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Do data on W and Z decays already constrain nonstandard physics?

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We study the predictions of several extensions of the standard model (SM) for the ratio $R=N(p\bar{p}\to W\to e\nu)/N(p\bar{p}\to Z\to e^+e^-)$. The starting point of our investigation is the observation that the SM prediction for R seems to be somewhat above the data, especially if the top quark is heavy. We first show that this situation is not changed qualitatively by the incorporation of full quark-mass-dependent QCD corrections for Γ_W and Γ_Z. However, if either gauginos or exotic E(6) leptons with suitably chosen masses and mixings are present, the prediction for R can be reduced by 0.5–0.7 units. A similar reduction is possible if the Z boson mixes with the Z' boson present in certain SO(10) and E(6) models. For large top masses the reduction of R can be even larger if the b quark has a sizable SU(2)-singlet component, which is possible in superstring-inspired E(6) models. However, sleptons or squarks cannot reduce the prediction for R, and in models with two Higgs doublets R is expected to be close to its SM value. We then show that existing data strongly disfavor a sequential down-type quark below 26 GeV, and derive limits on the V_{tb} element of the extended Kobayashi-Maskawa mixing matrix of four-generation models. We also show that the data on R together with existing bounds on the ρ parameter severely constrain models of four-generation quark mass matrices. Furthermore, we find that the simultaneous existence of a light photino and a chargino with mass below $M_Z/2$ is strongly disfavored. We finally discuss the possible effects of new physics on the bounds on the top-quark mass and the number of light neutrino species that can be derived from the experimental upper bound on R.

I. INTRODUCTION

The standard model (SM) of electroweak interactions[1] is by now known to describe nature at least at energies below the masses of the W and Z bosons. However, two particles necessary for the consistency of this model still have to be discovered. One is the elusive Higgs boson which is thought to be the relic of the spontaneous breaking of $SU(2)\times U(1)_Y$ to $U(1)_{em}$; the other is the top quark which is needed for the SM to be free of anomalies.

One way to search for these particles is to look for unusual decay modes of the electroweak gauge bosons; at present, however, the number of W and Z bosons produced[2,3] at the CERN $Sp\bar{p}S$ collider is too small for this approach. On the other hand, indirect information about the mass of the top quark can be gained[4] by measuring the ratio R of $W\to l\nu$ events to $Z\to l^+l^-$ events at the CERN collider. Theoretically this ratio is predicted[4] to be

$$R=\frac{N(W\to l\nu)}{N(Z\to l^+l^-)}=\frac{\sigma(p\bar{p}\to W)}{\sigma(p\bar{p}\to Z)}\frac{\Gamma_{\rm tot}(Z)}{\Gamma_{\rm tot}(W)}\frac{\Gamma(W\to l\nu)}{\Gamma(Z\to l^+l^-)}=R_\sigma R_\Gamma \ . \tag{1.1}$$

The last of the three factors in Eq. (1.1) is completely determined by the value of the electroweak mixing angle which by now is known[5] up to 3%. The ratio of the total cross sections, which we denote by R_σ, depends on the quark distribution functions within the proton, especially[4] on the ratio of u to d valence quarks. Recently it has been pointed out[6,7] that this ratio can be extracted from a measurement[8] of F_2^n/F_2^p in deep-inelastic μp and μD scattering. By this method the error in R_σ can be reduced from 0.2 (Ref. 4) to 0.08 (Refs. 6 and 7), i.e., to about 2%. The only unknown in Eq. (1.1) is then the ratio of the total W and Z decay widths. This ratio does not only depend on the mass of the top quark but also on whether there exist additional particles lighter than the W or Z that couple to them. The "classical" example[4] is light neutrinos from a fourth or higher generation, and therefore a measurement of R is often called "neutrino counting." This is somewhat misleading since many different kinds of new physics can change the prediction for R, as we will discuss in this paper.

One outcome of the recent analyses[6,7] of the SM prediction for R is that even for a light top quark, $m_t<M_Z/2$, and only three light neutrino species the prediction is about 0.8 units above the UA1/UA2 average[9] of

$$R^{\text{expt}}=8.4^{+1.2}_{-0.9} . \tag{1.2}$$

This is not really a reason to worry since the discrepancy is less than one standard deviation. There are, however, other indications that might favor a heavier top quark. Most recently the UA1 Collaboration has reported[10] the bound $m_t \gtrsim 40$ GeV, and the large observed[11] B^0-$\bar{B}^0$ mixing might favor[12] even larger values of m_t. Neither of these bounds is really free from uncertainties since the UA1 result heavily relies on the absolute normalization of the QCD $2\to 3$ cross section as well as fragmentation models, while the observed B^0-$\bar{B}^0$ mixing may well be compatible[13] with a lighter top quark even within the SM. In view of the more accurate measurements of R that will be available from the Fermilab Tevatron and with the advent of ACOL at CERN, it is nevertheless intriguing to investigate what kind of new physics could help to reduce the prediction for R. The first part of our paper is devoted to a comprehensive study of this question.

The most straightforward approach would be to increase Γ_W without changing Γ_Z. This is, however, not so easy since, by SU(2) invariance, particles that couple to the W usually also couple to the Z. In fact, in most cases nonstandard physics tends to *increase* the prediction for R, the classical example again being additional light neutrinos. Since the prediction for R is already somewhat too large, any new physics that increases R is obviously tightly constrained. These constraints are discussed in the second part of our paper.

The remainder of this paper is structured as follows. In Sec. II we reanalyze the SM prediction for R, properly taking into account the QCD corrections for W and Z decays into heavy quarks. Although these corrections can reduce the prediction for R by up to 0.2 units the main results of Refs. 6 and 7 are only slightly changed. In Sec. III we show that existing experimental bounds make it impossible to reduce the prediction for R by allowing W's and Z's to decay into squarks or sleptons. Nonstandard doublet Higgs bosons can reduce R by less than 0.1 units. On the other hand in certain regions of parameter space electroweak gaugino-Higgsino eigenstates can reduce R by as much as 0.5 units which can cancel the effect of 1.2 neutrino species. Even larger reductions of R are possible in the framework of superstring-inspired E(6) models.[14] R can be reduced by up to 0.7 units by allowing the W to decay into exotic colorless fermions. Also, if the b quark, due to large mixings, consists mainly of an exotic SU(2)-singlet quark, which is not ruled out by present data, the bound on m_t vanishes altogether. Finally, if the mixing of the standard Z with a new Z' gauge boson is the maximum allowed by present neutral-current data,[5] R can be reduced by as much as 0.7 units.

In Sec. IV we discuss constraints on new physics from the existing measurement of R. We show that a sequential fourth-generation down-type quark cannot be lighter than 26 GeV, and that its mixing with the b quark is tightly constrained, thereby giving the first direct bounds on the V_{tb} element of the Kobayashi-Maskawa mixing matrix. We also show that provided $m_t > 40$ GeV, our bound on the mass of the fourth down quark, when combined with the bound on the mass splitting within the new quark doublet that comes from an analysis[5] of electroweak radiative corrections to various processes, strongly disfavors several simple models of quark mass matrices with four generations. We then demonstrate that the combination of bounds from the measurement of R and from UA1 monojets[15] disfavor charginos with masses below $M_Z/2$ if the lightest neutralino is dominantly a photino. We conclude Sec. IV with a discussion of the bounds on m_t and the number of light neutrino species that emerge in various nonstandard scenarios. Finally, in Sec. V, we summarize our findings and draw some conclusions.

II. QCD CORRECTIONS TO Γ_W AND Γ_Z

Since W and Z bosons dominantly decay into quarks their total decay widths are substantially altered by strong-interaction radiative corrections. In most preceding investigations[4,6,7] of the quantity R it has been assumed that these corrections can be written as

$$\Gamma(V\to q\bar{q})=\Gamma_0(V\to q\bar{q})(1+\alpha_s/\pi) , \tag{2.1}$$

where Γ_0 is the width for $\alpha_s=0$ [see, however, Hikasa (Ref. 4)]. This is, however, only true for massless quarks. As pointed out in Ref. 16, Eq. (2.1) *underestimates* the widths for $Z\to b\bar{b}$ and especially $Z\to t\bar{t}$ and $W\to t\bar{b}$.

Simple parametrizations are available[17] for the QCD corrections to $Z\to t\bar{t}$, $b\bar{b}$; however, for the $W\to t\bar{b}$ decay, where the masses of the final-state quarks are different, it is necessary to use the general formula of Chang *et al.*[16] [Note that the relevant formula (3.11) of that reference has to be multiplied with $3/M_i^2$.] We also took the quark-mass dependence of α_s into account, using the parametrization of Halzen;[18] we checked that the parametrization of De Rújula and Georgi[17] reproduces this result up to less than 0.1%.

The importance of the radiative corrections is demonstrated by Fig. 1, where we show the QCD corrections to R as a function of m_t for $m_b=5$ GeV, $\sin^2\theta_W=0.22$, $N_\nu=3$, and[7] $R_\sigma=3.41$. The dotted-dashed curve has been obtained using Eq. (2.1); in this case the QCD corrections have almost the same effect on Γ_W and Γ_Z,which leads to an almost complete cancellation in R. That led the authors of Refs. 6 and 7 to the conclusion that R is insensitive to the QCD scale parameter Λ. Figure 1 shows that the uncertainty in R that results from the uncertainty in Λ is still much smaller than the uncertainty[6,7] that emerges from errors in the quark distribution functions. However, for $m_t \sim 50$–65 GeV, Eq. (2.1) overestimates R by about 0.2; this error is as large as the theoretical uncertainty[6,7] on R and thus clearly not negligible. The incorporation of the full QCD corrections increases the upper bounds on m_t quoted in Refs. 6 and 7 by about 3 GeV.

