{d^2B}{d\tau^2} = -4k^2 B , \tag{28}$$

and thus

$$B(\tau) = M\cos(2k\tau + \alpha) , \tag{29}$$

where M is the amplitude and α is a phase.

We have evaluated the moment equations (26) for a specific choice of thermal-fluctuation terms motivated by considering the coupling of ϕ to N scalar fields ψ_i which we assumed to be homogeneous.

The interaction Lagrangian is

$$\mathcal{L}_I = \tfrac{1}{2}\lambda\phi^2 \sum_{i=1}^{N} \psi_i^2 . \tag{30}$$

Incorporating this in the equation of motion leads to

$$\ddot{\phi} + 3H\dot{\phi} - a^{-2}(t)\nabla^2\phi = -\tfrac{1}{6}R\phi - \lambda N\phi\psi^2 . \tag{31}$$

In Fourier space the equation is identical after replacing ∇^2 with $-k^2$. Using the same argument as the one preceding (28) it follows that the random force term scales in time as $\ddot{\phi}$ and $a^{-2}\nabla^2\phi$, namely, as $a^{-3}(t)$.

Comparing (31) and (15) it follows that $\delta\phi = 0$ and $\delta\chi \sim \lambda N\phi\psi^2$. Hence $S = 0$. Since the ratio of the random force to the other forces is time independent, $a^5\hat{S}$ will be time independent. Its amplitude at the initial time t_0 (temperature T_0) can be obtained from (27):

$$\hat{S} = \lambda^2 N^2 \phi^2 \psi^4 \Delta\tau , \tag{32}$$

where $\Delta\tau$ is the typical interaction time. In terms of the number density n of one particle species and the interaction cross section $\sigma_I \sim \lambda l_{\text{phys}}^2$ (l_{phys} is the physical wavelength of the ϕ mode),

$$\Delta\tau \sim n^{-1}\sigma_I^{-1} . \tag{33}$$

From (32) and (33) it follows that $\hat{S} \sim a^{-5}$ as we showed above must be the case. The initial amplitude is

$$\hat{S}(t_0) \sim \lambda(Ak)^2 T_0 , \tag{34}$$

where A is the amplitude for ϕ at T_0.

We solved the system of moment equations (26) numerically, given the above choice of S and $\hat{S}$, i.e., $S(t) = 0$ and

$$\hat{S}(t) = \lambda(Ak)^2 T_0 \left[\frac{a(t)}{a(t_0)} \right]^{-5} .$$

We integrated starting at the Planck time t_{Pl} up to $T = T_c$ when inflation is expected to start. T_c is given by

$$V(0) = \frac{\pi^2}{30} N T_c^4 . \tag{35}$$

We considered a range of values for λ. Figure 2 presents results for $\lambda = 10^{-1}$ and $\sigma = 5\times 10^{-1}$. As can be seen in the plot, the thermal fluctuations have a negligible effect on the evolution of the moments. In Fig. 3 we show a three-dimensional plot where we vary λ through

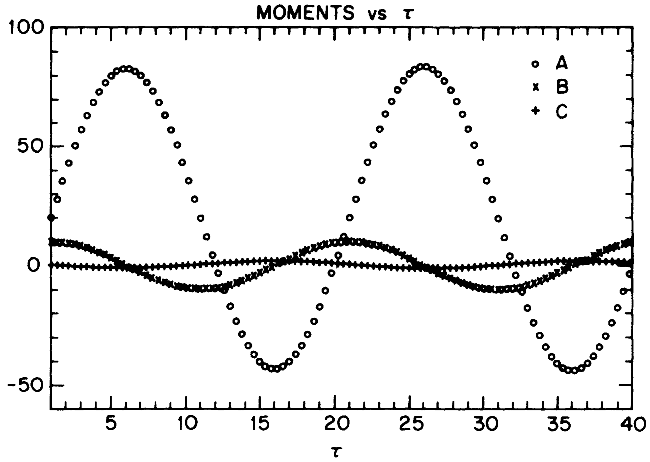

FIG. 2. Moments A,B,C, vs τ the conformal time, for $\lambda_\phi=10^{-2}$, $\lambda=10^{-1}$. The parameters are as given at the end of Sec. III.

its critical value to show the effect large λ has on the evolution of the moments. The critical value of λ is about 5×10^{-1}. As can be seen, for values of λ below λ_c thermal fluctuations are negligible. In Sec. III we shall give analytical arguments for the critical value of λ.

