Constraints from the Drell-Hearn-Gerasimov sum rule in chiral models of composite fermions

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(Received 3 November 1987)

We analyze a dispersion relation for spin-flip Compton amplitude in a chiral model of composite quarks and leptons. We discuss the asymptotic behavior of the amplitude and show under what conditions the Drell-Hearn-Gerasimov sum rule holds. Using a small-mass expansion we derive a consistency condition that relates the photoproduction cross sections for different helicities. We show that this condition suggests the existence of higher-spin excited fermions, as well as relations among *a priori* independent photoproduction amplitudes.

I. INTRODUCTION

There is considerable interest in chiral theories where anomalies associated with flavor symmetries allow certain composite fermions to be massless. In those models the physical masses of the lightest composite fermions come from small symmetry-breaking terms present in the fundamental Lagrangian. Perhaps a successful model of composite quarks and leptons will be based on an unbroken non-Abelian gauge theory, and if so it will be confining at low energies. In this case, experience from hadron physics suggests that an analysis based on dispersion techniques and symmetries may yield important information about the theory.

In this paper we study the Drell-Hearn-Gerasimov[1] (DHG) sum rule for a chiral composite model. In the chiral limit the sum rule places a nontrivial dynamical constraint on the model which goes beyond the results of a standard effective-Lagrangian analysis. For example, it appears to require the existence of higher-spin fermions in the spectrum of the excited states.

For definiteness, we adopt the strongly coupled standard model[2,3] (SCSM) but our results should apply to any chiral composite model in which the DHG sum rule is satisfied. The SCSM is a composite chiral model based on the Lagrangian of the standard model of Glashow, Salam, and Weinberg (GSW). In the SCSM, the physical left-handed fermions, intermediate vector bosons, and Higgs particles are composite and weak interactions correspond to residual forces among composite particles. The model has a long-distance behavior identical to the GSW model but predicts completely different physics (new resonances, higher-dimension interactions) at the Fermi scale $G_F^{-1/2}$ (Ref. 3). The new effects are usually referred to as the "exotic sector" of the theory. In the SCSM the photon couples to preon hypercharges with the coupling constant e—the QED coupling constant. The effects of electromagnetism can be studied perturbatively and we shall work in the lowest order in α.

In the subsequent sections we derive the DHG sum rule for SCSM (allowing for parity violation). We discuss the possible asymptotic behavior of the Compton scattering amplitude, which is crucial to the sum rule. Then we analyze the DHG sum rule near the chiral limit. We derive consistency conditions and study the constraints they place on the phenomenology of the model. At the end we conclude and discuss our results.

II. THE DHG SUM RULE

Consider a Compton scattering process on a composite lepton to lowest order in electromagnetism. The most general form of the *CP*-invariant forward-scattering amplitude in the laboratory frame (lepton at rest) is

$$f(\omega)=\chi_f^\dagger[\boldsymbol{\epsilon}_f^*\cdot\boldsymbol{\epsilon}_i f_1(\omega)-i\boldsymbol{\sigma}\cdot(\boldsymbol{\epsilon}_f^*\times\boldsymbol{\epsilon}_i)f_2(\omega)+\boldsymbol{\epsilon}_f^*\cdot\boldsymbol{\epsilon}_i\mathbf{n}\cdot\boldsymbol{\sigma}g_1(\omega)+i\mathbf{n}\cdot(\boldsymbol{\epsilon}_f^*\times\boldsymbol{\epsilon}_i)g_2(\omega)]\chi_i\ , \quad (1)$$

where ω is the photon energy, χ_i (χ_f) and $\boldsymbol{\epsilon}_i$ ($\boldsymbol{\epsilon}_f$) are initial (final) Pauli spinors and polarization vectors, and $\mathbf{n}$ is a unit vector in the direction of the photon momentum. Note that $g_1=g_2=0$ in hadron physics where one imposes parity conservation.

