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E_8 family unification, mirror fermions, and new low-energy physics

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The group E_8 is investigated from the point of view of family unification. A Peccei-Quinn symmetry is used to protect the light fermions from acquiring a superlarge mass. It is found that if this protection is to be maintained without destroying perturbative unification *three* families are uniquely picked out with a family group $SU(3)_{fam} \subset E_6 \times SU(3)_{fam} \subset E_8$. A relation between the $(V-A)/(V+A)$ hierarchy and the hierarchy of M_{PQ}/M_{GUT} is found. It is also found that, in addition to mirror families, several exotic fermions characteristically remain light.

I. INTRODUCTION

There has been much interest in the group E_8 recently in the context of superstring theories. This group contains the subgroup $E_6 \times SU(3)$. In many versions of superstring theory this SU(3) becomes associated with the SU(3) holonomy group of a Calabi-Yau manifold in such a way that, after compactification, E_6 or some subgroup of it is left unbroken.[1] E_6 has long been regarded as an elegant group for grand unification,[2] particularly in the framework of supersymmetry (since the fermions and bosons can be in the same 27-dimensional representation). Now, interestingly, while the SU(3) factor is connected indirectly to the number of families that arise in superstring theories, there is no direct connection between the **3** of SU(3) and the apparent fact that there are 3 families. Nor do the families that arise belong to representations of SU(3). (Though in some orbifold compactifications this can happen.) In this paper we consider the older and more modest approach which is not based on superstrings, but seeks to explain the triplet of families more simplistically in terms of the group structure $E_8 \supset E_6 \times SU(3)$ where the SU(3) is regarded as a family group. E_8 has been proposed before for family unification,[3] but it is a bit surprising that closer attention has not been paid to it. There are a number of interesting facets of this idea which have not been fully explored.

In several ways the models proposed here are to be classed with attempts to unify the families with orthogonal groups. There are in fact five viable (and reasonably small) groups that allow all of the families to be unified in a single representation: O(14), SO(16), SO(18), E_7, and E_8. All of these groups predict the existence of mirror fermion families. This is unfortunate from two points of view. First, there is no technically natural or simple way to explain why the mirror $(V+A)$ fermion families should be heavier than the ordinary $V-A$ families. Second, one cannot combine mirror families and low-energy supersymmetry without destroying the asymptotic freedom of the non-Abelian gauge couplings and hence perturbative unification. Since low-energy supersymmetry may be necessary to solve the gauge hierarchy problem this is a very high price to pay. Nevertheless these groups are sufficiently interesting both theoretically and experimentally that they are worth investigating despite these serious objections. SO(18) has been most studied[4] because the unifying representation for the fermions is complex and hence the fermions are protected by SO(18) itself from developing a superlarge mass. Unfortunately, from the point of view of elegance, this theory has eight families (and eight mirrors). If one is to get down to three or four light families some rather complicated symmetry breaking has to occur. (Just the right subgroup of the gauge group must remain unbroken until low energies to protect four families. This is nontrivial.) SO(14) also has complex spinors but they give only two families (and two mirrors). In order to get enough fermions one must go to the group O(14) which has irreducible but real spinors that are twice as big.[5] On the other hand, one has then four (or fewer) light families as a prediction. SO(16) has been studied[6] little because its spinors are real so that one must introduce a global Peccei-Quinn-type symmetry to protect the light fermions. Like O(14) this group naturally predicts four or fewer light families and, to our view, more elegantly. The price of introducing a Peccei-Quinn symmetry is not very high if one considers that it may be used to solve the strong *CP* problem.

The group E_8 is most like SO(16) [indeed it has a maximal SO(16) subgroup]. The representations are real, requiring the introduction of a Peccei-Quinn-type symmetry. The fundamental representation (which is the same as the adjoint) is large enough to give even *five* families and, since E_8 has a maximal SO(16) subgroup, one can get *four* families in a group-theoretically simple way. However, as we shall show, if one is not to destroy the natural protection which the Peccei-Quinn symmetry affords the light fermions, the case of *three* light families is uniquely picked out. These three are in a triplet of the SU(3) family group arising from the branching $E_8 \supset E_6 \times SU(3)$. SU(3) has long been regarded as an obvious and desirable choice for a family group.[7] Unfortunately this ordinarily leads to anomalies. The price we have paid by accepting mirror fermions has freed us from this difficulty and allowed us to realize a gauged SU(3) family group.