III. DIRECT SOLUTION OF THE EQUATIONS OF MOTION

An alternate way to describe the influence of thermal fluctuation terms on the equation of motion of ϕ is by performing direct estimates of the effect, based on the equation of motion. Such approaches will be described in detail in this section. We want to determine the critical value λ_c of the coupling constant λ, below which the effects of thermal fluctuations can be neglected. As in the previous section we shall take the interaction Lagrangian $\mathcal{L}_I$ to be given by

$$\mathcal{L}_I=\tfrac{1}{2}\lambda N\phi^2\psi^2 \ . \tag{36}$$

Again we assume that ϕ is weakly self-coupled so that $V'(\phi)=\partial V(\phi)/\partial\phi$ is negligible in the equations of motion. The conditions for this approximation to be valid were discussed at length in Refs. 6 and 7. For

$$V(\phi)=\lambda_\phi(\phi^2-\sigma^2)^2 \ , \tag{37}$$

the condition that $V'(\phi)$ does not prevent dynamical relaxation of ϕ is[7]

$$\lambda_\phi\simeq\sigma^2 \ . \tag{38}$$

The result for a Coleman-Weinberg potential[25] is similar.

It is quite easy to obtain a lower bound for λ_c. If the strength of the thermal force F_R is smaller than the Hubble damping force F_H for all temperatures above the critical temperature T_c, then thermal fluctuations will not prevent dynamical relaxation. Since

$$F_H\sim 3H^2\phi \tag{39}$$

and

$$F_R\sim\lambda N\phi\psi^2 \ , \tag{40}$$

the criterion for λ is

$$\lambda<3\left[\frac{H}{\psi}\right]^2 N^{-1}\sim 8\pi\psi^2 \ . \tag{41}$$

Since the ψ fields are assumed to be in thermal equilibrium, we get

$$\lambda<\frac{8\pi^2}{\sqrt{30}}T_c^2\sim\sigma^2 \ . \tag{42}$$

Thus the constraint on the coupling of ϕ to other fields is not any stronger than the constraint on the self-coupling of ϕ for the dynamical relaxation mechanism to work. Both constraints are weaker than the ones which follow[26]