The curves of Fig. 1 show steps at $m_t=M_Z/2$ and $m_t=M_W-m_b$; this is due to the fact that the QCD-corrected decay widths of gauge bosons into heavy quarks are finite at the boundary of phase space. However, our formulas cannot be trusted at the very edge of phase space, since here nonperturbative effects are important; here the gauge bosons mix with heavy-quark bound

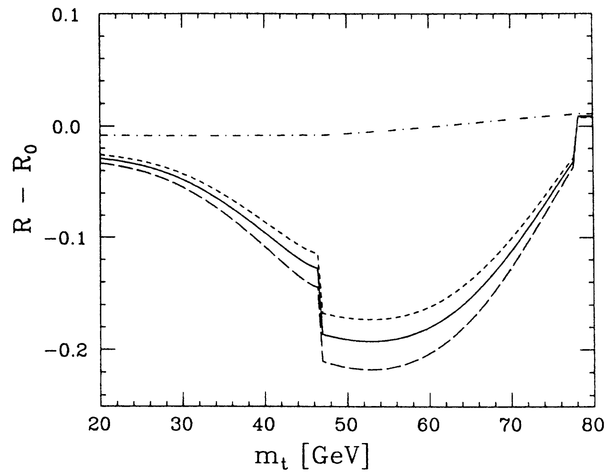

FIG. 1. The difference $R-R_0$ as a function of the mass of the top quark; R_0 is the tree-level prediction, whereas R includes QCD radiative corrections. The dotted-dashed curve has been computed under the assumption that these corrections only amount to multiplying all hadronic gauge-boson decay widths with $(1+\alpha_s/\pi)$, whereas the other three curves include full quark-mass-dependent corrections (Ref. 16), with $\Lambda_{\rm QCD}=0.1$ GeV (short-dashed), 0.2 GeV (solid), and 0.4 GeV (long-dashed), respectively. For the remaining figures we have chosen $\Lambda_{\rm QCD}=0.2$ GeV. Here and in the following figures we have assumed $\sin^2\theta_W=0.22$, $m_b=5$ GeV, $M_W=83$ GeV, and $M_Z=M_W/\cos\theta_W$. All results are for $R_\sigma=3.41$.

states.[19]

Finally it should be mentioned that the fully QCD-corrected standard-model prediction for R is almost insensitive to m_t as long as $m_t \lesssim 45$ GeV; i.e., a 23-GeV top quark is *not* favored over a 40-GeV top quark. However, the main conclusion of Refs. 6 and 7 is unaltered by the incorporation of full QCD corrections; the standard-model prediction for R is still somewhat above the measured central value although for $m_t \lesssim 56$ GeV the discrepancy is less than one standard deviation. In the next section we investigate how nonstandard physics can help to reduce the prediction for R. Especially it is interesting to investigate whether additional generations of (essentially) massless neutrinos can be allowed by the data on R if such new physics is present.

III. NONSTANDARD WAYS TO REDUCE THE PREDICTION FOR R

In this section we consider various extensions of the standard model that can bring down the prediction for R. We start with the simplest extension and proceed to investigate scenarios of increasing complexity.

A. Two Higgs doublets

The introduction of a second Higgs doublet[20] is probably the simplest extension of the standard model. At the first glance this seems to be an attractive possibility to reduce R since the Z cannot decay into a pair of identical neutral scalar particles. By choosing the mass of the charged Higgs boson to be $M_Z/2$ one can thus easily construct a model where Γ_W is larger than in the standard model whereas Γ_Z is unaltered.

The width for the decay of a gauge boson V into two scalars H_1, H_2 with masses m_1, m_2 is given by

$$\Gamma(V\to H_1H_2)=\frac{|\mathbf{p}|^3}{6\pi M_V^2}\,|a|^2\,, \tag{3.1}$$

where $\mathbf{p}^2=[(M_V^2-m_1^2-m_2^2)^2-4m_1^2m_2^2]/4M_V^2$ and a is the VH_1H_2 coupling constant. These constants are functions of two angles α and β, where α describes the mixing of the two neutral scalar bosons, and $\tan\beta$ is the ratio of the two vacuum expectation values.[20] The model contains two neutral scalars (H_h, H_l, with H_l being the lighter one), one neutral pseudoscalar $H_{\rm PS}$ and the charged scalar $H^\pm$. There are thus six $V\to H_1H_2$ decay modes, with coupling constants:[21]

$$\begin{aligned}
&a_{WH_hH^+}=\frac{g}{2}\sin(\alpha-\beta)\,,\quad a_{WH_lH^+}=\frac{g}{2}\cos(\alpha-\beta)\,,\\
&a_{WH_{\rm PS}H^+}=\frac{g}{2}\,,\quad a_{ZH^+H^-}=\frac{g}{2}\frac{\cos(2\theta_W)}{\cos\theta_W}\,,\\
&a_{ZH_hH_{\rm PS}}=\frac{g}{2}\frac{\sin(\alpha-\beta)}{\cos\theta_W}\,,\quad a_{ZH_lH_{\rm PS}}=\frac{g}{2}\frac{\cos(\alpha-\beta)}{\cos\theta_W}\,,
\end{aligned} \tag{3.2}$$

where g is the SU(2) gauge coupling. In the most general case we can treat $\alpha-\beta$ and the four Higgs-boson masses all as free parameters. We used the program MINUIT of the CERN software library to search for the set of parameters that minimizes R, imposing the experimental constraint[22] $m_{H^+}\geq 19$ GeV. The result is that there are two situations that lead to an equal decrease in R. In both cases m_{H^+} is at its boundary value, i.e., $m_{H^+}=19$ GeV; then one can either choose $m_{\rm PS}=0$, m_{H_h}, $m_{H_l}>M_Z$, or $m_{H_h}=m_{H_l}=0$, $m_{\rm PS}>M_Z$. In both cases R is independent of $\alpha-\beta$. The first case might be more interesting since it describes an axionlike scenario. However, even in this extreme case Γ_W can only be raised by 52 MeV, while Γ_Z simultaneously increases by 20 MeV. Therefore, R can at best be lowered by 0.1 by this approach. That means that even if the top quark is light one would have to introduce eight new Higgs doublets in order to bring the prediction of R down to its measured central value.

B. Supersymmetry

Here we discuss the minimal supersymmetric extension of the standard model.[23] We first observe that the two Higgs-boson doublets of this model cannot decrease R since the charged Higgs bosons are always heavier[24] than the W boson. Furthermore, the recent UA1 bound on squark masses,[25] $m_{\tilde q}\gtrsim 45$ GeV, excludes the possibility that squarks contribute to the total W decay width. This leaves us with sleptons and Higgsinos and gauginos, which we will discuss in turn.

1. Sleptons

We will start with a discussion of the sleptons. The width for the decay of a gauge boson into two sleptons is given by Eq. (3.1), where the relevant coupling constants are[26,27]

$$a_{W\tilde{e}_L\tilde{\nu}_L}=\frac{g}{\sqrt{2}}\ ,\quad a_{Z\tilde{e}_R\tilde{e}_R}=g\frac{\sin^2\theta_W}{\cos\theta_W}\ ,$$
$$a_{Z\tilde{e}_L\tilde{e}_L}=\frac{g}{2\cos\theta_W}(1-2\sin^2\theta_W)\ ,\quad a_{Z\tilde{\nu}_L\tilde{\nu}_L}=\frac{g}{2\cos\theta_W}\ . \tag{3.3}$$

Since we want to minimize R we clearly need $m_{\tilde{e}_R}>M_Z/2$. This leaves $m_{\tilde{e}_L}$ and $m_{\tilde{\nu}_L}$ as the only free parameters, subject only to the DESY PETRA bound[28] $m_{\tilde{e}_L}\gtrsim 22$ GeV. Scanning the whole allowed parameter space we found that it is *impossible* to reduce R from its SM prediction by introducing slepton decay modes for gauge bosons. The crucial difference to the case of a model with two Higgs doublets is that the real and imaginary parts of the $\tilde{\nu}_L$ field necessarily have the same mass; it is therefore impossible to close the $Z\to\tilde{\nu}_L\tilde{\nu}_L$ channel without sharply reducing the width of $W\to\tilde{e}_L\tilde{\nu}_L$.

2. *Gauginos and Higgsinos*

In minimal supersymmetric models the gauge-Higgs-fermion sector contains four neutral Majorana fermions and two charged Dirac fermions. The masses and mixings of these fields are determined by three parameters:[23] the gluino mass $|\mu_3|$, the supersymmetric Higgsino mass $2m_1$, and the ratio of the two vacuum expectation values (VEV's) of the neutral Higgs fields, $\langle\overline{H}^0\rangle/\langle H^0\rangle\equiv\tan\beta$, where $\langle\overline{H}^0\rangle$ and $\langle H^0\rangle$ give masses to $I_3=+\frac{1}{2}$ and $I_3=-\frac{1}{2}$ quarks, respectively. We denote the neutral mass eigenstates by $\tilde{Z}_i$, $i=1$–4, where $i=1$ labels the lightest neutralino, and the charginos by $\tilde{W}_+,\tilde{W}_-$, where the former is the heavier chargino.

The couplings of the gauge bosons to gauginos can be found in Refs. 26 and 29–31. The decays of gauge bosons into light gauginos have also been extensively studied in the literature.[30–32] We, therefore, present only the results on how the existence of light gauginos can change the prediction for R.

In Fig. 2 we present lines of constant R in the $2m_1-\mu_3$ plane, for $m_t=40$ GeV, $\Lambda=200$ MeV, $R_\sigma=3.41$, and $\tan\beta=1(2a)$ and $\tan\beta=2.5(2b)$, respectively. The region in between the dotted lines is excluded experimentally[28] since there the $\tilde{W}_-$ is lighter than 22 GeV. The solid, long dashed, dotted-dashed, and short-dashed curves are lines with $R=8.75$, 9, 9.5, and 10, respectively; for the given set of parameters the standard model predicts $R=9.2$. We see that the decay of gauge bosons into gauginos can significantly decrease the prediction for R, but only in a small part of parameter space; in other words, a precise measurement of R might tightly constrain the allowed values of $2m_1$ and μ_3. These constraints can be tightened even further at the Z^0 factories, the Stanford Linear Collider (SLC) and the CERN collider LEP I, since in the tiny region (bounded by the solid line) around $2m_1=40$ GeV, $\mu_3=-80$ GeV [see Fig. 2(b) and the short-dashed curve in Fig. 3] the decay $Z\to\tilde{Z}_2\tilde{Z}_2$ occurs with a branching ratio of 1–2%, whereas in the crescent-shaped region above $\mu_3=300$ GeV no nonstandard Z decays are observable. It is also interesting to note that in the former case the increase in Γ_W is mostly due to $W\to\tilde{W}_-\tilde{Z}_2$ decays, whereas in the region with $\mu_3>300$ GeV only $W\to\tilde{W}_-\tilde{Z}_1$ contributes. In this last case the $\tilde{Z}_1$ eigenstate is a complicated mixture[31] of all four current neutralino states; the large $W\to\tilde{W}_-\tilde{Z}_1$ width is partly due to the large Z-ino component of the $\tilde{Z}_1$, and partly due to the Higgsino components of both the $\tilde{W}_-$ and the $\tilde{Z}_1$. Finally it should be pointed out that in this region of parameter space weak gaugino signals will be the main supersymmetry (SUSY) signal at the Fermilab Tevatron, since gluino masses of more than 300 GeV imply[23] squark masses of more than 250 GeV; both masses are beyond the detectability limit[33] of this accelerator.