We designate the photon helicity by h (where $h=\pm1$) and the projection of the lepton spin on $\mathbf{n}$ by s (where $s=\pm\frac{1}{2}$). In obvious notation,

$$f_2(\omega)=\frac{1}{2}\left[\frac{A_{1,1/2}+A_{-1,-1/2}}{2}-\frac{A_{1,-1/2}+A_{-1,1/2}}{2}\right]. \quad (2)$$

The helicity amplitudes $A_{h,s}$ are related by the optical theorem to total cross sections for the reaction $\gamma+l\to$ anything. From (2) one gets

$$\mathrm{Im} f_2(\omega)=\frac{\omega}{8\pi}\left[\frac{\sigma_{1,1/2}+\sigma_{-1,-1/2}}{2}-\frac{\sigma_{1,-1/2}+\sigma_{-1,1/2}}{2}\right]\equiv\frac{\omega}{8\pi}\Delta\sigma\ . \tag{3}$$

The normalization of the spin-flip amplitude $f_2(\omega)/\omega$ at $\omega=0$ was established by Low and Gell-Mann and Goldberger:[4]

$$\lim_{\omega\to 0}\frac{f_2(\omega)}{\omega}=\frac{\alpha\kappa^2}{2m_l^2}\ ,$$

where m_l is the lepton mass, κ is the value of $(g-2)/2$ for the lepton, and α is the QED fine-structure constant. To obtain the DHG sum rule we assume that $f_2(\omega)$ obeys an unsubtracted dispersion relation:

$$\mathrm{Re} f_2(\omega)=\frac{2\omega}{\pi}P\int_{\omega_{\mathrm{th}}}^{\infty}\frac{d\omega'}{\omega'^2-\omega^2}\mathrm{Im} f_2(\omega')\ , \tag{4}$$

where we have used the fact that $f_2(\omega)$ is odd under crossing. We postpone until Sec. III a discussion of the crucial issue of whether $f_2(\omega)$ indeed obeys the unsubtracted dispersion relation.

Finally then, the DHG sum rule reads

$$\frac{\alpha\kappa^2}{2m_l^2}=\frac{1}{4\pi^2}P\int_{\omega_{\mathrm{th}}}^{\infty}\frac{d\omega}{\omega}\Delta\sigma\ . \tag{5}$$

[Note that $\Delta\sigma$ is $O(\alpha)$.]

We assume that our composite model mimics the standard model at low energies. To be precise, we assume that the dimension ≤ 4 terms in the effective Lagrangian for the lightest fermions and vector bosons coincide with the GWS model. This was shown to be the case for the SCSM under certain dynamical assumptions.[3] Additional contributions come from the "exotic" sector of the theory, i.e., from the higher-dimension operators and higher-mass intermediate states. Thus κ may be decomposed into a piece identical to the standard model and a piece that comes from the exotic sector, $\kappa=\kappa_{\mathrm{SM}}+\kappa_R$. If the photoproduction *amplitude* is likewise decomposed one is led to decompose $\Delta\sigma$ into a standard-model term $\Delta\sigma_{\mathrm{SM}}$, an exotic sector term $\Delta\sigma_E$ (both positive), and an interference term Δ_I between exotic and standard-model amplitudes. $\Delta\sigma_E$ corresponds to the photoproduction cross section calculated with amplitudes that contain only the higher-dimension operators and/or the higher-mass exotic states. Δ_I is the interference term between the "exotic" and standard-model amplitudes.

With $\alpha=0$, the leading contribution to κ_{SM} comes from the W and Z and is numerically very small. If $\kappa_{\mathrm{SM}}\lesssim|\kappa_R|$, then $|\Delta_I|\lesssim|\Delta\sigma_E|$ and κ_R reflects the compositeness of the lepton. $\Delta\sigma$ would then be dominated by the photoproduction of excited states (very much like in hadron physics) and it is convenient to analyze the sum rule that way. If, on the other hand, $|\kappa_R|\ll\kappa_{\mathrm{SM}}$ then $|\Delta_I|\gg|\Delta\sigma_E|$ and κ_R comes from the radiative corrections due to the exchange of the virtual intermediate vector bosons, where one of the vertices comes from the exotic sector and the other is the standard model vertex. [In other words, $\kappa_{\mathrm{SM}}\gg|\kappa_R|$ means that $\Lambda_c\gg m_W$. Thus the exotic resonances must be very broad and heavy, and it is not particularly advantageous to analyze the sum rule (5) in terms of the higher-mass states. Also it seems that the models of composite quarks, leptons, and intermediate vector bosons become unnatural in the limit $\Lambda_c\gg m_W$ (Ref. 3), and from now on we shall stick with the $\Lambda_c\gtrsim m_W$ case.]