There are five features of E_8 as an ordinary grand unified group that we regard as attractive. (1) It is in some sense a unique choice as being the largest of the exceptional series. (2) It predicts three families, unifies in a single irreducible representation all the light fermions, gives an SU(3) family group, and unifies this group. (3) It gives characteristic extra structure (beyond the mirror families) at low energy, as we shall see. (4) The simplest breaking pattern gives a gauge coupling at the unification scale of order unity, which seems to us to increase the technical naturalness of the model. (5) There is a relation between the Peccei-Quinn breaking scale and the $(V-A)$ to $(V+A)$ mass hierarchy: $(M_{\rm PQ}/M_{\rm GUT})\sim(M_{V-A}/M_{V+A})$. We feel that these features make the group E_8 the most interesting of the family unification groups.

II. PROTECTING THE LIGHT FERMIONS

The **248** of E_8 is real so that E_8 by itself will not forbid a mass term for the fermions, which we take to be in this representation. Therefore we impose a global U(1) (quasi)symmetry under which the **248** of fermions has a charge of $+1$. We could also impose some discrete symmetry for the same purpose, but the U(1) has the advantage of serving as a Peccei-Quinn symmetry. Now, it would be bad to have all the fermions in the **248** remain light as the perturbative unification would be destroyed. The question is whether it is possible that enough of the **248** become superheavy to retain perturbative unification while at the same time a remnant of $E_8\times U(1)_{\rm PQ}$ is left to protect the light fermions. The answer is that there is a unique way of doing this, and that therefore certain features of low-energy physics are predictable.

To analyze this question and give us a notation in which to discuss these models let us decompose the representations of E_8 under an SU(5) $\times U(1)\times SU(3)$ subgroup as follows:

$$E_8\supset E_6\times SU(3)_{\rm fam}\supset SO(10)\times U(1)_z\times SU(3)_{\rm fam}$$
$$\supset SU(5)\times U(1)_x\times U(1)_z\times SU(3)_{\rm fam}\ .$$

A representation which is an r of SU(5), an s of $SU(3)_{\rm fam}$, and with charge p and q of $U(1)_x$ and $U(1)_z$, respectively, we will denote $(r^{p,q},s)$. Then

$$\begin{aligned}\mathbf{248}\rightarrow&[(24^{0,0},1)+(10^{-4,0},1)_A+(\overline{10}^{4,0},1)_{\overline{A}}+(1^{0,0},1)_\alpha]\\&+[(10^{1,-3},1)_B+(\overline{5}^{3,-3},1)_b+(1^{5,-3},1)_\beta]+[(\overline{10}^{-1,3},1)_{\overline{B}}+(5^{3,3},1)_{\overline{b}}+(1^{-5,3},1)_{\overline{\beta}}]\\&+[(1^{0,0},1)_\delta]+[(1^{0,0},8)]+[(10^{1,1},3)_C+(\overline{5}^{-3,1},3)_c+(1^{5,1},3)_\gamma]+[(5^{-2,-2},3)_{\overline{d}}+(\overline{5}^{2,-2},3)_e]+[(1^{0,4},3)_\epsilon]\\&+[(\overline{10}^{-1,-1},\overline{3})_{\overline{C}}+(5^{3,-1},\overline{3})_{\overline{c}}+(1^{-5,-1},\overline{3})_{\overline{\gamma}}]+[(\overline{5}^{2,2},\overline{3})_d+(5^{-2,2},\overline{3})_{\overline{e}}]+[(1^{0,-4},\overline{3})_{\overline{\epsilon}}]\ .\end{aligned}$$

The representations in the square brackets come from irreducible representations of SO(10). Note that the SU(3) singlets make up a **78** of E_6, the triplets a **27**, and the antitriplets a $\overline{\mathbf{27}}$. We denote, for convenience, certain representations by letters which appear as subscripts above. We emphasize that we are not breaking the group E_8 down to SU(5) $\times U(1)\times U(1)\times SU(3)$ but only classifying particles under this subgroup.