As shown in Fig. 3 the reduction in R is only sizable if $M_{\tilde{W}_-}$ lies between $M_Z/2$ and 55 GeV. It is also interesting to note that the absolute minimum of R, which is reached for large and positive μ_3, only increases from 8.62 to 8.7 when $\tan\beta$ is raised from 1 to 2.5. This brings

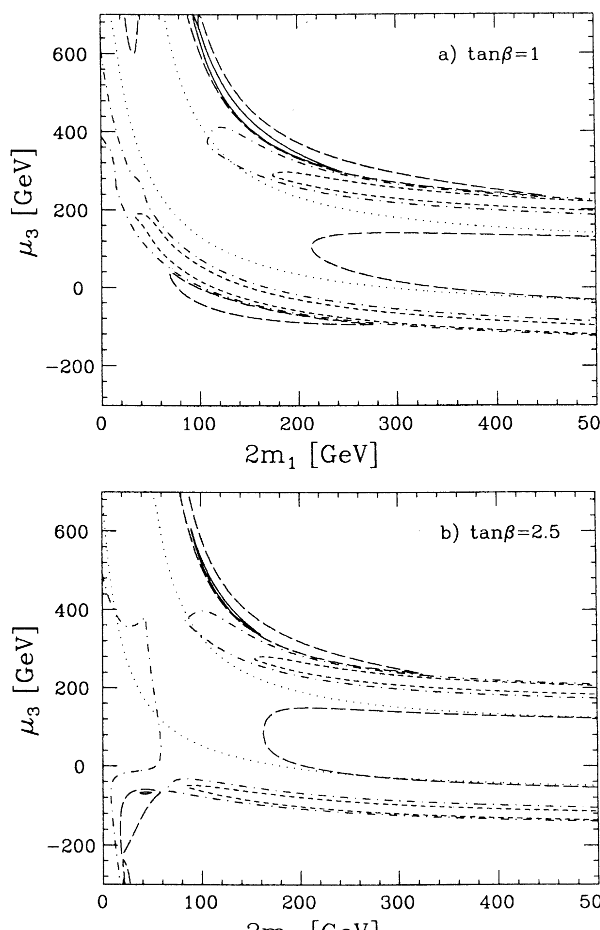

FIG. 2. Contours of constant R in the μ_3-2m_1 plane in minimal supergravity, with (a) $\tan\beta=1$ and (b) $\tan\beta=2.5$. Squarks and sleptons are assumed to be too heavy to contribute. On the solid, long-dashed, dotted-dashed, and short-dashed lines R has the value 8.75, 9, 9.5, and 10, respectively. We have taken $m_t=40$ GeV and $R_\sigma=3.41$. The region between the dotted lines is excluded experimentally since here $M_{\tilde{W}_-}<22$ GeV.

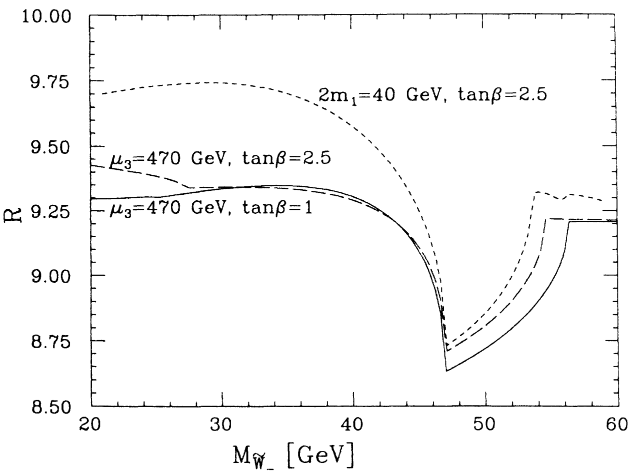

FIG. 3. R as a function of $M_{\tilde{W}_-}$ in minimal supergravity, for $m_t=40$ GeV and $R_\sigma=3.41$, where it has again been assumed that sfermions do not contribute. Note that on the two curves for $\mu_3=470$ GeV, $2m_1$ is always positive and lies between 100 GeV and 250 GeV, while on the curve for $2m_1=40$ GeV, μ_3 varies between $+180$ GeV ($M_{\tilde{W}_-}=22$ GeV) and -200 GeV ($M_{\tilde{W}_-}=59$ GeV); $M_{\tilde{W}_-}>59$ GeV is not possible for the given choice of $2m_1$ and $\tan\beta$.

the prediction of R very close to the measured[9] central value of 8.4. The effects on the bounds on m_t and N_ν will be discussed in Sec. IV.

C. Superstring-inspired E(6) models

E(6) supergravity models[14] offer several possibilities to reduce the prediction of R. The exotic SU(3)-singlet fermions that are part of the 27-dimensional representation of E(6) can decrease Γ_Z/Γ_W while the mixing between exotic quarks and standard $q=1/3$ quarks as well as the mixing of the standard Z boson with a new Z' boson can change both R_Γ and R_σ [see Eq. (1.1)]. We will investigate each of these three possibilities in the following subsections.

1. Exotic leptons

Besides the standard leptons, each 27-dimensional representation of E(6) contains a right-handed neutrino ν_{R_i}, another $SU(2)_L\times U(1)_Y$ singlet N_i, and two $SU(2)_L$ doublets H_i and $\bar{H}_i$ where $Y(\bar{H}_i)=-Y(H_i)=\frac{1}{2}$ (i is a generation index). The ν_{R_i} are not of interest here since they couple neither to the W nor to the Z. In the most general case[34,35] the fermions of the N_i, H_i, and $\bar{H}_i$ superfields mix with each other and with the superpartners of the gauge bosons. However, if one works in the basis where only one of the three (or more) H_i^0 scalars, one of the $\bar{H}_i^0$, and one of the N_i acquire a nonvanishing VEV, it is natural[34,36] to assume that the superpartners of those scalars that do *not* have a VEV decouple from the gaugino-Higgsino sector. Here we furthermore assume that there is no fourth generation and no "survivors" from additional $27+\overline{27}$ representations that mix with the exotic fermions. In this minimal case[37,38,34] the model contains six exotic neutral Majorana fermions χ_i and two charged exotic Dirac fermions L_i.

Following Ref. 35 we note that the two lightest of the neutral exotic fermions, $\chi_{1,2}$, cannot be much heavier than the Z boson. Since no such bound exists for the L_i one naively expects that the introduction of these exotic leptons increases Γ_Z/Γ_W and thus R; this has been verified in Ref. 39 for a simple scenario [no mixing between the $SU(2)_L$ doublets and the singlets]. Here we will show, however, that R can also be substantially *decreased* provided the parameters of the 6×6 neutralino mass matrix have certain values.

Following Ref. 34 we will work in the basis where the mass matrix of the charged exotic leptons is already diagonal and positive; this can be achieved by two rotations, one each in the H_1-H_2 and $\bar{H}_1$-$\bar{H}_2$ spaces, which do not change any gauge couplings. After these rotations each L_i^+ Dirac spinor consists of the Weyl spinor of the $\bar{H}_i$ superfield and the conjugate Weyl spinor of the H_i superfield. In this basis the mass matrix for the six χ_i contains ten free parameters,[34,35] two of which are m_{L_1} and m_{L_2}; if one requires all Yukawa couplings to remain in the perturbative regime all the way up to the grand-unified-theory (GUT) scale $M_X\gtrsim10^{16}$ GeV, the absolute value of the eight remaining parameters of the χ_i mass matrix cannot be larger[35] than 100 or 150 GeV.

The width for the decay of a vector boson V into two fermions f_1 and f_2 with masses m_1 and m_2 can be written as

$$\Gamma(V\to f_1f_2)=\frac{\Delta_{12}}{12\pi M_V^2}\,|\mathbf{p}|\left[(a^2+b^2)\left[2M_V^2-m_1^2-m_2^2-\frac{(m_1^2-m_2^2)^2}{M_V^2}\right]+6(a^2-b^2)m_1m_2\right], \quad (3.4)$$

where $|\mathbf{p}|$ is the same as in Eq. (3.1) and a and b are vector and axial-vector couplings, respectively; the statistics factor Δ_{12} is $\frac{1}{2}$ if f_1 and f_2 are identical Majorana fermions and 1 otherwise. For the case of Z bosons decaying into exotic L_i leptons we have

$$a_{ZL_iL_j}=\frac{g}{2}\frac{1-2\sin^2\theta_W}{\cos\theta_W}\delta_{ij}\ ,\quad b_{ZL_iL_j}=0\quad (i=1,2)\ . \quad (3.5a)$$

The $Z\chi_i\chi_j$ coupling is purely vector (axial vector) if the product of the ith and jth eigenvalue of the neutral fermion mass matrix is negative (positive); in either case its absolute value t_{ij} is given by

$$t_{ij}=\frac{g}{2\cos\theta_W}\left| n_1^{(i)}n_1^{(j)}+n_4^{(i)}n_4^{(j)}-n_2^{(i)}n_2^{(j)}-n_5^{(i)}n_5^{(j)}\right| , \tag{3.5b}$$

where $n_1^{(i)}$, $n_2^{(i)}$, $n_4^{(i)}$, and $n_5^{(i)}$ are the H_1^0, $\overline{H}{}_1^0$, H_2^0, and $\overline{H}{}_2^0$ components of the ith eigenvector. Finally, for the decay $W\to L_i\chi_j$ we find

$$\begin{aligned} a_{WL_i\chi_j}&=\frac{g}{2\sqrt{2}}(n_j n_{3i-1}^{(j)}-n_{3i-2}^{(j)}) ,\\ b_{WL_i\chi_j}&=\frac{g}{2\sqrt{2}}(-n_j n_{3i-1}^{(j)}-n_{3i-2}^{(j)}) , \end{aligned} \tag{3.5c}$$

where n_j is the sign of the jth eigenvalue of the χ mass matrix.

