On the Wilson coefficient of the penguin operator

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Recently an additional contribution to the real part of the Wilson coefficient function c_5 of the dominant penguin operator O_5 in $K \to 2\pi$ decay was found by Bardeen, Buras, and Gérard. We show that in the effective-Hamiltonian approach used by Gilman and Wise with the "heavy"-charm-quark approximation, the additional contribution to c_5 originates from the penguin diagrams with u-quark loops which are left after the charm quark is integrated out. For $\mu \sim 1$ GeV (μ being the renormalization scale), the inclusion of the u-quark loop diagram amounts to adding a contribution $\frac{5}{3}\alpha_s(\mu)/12\pi$ to the standard estimate of c_5. Penguin diagrams at the low-momentum scale are briefly discussed.

Recently Bardeen, Buras, and Gérard[1] (BBG) have identified an additional enhancement of the real part of the Wilson coefficient function c_5 (or $\tilde{z}_6$ in the notation of Ref. 1; we follow the notation of Ref. 2) of the penguin operator O_5 (or Q_6) with respect to standard estimates which comes from an incomplete Glashow-Iliopoulos-Maiani (GIM) cancellation above the charm-quark mass. However, this does not mean anything is wrong with the effective-weak-Hamiltonian approach of Gilman and Wise[3] with heavy quarks integrated out or their calculation of c_5. In this paper we will show that an additional contribution to $\mathrm{Re}c_5$ stems from the u-loop penguin diagrams which are left after the charm quark is integrated out from the effective Hamiltonian. Adding the contribution $\frac{5}{3}\alpha_s(\mu)/12\pi$ back into the Gilman-Wise estimate of c_5, it is in agreement with the values obtained by Bardeen, Buras, and Gérard in the $1/N_c$ approximation.

To begin with, the anomalous dimension relevant for the calculation of $\mathrm{Re}c_5$ is computed to be[1] [a simple way of deriving this equation is discussed after Eq. (11)]

$$\gamma(Q^2,m_u^2,m_c^2)=\frac{\alpha_s(Q^2)}{6\pi}\int_0^1 dx\, 6x^2(1-x)^2Q^2\left[\frac{1}{m_u^2+x(1-x)Q^2}-\frac{1}{m_c^2+x(1-x)Q^2}\right] . \tag{1}$$

For a realistic value of m_c, it is evident that GIM cancellation is not complete for $Q^2 \geq m_c^2$, in contrast with the usual assumption[3] of an exact GIM compensation above the charm-quark mass. Therefore, the coefficient $c_5(\mu)$ receives contributions not only from the range $\mu^2 < Q^2 < m_c^2$ (μ being the renormalization scale, i.e., the QCD subtraction scale) but also from $Q^2 > m_c^2$. As a consequence, $\mathrm{Re}c_5$ is enhanced relative to standard estimates by a factor of 2 and 3 for $\mu = 0.8$ and 1.0 GeV, respectively.[1]

In the Gilman-Wise (GW) approach[3] of the weak effective Hamiltonian, the W boson, and t, b, c heavy quarks are successively removed from explicitly appearing in the theory via the penguin diagram. Without loss of generality in the ensuing discussions, we shall neglect CP violation and only consider the four-quark case. When the W field is integrated out from the theory, it leads to the following $\Delta S = 1$ effective Hamiltonian:

$$\mathcal{H}_{\mathrm{eff}}^{\Delta S=1}=\sqrt{2}G_F \sin\theta_C \cos\theta_C (c_+O_+ + c_-O_-) , \tag{2}$$

where

$$O_\pm = (\bar{s}u)(\bar{u}d) \pm (\bar{s}d)(\bar{u}u) - (u \to c) ,$$

$(\bar{q}_1 q_2) \equiv \bar{q}_1\gamma_\mu(1-\gamma_5)q_2/2$, and $c_\pm$ are QCD-corrected coefficients. Also, the W line in the penguin diagram [Fig. 1(a)] is contracted to a point, and u,c quarks propagate in the loop [Fig. 1(b)]. The next step is to integrate the charm quark out by treating m_c as heavy. It follows from Eq. (1) that, in the "heavy"-charm-quark approximation (whether this is a reliable approximation is another issue),

$$\gamma(Q^2,m_u^2,m_c^2) \simeq \begin{cases} \dfrac{\alpha_s(Q^2)}{6\pi}, & m_u^2 \ll Q^2 \leq m_c^2 , \\ 0, & Q^2 \geq m_c^2 . \end{cases} \tag{3}$$