Suppose that the $(24^{0,0},1)$ acquires a superlarge mass with itself. That is, there is a mass term of the form $(24^{0,0},1)\times(24^{0,0},1)$. The relevant Higgs boson has PQ charge (-2) and gauge quantum numbers $(r^{0,0},1)$ where $r\subset 24_s^2=\mathbf{1}+\mathbf{24}+\mathbf{75}+\mathbf{200}$. It does not appear that there would remain in this case any symmetry to protect any of the fermions against acquiring a superlarge mass. In the first place, $U(1)_{\rm PQ}$ is broken, leaving $U(1)_x\times U(1)_z\times SU(3)_{\rm fam}$. But as the fermions are real under this unbroken group no protection is afforded by it. Nor do we see any other symmetry argument, such as that we shall use below for the case of the $E_8\rightarrow E_7$-breaking pattern, which can guarantee the lightness of any of the fermions. We must reject this possibility therefore. Then let us consider making some of the $\mathbf{10}+\overline{\mathbf{10}}$ fermions heavy. Using the notation described above, we can rule out superheavy masses of the form $\overline{A}A$, $\overline{B}B$, and $\overline{C}C$ on the same grounds as we did a superheavy mass for the (24,1). There remain then only three cases to consider, namely, superheavy masses of the form $\overline{A}B$, $\overline{A}C$, or $\overline{B}C$. (Clearly $\overline{B}A$, $\overline{C}A$, and $\overline{C}B$ are the same.) If $\overline{A}B$ and $\overline{B}A$ superlarge masses exist, the relevant Higgs fields will have a PQ charge of -2 and gauge quantum numbers $(r^{-5,3},1)$ and $(r^{5,-3},1)$, where $r\subset\mathbf{1}+\mathbf{24}+\mathbf{75}$. Then the following mass terms are allowed to form by the unbroken subgroup of $E_8\times U(1)_{\rm PQ}$: $\overline{A}B$, $\overline{B}A$, $c\overline{e}$, $\overline{c}e$, and the singlet masses $\alpha\beta$, $\alpha\overline{\beta}$, $\delta\beta$, $\delta\overline{\beta}$, $\gamma\overline{\epsilon}$, and $\overline{\gamma}\epsilon$. This leaves light the $C\equiv(10^{1,1},3)$, $\overline{C}\equiv(\overline{10}^{-1,-1},\overline{3})$, $d\equiv(\overline{5}^{2,2},\overline{3})$, and $\overline{d}\equiv(5^{-2,-2},3)$, which make up three families and three mirror families, together with the $(24^{0,0},1)$, $(1^{0,0},8)$, $b\equiv(\overline{5}^{-3,-3},1)$, $\overline{b}\equiv(5^{3,3},1)$, and one singlet. All together 133 fermions remain light. The vacuum expectation value (VEV) of the Higgs fields in the $(1^{-5,3},1)$ and $(1^{5,-3},1)$ will break E_8 to E_7. The adjoint of E_7 is 133 dimensional. One can understand the low-energy fermion spectrum simply in terms of a decomposition of the representations of E_7 under the subgroups: $E_7\supset SU(6)\times SU(3)\supset SU(5)\times U(1)\times SU(3)$. Under this

$$\begin{aligned}\mathbf{133}\rightarrow&(15,3)+(\overline{15},\overline{3})+(35,1)+(1,8)\\\rightarrow&(10,3)+(5,3)+(\overline{10},\overline{3})\\&+(\overline{5},\overline{3})+(24,1)+(5,1)+(\overline{5},1)+(1,1)+(1,8)\ .\end{aligned}$$

(We should remark here that there are models based on this group breaking in the literature.[8]) Now, it is easily shown in the other two cases we mentioned, where there is a superheavy mass of the form $\overline{A}C$ or $\overline{B}C$, that E_8 is also broken to an E_7 subgroup and 133 fermions remain light. Group theoretically these are all equivalent under a group automorphism (though they appear different here since our notation is based upon a particular decomposition of E_8). Suppose that *no* $\mathbf{10}+\overline{\mathbf{10}}$ pairs become superheavy. Then we will have at least **154** light states [of SU(5): a **24**, five $\mathbf{10}+\overline{\mathbf{10}}$ pairs, and at least three (for three families) $\mathbf{5}+\overline{\mathbf{5}}$ pairs], with a group index of at least 46. This is like having 23 flavors of light quark which would completely destroy the perturbative unification of the theory. Hence the only viable possibility is the breaking we have discussed which leaves the **133** (adjoint of E_7) of fermions light. The index of these light fermions is **36** which is equivalent to having 18 light families, to be compared to the critical number for the asymptotic freedom of SU(3) color of $16\frac{1}{2}$. So above the scale (presumably roughly in the 100 GeV–TeV range) where the extra light fermions are, the non-Abelian gauge couplings start growing again, but very slowly. They will become of order unity only at or near the unification scale. We regard this as a happy accident, since it is in line with the "naturalness" criterion that the fundamental gauge couplings be of order unity.