Note that the $Z\chi_i\chi_j$ coupling can be tuned to zero by arranging for an explicit cancellation among the terms of Eq. (3.5b); the decay of the Z into charged exotic fermions can, however, only be suppressed by making the latter sufficiently heavy. Performing a MINUIT search in the ten-dimensional parameter space, we do, in fact, find that the prediction for R can be minimized by choosing $m_{L_1}=m_{L_2}=M_Z/2$; the other eight parameters of the 6×6 neutral-fermion mass matrix M have to be chosen as [in the basis $(H_1^0,\overline{H}{}_1^0,N_1,H_2^0,\overline{H}{}_2^0,N_2)$]

$$\begin{aligned} -M_{13}=M_{23}=M_{16}=M_{26}=-M_{34}=-M_{35}\\ =-M_{46}=M_{56}\equiv m . \end{aligned} \tag{3.6}$$

As discussed earlier the requirement of perturbative unification leads to an upper bound[35] on m; however, the prediction for R depends only very weakly on m, falling from 8.46 to 8.44 (for $m_t=40$ GeV, $\Lambda=200$ MeV, and $R_\sigma=3.41$) when m is raised from 100 GeV to 150 GeV. Note that the choice (3.6) leads to $m_{\chi_1}=m_{\chi_2}=0$, but since also $t_{11}=t_{12}=t_{22}=0$, Γ_Z is predicted to be the same (up to radiative corrections) as in the standard model. This extreme scenario may be cosmologically unacceptable[40] since the introduction of two additional neutrinolike states would increase the predictions for primordial ^{4}He production; however, introducing masses for χ_1 and χ_2 of 0.5 GeV circumvents this problem but increases the prediction for R by less than 0.2%.

On the other hand, the amount by which the prediction for R can be reduced does depend quite strongly on the masses of the two charged exotic fermions. This is demonstrated by Fig. 4 where we show the minimal possible value of R as a function of m_{L_1} for $m_{L_2}=20$ GeV (long-dashed), 47 GeV (solid), and 100 GeV (short-dashed). These curves have been obtained by fixing m_{L_1} and m_{L_2} and fitting the remaining eight parameters such that R is minimized. It is amusing to see that, for

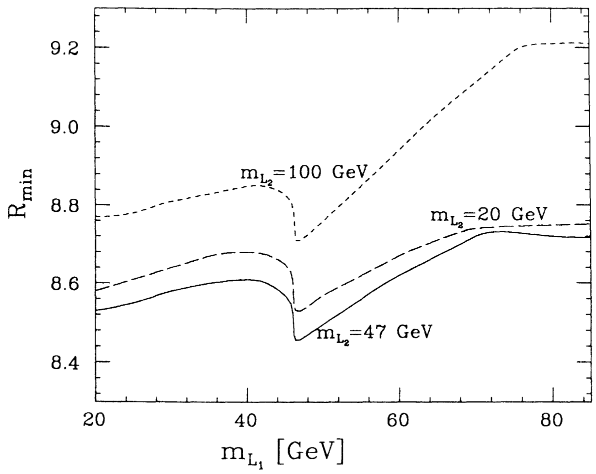

FIG. 4. The minimal value of R that can be obtained by the introduction of two generations of exotic E(6) leptons in superstring-inspired models; the third-generation exotic leptons are the superpartners of Higgs bosons and mix with the gauginos, and are thus not included. For each pair of charged lepton masses m_{L_1} and m_{L_2} the parameters of the 6×6 neutral Majorana fermion mass matrix have been chosen such that R is minimal. The results are for $m_t=40$ GeV and $R_\sigma=3.41$.

$m_{L_2}=47$ GeV and 47 GeV $\le m_{L_1}\le 50$ GeV, $R_{\min}$ is less than 0.1 units above the measured central value[9] of R.

2. *Exotic quarks*

Besides the exotic leptons discussed in Sec. III C 1, the 27-dimensional representation of E(6) also contains the exotic $q=-\frac{1}{3}$ quark D and the $q=+\frac{1}{3}$ quark D^c. These fields are singlets under $SU(2)_L$ and can thus not contribute to the total W decay width. They can nevertheless decrease the prediction for R if they mix[41] with standard down-type quarks.

Here, we will assume that each standard d quark mixes with just one exotic D quark. However, the mixing of left- and right-handed quarks will, in general, be different.[41] The light physical down-quark states can thus be written as

$$\begin{aligned} d_{iL}^p&=\cos\alpha_i^{(L)}d_{iL}+\sin\alpha_i^{(L)}D_i ,\\ d_{iR}^p&=\cos\alpha_i^{(R)}d_{iR}+\sin\alpha_i^{(R)}D_i^c , \end{aligned} \tag{3.7}$$

where $i=1,2,3$ is a generation index. The vector and axial-vector couplings for the $W\to ud$ and $Z\to d\bar{d}$ decays [see Eq. (3.4)] are then given by

$$a_{Wd_j^p\bar{u}_i}=-b_{Wd_j^p u_i}=\frac{g}{2\sqrt{2}}V_{ij}\cos\alpha_i^{(L)}\equiv\frac{g}{2\sqrt{2}}U_{ij} , \tag{3.7a}$$

where V_{ij} is the relevant element of the Kobayashi-Maskawa (KM) matrix;[42] and

$$\begin{aligned} a_{Zd_i^p\bar{d}_i^p}&=\frac{g}{2\cos\theta_W}(\tfrac{2}{3}\sin^2\theta_W-\tfrac{1}{2}\cos^2\alpha_i^{(L)}) ,\\ b_{Zd_i^p\bar{d}_i^p}&=\frac{g}{2\cos\theta_W}\tfrac{1}{2}\cos^2\alpha_i^{(L)} . \end{aligned} \tag{3.7b}$$

Note that none of the couplings in Eqs. (3.7) depends on $\alpha_i^{(R)}$, since d_R and D^c have the same SU(3) $\times$SU(2)$_L\times$U(1)$_Y$ quantum numbers. The couplings (3.7b) would depend on $\alpha_i^{(R)}$ if the Z mixes with an additional E(6) Z' boson[14] (see Sec. III C 3); here we have neglected this small effect.

We see from Eq. (3.7a) that the $V-A$ structure of the charged current is preserved; however, its overall strength is reduced by the factor $\cos\alpha_i^{(L)}$. Experimentally this would lead to a nonunitary U matrix, with $\sum_{j=1}^{3}|U_{ij}|^2=\cos^2\alpha_i^{(L)}$. Existing limits[43] on the nonunitarity of the U matrix thus force the d-D mixing for the first two generations to be negligibly small.[44] However, since the measured charged-current couplings of the physical b quark are all very small[43] and with the given *Ansatz* the neutral-current couplings of the d and s quarks are automatically flavor diagonal, there is almost *no* experimental bound on $\alpha_3^{(L)}$.

Note that the measured forward-backward asymmetry[45] of the process $e^+e^-\to b\bar b$ or the observed mixing in the K^0-$\bar K^0$ (Ref. 46) and B^0-$\bar B^0$ (Ref. 11) systems does not directly measure the couplings in Eqs. (3.7). In Ref. 47 it was suggested that the asymmetry can be explained by the exchange of nonstandard scalars whose Yukawa couplings are not proportional to fermion masses and thus essentially unconstrained, whereas the exchange of superpartners and/or exotic quarks can account for the observed mixings of K^0 and B^0 mesons. Furthermore the analysis of Ref. 48 does not apply here. There it was assumed that there is no top quark and that the b quark decays via both charged and neutral currents by virtue of its mixing with d and s quarks. In this case it can be shown[48] that $\Gamma(b\to l^+l^-X)/\Gamma(b\to l^+\nu_l X)>0.12$, which is already excluded by experiment.[49] In our case, however, the decay of b^p is via charged currents only as in the standard model.

Finally, the nonstandard contributions to the forward-backward asymmetry in $e^+e^-\to b\bar b$ mentioned above do not necessarily imply the existence of nonstandard contributions to b decays, since, for example, the former might be entirely due to scalar SU(2)-singlet leptoquark exchange with flavor-diagonal couplings.

On the other hand, Eqs. (3.7b) show that a nonvanishing $\alpha_3^{(L)}$ can greatly reduce the $Z\to b\bar b$ decay width. Note that the couplings (3.7b) depend quadratically on $\cos\alpha_3^{(L)}$ while the charged-current couplings (3.7a) are linear in that quantity. Therefore a nonvanishing $\alpha_3^{(L)}$ can reduce Γ_Z/Γ_W and thus R even if the top quark is light. This is demonstrated by the solid curve in Fig. 5 which shows R as a function of $\alpha_3^{(L)}$ for $m_t=40$ GeV, $R_\sigma=3.41$, and $\Lambda=200$ MeV. However, even in the extreme case $\alpha_3^{(L)}\simeq0.7$ the effect is not too big, reducing R from 9.2 to 8.95. Note that for $\alpha_3^{(L)}>1$, R is even larger than in the standard model since in this region the $W\to t\bar b$ decay width is suppressed while $Z\to t\bar t$ still contributes with full strength. Furthermore Eqs. (3.7b) show that the $Z\to b\bar b$ width never vanishes completely. If one neglects the small effects due to the finite b mass this width is proportional to $a_{Zb\bar b}^2+b_{Zb\bar b}^2$, which from Eqs. (3.7b) is bounded from below by

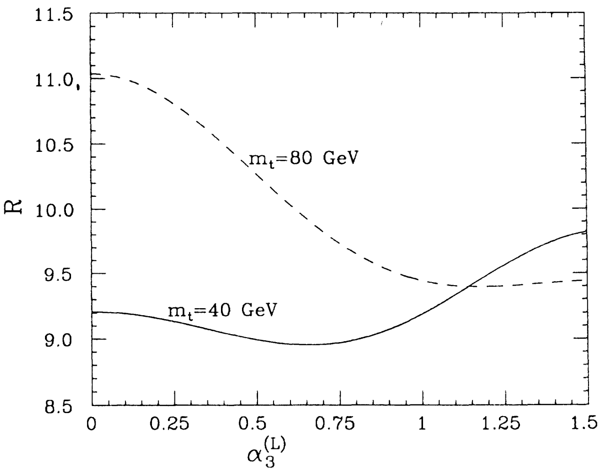

FIG. 5. The effect of b-D mixing, parametrized by the mixing angle $\alpha_3^{(L)}$ [see Eq. (3.7)], on R; D is an exotic SU(2)-singlet quark as present in the **27** of E(6). The heavier b-D eigenstate is assumed to be heavier than $M_Z-m_{b_l}$. The results are for $R_\sigma=3.41$.

$$a_{Zb\bar b}^2+b_{Zb\bar b}^2\geq\frac{g^2\sin^4\theta_W}{18\cos^2\theta_W}\ , \tag{3.8}$$

which is about 1/35 of the corresponding standard-model value; the bound (3.8) is saturated for $\cos^2\alpha_3^{(L)}=\frac{2}{3}\sin^2\theta_W$, which is much larger[43] than the experimental bound $\cos^2\alpha_3^{(L)}\geq|U_{bu}|^2+|U_{bc}|^2$.