The anomalous dimension at $Q^2 < m_c^2$ is precisely the coefficient of the usual leading-logarithmic term resulting from the evaluation of the c-quark loop (see below). Because of the nearly complete GIM cancellation, the Wilson coefficient c_5 will not receive contributions from $Q^2 \geq m_c^2$ with the approximation of a heavy-charm-quark

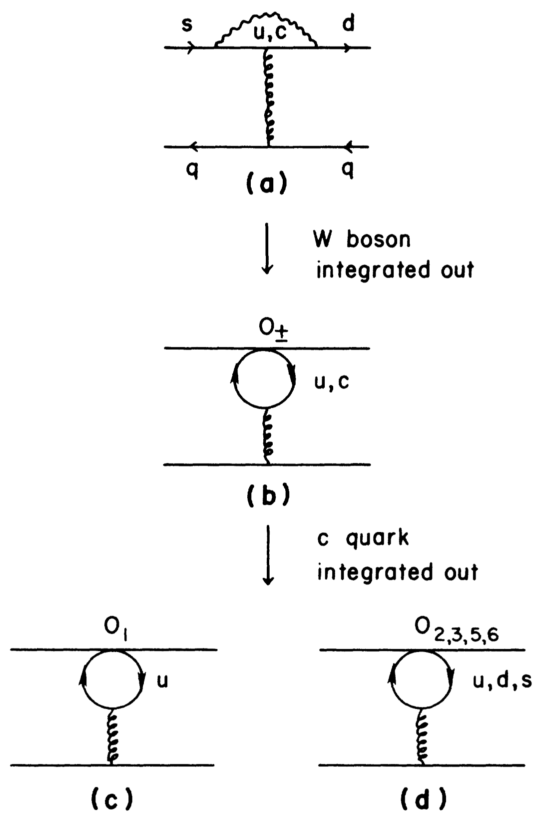

FIG. 1. Three stages of evolution of the penguin diagram. To leading order in $1/M_W^2$, the W line in (a) is contracted to a point, u and c quarks propagate in the loop of (b). After the charm-quark loop is integrated out, what are left are the penguin diagrams (c) and (d). The dominant contribution is (c) owing to the largest Wilson coefficient c_1. The notation q represents u, d, s quarks and the four-quark operators O_i are given in Ref. 2.

mass. Since in the real world m_c is not very large, the realistic value of $\mathrm{Re}c_5$ is the one obtained from Eq. (1) rather than from Eq. (3). The question is then how to account for the enhancement of c_5 within the approach of Gilman and Wise. We shall see that there is nothing wrong with the short-distance expansion of Gilman and Wise; the enhancement of c_5 is due to diagrams with u quark loops.

When the charm-quark loop is integrated out (the divergent parts of u- and c-quark loops are canceled out by the GIM mechanism), a new effective Hamiltonian [we have neglected leading logarithmic corrections from M_W down to m_c, that is we start from Fig. 1(a)]

$$\mathcal{H}_{\text{penguin}}^{\Delta S=1}=\frac{G_F}{\sqrt{2}}\sin\theta_C\cos\theta_C c_5[\bar{s}\gamma_\mu(1-\gamma_5)\lambda^a d]\times(\bar{u}\gamma^\mu\lambda^a u+\bar{d}\gamma^\mu\lambda^a d+\bar{s}\gamma^\mu\lambda^a s) \tag{4}$$

is induced with

$$c_5(\mu)=-\frac{\alpha_s(|k^2|)}{2\pi}\int_0^1 dx\,x(1-x)\ln\frac{m_c^2-k^2x(1-x)}{\mu^2}, \tag{5}$$

where k^2 is the momentum squared of the virtual gluon and μ^2 will be set equal to $|k^2|$ in the final step. If the charm quark is treated as heavy

$$c_5(\mu)\simeq-\frac{\alpha_s(\mu)}{12\pi}\ln\frac{m_c^2}{\mu^2}. \tag{6}$$

The $(V-A)(V-A)$ parts of the penguin Hamiltonian (4) contribute to the operators O_1 and O_2 (Ref. 2), whereas the $(V-A)(V+A)$ parts induce new penguin operators $O_{5,6}$, where

$$O_5=\bar{s}_L\gamma_\mu\lambda^a d_L(\bar{u}_R\gamma^\mu\lambda^a u_R+\bar{d}_R\gamma^\mu\lambda^a d_R+\bar{s}_R\gamma^\mu\lambda^a s_R) \tag{7}$$

with $q_{L,R}\equiv\frac{1}{2}(1\mp\gamma_5)q$. A sum of leading logarithms $\ln(m_c^2/\mu^2)$ and $\ln(M_W^2/m_c^2)$ to all orders in strong interactions via renormalization-group technique accounts for all leading QCD logarithmic corrections (from M_W down to μ) to the real part of the Wilson coefficient $c_5(\mu)$ of the penguin operator O_5.