Notice that while there are enough fermions in the **248** to give even *five* families plus their mirrors, the above considerations uniquely pick out *three*. Furthermore these families are in representations of an SU(3) $\subset E_8$. Thus we conclude that the phenomenologically interesting decomposition of E_8 is not into SU(5) $\times$SU(5), or SO(10) $\times$SU(4), but into $E_6\times SU(3)_{\text{fam}}$.

Let us see in detail how the 133 light fermions are protected in the breaking. If Higgs fields with PQ charge (-2) and gauge quantum numbers $(1^{5,-3},1)$ and $(1^{-5,3},1)$ acquire VEV's, they will break $U(1)_x \times U(1)_z \times U(1)_{PQ}$ down to a gauge group $U(1)'$, whose charge, Q', is given by $Q'=\frac{1}{4}(3Q_x+5Q_z)$. Now consider the gauge group $U(1)''$ orthogonal to $U(1)'$ which is generated by $Q''=\frac{1}{4}(Q_x-Q_z)$. [It is orthogonal because $\operatorname{Tr}_{248}(Q_x)^2/\operatorname{Tr}_{248}(Q_z)^2=\frac{5}{3}$.] The charge Q'' of all 133 light fermions is zero, the charges of all the SU(5)-nonsinglet superheavy fermions is odd, and the charges of the Higgs fields that get expectation values are ± 2 [for the $\Omega\equiv(1^{5,-3},1)$ and the $\Omega'\equiv(1^{-5,+3},1)$]. Clearly the only way to get a superheavy mass term that connects a light fermion to a light fermion is to have a product of VEV's which has vanishing Q''. Such a combination would be $(\Omega)^m(\Omega)^{\dagger n}(\Omega')^p(\Omega')^{\dagger q}$ where $m-n-p+q=0$. This has PQ charge $2(m-n+p-q)=4(m-n)$. This is a multiple of 4, while in order to couple to a fermion bilinear it must be -2. (If there were another Higgs fields with $Q_{PQ}=-2$, that coupled to the fermions, and had a component with $Q''=0$ which received a superlarge VEV, then such mass terms would become possible at higher orders. But there is no need to introduce such a Higgs boson.) Next, it is even easier to see that no light fermion to heavy fermion mass terms involving only Ω and Ω' are possible since these Higgs fields have even Q'' and such a fermion bilinear [for SU(5)-nonsinglet fields] has odd Q''. So it is clear that symmetry protects the light fermions from getting a superlarge mass to any order. This is so even though there remains no unbroken continuous symmetry below the grand-unified-theory (GUT) scale under which the fermions are in a complex representation. This raises the question again of whether the $(24^{0,0},1)$ can get a superlarge mass with itself without destroying the symmetry protection of the other fermions. We do not see any way for this to happen.