The effects of a nonvanishing $\alpha_3^{(L)}$ are quite different if the top quark is heavy, $m_t>M_W-m_b$, as demonstrated by the dashed curve in Fig. 5. Since the $W\to t\bar b$ channel is now closed R is minimal where the bound (3.8) is saturated. Although even in this extreme case the predicted value of R is about one unit above the measured central value[9] it is well below the 90%-confidence-level (C.L.) upper bound. Note that, for large values of $\alpha_3^{(L)}$, R decreases as m_t increases from 0 to $m_Z/2$ and is almost independent of the top mass if $m_t>M_Z/2$.

3. Nonstandard neutral gauge bosons

Many superstring-inspired E(6) models predict[14] the existence of an additional neutral gauge boson with a mass of a few hundred GeV. This Z' boson will in general mix with the standard Z boson and thereby change its couplings to fermions. Note that this does not only change the ratio Γ_Z/Γ_W but also the partial $Z\to e^+e^-$ decay width as well as the $p\bar p\to Z+X$ production cross section.[50]

Following the notation of Ref. 5 we write the lighter physical Z boson as

$$Z_1=Z_1^0\cos\theta+Z_2^0\sin\theta\ , \tag{3.9}$$

where Z_1^0 and Z_2^0 are the standard SU(2)$_L\times$U(1)$_Y$ and nonstandard $\tilde{\mathrm{U}}(1)$ gauge boson. Here we consider three different possibilities for $\tilde{\mathrm{U}}(1)$; these are the U(1)$_\chi$, U(1)$_\psi$, and U(1)$_\eta$ of Ref. 5, where E(6)$\supset$SO(10)$\times$U(1)$_\psi$

TABLE I. The vector (a) and axial-vector (b) couplings of the standard fermions to the Z boson, see Eq. (3.4), in units of $g_Z \equiv g/2\cos\theta_W$, for the three nonstandard U(1) groups discussed in the text. Here $c \equiv \cos\theta$ [see Eq. (3.9)], $s \equiv \sin\theta\sin\theta_W$, and $x_W \equiv \sin^2\theta_W$. We have assumed that the coupling constant of the new U(1) is the same as that of $\mathrm{U}(1)_Y$, i.e., $g\tan\theta_W$.

	$\mathrm{U}(1)_\chi$		$\mathrm{U}(1)_\psi$		$\mathrm{U}(1)_\eta$	
	a/g_Z	b/g_Z	a/g_Z	b/g_Z	a/g_Z	b/g_Z
e	$c(2x_W-\frac{1}{2})+\frac{2}{\sqrt{6}}s$	$\frac{1}{2}c-\frac{s}{\sqrt{6}}$	$c(2x_W-\frac{1}{2})$	$\frac{1}{2}c-s\sqrt{5/24}$	$c(2x_W-\frac{1}{2})+\frac{1}{2}s$	$\frac{1}{2}c+\frac{1}{6}s$
ν	$\frac{1}{2}c+\sqrt{3/8}s$	$-\frac{1}{2}c-\sqrt{3/8}s$	$\frac{1}{2}c+\sqrt{5/72}s$	$-\frac{1}{2}c-\sqrt{5/72}s$	$\frac{1}{2}c+\frac{1}{6}s$	$-\frac{1}{2}c-\frac{1}{6}s$
u	$c(\frac{1}{2}-\frac{4}{3}x_W)$	$-\frac{1}{2}c+\frac{s}{\sqrt{6}}$	$c(\frac{1}{2}-\frac{4}{3}x_W)$	$-\frac{1}{2}c-\sqrt{5/24}s$	$c(\frac{1}{2}-\frac{4}{3}x_W)$	$-\frac{1}{2}c+\frac{2}{3}s$
d	$c(-\frac{1}{2}+\frac{2}{3}x_W)-\frac{2}{\sqrt{6}}s$	$\frac{1}{2}c-\frac{s}{\sqrt{6}}$	$c(-\frac{1}{2}+\frac{2}{3}x_W)$	$\frac{1}{2}c-\sqrt{5/24}s$	$c(-\frac{1}{2}+\frac{2}{3}x_W)-\frac{1}{2}s$	$\frac{1}{2}c+\frac{1}{6}s$

$\supset \mathrm{SU}(5)\times\mathrm{U}(1)_\chi\times\mathrm{U}(1)_\psi$ and $E(6)\supset\mathrm{SU}(3)_c\times\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y\times\mathrm{U}(1)_\eta$. $\mathrm{U}(1)_\eta$ emerges in rank-5 superstring models[37] whereas $\mathrm{U}(1)_\chi$ can obviously also be realized in an SO(10) grand-unified-theory (GUT) model. The neutral-current couplings of the standard-model fermions for these three cases are listed in Table I. As mentioned before, Z-Z' mixing also changes the production cross section $\sigma(p\bar{p}\to Z+X)$. This effect can be computed from the couplings of Table I, using the fact that in the standard-model $u\bar{u}$ annihilation contributes 2.67 times more to the total Z cross section at $\sqrt{s}=630$ GeV than $d\bar{d}$ annihilation.[51]

The mixing angle θ in Eq. (3.9) cannot be arbitrarily large.[5,52] Here we follow the recent analysis of Ref. 5 which gave the 90%-C.L. bounds:

$$\begin{aligned} &-0.07\le\theta_\chi\le 0.01\ , \\ &-0.08\le\theta_\psi\le 0.05\ , \\ &-0.2\le\theta_\eta\le 0.01\ . \end{aligned} \tag{3.10}$$

In Fig. 6 we show R as a function of m_t for the standard

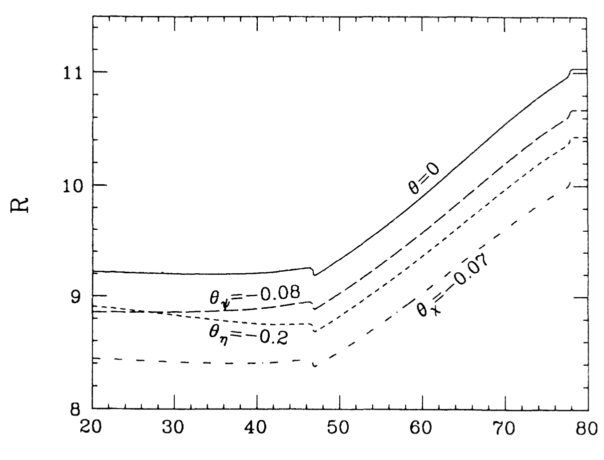

FIG. 6. The effect of Z-Z' mixing, parametrized by the mixing angle θ, on R. The subscripts ψ, η, and χ refer to different U(1) groups: $E(6)\supset\mathrm{SO}(10)\times\mathrm{U}(1)_\psi\supset[\mathrm{SU}(5)\times\mathrm{U}(1)_\chi]\times\mathrm{U}(1)_\psi$ and $E(6)\supset\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)_Y\times\mathrm{U}(1)_\eta$. The sign of θ is chosen such that R is reduced, whereas the magnitude of θ is the maximum allowed by present neutral-current (Ref. 5) data. It has been assumed that $R_\sigma=3.41$ in the standard model.

model (solid line) and the three models with additional $\tilde{\mathrm{U}}(1)$ gauge group, with θ chosen as small as allowed by the bound (3.10) since this always leads to the minimal prediction for R.

It is remarkable that although the bounds on θ are most restrictive for the $\mathrm{U}(1)_\chi$, this model leads to the maximal reduction of R. This is due to the fact that in this model one can simultaneously increase the partial $Z\to e^+e^-$ decay width by 6.4% and decrease Γ_Z and R_σ by 0.7% and 2%, respectively; taken together, this reduces the prediction for R by 9%. In the other two models these changes always tend to cancel each other.

IV. CONSTRAINTS ON NONSTANDARD PHYSICS FROM CURRENT MEASUREMENTS OF R

In the previous section we have discussed how nonstandard physics can reduce R below that expected in the standard model with three generations. On the other hand, the available experimental information on R has been used to limit the possible number of neutrino species[4,6,7] as well as masses of weak gauginos,[39] new quarks,[39,53] and Majorons.[53] Such discussions are possible because the value of R determined from the combined[9] results of UA1 (Ref. 2) and UA2 (Ref. 3) is saturated by the standard model and is even smaller than it for large values of m_t. In this section we discuss the prospects for limiting the masses of different kinds of new particles expected in various extensions of the standard model, and also the mixings of new quarks with the usual quarks. In particular we show that data on R in combination with bounds[5] on the splitting within quark doublets strongly disfavor several simple *Ansätze* for four-generation quark mass matrices. We finally discuss how various nonstandard-physics contributions to R, discussed earlier, affect the limit on the number of neutrino species and on the t-quark mass.

A. Fourth generation

Although the standard model of electroweak interactions is usually based on three generations of quarks and leptons, there exists no convincing argument which determines uniquely the number of generations. Proofs for or limits on the existence of a fourth generation (ν_E, E, t', b') are therefore very important. Various four-generation scenarios have been considered in the literature.[54] Nevertheless the masses of the members of the fourth generation and their weak mixing angles are still essentially unknown. If we assume that the new quarks (t',b') and leptons (ν_E,E) are sequential, then their neutral-current couplings are unaffected by any mixing with the first three generations. These are then given by the standard values and are (in units of $g/2\cos\theta_W$)

$$a_{Zf\bar{f}}=\pm(\tfrac{1}{2}-2\,|q_f|\sin^2\theta_W)\ \text{ for } q_f \gtrless 0\ ,\quad b_{Zf\bar{f}}=\mp\tfrac{1}{2}\ , \tag{4.1}$$

$$a_{Z\nu\bar{\nu}}=-b_{Z\nu\bar{\nu}}=\tfrac{1}{2}\ \text{ for neutrinos .}$$

The charged-current couplings depend, of course, on the elements of the extended Kobayashi-Maskawa (KM) matrix. In the following discussion we make the following standard assumptions. (i) The mixing between the first two and the fourth generation is negligibly small. This assumption can be justified by phenomenological analyses even in the presence of a fourth generation.[54] (ii) In a four-generation scenario the elements $V_{ij}(i,j=3,4)$ of the KM matrix are essentially unknown.[54] Even the unusual scenario with t dominantly coupled to b' is not excluded. We have therefore studied R as a function of $\cos\theta_M \equiv |V_{tb}|$. Note that the angle θ_M as defined here can be identified with a mixing angle in certain parametrizations of the extended KM matrix, with additional assumptions. We have assumed $|V_{td}|$ and $|V_{ts}|$ to be negligible. If we denote the mass eigenstates by b_l and b_h, respectively, ($m_{bl}\simeq 5$ GeV), then their charged-current couplings are given by (in units of $g/2\sqrt{2}$)

$$\begin{aligned} a_{Wtb_l}&=-b_{Wtb_l}=\cos\theta_M\equiv|y|\ ,\\ a_{Wtb_h}&=-b_{Wtb_h}=\sin\theta_M=\sqrt{1-|y|^2}\ . \end{aligned} \tag{4.2}$$

(iii) We also assume that the charge $\frac{2}{3}$ quark t' belonging to the fourth generation is heavy ($M_W<m_{t'}+m_{b_l}$) and hence does not contribute to W and Z decays.