III. DISPERSION RELATIONS IN THE SCSM

In this section we briefly review the assumptions which underlie the dispersion relation (4). The standard analyticity arguments (which we assume to hold) imply the validity of "fixed-t" dispersion relations:

$$\mathrm{Re} f_2(\omega,t)=\frac{2\omega}{\pi}P\int_{\omega_{\mathrm{th}}(t)}^{\infty}\frac{d\omega'}{\omega'^2-\omega^2}\mathrm{Im} f_2(\omega',t)+\sum_{i=0}^{N}A_i(t)\omega^{2i+1} \tag{6}$$

with real A_i's and q^2 dependence suppressed. For forward scattering $t=0$, and unitarity implies $\mathrm{Im} f_2=(\omega/8\pi)\Delta\sigma$. The convergence of the integral in (6) depends on the behavior of $\Delta\sigma$ as $\omega\to\infty$. Note that at high c.m. energies, leptons polarized along **n** become right handed and those polarized opposite to **n** become left handed. As $\omega\to\infty$ we expect the right-handed and left-handed cross sections in $\Delta\sigma$ to cancel in pairs. For the right-handed fundamental fermions, this follows from the fact that to order α, only Born graphs contribute to $\Delta\sigma$. In the case of the left-handed fermions $\Delta\sigma$ has to be calculated to all orders in the confining dynamics. Nevertheless, as in hadron physics, we expect that the total cross sections at high energies are dominated by diffractive scattering. Thus the cross sections for the scattering of ±1 helicity photons on composite left-handed leptons become equal as $\omega\to\infty$. We see that

$$\Delta\sigma\underset{\omega\to\infty}{\longrightarrow}0$$

so the integral in (6) converges.

The presence of the real polynomial in (6) in general may be required by the behavior of $\mathrm{Re} f_2(\omega,t)$ as $\omega\to\infty$. If the Compton amplitude Reggeized it would mean that $A_0=A_1=\cdots=A_N=0$. Unfortunately, $f_2(\omega,t)$ is calculated to the lowest order in α and the usual arguments based on t-channel partial-wave unitarity do not apply. On the other hand, the sum rule (4) is equivalent to $A_0=0$, which we would like to assume.

There exists an argument for $A_0=0$ in QCD and it appears that we may use it in the SCSM as well. Let us generalize the Compton amplitude to electroproduction processes, i.e., give the virtual photon a mass q^2. We assume that for some $q^2\gg G_F^{-1}$ both the gauge coupling constant $g_{\mathrm{SU}(2)}$ and the scalar quartic self-coupling λ are numerically small [it is justifiable if $\lambda(|q^2|\lesssim G_F^{-1})\ll 1$]. Then $f_2(\omega,q^2)$ should scale, i.e., in the limit $-q^2,\omega\to\infty$ (where $x=-2\omega m_l/q^2=\mathrm{const}$) $A_i\sim(-q^2)^{-(2i+1)}$, so the $A_i(q^2)$ are not polynomials, and must have singularities at finite q^2. Now singularities in q^2 correspond to physical intermediate states that couple to photon. It follows that the real fixed pole terms come from diagrams which have propagators of composite particles attached to the external photon lines. Those diagrams are related to the diagrams that describe scattering and/or photoproduction of composite particles. In other words, the fixed pole terms in (6) imply the existence of the fixed poles in the amplitudes for the scattering and photoproduction of composite particles. If scattering and photoproduction amplitudes have standard Regge behavior (as in hadron physics), then this forbids the existence of any real polynomials in (6) and completes our argument about the validity of the dispersion relation.

IV. THE SMALL-MASS EXPANSION

We are studying a contribution to the anomalous magnetic moment that is of order 1 in α and comes from the dynamics associated with the composite nature of the lepton (including weak interactions which are the long-distance effects of this dynamics). In any chiral composite model the right-handed lepton decouples from the composite left-handed lepton in the limit $m_l \to 0$ and the anomalous magnetic moment defined by

$$\mathcal{L}_I = \frac{1}{2}\left[\frac{e\kappa}{2m_l}\bar{l}_L \sigma_{\mu\nu} F^{\mu\nu} l_R + \text{H.c.}\right] \tag{7}$$

must be $O(m_l^2)$ for small m_l. Thus we define $\kappa = am_l^2/\Lambda_a^2$, where a is of order 1 in α and m_l, and Λ_c is the dynamical mass scale of the composite system. For example, a radiative contribution due to the exchange of the virtual W bosons in the standard model gives