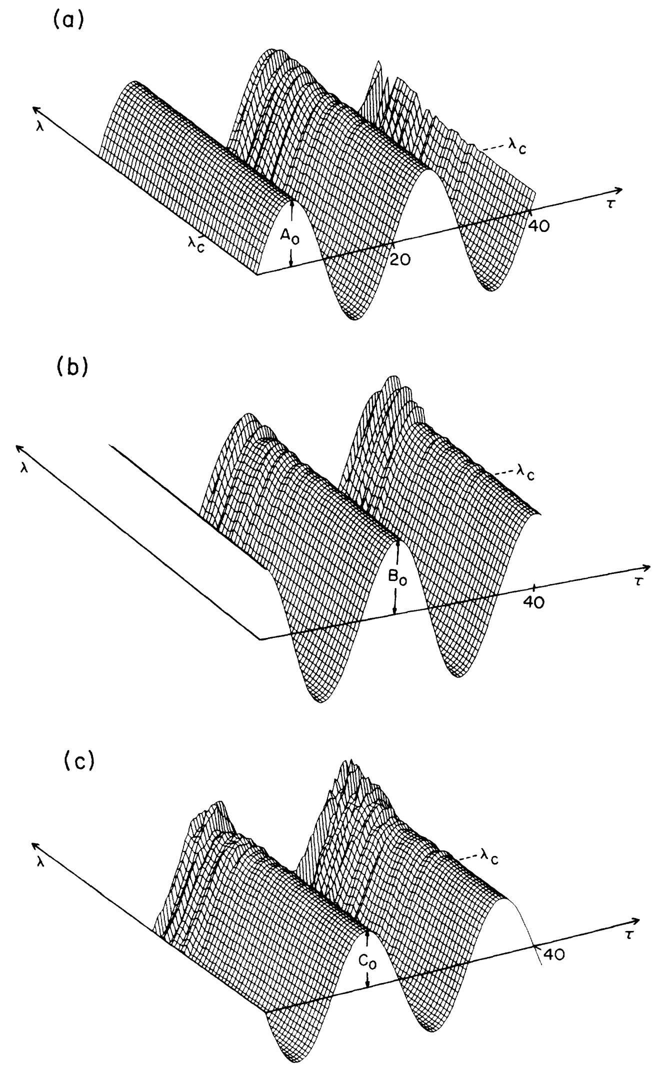

FIG. 3. A three-dimensional surface plot of the evolution of the moments A,B,C as a function of the conformal time τ as we vary λ through its critical value λ_c. Below λ_c we see no effect of the thermal fluctuation, whereas above we see clearly the contribution of the fluctuations. The parameters are as given at the end of Sec. III.

by demanding that the density perturbations not be too large.

The above is a severe underestimate of λ_c. Since F_R is a random force and thus not always opposing F_H, F_R can in fact be larger than F_H without preventing relaxation of ϕ. An improved estimate of λ_c can be obtained by a perturbative Green's-function method of the effects of F_R. We work with a conformally coupled free scalar field ϕ. In momentum space and conformal time, the equation of motion is

$$\phi'' = -\left[k^2 + \lambda N \left[\frac{\pi^2}{30}\lambda_\psi^{-1}\right]^{1/2} T_0^2\right]\phi \ , \tag{43}$$

where a prime stands for $d/d\tau$ and λ_ψ is a factor of order unity given by the initial amplitude of ψ:

$$\lambda_\psi \psi^4(T_0) = \frac{\pi^2}{30} T_0^4 \ . \tag{44}$$

From (43) we clearly get an upper bound on λ_c:

$$\lambda_c < N^{-1}\left[\frac{\pi^2}{30}\lambda_\psi^{-1}\right]^{-1/2}\left[\frac{k}{T_0}\right]^2 . \tag{45}$$

For larger λ the random force dominates and no dynamical relaxation is possible.

Now we shall treat the second term on the right-hand side of (43) as a small perturbation and evaluate its effect using a perturbative Green's-function method.[6] In terms of the fundamental solutions $\phi^{(1)}(\tau) = \sin(k\tau)$, $\phi^{(2)}(\tau) = \cos(k\tau)$, and the Wronskian $\epsilon(\tau) = k^{-1}$, the effect of the random force is

$$\phi^I(\tau) = -\phi^{(1)}(\tau)\int_0^\tau d\tau' \phi^{(2)}(\tau')\epsilon(\tau')I(\tau') + \phi^{(2)}(\tau)\int_0^\tau d\tau' \phi^{(1)}(\tau')\epsilon(\tau')I(\tau') \ , \tag{46}$$

where $I(\tau)$ is the source term

$$I(\tau) = \lambda N \left[\frac{\pi^2}{30}\lambda_\psi^{-1}\right]^{1/2} T_0^2 \phi^{(0)}(\tau) \equiv \mathcal{K} k \phi^{(0)}(\tau) \tag{47}$$

and $\phi^{(0)}(\tau) = A\sin(k\tau + \varphi)$ is the unperturbed solution. For time intervals smaller than the mean interaction time of ϕ with the thermal bath we get

$$\phi^I(\tau) \simeq A\mathcal{K}\tau \ . \tag{48}$$

Since the random force changes sign, (48) will not be true for large times. There will be a maximal conformal time $\hat{\tau}$ for which F_R is coherent. For $\tau > \hat{\tau}$ we must add up a contribution to $\phi^I(\tau)$ as a random walk, i.e.,

$$\phi^I(\tau) \simeq A\mathcal{K}\hat{\tau}\left[\frac{\tau}{\hat{\tau}}\right]^{1/2} . \tag{49}$$

We determine λ_c as the maximal value of λ for which $\phi^I(\tau) < A$ at the time when inflation would start, that is at $T = T_c$. The evaluation of this condition is straightforward, but depends in a crucial way on $\hat{\tau}$.