In the calculation of contributions to the W and Z widths due to the decays $Z\to b_h\bar{b}_h$, $Z\to t\bar{t}$, and $W\to tb_h$, the mass-dependent $O(\alpha_s)$ QCD corrections mentioned in Sec. II have been used.

1. Bounds on masses and coupling for fourth generation

As remarked earlier experimentally R is bounded from above ($R<9.9$ at 90% C.L.).[9,6] Even this bound is nearly saturated by the theoretical expectation in the standard model with three generations, for all values of the t-quark mass.[6,7,3] Hence any new contribution to the total Z width or any reduction in the total W width, as compared to the standard model, is highly constrained.

To study these constraints one can consider various possible cases. If one assumes that the 4×4 KM matrix follows the usual pattern (i.e., it is nearly diagonal) one can set $|\cos\theta_M|\simeq 1$. In this case, the Z width receives additional contributions due to $Z\to\nu_E\bar{\nu}_E$, $Z\to b_h\bar{b}_h$, and possibly $Z\to E\bar{E}$ compared to the standard model, whereas the W width is increased through $W\to E\nu_E$. Naturally the mass of b_h is then strongly constrained. This has already been considered.[39,53] However, any such analysis involves some uncertainty due to the use of the theoretical estimate of $R_\sigma=\sigma(p\bar{p}\to W)/\sigma(p\bar{p}\to Z)$ involved in the calculations of the ratio R given by Eq. (1.1). The uncertainties in the knowledge of R_σ are a reflection of uncertainties in the determination of the structure functions. R_σ is crucially controlled by u_V/d_V where u_V and d_V are the u,d valence-quark densities in the proton. Latest analyses[6,7] using the recent deep-inelastic-scattering (DIS) data,[8] give

$$\begin{aligned} R_\sigma&=3.41\pm0.08\ \ (\text{Ref. }3)\ ,\\ R_\sigma&=3.36\pm0.09\ \ (\text{Ref. }2)\ . \end{aligned} \tag{4.3}$$

For the purpose of obtaining the most conservative bound we use $R_\sigma=3.27$, $m_E=41$ GeV, and $m_t=40$ GeV. [Note that UA1 has reported lower bounds $m_E>41$ GeV (Ref. 55) and $m_t>40$–45 GeV (Ref. 10).] The solid curve in Fig. 7 shows R_Γ expected for this case as a function of m_{b_h}, along with the upper limits on R_Γ, at 90% confidence level, indicated by solid lines A and B, implied for the two different choices of $R_\sigma=3.41$ corresponding to the central value of Ref. 7 and the most conservative value $R_\sigma=3.27$ implied by Eq. (4.3). Thus we get $m_{b_h}>26$ GeV at 90% C.L. An increase in m_E or m_t will reduce the W width even further and hence make the bound even more stringent, as is shown by the dashed curve drawn for $m_E=70$ GeV.

In this connection, it should be mentioned that an excess of low-thrust events containing muons at the maximum PETRA energies was interpreted as possible evidence of low mass, charge $+\frac{1}{3}$ quark ($m_{b_h}\sim 23$ GeV) by some authors.[56] The above analysis shows that such a scenario is disfavored by the collider data. This conclusion was already reported in Refs. 39 and 53 using rather simple-minded estimates of R_σ. It is worthwhile

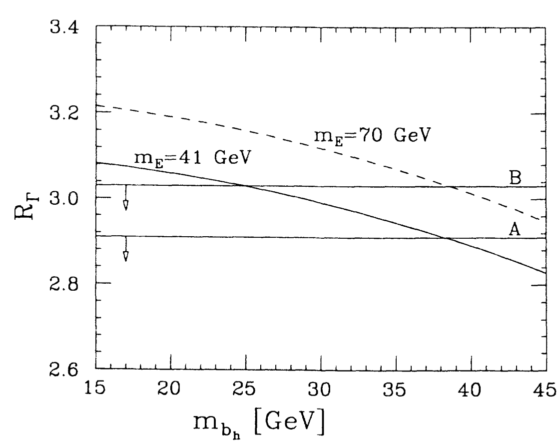

FIG. 7. R_Γ in models with a sequential fourth generation, for $m_t=40$ GeV. It has been assumed that $\cos\theta_M=1$, i.e., $b_h=b'$, and $m_{t'}>M_W-m_{b_h}$. The two lines A and B show the 90%-C.L. upper bounds on R_Γ that can be derived from the experimental bound on R for $R_\sigma=3.41$ and 3.27, respectively.

noting that the conclusion essentially remains the same, even with these new estimates of R_σ and inclusion of the correct mass dependence in the $O(\alpha_s)$ QCD corrections to W and Z widths.

Next we consider the case of a nonzero mixing between the third and fourth generation, i.e., $|y|<1$. From Eq. (4.2) it is clear that any deviation of θ_M away from zero will tend to decrease the W width since $m_{b_h}>m_{b_l}=5$ GeV. Since the neutral-current couplings are not affected by this mixing, the bounds on m_{b_h} are even stronger than shown in Fig. 7. For $m_{b_h}>M_Z/2$, the Z width receives a contribution only from $Z\to b_l\bar{b}_l$ and the effect of mixing is to reduce the W width. The solid curve in Fig. 8 shows the expected value of R as a function of θ_M, for $m_{b_h}=50$ GeV, $m_t=40$ GeV, $m_E=41$ GeV, and $\Lambda=0.2$ GeV. As can be seen, the data constrain θ_M strongly. The two solid lines A and B in Fig. 8 show upper limits at 90% C.L. on R_Γ, indicated by current data on R, obtained by using $R_\sigma=3.41$ (3.27). The bounds on θ_M as a function of m_t for $m_{b_h}=50$ GeV, with the most conservative choices of R_σ and m_E are shown by the solid curve in Fig. 9. The discontinuities in the curve at $m_t=M_Z/2$ and $m_t=M_W-m_{b_h}$ are, as discussed earlier, due to the fact that the QCD-corrected widths of gauge bosons into heavy quarks are finite at the boundary of phase space. As can be clearly seen from both the figures the scenario where the t is dominantly coupled to b' is strongly disfavored by the current data on R.

If $m_{b_h}<M_Z/2$ then of course for small values of m_{b_h} the expected value of R could be larger than the 90% C.L. for zero mixing, so any finite mixing will make matters worse. When we choose a value of m_{b_h} allowed by data on R by the above analysis, e.g., $m_{b_h}=30$ GeV, we again find that a large mixing between b and b' is not favored by the data, as shown by the dashed curves in Figs. 8 and 9, respectively. It should be emphasized here

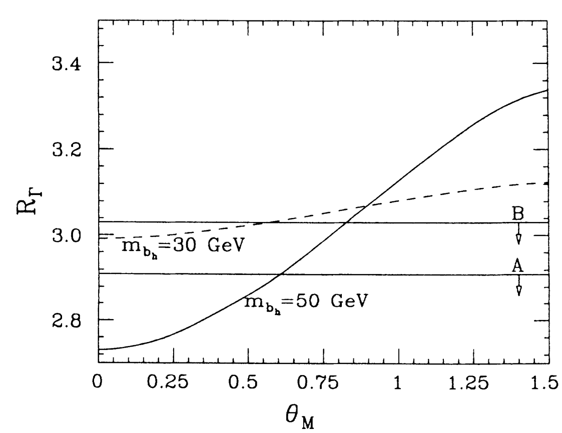

FIG. 8. R_Γ as a function of θ_M, where $\cos\theta_M=|V_{tb}|$, for $m_t=40$ GeV and $m_E=41$ GeV. The t' quark is assumed to be too heavy to contribute. The meaning of the lines A and B is as in Fig. 7.

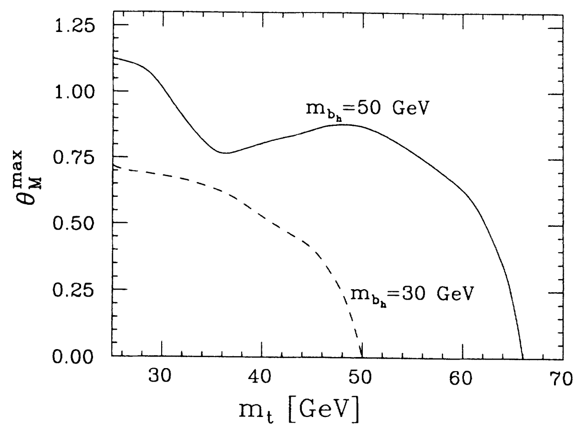

FIG. 9. The maximal allowed value of θ_M, corresponding to the bound $R<9.9$, for $R_\sigma=3.27$, $m_E=41$ GeV, and $m_{t'}>M_W-m_{b_h}$. Larger values of m_E or R_σ would lead to even stronger bounds.

that this method provides the only phenomenological bound on $|V_{tb}|^2$ at present.