III. THE STAGES OF SYMMETRY BREAKING

So far we have focused only on the fermion masses. We can use a **3875** of the Higgs fields to give the superlarge masses to the 115 unwanted fermions leaving the 133 remaining fermions light. This Higgs field must have, as noted, $Q_{PQ}=-2$. Alone this Higgs field would break E_8 only to E_7, whereas we would like to have E_8 broken to a smaller subgroup than that at superlarge energies. This can be achieved easily by Higgs fields that have $Q_{PQ}=0$ and which, therefore, do not couple directly to fermions. In particular, an adjoint Higgs field (**248**) would be the simplest choice. This can break the group all the way down to $[SU(3)_c\times SU(2)_L\times U(1)_y] \times U(1)'\times SU(3)_{\text{fam}}$ at the GUT scale, where the bracketed group is that of the standard model. These are the only superlarge breakings required. As we shall see later, however, one can obtain interesting and desirable patterns of light fermion masses if there is slightly more structure at high energies. At low energies we need two levels of breaking. First, the "extra" light fermions (24,1), (5,1), $(\overline{5},1)$, and (1,8) must be given large enough masses [which need not break SU(2) $\times$U(1)]. Second, SU(2) $\times$U(1) breaking must occur and the three light families and their mirrors must get mass. How large are the (24,1), (5,1), $(\overline{5},1)$, and (1,8) masses? One cannot, of course, say precisely. We have already argued, however, that if these fermions get superheavy masses there is no symmetry protection against large mass terms connecting ordinary fermions to their mirror partners, which could arise at higher order. Indeed if these "extra fermions" get mass from a Higgs boson in a "small" representation of E_8 (there are few such) at the tree level, then also at the tree level the ordinary and mirror fermions get large SU(2) $\times$U(1)-singlet masses, which is obviously unacceptable. One can show, however, that there is no group-theoretical objection, fundamentally, to having substantial (e.g., 100 GeV–1 TeV) masses for the (24,1), (5,1), $(\overline{5},1)$, and (1,8) and very small singlet masses between the ordinary and mirror fermions. This is because there are components in the **27 000** which couple at the tree level to give the former but not the latter. [We are thinking of components 1(1(2430(27 000))), 1(210(2430(27 000))) and 24(210(2430(27 000))) where we are denoting them by their SU(5) (SO(10) (E_6(E_8))) quantum numbers.] Doubtless there are more elegant ways to achieve this than introducing such a huge representation (even if it is the third smallest irrep of E_8). We envisage these masses arising in some way in higher order from products of smaller representations. This is an important issue which nevertheless we will not pursue further here. A similar and more difficult problem is the hierarchy between ordinary $(V-A)$ families and mirror $(V+A)$ families. This is a problem shared by the orthogonal group models, SO(18), SO(16), and O(14). In Sec. IV we will discuss some tentative but attractive ideas for dealing with this problem.

IV. THE $(V-A)/(V+A)$ HIERARCHY, THE PECCEI-QUINN SCALE, AND THE FAMILY HIERARCHY

In all groups that give mirror fermions, SO(18), SO(16), O(14), E_7, and E_8, there is a tendency for the $(V-A)$ and $(V+A)$ families to have the same masses since these fermions can be mapped into each other by a group automorphism. In all of these groups[4–6] this degeneracy can be lifted. But in none of them can a large hierarchy between the $(V-A)$ and $(V+A)$ masses be achieved in a technically natural way.[5,9] It always arises as the result of some other hierarchy of mass scales which is itself not technically natural. We will proceed now to show that in a simple treatment of this problem in E_8, the ratios M_{V-A}/M_{V+A} are suppressed by the ratio M_{PQ}/M_{GUT}, where M_{PQ} is the scale at which the Peccei-Quinn symmetry breaks ($\lesssim 10^{12}$ GeV) and M_{GUT} is scale at which the largest fermion masses arise. We use a version of Dimopoulos's idea[10] for explaining mass hierarchies among light fermions at the tree level from hierarchies among superheavy scales.

The superheavy fermion masses, as we have noted above, can come from components of a **3875** that transform as $(1^{5,-3},1)$ and $(1^{-5,3},1)$. Let us also suppose that the components of the **3875** that transform as $(1^{-5,-1},\bar{3})$ and $(1^{5,1},3)$ acquire large VEV's. [However we do not allow those components that transform as $(1^{0,0},1)$ or $(1^{0,4},3)$ or their conjugates to develop large VEV's. This is consistent with stability requirements.] Moreover, let us assume that the VEV's of these $SU(3)_{fam}$ triplet-Higgs fields are all parallel in SU(3); that is, that they break SU(3) down only to $SU(2)_{fam}$. This will give an interesting pattern of masses for the three families. It is also noteworthy that it gives the Ramond family group[11] SU(2) with the three families in a 2+1.