$$a_W = \frac{g^2_{Wf\bar{f}}}{8\pi^2}\frac{\Lambda_c^2}{m_W^2}\frac{10}{3} ,$$

where m_W is the mass of W and $g_{Wf\bar{f}}$ is the effective W-fermion-antifermion coupling constant. Let us now substitute κ into the DHG sum rule and change the variable of integration to the c.m. energy squared. The DHG sum rule becomes now

$$\frac{\alpha a^2 m_l^2}{2\Lambda_c^4} = \frac{1}{4\pi}\int_{s_{\text{th}}}^{\infty}\frac{ds}{s-m_l^2}\Delta\sigma \ . \tag{8}$$

Recall that in the SCSM the masses of the lightest fermions come from the chiral-symmetry-violating terms whose numerical coefficients are of the order $\beta = m/\Lambda_c \ll 1$. It follows that the physical observables are expandable in powers of β; in particular relationship (8) must hold to each order of the expansion. This gives a set of consistency conditions which the chiral composite model must obey. Since $\Delta\sigma$ starts at ~ 1, in the lowest order we get

$$0 = \int_0^{\infty}\frac{ds}{s}\overline{\Delta\sigma} \tag{9}$$

where $\overline{\Delta\sigma} = \lim_{m_l \to 0}\Delta\sigma$.

Again the contribution of the right-handed leptons cancels off and the sum rule (9) applies to the SCSM without the right-handed fields, i.e., to a pure composite theory. Note that Eq. (9) states that there should be a strict balance between the photoproduction of states with helicity $\frac{3}{2}$ and states with helicity $-\frac{1}{2}$ (with corrections of the order $m_l^2/\Lambda^2 \ll 1$). As a consequence, in the chiral composite model the experimental limits on the "composite" anomalous magnetic moment do not seriously constrain the photoproduction amplitudes. The leading terms that are ~ 1 in m_l^2/Λ_c^2 cancel off in Eq. (8), and it is necessary to calculate the first "nontrivial" terms $O(m_l^2/\Lambda_c^2)$ to establish the value of a (and κ). For a comparison, in a nonchiral composite model $\kappa = bm_l/\Lambda_c$ and the sum rule reads

$$\frac{ab^2}{2\Lambda_c^2} = \int_0^{\infty}\frac{ds}{s}\overline{\Delta\sigma} \tag{10}$$

for $m_l \ll \Lambda_c$. In these models, even present limits on κ require very large values of Λ_c (e.g., for $b=1$, $\Lambda_c \gtrsim 11\,800$ TeV), and, via Eq. (10) great suppression of photoproduction amplitudes.[5] In a chiral model, for $a=1$, present data on $(g-2)$ only require $\Lambda_c \gtrsim 1.08$ TeV and, via Eq. (8), do not severely constrain photoproduction of exotic states.

Since it is natural to expect that the DHG sum rule holds for the standard model, we have separate relationships:

$$0 = \int_0^{\infty}\frac{ds}{s}\overline{\Delta\sigma}_{\text{SM}} = \int_0^{\infty}\frac{ds}{s}(\overline{\Delta\sigma}_E + \overline{\Delta}_I) , \tag{11}$$

where $\overline{\Delta\sigma}_{\text{SM}} = \lim_{m_l \to 0}\Delta\sigma_{\text{SM}}$, $\overline{\Delta\sigma}_E = \lim_{m_l \to 0}\Delta\sigma_E$, and $\overline{\Delta}_I = \lim_{m_l \to 0}\Delta_I$. Equation (11) constrains the exotic sector contribution to the photoproduction cross section. To analyze Eq. (11) we assume $\overline{\Delta\sigma}_{\text{E}} \gg \overline{\Delta}_I$ and that it is saturated by a single spin-$\frac{3}{2}$ excited state. We can introduce two amplitudes for the reaction $\mu^*_{3/2} \leftrightarrow \mu + \gamma$ in terms of two effective couplings:

$$\mathcal{L}_1 = \frac{e\kappa_1}{M_{3/2}}\bar{\phi}^{\mu}\gamma^{\nu}\psi_L \mathcal{F}_{\mu\nu} + \text{H.c.} \tag{12}$$

and

$$\mathcal{L}_2 = \frac{e\kappa_2}{M_{3/2}^2}\bar{\phi}^{\alpha}\sigma^{\mu\nu}\psi_L \partial_{\alpha}\mathcal{F}_{\mu\nu} + \text{H.c.} \ , \tag{13}$$