2. *Implications for model building*

As pointed our earlier,[39] such lower bounds on the mass m_{b_h} have serious implications for four-generation model building. Here we discuss this question in further details. It has been established by experiment that, at least for the three-generation case, the off-diagonal elements of the KM matrix are small.[43] The discussion above, about the limits on $V_{tb'}$, indicates that this pattern is likely to persist in the case of the four-generation scenario also. One simple way to incorporate this pattern of quark mixing is to assume that the up and down mass matrices are nearly proportional:[57]

$$M_u=\alpha M_d+M' , \tag{4.4}$$

where α is a constant and M' is a perturbation. The Stech *Ansatz*[58] for the quark mass matrices gives a very similar form with additional constraints on M'. Note that for $M'=0$ the KM matrix reduces to the identity matrix, and we have the relations[58]

$$m_i^u/m_i^d=\text{const} . \tag{4.5}$$

Here m_i^u (m_i^d), $i=1,2,3,4$, is the mass of the up (down) quark belonging to the ith generation. If m_t is found to be in the neighborhood of 50 GeV this relation will be satisfied, to a good approximation, for the second and third generations. Equation (4.5) is expected to be invalid for the light first-generation quarks since M' is not negligible in this case. On the other hand, we may expect it to be a good approximation for the heavier generations. We thus have

$$\frac{m_{t'}}{m_{b'}}=\frac{m_t}{m_b} \tag{4.6a}$$

or

$$m_{t'}-m_{b'}=\left[\frac{m_t-m_b}{m_b}\right]m_{b'} . \tag{4.6b}$$

Equation (4.6b) above is interesting, because the lower limits on m_t indicated by UA1 data[10] and the limit on $m_{b'}$ implied by our analysis of the previous section imply a lower limit on $\Delta m_4=m_{t'}-m_{b'}$. On the other hand, the radiative-correction parameters $\rho^{\rm NC}$ or Δr (Ref. 59) are sensitive to isospin breaking and, hence limit Δm_4 from above. A recent analysis[5] gives (for $m_H<1000$ GeV)

$$\Delta m_4<200 \text{ GeV} . \tag{4.7}$$

If we use $m_t>40$ GeV and $m_E=41$ GeV current data on R imply $m_{b'}>26$ (40) GeV at 90% C.L. for $R_\sigma=3.27$ (3.41) using $m_b=5$ GeV. Equation (4.6b) above then gives $\Delta m_4>185$ (280) GeV for the two choices of R_σ. Thus the simple model leading to Eq. (4.4) is likely to be in trouble if a fourth generation exists. Note that this and the following results describe the most conservative bounds. Any increase in m_t or m_E will worsen the disparity between the upper and lower bounds on Δm_4 given here.

Other four-generation models have been considered by several authors.[60–62] Using the Gronau-Johnson-Schechter *Ansatz*[63] it was shown[60] that $m_{t'}$ is given by

$$\frac{m_{t'}}{m_{b'}}\simeq\left[\frac{m_t}{m_b}\right]\left[\left[\frac{m_b}{m_s}\right]\left[\frac{m_c}{m_t}\right]\right]^{1/2},$$

i.e.,

$$\Delta m_4=\left[\frac{(m_tm_c)^{1/2}-(m_bm_s)^{1/2}}{(m_bm_s)^{1/2}}\right]m_{b'} . \tag{4.8}$$

Using the lower bounds on m_t and $m_{b'}$, $m_b=5$ GeV, $m_c=1.3$ GeV, and $m_s=0.150$ GeV, we get $\Delta m_4\gtrsim 195$ (295) GeV for $R_\sigma=3.27$ (3.41), again barely consistent with the bound (4.7).

The simple relation (4.5) is also obtained if an S_4 permutation symmetry is imposed on the standard $SU(2)\times U(1)$ model.[61] Moreover, in this model the charged-lepton masses also have a similar hierarchy. Consequently, one also gets

$$\Delta m_4=\frac{m_E}{m_\tau}(m_t-m_b) . \tag{4.9}$$

Using the lower bounds stated above, $m_\tau=1.78$ GeV, and[55] $m_E\geq 41$ GeV we get $\Delta m_4>920$ GeV, in gross conflict with the bound (4.7).

The existence of a horizontal symmetry among the four generations has been used[62] to predict $m_{b'}/m_{t'}$. This model gives

$$\frac{m_c}{m_s}=\frac{m_t+m_{t'}}{m_b'+\eta m_b} .$$

Hence

$$\Delta m_4=\left[\frac{m_c-m_s}{m_s}\right](m_{b'}+\eta m_b)-(m_t-\eta m_b) . \tag{4.10}$$

Here $\eta=\pm1$. Using the limits on and values of quark masses stated above, with $\eta=-1$ we get, for $R_\sigma=3.27$ (3.41),

$$\Delta m_4>116 \; (225) \text{ GeV} ,$$

which is only marginally consistent with (4.7) unless R_σ is close to its lower bound. Of course, the results derived from Eqs. (4.8) and (4.10) are sensitive functions of m_s and hence should be treated with caution. But it is noteworthy that a lower bound on $m_{b'}$ obtained from an analysis of R already is capable of putting rather severe constraints on four-generation model building.

B. Supersymmetry

In this subsection we discuss if the current data on R can give us any constraints on sparticle masses. We consider the minimal supersymmetric extension of the standard model[23] mentioned earlier. We first recall the mass of the charged Higgs boson in these models is always expected[24] to be greater than M_W, hence obviously this analysis can say nothing about it. On the hand, at least one neutral scalar boson is expected to be lighter than M_Z (Refs. 24 and 64). If the neutral pseudoscalar is also sufficiently light in principle its contribution to the width via $Z\to H_2^0H_{\rm PS}$ might be able to bound the sum of their masses from below. We find, however, that this width is too small to lead to any significant constraint.[65] Note that in this case $\cos(\alpha-\beta)$ of Eq. (3.2) can be expressed in terms of the masses of the neutral Higgs bosons and M_Z (Ref. 21). The squark masses are already bounded,[25] $m_{\tilde q}\gtrsim M_Z/2$. Hence in the following we will discuss only the slepton and gaugino-Higgsino sectors.

1. Sleptons

The couplings of sleptons to Z an W have already been given in Eq. (3.3). It was already remarked earlier that it is *impossible* to reduce R by contributions to the gauge boson decay widths involving sleptons. On the other hand, the increase in R caused by Z/W decays into sleptons is not large enough to give any meaningful bound beyond the current bound[28] of $m_{\tilde e_{L,R}}\gtrsim 22$ GeV.

2. Gauginos and Higgsinos

The details of the gauge-Higgs-fermion sector have already been discussed in Sec. III B 2. There we have seen that the effects on R due to W and Z decays into final states involving gauge-Higgs fermions can be large and depend on the three parameters discussed earlier. While a reduction of R is possible only for a very small part of the parameter space, it is relatively easy to increase R beyond its standard-model value.

The solid (dashed) curve in Fig. 10 shows R_Γ as a function of $M_{\tilde W_-}$ for a small gluino mass $\mu_3=55$ GeV with

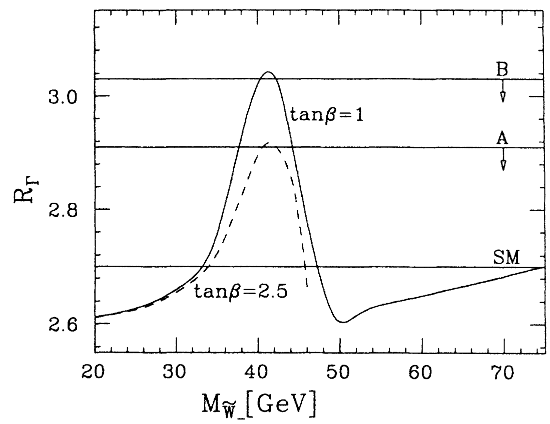

FIG. 10. R_Γ in minimal supergravity as a function of $M_{\tilde{W}_-}$, for $\mu_3=55$ GeV and $m_t=40$ GeV. Note that for $\tan\beta=2.5$ and the given value of μ_3, $M_{\tilde{W}_-}$ cannot be larger than about 46 GeV. The A and B lines are the same as in Fig. 7, whereas the line labeled SM shows the standard-model prediction.

$\tan\beta=1$ (2.5), $\Lambda=0.2$ GeV, and $m_t=40$ GeV. The choice of μ_3 corresponds to the lower bound reported[25] by the UA1 Collaboration from the collider data. Note that there are two possible values of $2m_1$ that lead to a given value of $M_{\tilde{W}_-}$ for given μ_3 and $\tan\beta$, but of these two possible solutions we choose the one where $\tilde{Z}_2$ is dominantly Z-ino. In this range $\tilde{Z}_1$ is essentially a photino.[66] The horizontal line A and B indicate the upper bound at 90% C.L. for $R_\sigma=3.41$ and for the most conservative value of $R_\sigma=3.27$, respectively. The horizontal line labeled SM indicates the R_Γ expected in the standard model with three generations. The values of R_Γ expected including the effect of decays into gauginos have also been calculated for three generations. As has been observed in Ref. 30, the closing up of the channel $W\to\tilde{W}_-\tilde{Z}_2$, when $Z\to\tilde{W}^-_-\tilde{W}^+_-$ is still open, causes R to rise appreciably above the standard-model value and experimental upper limits at 90% C.L. For example, even with the most conservative choice of $R_\sigma=3.27$, $M_{\tilde{W}_-}$ between 39 and 44 GeV is ruled out for $\tan\beta=1$. For the choice of $R_\sigma=3.41$, this region extends from 37 to 45 GeV (40–44) for $\tan\beta=1$ (2.5). This indicates what range of $\tilde{W}_-$ masses is disfavored by current data on R if the $\tilde{Z}_1$ is a light photinolike state. We note, however, that the same values of $M_{\tilde{W}_-}$ can also be obtained for very different model parameters (e.g., large μ_3, as can be seen in Fig. 3); the couplings of the gauge-Higgs fermions to the gauge bosons, and hence the prediction for R, can therefore be quite different from the light-photino case, so that one has to be very careful in ruling out ranges of $\tilde{W}_-$ and $\tilde{Z}_{1,2}$ masses.[31] For smaller values of $M_{\tilde{W}_-}$ a large number of missing-energy events are expected at the collider[30,67] and their presence can be confirmed or ruled out by current UA1 data. The implications of these supersymmetric contributions for the number of neutrino species will be discussed at the end of this section.

C. Superstring-inspired E(6) models

These models of course predict the existence of different kinds of exotic particles and offer a variety of ways to change R. With the large amount of freedom that is available it is clear that current data cannot really constrain this class of models very seriously. In the absence of any mixing the exotic neutrals just add to the Z width as extra neutrino species, and the exotic charged leptons add both to the W and the Z width. Hence in that situation some kind of limits on the masses of exotic neutrals are possible. But as discussed extensively a complicated mixing pattern will invalidate any such statements.

1. *Exotic quarks*

As stated earlier the 27-dimensional representation of E(6) also contains the exotic charge $-\frac{1}{3}$ quark D and charge $+\frac{1}{3}$ quark D^c. If these do not mix with any other down-type quarks, then due to their singlet nature they do not contribute much to the Z width. Thus as noted earlier[39] we have an interesting conclusion that if the e^+e^- experiments should reveal the existence of a light ($m\leq 23$ GeV) charge $\frac{1}{3}$ quark it can be consistent with the present data on R only if it is an $SU(2)_L$ singlet.

Of course the presence of nonzero mixing of D,D^c with standard d-type quarks can in principle affect R. Equations (3.7) give the couplings of the lighter physical down-quark state, in the presence of mixing between D, D^c, and d quarks. For instance, we saw in Sec. III C 3 that it is possible to decrease R somewhat assuming that the heavier physical state had a mass $m_{b_h}>M_Z-m_{b_l}$.