Now let us introduce a hierarchy among these large VEV's. Suppose $\langle(1^{-5,3},1)\rangle$ is of order M_{GUT}, while $\langle(1^{5,-3},1)\rangle$, $\langle(1^{5,1},3)\rangle$, and $\langle(1^{-5,-1},\bar{3})\rangle$ are of order $M_{PQ} \ll M_{GUT}$. [Remember that the **3875** of the Higgs field, having $Q_{PQ}=-2$, is *complex* so that, in general, $\langle(1^{5,-3},1)\rangle \neq \langle(1^{-5,3},1)\rangle^*$. See the Appendix where it is shown that such a hierarchy is easy, if not technically, natural to implement.] At M_{GUT}, $U(1)'\times U(1)''\times U(1)_{PQ}$ breaks down to $U(1)'\times\tilde{U}(1)_{PQ}$, where $\tilde{U}(1)_{PQ}$ is a global (quasi)symmetry generated by $(Q''-Q_{PQ})$. At this stage exactly half of the SU(5)-nonsinglet superheavy fermions get their masses, namely, $\bar{A}$, B, $\bar{c}$, and e, but *not* A, $\bar{B}$, c, and $\bar{e}$. At M_{PQ}, when $(1^{+5,-3},1)$ acquires a VEV, the Peccei-Quinn symmetry $\tilde{U}(1)_{PQ}$ breaks completely (except for possible discrete remnants). Therefore $10^9 \lesssim M_{PQ} \lesssim 10^{12}$ GeV. Moreover, the rest of the superheavy fermions then pick up their masses, namely, A, $\bar{B}$, c, and $\bar{e}$. The role of $O(M_{PQ})$ $SU(3)_{fam}$-triplet Higgs VEV's is to mix $SU(3)_{fam}$-singlet and -triplet fermions, breaking $SU(3)_{fam}$, and allowing some interesting structure to emerge at low energy. The $SU(2)\times U(1)$-breaking masses arise from an H, which is a mixture of $(5^{-2,2},\bar{3})$ and $(5^{3,-1},\bar{3})$, and an H' which is a mixture of $(\bar{5}^{2,-2},3)$ and $(\bar{5}^{-3,1},3)$. Each gets a VEV of the order of the weak scale.

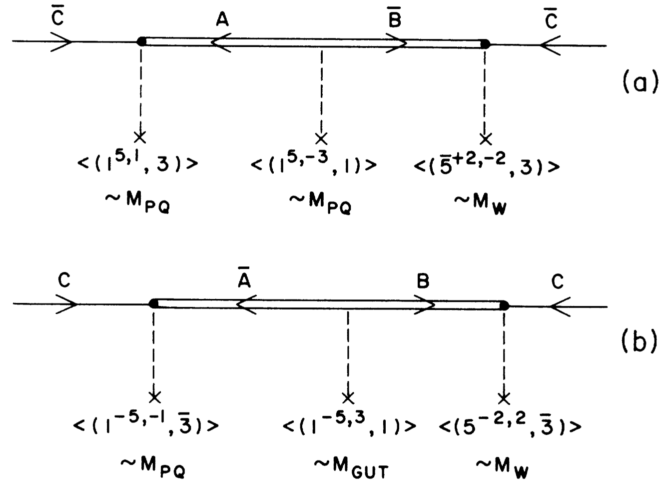

FIG. 1. Tree-level diagrams showing mass terms for (a) charge $\frac{2}{3}$ mirror quarks and (b) ordinary quarks. The letters A, C, etc., refer to fermion representations (see the text). The ordinary quark mass is of order $(M_{PQ}/M_{GUT})M_W$ since the mass of $\bar{A}$, B is of order M_{GUT}. The mirror quark mass is of order M_W since the mass of A , $\bar{B}$ is of order M_{PQ}.

Figures 1 and 2 show how the light fermion masses arise. Note that the charge $\frac{2}{3}$ quark masses which arise from Fig. 1(b) are of order $(M_{PQ}/M_{GUT})M_W$ whereas the *mirror* charge $+\frac{2}{3}$ quark masses from Fig. 1(a) are of

