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Nucleon distribution amplitudes from a relativistic quark model

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We show that the essential features of the Chernyak-Zhitnitsky distribution amplitudes for the nucleon can be understood with the basic concepts of the constituent quark model formulated in the light-cone Fock approach. The key ingredient of the model is the relativistic nonstatic spin wave function together with the fact that quarks bound in light hadrons are highly relativistic (small-valence-size hypothesis). The picture is consistent with that of an analogous study for the $q\bar{q}$ system.

A basic part of perturbative quantum chromodynamics (QCD) predictions are the hadronic wave functions which describe the hadron in terms of its constituent quarks and gluons. In the case of exclusive processes at large momentum transfer via the factorization theorem,[1] it is possible to separate the short-distance dynamics governed by perturbative QCD from the soft nonperturbative contributions responsible for quark and gluon confinement at large distances. All binding effects from soft, low-momentum-transfer interactions are summarized by the valence-quark distribution amplitudes

$$\phi(x_i,Q)=\int [d^2k_\perp]\psi(x_i,\mathbf{k}_{\perp i})\theta(\mathbf{k}_{\perp i}^2 \lesssim Q^2) \ , \qquad (1)$$

which are computed from the valence wave function of the hadron at equal time $x^+=x^0+x^3$ on the light cone and gives the probability amplitude for the valence quarks with light-cone fractions $x_i=p_i^+/p^+=(p_i^0+p_i^3)/(p^0+p^3)$ to combine into the hadron with relative momenta $\mathbf{k}_{\perp i}$ up to the scale Q^2.

In the past few years, quite substantial progress has been achieved in nonperturbative methods for calculation of hadron properties. Suggestions have been made for the form of the distribution amplitudes of mesons and nucleons on the basis of the method of QCD sum rules[2-4] and from lattice QCD calculations.[5] The following surprising picture of the longitudinal-momentum-space structure of the hadrons has emerged. There is a large asymmetry between quarks, i.e., for both the pion and nucleon a large part of the momentum is carried by one quark. The valence quarks in light hadrons are highly relativistic. All longitudinal-momentum distributions are broad and very different from the nonrelativistic δ-function form centered at $x_i=\frac{1}{3}$. At the same time attempts have been made to calculate distribution amplitudes directly from hadron momentum-space wave functions and to understand at least qualitatively the physics behind the observed features of amplitudes calculated from QCD sum rules and lattice techniques. With the basic concepts of the constituent quark model formulated in the light-cone approach we presented a relativistic model of the pion valence-quark wave function and found that a nonstatic relativistic wave function and the small size of the valence configuration are essential to reproduce the basic features of the Chernyak-Zhitnitsky amplitude for the pion.

It also remains to demonstrate that the same physical idea will succeed in describing the nucleon distribution amplitude, thereby unifying mesons and baryons in a single consistent relativistic framework. The demonstration of this fact is the main subject of this paper.

Our program for the nucleon invokes essentially the same basic phenomenlogical constraints from the quark model as the approach of Ref. 6. Details are given in Ref. 7. We nevertheless briefly describe the basic concepts again here for completeness. (i) Nucleon states are dominated by the valence-quark configuration with typical constituent quark masses, $m\approx 330$ MeV. (ii) The valence component is a system with substantial relativistic motion described, for analytical simplicity, by the Gaussian momentum-space wave function

$$\phi^N(x_i,\mathbf{k}_{\perp i})=A\exp\left[\frac{1}{6\alpha^2}\left(m_N^2-\sum_{i=1}^{3}(\mathbf{k}_{\perp i}^2+m_i^2)/x_i\right)\right] . \qquad (2)$$

The Gaussian parameter α is determined by the value of the average quark transverse momentum, viz., $\alpha^2\approx\langle k_\perp^2\rangle$. The QCD sum-rule implication that the hadron valence-quark wave functions are broad in longitudinal momentum also suggests a broad transverse-momentum distribution. Moreover, there are suggestions made by Brodsky, Huang, and Lepage[8] that the proton transverse size is even smaller than that of the pion. In particular, a small size for the proton valence wave function, $R_{qqq}\approx 0.2$–0.3 fm, would correspond to the Gaussian parameter $\alpha\approx 660$–1000 MeV. (iii) The three-quark valence system in the nucleon is an interacting particle state with the standard quark-model spin-parity and isospin assignments. To determine the relativistic spin wave function we need some approximation to deal with the problem of the angular momentum in the light-cone dynamics. For free spin-$\frac{1}{2}$ constituents, the one-particle instant (equal-t) and light-cone (equal-x^+) states are related by a unitary transformation called the Melosh transformation.[9] We use the relation together with a light-cone analog of the mock-hadron method by Isgur.[10] Namely, we assume that the $S=\frac{1}{2}$ spin wave function is constructed by the Clebsch-Gordan prescription for a collection of free quarks, but with the mean total invariant mass of the free quarks equal to the nucleon mass m_N, i.e., $\sum_{i=1}^3 p_i^\mu\rightarrow p_N^\mu$. Obviously it is not a completely satisfactory technique but it at least allows one to deal with a system of interacting quarks by assuming a correspondence of real hadrons with weakly bound ones.[11] Using the prescription, we get the following model for the Lorentz-invariant light-cone wave function (1):