where ϕ^{μ} is a Rarita-Schwinger field creating $\mu^*_{3/2}$, ψ_L is a Weyl field creating μ_L, and $M_{3/2}$ is the mass of $\mu^*_{3/2}$. We can calculate the cross sections in (11) using (12) and (13) in the limit when the width of $\mu^*_{3/2}$ is much smaller than its mass. Then the DHG sum rule tells us that

$$\kappa_1^2 = \tfrac{4}{3}\kappa_2^2 \ .$$

On the other hand, it is obvious that a single spin-$\frac{1}{2}$ excited state cannot saturate Eq. (11). It follows from the fact that a spin-$\frac{1}{2}$ particle cannot be produced with helicity $\frac{3}{2}$, and $\overline{\Delta\sigma}_E < 0$ over the whole kinematic region. We are forced to conclude that there must exist resonances of spin $\geq \frac{3}{2}$ which are the orbital excitations of the lightest particles and whose masses and/or couplings are of the same order as masses and/or couplings of the radial spin-$\frac{1}{2}$ excitations. Note that without the sum rule (11) (in other words, in a composite model that is not chiral) it would be possible to assume that only spin-$\frac{1}{2}$ resonances contribute to the DHG sum rule. [After all $\Delta\sigma_E < 0$ of the order 1 means $\kappa_R < 0$ of the order $O(m_l/\Lambda_c)$ and this is not prohibited in a nonchiral composite model.]

Let us study now what happens if the sum rule (11) is saturated by one spin-$\frac{1}{2}$ and one spin-$\frac{3}{2}$ state. There is only one helicity amplitude for the transition $\mu^*_{1/2} \leftrightarrow \mu + \gamma$ and again we write it in terms of an effective coupling:

$$\mathcal{L}_{1/2} = \frac{e\kappa_{1/2}}{M_{1/2}} \bar{\chi} \sigma^{\mu\nu} \psi_L \mathcal{F}_{\mu\nu} + \text{H.c.} \tag{14}$$

Here χ is a Dirac field creating $\mu^*_{1/2}$ and $M_{1/2}$ is the mass of $\mu^*_{1/2}$. A constraint due to (14) in the narrow-width approximation is

$$\kappa_1^2 = \tfrac{4}{3}\kappa_2^2 + 8\kappa_{1/2}^2 \frac{M_{3/2}^2}{M_{1/2}^2} \ . \tag{15}$$

If one assumes that $\kappa_{E1} \sim \kappa_{M2} \sim \kappa_{1/2}$ then Eq. (15) requires that the mass $M_{1/2}$ must be a couple of times bigger than the mass $M_{3/2}$.

Note that a proper interpretation of the results such as (15) can be obtained only in terms of models which describe the dynamics of bound states. Remember that an important test of the SU(6)⊗O(3) model of hadrons was to check its predictions for various photoproduction rates. Similarly for the SCSM there should exist an effective description of the composite particles whose symmetries and dynamics guarantee relations such as (15).

V. CONCLUSIONS

In this paper we have analyzed the DHG sum rule for the spin-flip Compton amplitude in the chiral composite model. We have shown that one can obtain an unsubtracted dispersion relation provided the model has diffractive high-energy behavior, scales in some high-q^2 region and the scattering and photoproduction amplitudes for the pairs of composite particles Reggeize without fixed poles. We have analyzed the exotic sector contribution to the DHG sum rule in terms of the excited states. Our discussion applies for the nearby compositeness, i.e., when Λ_c is not orders of magnitude bigger than m_W. As we show, in this case the DHG sum rule requires the existence of the higher-spin excited states. Of course exotic contributions to the anomalous magnetic moment must be comparable to or exceed the weak-interaction contribution if nearby compositeness is correct. These facts suggest that nearby compositeness will soon be tested experimentally. For example, in the high-energy collider experiments, one will be able to search for the higher-spin, higher-mass excited states. On the other hand, high-accuracy magnetic moment measurements will look for the weak-interaction contribution to κ and will check if those are consistent with the standard model.

ACKNOWLEDGMENTS

This work was supported in part by funds provided by the U.S. Department of Energy under Contract No. DE-AC02-76ER03069 and NSF Grant No. PHY-8414643.

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