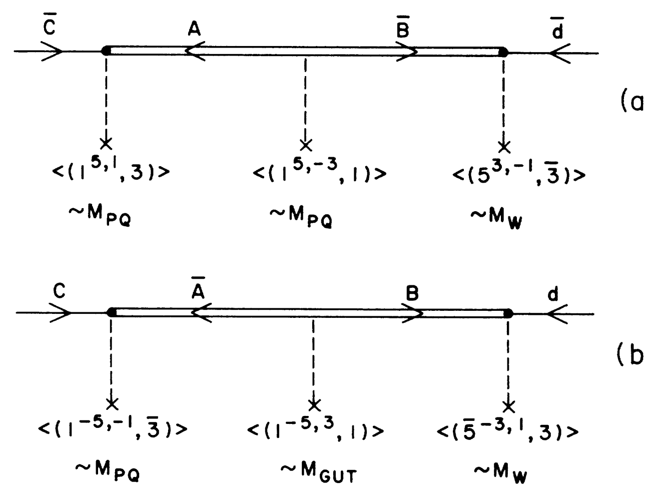

FIG. 2. Diagrams analogous to Fig. 1 but for charge $-\frac{1}{3}$ quarks and charged leptons. (a) and (b) contribute to mirror and ordinary fermions, respectively.

order $(M_{\rm PQ}/M_{\rm PQ})M_W = M_W$. This gives the rough relation $M_{V-A}/M_{V+A} \sim M_{\rm PQ}/M_{\rm GUT}$. The diagrams of Fig. 2 give the same result for charge $-\frac{1}{3}$ quarks and charged leptons. We find this very appealing. The two most puzzling features of the model, that the fermions do not all get superheavy mass and that there is a $(V-A)/(V+A)$ hierarchy, are accounted for by two puzzling features that were already needed in axion models: the existence of a Peccei-Quinn symmetry and the fact that it must be broken at an intermediate scale.

Notice that our account of the fermion masses has an interesting bonus. The diagrams of Figs. 1 and 2, together with the assumption that the $SU(3)_{\rm fam}$ triplet breaks $SU(3)_{\rm fam} \to SU(2)_{\rm fam}$, imply that one of the families [the $SU(2)_{\rm fam}$-singlet one] is heavier than the others. This is also a realization of the kind of idea explored by Ramond.[11] There are also diagrams which give tree-level masses to the d, s, e, and μ, which we have not shown. Undoubtedly the story of the light fermion masses has more to it than told here, but we have already gotten more specific about the fine details of models than perhaps we should. There is an apparent difficulty, though it turns out not to be real, with the kind of mechanism for obtaining structure in the light fermion masses that we illustrate in Figs. 1 and 2. Since all fermions belong to the same representation of E_8 there is inevitably present in the model a component of a Higgs boson which can give any possible mass term. In particular, we can close the fermion lines in Figs. 1 and 2 by coupling to a Higgs boson. That Higgs boson manifestly must develop a nonvanishing VEV since by closing the fermion line we construct a tadpole term for it. How big is that VEV? If the (mass)^2 of this Higgs component (call it ϕ) is $O(M_{\rm GUT}^2)$ then we have a potential of the form $O(M_{\rm GUT}^2)\phi^2 + O(M_W M_{\rm PQ}\tilde{M})\phi$, where $\tilde{M}$ is $M_{\rm GUT}$ or $M_{\rm PQ}$. Then $\langle\phi\rangle \sim M_W(M_{\rm PQ}/M_{\rm GUT})(\tilde{M}/M_{\rm GUT})$. This is not large enough to disturb the $(V-A)/(V+A)$ hierarchy.

There is a final potential difficulty with the type of E_8 model we have been describing, and that pertains to neutrino masses. In order not to conflict with astrophysical bounds[12] from He abundance we must have three or at most four light neutrinos. Among the 133 light fermions there are eight SU(2)-doublet neutrinos and ten SU(2)-singlet neutrinos. The eight doublet neutrinos are in the three families $(\bar{5},\bar{3})$, and three mirror families (5,3) and the extra $(5,1)+(\bar{5},1)$ of fermions. The ten singlet neutrinos are in a (1,1), the (1,8), the (24,1). Naively it would seem that, with all these singlet neutrinos with weak-scale masses, one would have no difficulty in making as many neutrinos as necessary heavy enough to escape the bounds from ^{4}He abundance. Unfortunately Dirac masses for these neutrinos do not arise at the tree level from $SU(2) \times U(1)$-breaking Higgs bosons H and H'. While it is certainly *possible* to "arrange" for large dirac neutrino masses (one might introduce some E_8-singlet fermions to play the role of the singlet neutrinos) they do not seem to emerge in a particularly simple or elegant way. In particular, it is not obvious why the e, μ, and τ neutrinos should be relatively light while the neutrinos of the mirror families should be much heavier. Clearly there is much more work to be done on the mass spectrum of the light fermions. The set of such fermions is rich and there is a lot of structure to their masses. The main focus of this paper has been on three "grosser" features of the light fermion spectrum: *which* set of fermions can remain light in a technically natural way, the family structure, and the $(V-A)/(V+A)$ hierarchy.