$$\psi_\uparrow(x_i,\mathbf{k}_{\perp i},\lambda_i) = \phi^N(x_i,\mathbf{k}_{\perp i})\chi_\uparrow^N(x_i,\mathbf{k}_{\perp i},\lambda_i)\Big/\left[\prod_i x_i\right]^{1/2}, \tag{3}$$

where

$$\chi_\uparrow^N(\hat{1},\hat{2},\hat{3}) = J_\uparrow(\hat{1},\hat{3},\hat{2}) + J_\uparrow(\hat{2},\hat{3},\hat{1}),$$

with

$$J_\uparrow(\hat{1},\hat{2},\hat{3}) = \bar{u}_{\lambda_1}(m_N + p_\mu\gamma^\mu)\gamma_5 v_{\lambda_2}\bar{u}_{\lambda_3}u_\uparrow .$$

$\hat{1}$, $\hat{2}$, and $\hat{3}$ are collective momentum-helicity indices $(x_i,\mathbf{k}_{\perp i},\lambda_i)$, $i=1,2,3$. u_λ and v_λ are the light-cone spinors of Ref. 1. The relativistic nonstatic spin wave functions χ^H are given in Table I.[12]

With the valence-quark-dominance assumption, any proton state $|\psi\rangle$ with momentum $p_N = (p^+,p^-,\mathbf{p}_\perp)_N = (p^0+p^3,(m^2+p_\perp^2)/p^+,\mathbf{p}_\perp)_N$ and helicity $\uparrow$ is described by

$$|p_N\uparrow\psi\rangle = \sum_{\lambda_i}\int\frac{[dx][d^2k]}{(x_1x_2x_3)^{1/2}}\psi_\uparrow^N(\hat{1},\hat{2},\hat{3})\, u^\dagger_{p_1\lambda_1}u^\dagger_{p_2\lambda_2}d^\dagger_{p_3\lambda_3}|0\rangle , \tag{4}$$

where $p^\dagger_{p_i\lambda_i}$ and $d^\dagger_{p_i\lambda_i}$ are the creation operators of the u and d quarks, respectively, with momentum $p_i^+ = x_ip^+$, $\mathbf{p}_{\perp i} = x_i\mathbf{p}_{\perp N} + \mathbf{k}_{\perp i}$. We keep color implicit. Notice that the state (4) is written in the so-called *uds basis.*[13] In the *uds* basis one carries out only a part of the antisymmetrization that would be required by the full S_3 group; the rest is insured by the anticommutation properties of the quark operators. In particular, the wave function (3) is symmetric under the exchange of the first two quarks, i.e., $\psi_\uparrow^N(\hat{1},\hat{2},\hat{3}) = \psi_\uparrow^N(\hat{2},\hat{1},\hat{3})$.

Since distribution amplitudes $\phi(x_i,Q)$ [Eq. (1)] are the $L_z=0$ projection of the wave function (3), they are determined by the wave-function components with total quark helicity $+\frac{1}{2}$, viz.,

$$\psi_\uparrow(1,2,3;\uparrow\uparrow\downarrow) = -2T(1,2,3),$$

$$\psi_\uparrow(1,2,3;\uparrow\downarrow\uparrow) = \varphi(1,2,3),$$

$$\psi_\uparrow(1,2,3;\downarrow\uparrow\uparrow) = \varphi'(1,2,3),$$

TABLE I. The nucleon light-cone spin wave function $\chi^N(\hat{1},\hat{2},\hat{3})$ ($L_z=0$ components only), $a_i = m_Nx_i + m$, $k_i^{L,R} = k_i^1 \mp ik_i^2$.

λ_1	λ_2	λ_3	$\chi_\uparrow^N(\hat{1},\hat{2},\hat{3})\sqrt{x_1x_2x_3}$
↑	↓	↑	$a_1a_2a_3 + k_3^R(2a_1k_2^L - a_3k_1^L)$
↓	↑	↑	$a_1a_2a_3 + k_3^R(2a_2k_1^L - a_3k_2^L)$
↑	↑	↓	$-2a_1a_2a_3 - k_3^R(a_1k_2^L + a_2k_1^L)$

where 1, 2, and 3 are collective momentum variables $(x_i,\mathbf{k}_{\perp i})$, $i=1,2,3$. As shown in Ref. 7, the requirements that the nucleon is the $I=\frac{1}{2}$ color-singlet representation of three quarks leads to the following relations:

$$2T(1,2,3) = \varphi(1,3,2) + \varphi(2,3,1), \tag{5a}$$

$$\varphi(1,2,3) = V(1,2,3) - A(1,2,3), \tag{5b}$$

$$\varphi'(1,2,3) = V(1,2,3) + A(1,2,3), \tag{5c}$$

where $V \cong [\varphi(1,2,3)+\varphi(2,1,3)]/2$ and $A = [\varphi(2,1,3) - \varphi(1,2,3)]/2$. Thus, in the $L_z=0$ sector we are left with only one independent wave-function-helicity component: $\varphi(1,2,3)$. Note, however, that the requirement does not specify the $(x_i,\mathbf{k}_{\perp i})$ permutation properties of the independent amplitude $\varphi(1,2,3)$. Even if the momentum wave function $\phi^N(x_i,\mathbf{k}_{\perp i})$ is totally symmetric to an exchange of the individual momenta $(x_i,\mathbf{k}_{\perp i})$, the nonstatic spin wave function χ^N provides the asymmetric contribution: $A(1,2,3)$.