In order to be consistent with the analysis in Sec. II we choose the real time interval $\hat{t}$ corresponding to $\hat{\tau}$ to be [see (33)]

$$\hat{t} = n^{-1}\lambda^{-1} l_{\rm phys}^{-2} \sim a(t)\lambda^{-1} T_0^{-3} k_0^2 \ , \tag{50}$$

which implies a constant $\hat{\tau}$:

$$\hat{\tau} = \lambda^{-1} T_0^{-3} k_0^2 \ . \tag{51}$$

Inserting this into (49) and using the initial conditions $T_0 = k_0 = 1$ we obtain

$$\lambda_c \sim \sigma \ , \tag{52}$$

a significantly weaker condition than (42).

We have checked our results by numerical simulations. We solved the full nonlinear Klein-Gordon equation (3), including the double-well potential $\lambda_\phi(\phi^2 - \sigma^2)^2$ and the random force term ϵF_R where F_R is given by Eq. (40) and ϵ is a random number in the interval $[-1,1]$. Figure 4 shows the results for the particular choice $\lambda_\phi = 10^{-2}$ and $\sigma = 5\times 10^{-1}$. The initial scalar-field configuration was a plane wave with amplitude $A = \lambda_\phi^{-1/4}$ and wave number $k = \lambda_\phi^{1/4}$. The coupling strength to the thermal bath was $\lambda = 10^{-1}$ and we took $N = 1$. We let the system evolve up to $\tau = 40$. We see that for the length of this run the effects of thermal fluctuations are negligible, in agreement with our expectations. The numerical analysis will be described in detail in a subsequent paper[8] by one of us (H.A.F.). In Ref. 8 we shall also discuss the agreement between numerical and analytical results.

IV. CONCLUSIONS

We have studied the influence of thermal fluctuations on the evolution of the scalar field ϕ during the period of dynamical relaxation of ϕ before the onset of inflation. We conclude that provided the constant λ, which couples ϕ to the particles in the radiation bath, is smaller than a critical value λ_c, then the effect of thermal fluctuations is negligible. In a typical double-well $\lambda_\phi\phi^4$ model with a symmetry-breaking scale $\sigma \ll 1$, we get

$$\lambda_c \sim \sigma \ . \tag{53}$$

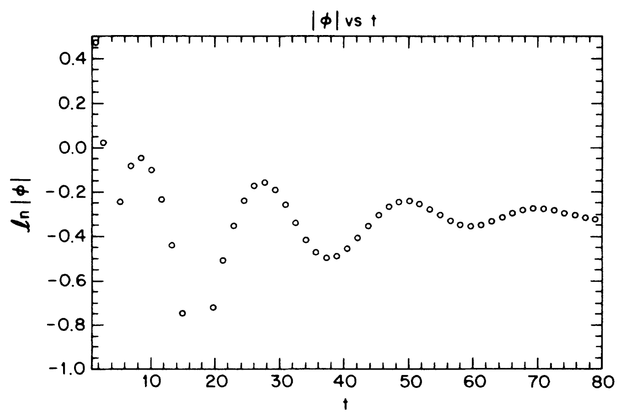

FIG. 4. The absolute value of the scalar field $|\phi|$ as a function of the time t. The time units are normalized to be increments of the initial Hubble constant H_0^{-1}. The parameters are as given at the end of Sec. III.

This constraint is weaker than the constraint on the self-coupling constant λ_ϕ required in order to achieve dynamical relaxation; it is much weaker than the constraints coming from the magnitude of the energy-density fluctuations.

Our results are based both on direct analytical estimates of the effects of the random-force terms in the equation of motion for ϕ and on the evolution of the equations for the lowest moments of the probability distribution $P(\phi,\dot{\phi},t)$. These moment equations are obtained by generalizing the usual stochastic approach to inflation which is only valid if ϕ is homogeneous and slowly moving. We also derived the differential equation for $P(\phi,\dot{\phi},t)$ in the presence of thermal excitations.

In a following paper[8] one of us (H.A.F.) will summarize a numerical analysis of thermal fluctuations in new inflation based on Sec. III. We have not attempted to solve the equation for $P(\phi,\dot{\phi},t)$ numerically; this is left for future work.

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