Therefore, after the charm quark is integrated out from Fig. 1(b), we are left with the penguin diagrams Figs. 1(c) and 1(d) (the so-called "eye" graphs in lattice QCD calculations[4]). Figures 1(c) and 1(d) arise from the effective Hamiltonian[2]

$$\mathcal{H}_{\text{eff}}^{\Delta S=1}=\sqrt{2}G_F\sin\theta_C\cos\theta_C\sum_{i=1}^{6}c_i(\mu)O_i(\mu) \tag{8}$$

in which only the light quarks are explicitly presented. Therefore, contributions from Figs. 1(c) and 1(d) should be included after the use of the heavy-charm-quark approximations. Since the four-quark operator O_1 has the largest Wilson coefficient,[5] it is evident that the penguin diagram induced by the effective Hamiltonian (8) is dominated by Fig. 1(c) in which only the u quark circulates in the loop.

As before, we first consider the u-loop penguin diagram in Fig. 1(a). The one-gluon approximation of Fig. 1(a) with the u quark in the loop induces the same penguin Hamiltonian (4) and contributes to c_5 by the amount

$$\frac{\alpha_s(|k^2|)}{2\pi}\int_0^1 dx\,x(1-x)\ln\frac{m_u^2-k^2x(1-x)}{\mu^2} \tag{9}$$

which can be evaluated exactly:[6]

$$\frac{\alpha_s(|k^2|)}{2\pi}\left[\frac{1}{6}\ln\frac{m_u^2}{\mu^2}-\frac{5}{18}-\frac{2}{3}\frac{m_u^2}{k^2}+\frac{1}{6}\frac{1+2m_u^2/k^2}{(1-4m_u^2/k^2)^{1/2}}\ln\frac{1+(1-4m_u^2/k^2)^{1/2}}{-1+(1-4m_u^2/k^2)^{1/2}}\right]. \tag{10}$$

For $\mu^2=|k|^2>m_c^2$ (Ref. 7), Eq. (10) reduces to

$$-\frac{\alpha_s(\mu)}{12\pi}\frac{5}{3} \tag{11}$$

which is a leading-nonlogarithmic contribution. Before proceeding, we note that Eq. (1) is easily recovered by virtue of the relation $\gamma\sim Q^2(d/dQ^2)$ [Eq. (5) + Eq. (9)] for $Q^2=\mu^2=-k^2$. When $Q^2\gg m_u^2$ (i.e., Q^2 is confined to

TABLE I. The values of the Wilson coefficient function $\mathrm{Re}c_5(\mu)$ for different μ and $\alpha_s(\mu)$. Values of $\mathrm{Re}c_5$ in the analyses of Bardeen, Buras, and Gérard, and of Gilman and Wise are taken from Table I of Ref. 1; $\mathrm{Re}\tilde{c}_5$ is defined in Eq. (12).

μ (GeV)	0.8	1.0	0.8	1.0	0.8	1.0
$\alpha_s(\mu)$	0.54	0.47	0.77	0.63	1.09	0.82
$\mathrm{Re}c_5(\mu)_{\mathrm{GW}}$	−0.020	−0.010	−0.030	−0.015	−0.045	−0.020
$\mathrm{Re}\tilde{c}_5(\mu)$	−0.044	−0.031	−0.064	−0.043	−0.093	−0.056
$\mathrm{Re}c_5(\mu)_{\mathrm{BBG}}$	−0.045	−0.033	−0.065	−0.045	−0.094	−0.060

the region where perturbative QCD is applicable), it is easily seen that $\gamma=0$ for the u-loop penguin diagram. [Indeed, Eq. (11) does not contain logarithmic terms.] This implies that in this case hard-gluon corrections can be neglected (at least for the range $m_c^2>Q^2>\mu^2$). Therefore, in spite of the fact that the result (11) is derived at the level of Fig. 1(a), it represents the contributions of Figs. 1(c) and 1(d). Consequently, it is expected that the following relation is established:

$$\mathrm{Re}c_5(\mu)_{\mathrm{GW}}-\frac{\alpha_s(\mu)}{12\pi}\frac{5}{3}\equiv \mathrm{Re}\tilde{c}_5(\mu)=\mathrm{Re}c_5(\mu)_{\mathrm{BBG}}\,, \qquad (12)$$