After $[dk_\perp]$ integration[14] in (1), one obtains from the model (3)

$$V(x_1,x_2,x_3) = \hat{\phi}^N\left[\prod_i\tilde{a}_i + x_1x_2\tilde{a}_3 - x_1x_3\tilde{a}_2 - x_2x_3\tilde{a}_1\right], \tag{6a}$$

$$A(x_1,x_2,x_3) = \hat{\phi}^N[(x_2-x_1)x_3\tilde{m}], \tag{6b}$$

$$\varphi(x_1,x_2,x_3) = \hat{\phi}^N\left[\prod_i\tilde{a}_i + x_1x_2\tilde{a}_3 - x_1x_3(\tilde{a}_2-\tilde{m}) - x_2x_3(\tilde{a}_1+\tilde{m})\right], \tag{6c}$$

TABLE II. Moments of distribution amplitudes V and $\varphi = V - A$ obtained from the relativistic wave function for $a=660$ and 725 MeV. They are compared with the moments of Ref. 2.

			V		φ	
n_1	n_2	n_3	Model	Ref. 2	Model	Ref. 2
1	0	0	0.41–0.46	0.38–0.42	0.52–0.64	0.60–0.75
0	1	0	0.41–0.46	0.38–0.42	0.30–0.27	0.09–0.16
0	0	1	0.19–0.09	0.18–0.24	0.19–0.09	0.18–0.24
2	0	0	0.20–0.24	0.18–0.25	0.28–0.37	0.25–0.40
0	2	0	0.20–0.24	0.18–0.25	0.13–0.12	0.03–0.08
0	0	2	0.05–0.00	0.08–0.12	0.05–0.00	0.08–0.12
1	1	0	0.14–0.16	0.07–0.12	0.14–0.16	0.07–0.12
1	0	1	0.07–0.05	0.04–0.08	0.10–0.11	0.09–0.14
0	1	1	0.07–0.05	0.04–0.08	0.03–0.01	−0.03–0.03

where $\hat{\phi}^N = \exp(\sum \tilde{m}_i^2/x_i)$, $\tilde{a}_i = x_i \tilde{m}_N + \tilde{m}$, $\tilde{m}_N = m_N/\sqrt{6}a$, $\tilde{m} = m/\sqrt{6}a$. The distribution amplitudes we derived from the relativistic quark model contain the common factor $\hat{\phi}^N$ which comes from the Gaussian momentum function. If the valence state has a broad transverse-momentum distribution, i.e., $\alpha \gg m$ (the Brodsky-Huang-Lepage small-valence-radius hypothesis[8]) then $\hat{\phi}^N$ is also broad and very close to $\phi_{as}(x_i) = \prod_i x_i$, i.e., to the asymptotic form of the nucleon distribution amplitude.[1] On the other hand, if $m \gtrsim \alpha$ the $\hat{\phi}^N$ imitates the nonrelativistic distribution amplitude for which $x_i \approx \frac{1}{3}$, i.e., each quark carries an equal fraction of the nucleon momentum. But this picture is still modified by the presence of the square brackets in (6) which come from the nonstatic spin wave function. We find that for $\alpha \approx (660-725)$ MeV, the spin-wave-function factor produces in the distribution amplitudes a complex "bump-dip" structure similar to those reported in Refs. 2-4. To illustrate this we calculate moments of normalized amplitudes V and φ. The results are given in Table II in comparison with those obtained from QCD sum rules (Ref. 2).

The main purpose of this work was to check whether the description of basic features of both pion and nucleon distribution amplitudes could be unified in a single-wave-function approach. We have found that the basic physics behind the observed properties of the amplitudes as calculated from QCD sum rules is simple and can be understood essentially with the basic concepts of the relativistic quark model.

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[11]We stress once again the role played by the quark interaction in the derivation of the Chernyak-Zhitnitsky results. If one turns off the interaction in the spin wave function χ^H, assuming $p_N = \sum_{i=1}^{3} p_i$, then the resultant spin wave function leads to distribution amplitudes which are unable to reproduce the Chernyak-Zhitnitsky results. Compare an analogous remark made in Ref. 6.
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[14]Because of the presence of the damping Gaussian factor in (2) we can perform the integration up to infinity. With the values of χ quoted above, it corresponds to the scales $Q \approx 1.5-2$ GeV. We do not consider here the problem of the normalization point of the distribution amplitude (1). As is noticed in Ref. 2, variations of the moments with Q in the interval 1-2 GeV are smaller than the uncertainties with which they are determined from the sum rules.