Finally we should point out one more interesting area of potential low-energy phenomenology. We saw above that the breaking that gave 115 of the 248 fermions superlarge mass also broke the rank of the group by one from E_8 to E_7. The extra *three* units of rank comprise *two* for $SU(3)_{\rm fam}$ group and *one* for the $U(1)'$ group. In the particular scheme we discussed in this section we broke $SU(3)_{\rm fam}$ down to $SU(2)_{\rm fam}$ at large energies. This leaves not $SU(3)_{\rm fam} \times U(1)'$ but $SU(2)_{\rm fam} \times \tilde{U}(1)'$, where $\tilde{U}(1)'$ is a mixture of $U(1)'$ and the "hypercharge" U(1) of $SU(3)_{\rm fam}$. In any event there is no reason to break the extra U(1) gauge group [$U(1)'$ or $\tilde{U}(1)'$] at very high scales. It is quite reasonable that it could remain at low enough energies to give interesting extra Z^0 phenomenology. Notice that the family-independent part of this U(1), namely, $U(1)'$, is uniquely picked out by the considerations of Sec. II.

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APPENDIX

In order to break the PQ symmetry at a much lower scale than the GUT scale it is necessary to have $\langle\Omega\rangle \gg \langle\Omega^*\rangle$, where Ω^* and Ω have opposite gauge quantum numbers, but are not just complex conjugates of each other since they are in a *complex* **248**. To show that this kind of hierarchy is not difficult to achieve (though it is not technically natural) let us examine a simple analogue. Consider an SO(3) model with a complex 3 of the Higgs field: $\Omega^+ = \Omega_1 + i\Omega_2 \equiv a$, $\Omega^- = \Omega_1 - i\Omega_2 \equiv b$, $\Omega_3 \equiv (1/\sqrt{2})c$. There are two SO(3)-invariant quadratic combinations $\sum_{j=1}^3 \Omega_j^* \Omega_j$ and $\sum_{j=1}^3 \Omega_j \Omega_j$; the first being a real quantity and the second complex: $\sum_j \Omega_j^* \Omega_j = \frac{1}{2}(|a|^2 + |b|^2 + |c|^2)$ and $\sum_j \Omega_j \Omega_j = ab + \frac{1}{2}c^2$. The potential will depend only on these combinations and will thus determine three real quantities. Altogether a, b, and c which are complex have six real quantities. So three quantities are undetermined. This is just three-parameter degeneracy due to the SO(3) symmetry of the minimum. Really all the minima are equivalent. Consider

$$V = \lambda \left| \sum_j \Omega_j^* \Omega_j - \frac{\mu^2}{2\lambda} \right|^2 + \lambda' \left| \sum_j \Omega_j \Omega_j \right|^2 .$$

This is clearly minimized when $|a|^2 + |b|^2 + |c|^2 = (\mu^2/\lambda)$ and $ab + \frac{1}{2}c^2 = 0$. One solution is

$(\lambda/\mu^2)^{1/2}(a,b,c) = (1,0,0)$ which gives $\langle \Omega^+ \rangle \neq 0$ and $\langle \Omega^- \rangle = 0$. Obviously a slight perturbation on this potential can make $\langle \Omega^- \rangle$ nonzero but still small compared to $\langle \Omega^+ \rangle$. This can also be written $(\lambda/\mu^2)^{1/2}(\Omega_1,\Omega_2,\Omega_3) = (\frac{1}{2}, -i/2, 0)$. One can see the equivalence of the other minima easily. Just to illustrate, consider $(\lambda/\mu^2)^{1/2}(a,b,c) = (\frac{1}{2},\frac{1}{2},-i/\sqrt{2})$ which also minimizes the potential. This gives $(\lambda/\mu^2)^{1/2}(\Omega_1,\Omega_2,\Omega_3) = (\frac{1}{2}, 0, -i/2)$, which is the same as the previous case under a rotation in group space. Thus, perhaps surprisingly, there is no tendency for Ω^+ and Ω^- to have the same magnitude even in a very simple case. One expects the same to hold for E_8.

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