where $c_5(\mu)_{\mathrm{GW}}$ and $c_5(\mu)_{\mathrm{BBG}}$ are the values of c_5 obtained by GW and BBG, respectively. From Table I it is evident that the agreement between $\mathrm{Re}\tilde{c}_5$ and $\mathrm{Re}c_5(\mu)_{\mathrm{BBG}}$ is excellent and remarkable, in view of the fact that the BBG values are obtained in the $1/N_c$ approximation. To see the importance of the new contributions we note that, to first order in QCD,

$$\mathrm{Re}\tilde{c}_5(\mu)\approx-\frac{\alpha_s(\mu)}{12\pi}\left[\ln\frac{m_c^2}{\mu^2}+\frac{5}{3}\right]. \qquad (13)$$

It is clear that the contribution $\frac{5}{3}$, which comes from the u-loop penguin diagram,[8] becomes dominant when μ^2 is in the perturbative regime (i.e., $\mu\gtrsim 1$ GeV): It will enhance the naive estimate of $\mathrm{Re}c_5$ by a factor of 2–3 for $\mu=0.8-1.0$ GeV (see Table I).

Thus far we have only focused on the region $\mu\gtrsim 1$ GeV where perturbative QCD is applied. In principle, the short-distance $\Delta S=1$ effective Hamiltonian given by Eq. (8) will suffice to describe kaon nonleptonic decays if we are able to evaluate the hadronic matrix elements at $\mu\sim 1$ GeV. Unfortunately, an explicit μ dependence is not seen in the usual matrix-element calculations.[9] Normally, it is argued that the on-shell $K-2\pi$ matrix elements of four-quark operators correspond to the scale $\mu=O(m_K,m_\pi)$. In the conventional methods of calculations, such as vacuum insertion and the bag model, the low-momentum eye graphs, Figs. 1(c) and 1(d), are entirely neglected. At $\mu=O(m_K,m_\pi)$, the penguin diagrams Figs. 1(c) and 1(d) are no longer dominated by the one-gluon exchange and all soft-gluon effects are involved. The inclusion of the low-energy eye graphs amount to adding long-distance contributions to the $\Delta I=\frac{1}{2}$ hadronic matrix elements $\langle\pi\pi\,|\,O_{1-3,5,6}\,|\,K\rangle$ and hence could play an essential role for the explanation of the $\Delta I=\frac{1}{2}$ rule in kaon decay.[10] Unfortunately, the estimate of these graphs is very difficult, although efforts have been made in passing.[11] Finally, a formidable task we need to deal with is how to continue the Wilson coefficients $c_i(\mu)$ from the perturbative domain down to the low-momentum regime.[12]

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[5]The characteristic values of Wilson coefficients are (Ref. 3) $c_1=-2.11$, $c_2=0.12$, $c_3=0.09$, $c_4=0.45$, $c_5=-0.025$, $c_6=-0.003$ for $\mu=0.86$ GeV, $\alpha_s=0.75$, $m_t=40$ GeV. Since the operator O_4 is pure $\Delta I=\frac{3}{2}$, it does not contribute to penguin diagrams.

[6]I wish to thank J. O. Eeg for pointing out that the same analytic result was also obtained in J. Finjord, Nucl. Phys. **B181**, 74 (1981); M. Galić, Report No. SLAC-PUB-2602, 1980 (unpublished).

[7]If k^2 is timelike (i.e., $k^2>0$), there will be a dynamic phase induced in the penguin diagram, see M. Bander, D. Silverman, and A. Soni, Phys. Rev. Lett. **43**, 242 (1979).

[8]It should be stressed that in the perturbative region of QCD, Figs. 1(c) and 1(d) should not be included in the analysis of BBG: the effect of the u quark is already taken into consideration in Eq. (1).

[9]C. T. Hill and G. G. Ross [Phys. Lett. **94B**, 234 (1980)] pointed out that a μ dependence in the penguin matrix elements arises from the momentum dependence of the quark masses. The usually quoted current-quark masses correspond to $\mu\sim 1$ GeV. From our discussions below, it is necessary to include Fig. 1(d) to account for the full μ dependence of the matrix elements of penguin operators at the low-energy μ scale.

[10]It is known that the perturbative evaluation of penguin dia-

grams is far from accounting for the $\Delta I = \frac{1}{2}$ rule. For recent estimates, see H. Y. Cheng, Phys. Rev. D **36**, 2056 (1987); **37**, 1338(E) (1988); and Ref. 12.

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