For smaller values of m_{b_h} the decays $Z\to b_l b_h$, $Z\to b_h b_h$ can increase the Z width and $W\to t b_h$ contributes to the W width. The couplings of b_h to W and Z are given by Eq. (3.7) replacing $\cos^2\alpha_i^{(L)}$ by $\sin^2\alpha_i^{(L)}$; and

$$a_{Z_{b_l b_h}}=-b_{Z_{b_l b_h}}=\tfrac{1}{2}\cos\alpha_3^{(L)}\sin\alpha_3^{(L)} \tag{4.11}$$

in units of $g/2\cos\theta_W$. One finds that even when $m_{b_h}<M_Z-m_{b_l}$, over most of the range in $\alpha_3^{(L)}$ the ratio R is almost independent of $\alpha_3^{(L)}$ and also of m_{b_h}, most of the time quite close to the SM prediction. Only for very small values of $\cos^2\alpha_3^{(L)}$ (<0.15), for $m_{b_h}\approx M_W-m_t$, the effect of the closing of the channel $W\to t b_h$ can cause a rise in R beyond even the 90%-C.L. upper limit, and hence disfavor a narrow range of $m_{b_h}\approx M_W-m_t$. In the above, any dependence on $\alpha_i^{(R)}$ coming from Z-Z' mixing has been neglected, following the discussion in the earlier section.

D. Limits on the number of light neutrino species and the top mass

Since each extra neutrino species contributes one unit to the numerator in R, classically R has been used[4] for limiting the possible number of light neutrino species N_ν. Because of the very strong rise in the expected value of R as the $W \to t\bar{b}$ channel closes with increasing value of m_t the allowed number of neutrino species depends critically on m_t (Refs. 3, 6, and 7). Hence no independent statement about either can be made using the data on R. The discussions in the earlier sections show that R can be considerably reduced from the value expected in the standard model with three generations (R_{std}), due to the nonstandard physics contributions to W/Z decays. It will therefore certainly change the allowed values of m_t and N_ν by the current, combined data of UA1 and UA2 on R. In the following discussion we assume that the b' and t' are heavy enough not to contribute to the gauge-boson decay widths. But we consider two cases $m_E = 41$ GeV, the lowest allowed by existing data,[55] and $m_E > M_W$. Clearly the presence of a light E helps to increase the allowed range of m_t and N_ν a little further. In Table II we give a comparison of upper limits on m_t for each value of $N_\nu = 3,4,5,6$ for the standard model and for various nonstandard scenarios discussed in Sec. III. We quote the values for the most conservative choice of R_σ, i.e., $R_\sigma = 3.27$, as well as the standard value used by us throughout,[7] i.e., $R_\sigma = 3.41$. Also shown is the dependence of these results on the mass of the heavy lepton. Of course we use the minimum value of R that can be obtained in the case of supersymmetric models and models with Z-Z' mixing. In the case of supersymmetric models the results presented are for $\tan\beta = 1$, $\mu_3 \simeq 470$ GeV and $M_{\tilde{W}} = M_Z/2$. An asterisk in the table means that this combination of parameters is ruled out even if $m_t = 22.5$ GeV, whereas a blank space indicates that no bound on m_t can be given.

Note that a lighter lepton E increases the allowed range of m_t for a given N_ν considerably. The values given for $R_\sigma = 3.27$ and $m_E = 41$ GeV represent the most conservative limits from the current experimental value. We also see that supersymmetry decreases R appreciably to allow roughly one more neutrino species, and for a given N_ν it increases the allowed range of m_t-values by about 10–15 GeV.

Even larger values of m_t and N_ν are allowed if the Z mixes with the $\mathrm{U}(1)_\chi$ Z' with $\theta = -0.07$, the minimal allowed value.[5] For $N_\nu = 3$, no bound on m_t can be given if R_σ in the standard model is less than 3.35, and if $R_\sigma(\text{SM}) = 3.27$, even three additional massless neutrinos are compatible with the upper bound on R if $m_t \leq 52$ GeV, even if all new charged leptons are heavier than the W. Recall that this particular Z' boson can be embedded into any SO(10) GUT.

The upper bound on R cannot constrain N_ν in case of the E(6) models considered in Sec. III C. If there are more than three generations the model can only be consistent[68] if all exotic quarks and leptons get masses $\gtrsim 10^{11}$ GeV; otherwise the $\mathrm{SU}(2)_L$ and $\mathrm{SU}(3)_c$ gauge couplings would become large well below the unification scale. We, therefore, only quote the emerging bounds on m_t. The exotic leptons discussed in Sec. III C 1 suffice to push the bound on m_t up to 76 GeV even for $R_\sigma = 3.41$; m_t is unbounded if $R_\sigma \leq 3.37$. An arbitrarily heavy top quark is also allowed by the upper bound on R if the physical b quark is a mixture of SU(2)-doublet and -singlet quarks; if $m_t > M_W - m_b$, we find that $\alpha_3^{(L)}$ has to be larger than 0.66 (0.48) if $R_\sigma = 3.41$ (3.27). This means that the doublet component of the b quark can be as large as 0.88 even for a very heavy top quark.

V. SUMMARY AND CONCLUSIONS

The starting point of this investigation was the observation[6,7] that the SM prediction for R is somewhat above the measured value even if the top quark is light. In Sec. II we have shown that the incorporation of full quark-mass-dependent QCD corrections[16] do not change this situation qualitatively. Although this discrepancy is less than one standard deviation for $m_t \lesssim 55$ GeV, large top-quark masses already seem to be ruled out. This led us (in Sec. III) to examine the extent to which R could be reduced within the framework of various popular extensions of the standard model.

We found that the addition of a new Higgs doublet can reduce R by at most 0.1 units. Minimal supersymmetric models[23] on the other hand allow to reduce R by up to 0.5 units; this reduction is entirely due to the gaugino-Higgsino sector of the theory. Existing bounds on squark and slepton masses preclude the possibility of reducing R via their production in gauge-boson decays. The effect on

TABLE II. Upper bounds on the top-quark mass (in GeV) from the experimental bound $R > 9.9$ (90% C.L.). We show results for $N_\nu = 3$, 4, 5, and 6 neutrino species, for $R_\sigma = 3.27$ and 3.41, and for the mass of the fourth charged lepton, m_E, equal to 41 GeV and $m_E > M_W$. For the supersymmetric models we chose the parameters of the gaugino-Higgsino sector such that R is minimized, e.g., $\mu_3 \simeq 470$ GeV, $2m_1 \simeq 140$ GeV, and $\tan\beta = 1$. The last four columns show bounds on m_t for the case where the Z boson mixes with the $\mathrm{U}(1)_\chi Z'$, with $\theta_\chi = -0.07$.

	Standard model				Supersymmetry				Z-Z' mixing model			
N_ν	$R_\sigma = 3.27$		$R_\sigma = 3.41$		$R_\sigma = 3.27$		$R_\sigma = 3.61$		$R_\sigma = 3.27$		$R_\sigma = 3.41$	
	$m_E = 41$	$m_E > M_W$	$m_E = 41$	$m_E > M_W$	$m_E = 41$	$m_E > M_W$	$m_E = 41$	$m_E > M_W$	$m_E = 41$	$m_E > M_W$	$m_E = 41$	$m_E > M_W$
3		66		60		78		72				76
4	65	56	59	49	78	69	72	62		71	76	64
5	55	30	44	*	67	59	60	50	72	61	63	55
6	*	*	*	*	58	46	50	*	62	52	55	*

R is maximal if the lightest neutralino $\tilde{Z}_1$ is not a photino or Higgsino but a complicated mixture of all four current states. Our findings are in sharp contrast to the results of Ref. 69 where it was found that in supersymmetric models R is always larger than or at best just below the SM prediction. This apparent discrepancy results from the assumptions made in Ref. 69 that the $\tilde{Z}_1$ is a very light photino, and that gaugino and sfermion masses are related, so that all nonsupersymmetric terms are uniquely determined by one mass parameter. In Sec. IV B 2 we have shown that if the $\tilde{Z}_1$ is indeed a light photino the data on monojets and on R together strongly disfavor chargino masses below $M_Z/2$.

In superstring-inspired E(6) models[14] R can be reduced even further. This is not surprising since they contain many new particles and even more free parameters. The introduction of either new exotic SU(3)-singlet fermions (with properly chosen masses and couplings) or a new Z' boson that mixes with the standard Z boson suffices to circumvent any bound on m_t if R_σ in the standard model is less than 3.35, which is perfectly possible;[6,7] this latter possibility can also be realized in a conventional SO(10) GUT. The bound on m_t also goes away if the b quark mixes with an exotic SU(2)-singlet quark if the mixing angle is larger than 38°; to the best of our knowledge this possibility is not ruled out by existing data.

In Sec. IV we discussed what constraints on extensions of the SM can be derived from existing data on R. We found that a sequential fourth-generation down-type quark with mass below 26 GeV is ruled out. This bound does not only forbid to interpret[56] the anomalous low thrust PETRA events containing muons as the production and subsequent decay of a pair of sequential down-type quarks; in many four-generation models that use simple *Ansätze*[58,60–62] for the quark mass matrices it translates into lower bounds on the difference of fourth-generation quark masses barely consistent[5] with existing bounds. We also showed that models where the b quark dominantly couples to a heavy t' quark are strongly disfavored by the data. As far as we know our Fig. 9 shows the first published bound on the V_{tb} element of the extended Kobayashi-Maskawa matrix for the case of four generations. This is in sharp contrast to the case of exotic E(6) weak isosinglet quarks; no lower bound on their masses can be derived, so that they could possibly be[56] the source of the low-thrust events mentioned above, and as discussed earlier at least for heavy top quarks R actually favors a large mixing between the b and the new quarks. We finally give (in Table II) the bounds on m_t and the number of light neutrino species that can be derived in nonsupersymmetric or supersymmetric models with additional generations, and in models with Z-Z' mixing.

Many of the scenarios discussed in this paper can be readily tested as soon as the Z factories SLC and LEP have produced a few thousand Z bosons. These include light charged Higgs bosons (Sec. III A), some choices of gaugino-Higgsino parameters, see Sec. III B 2, b-D and Z-Z' mixing (Secs. III C 2 and III C 3), and a sequential fourth generation. However, in some regions of the gaugino-Higgsino parameter space, as well as for the favored choice of exotic E(6) leptons, the properties of the Z boson are the same as in the standard model. Moreover, the observation of anomalous Z decays might not suffice to discriminate between the competing models. A high-precision measurement of R at future runs of the CERN Sp$\bar{p}$S or the Tevatron is, therefore, of the utmost importance.

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